Theory perspective on the Flavour Anomalies

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Overview

Contents

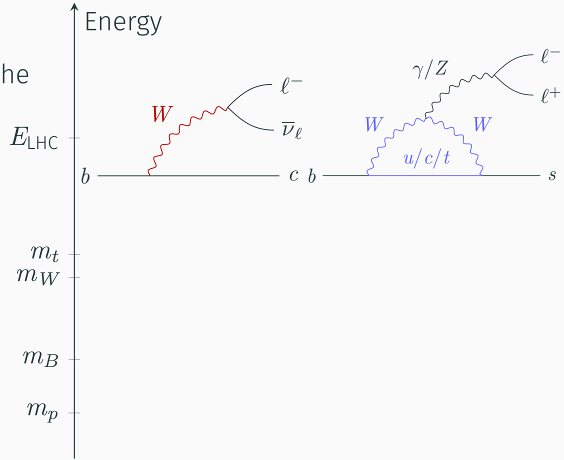
- this is not a comprehensive discussion of the flavour anomalies (substantial tensions shy of $5\,\sigma$ individually)
  - aiming for an overview of a subjective selection of flavour anomalies
  - off the menu:
    - $(g-2)_\mu$ — see following talks by M. Lancaster & A. El-Khadra
    - Cabibbo anomaly, ...
  - provide an idea of current status of and complexity behind the flavour anomalies
- concentrating on longstanding b anomalies

  $b \to c\tau^-\overline{\nu}$ driven by BaBar ’12 & LHCb ’15&’18 measurements

  $b \to s\mu^+\mu^-$ driven by LHCb ’13...’21 analyses (& consistent with ATLAS, Belle, CMS)
- after overviews by L. Grillo and me: more in-depth discussions Tuesday afternoon and Wednesday morning
  - semileptonic decays at LHCb — G. Wormser
  - rare decays with Belle II — S. Stefkova
  - probing LFU violation with LHCb measurements — C. Benito
  - first semileptonic measurements at Belle II — F. Bernlochner

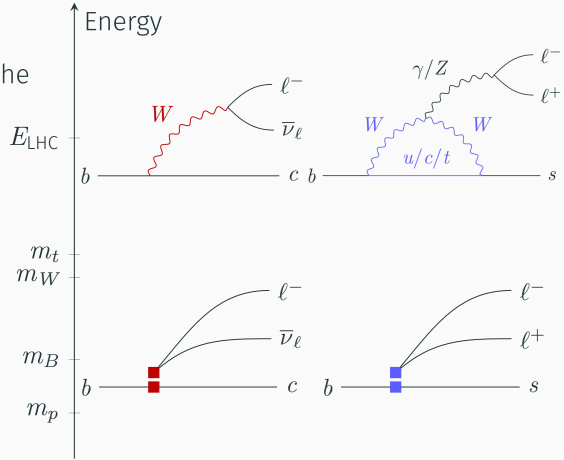
- theory predictions for b decays require an elaborate framework
- multiscale problem: m_t, $m_W \gg m_b \gg \Lambda_{\text{had}}$
- “divide and conquer“ approach:
  - introduce weak effective theory (WET), separating high-energy scale m_t, m_W from low-energy scales m_b, Λ_{had}
  - use renormalization group equations to understand WET at low scale $\simeq m_b$
  - WET simplifies hadronic matrix elements, compute at low scale $\simeq m_b$
    - from lattice QCD (if possible)
    - in power expansion of Λ_{had}/m_b using heavy-quark effective theory (HQET) and/or soft-collinear effective theory (SCET)
    - in QCD sum rules: Shifman-Vainshtein-Zakharov sum rules (SVZSR) for and more importantly light-cone QCD sum rules (LCSR)

- low-energy description of both the SM and BSM models*
- removes W and t, Z fields



*: under weak assumption that BSM physics ”lives” at or above the electroweak scale

- low-energy description of both the SM and BSM models*
- removes W and t, Z fields
- introduces dim-6 effective operators
  - dim-8 suppressed by $m_b^2/m_W^2 \sim 0.4\%$



$$\mathcal{L}^{\text{eff}} = \sum_i \mathcal{C}_i \times [\bar{c}\,\Gamma_i\, b] \times [\bar{\ell}\,\tilde{\Gamma}_i\, \nu] + \sum_j \mathcal{C}_j \times [\bar{s}\,\Gamma_j\, b] \times [\bar{\ell}\,\tilde{\Gamma}_j\, \ell]$$

*: under weak assumption that BSM physics "lives" at or above the electroweak scale

Framework — Operator Bases

$b \to c\tau^- \overline{\nu}$

- 10 operators for$\ell = \tau$ flavour

$$[\overline{s}\,\Gamma_i\,b] \times [\overline{\ell}\,\tilde{\Gamma}_i\,\nu]$$

$$O_{V_L} : \gamma^\mu P_L \otimes \gamma_\mu P_L$$

- reduces to 5 if left-handed neutrinos assumed
  - very manageable in fits

$b \to s\mu^+\mu^-$

- 10 $b \to s\mu\mu$ operators

$$[\overline{s}\,\Gamma_j\,b] \times [\overline{\ell}\,\tilde{\Gamma}_j\,\ell]$$

$$O_9 : \gamma^\mu P_L \otimes \gamma_\mu$$

$$O_{10} : \gamma^\mu P_L \otimes \gamma_\mu \gamma_5$$

- additional operators required for consistent description at $O(\alpha_e)$
- $b \to s\{\gamma, g, \overline{q}q\}$ can all contribute to $b \to s\ell^+\ell^-$ processes
  - $b \to s\overline{q}q$ operators are typically assumed to be SM-like

Framework — SM Values of WET Coefficients

To probe BSM physics, we need accurate knowledge of SM contributions!

$b \to c\tau^- \bar{\nu}$

- matching at tree-level
- only one non-zero coefficient
- no QCD-induced scale evolution
- e.m. radiative corrections under control

  [A. Sirlin '90]

$b \to s\mu^+\mu^-$

- matching starts at one-loop

  [Adel,Yao hep-ph/9308349]
- QCD-induced scale dependence
- NNLO QCD matching

  [Greub et al. hep-ph/9703349]

  [Bobeth et al. hep-ph/9910220]
- partial NNLL evolution

  [Chetyrkin et al. hep-ph/9612313]

  [Bobeth et al. hep-ph/0312090]

  [Gorbahn,Haisch hep-ph/0411071]

  [Gorbahn et al. hep-ph/0504194]

- working dominantly to leading order in α_e

$\Rightarrow$ matrix elements of semileptonic operators factorize

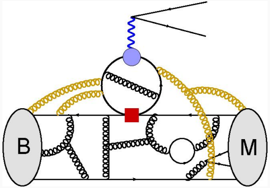
- hadronic matrix elements are discussed in terms of scalar-valued hadronic form factors

$b \to c\tau^-\bar{\nu}$ & $b \to s\mu^+\mu^-$

- number of indep. form factors depends on hadrons involved
- 3 for $P \to P\ell\ell'$
  e.g. $\overline{B} \to D\tau^-\bar{\nu}$ or $B \to K\mu^+\mu^-$
- 7 for $P \to V\ell\ell'$
  e.g. $\overline{B} \to D^*\tau^-\bar{\nu}$ or $B \to K^*\mu^+\mu^-$
- ≥ 10 for baryonic processes

$b \to s\mu^+\mu^-$ only

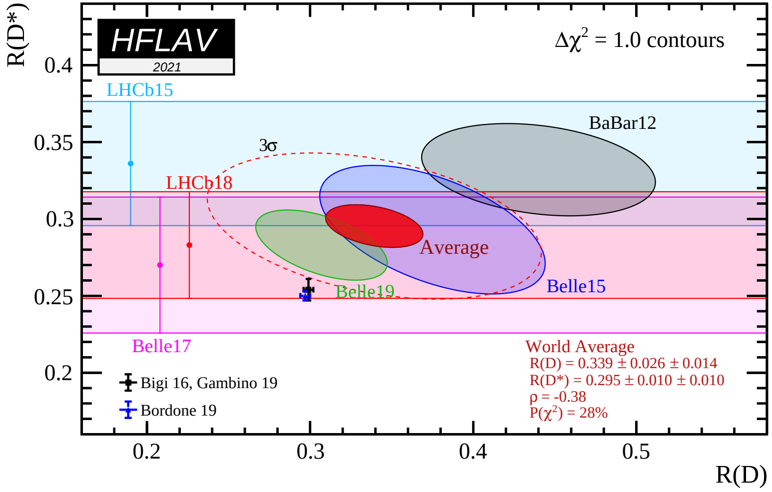
- non-local contributions pollute local $b \to s\mu^+\mu^-$ interactions
- dominant: intermediate on-shell vector $\bar{c}c$



$$b \to c\tau^-\overline{\mu}$$

Test of Lepton-Flavour Universality (LFU)

[HFLAV 1909.12524; Spring '21 update]



$\overline{B} \to D^{(*)}$ Form Factors are special

- heavy-quark expansion very effective if both quark flavours b & c are heavy [Isgur,Wise '89]
  - simultaneous expansion in α_s up to NLO and $\Lambda_{\text{had}}/m_{b,c}$ up to 2nd power [Falk,Neubert hep-ph/9209268 & hep-ph/9209269]
  - yields relations between form factors across both different currents and processes
  - relates BSM-only (tensor) FF to SM-like matrix elements [Bernlochner et al. 1703.05330]
- precise lattice QCD results for $\overline{B}_{(s)} \to D_{(s)}$ form factors in large parts of phase space [FNAL/MILC 1503.07237; HPQCD 1505.03925]
- first lattice QCD results for $\overline{B}_{(s)} \to D^*_{(s)}$ form factor [HPQCD 2105.11433; FNAL/MILC 2105.14019]
- consistent picture of all theory inputs to NLO in α_s & $1/m^2$ [Bordone et al. 1908.09398 & 1912.09335]

global fit to $b \to c\tau^-\overline{\nu}$ data

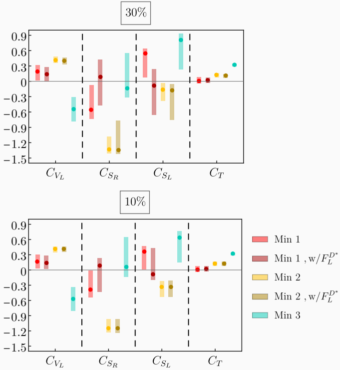
[Murgui et al. 1904.09311]

- measurements
  - R_D, R_{D^*}
  - D^* polarisation (optional)
- assumptions:
  - $\Gamma(B_c^- \to \tau^-\overline{\nu})/\Gamma(B_c^-) < X\%$
  - semi-tau. width cannot dominate $\Gamma(B_c^-)$

    [Alonso et al. 1611.06676]

  - no r.h. $b \to c$ vector current, since it is lepton-flavour universal

    [Cata,Jung 1505.05804]

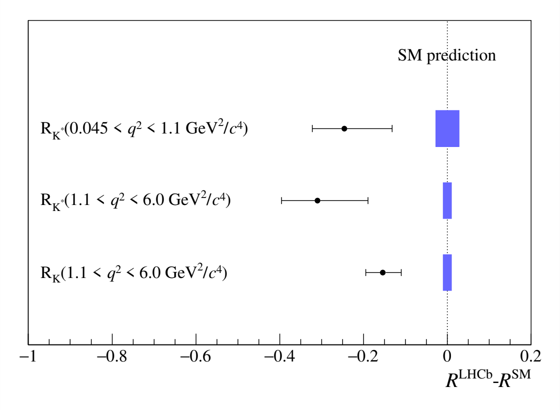


What's Next?

- global fits need updating, due to new measurements and predictions
  - $R_{J/\psi}$ from semileptonic B_c decays
- LHCb is working hard on new measurements
  - R_D / combined $R_D \& R_{D^*}$ measurements
  - R_{Λ_c} will test complementary WET constraints [Böer et al. 1907.12554]
- Belle II in excellent position to contribute in near future
- a lot of work before LFU violation can be claimed!
  - anomalies tend to vanish
  - theory under good control; need more measurements!

$$b \to s\mu^+\mu^-$$

Anomalies in Plots

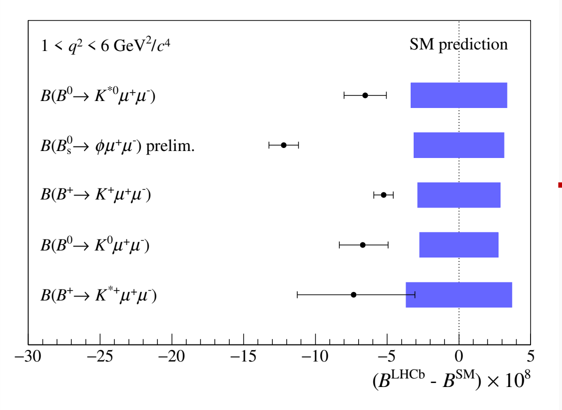


$$R_X \equiv \frac{\langle \mathcal{B}(B \to X\mu^+\mu^-)\rangle_{1,6}}{\langle \mathcal{B}(B \to Xe^+e^-)\rangle_{1,6}}$$

[plot by C. Langenbruch]

- SM predictions ~ 1 if $1\,\mathrm{GeV}^2 \leq q^2 = m_{\ell\ell}^2 \leq 6\,\mathrm{GeV}^2$
- LHCb meas. consistently lower, with $\geq 3\,\sigma$ tensions in R_K

see talks by L. Grillo & C. Benito



$$\langle \mathcal{B}(B \to X\mu^+\mu^-) \rangle_{1,6}$$

[plot by C. Langenbruch]

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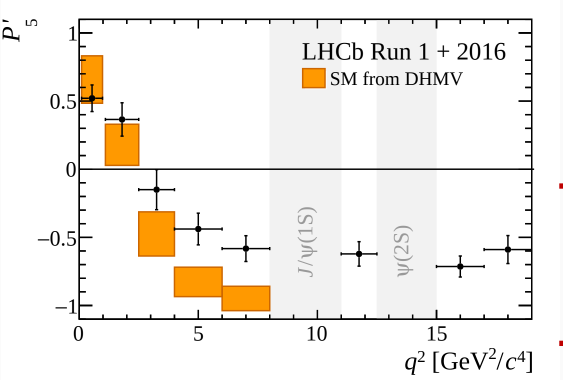
- larger th. uncertainties for $\mathcal{B}$
- muonic $\mathcal{B}$ systematically below SM pred.

Anomalies in Plots



representing full kinematic distribution of $B \to K^*(\to K\pi)\mu^+\mu^-$

- SM predictions ~ 1 if $1\,\text{GeV}^2 \leq q^2 = m_{\ell\ell}^2 \leq 6\,\text{GeV}^2$
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- larger th. uncertainties for $\mathcal{B}$
- muonic $\mathcal{B}$ systematically below SM pred.

- angular observables compared in bins of q^2
- deviations significant and consistent with R_X, $\mathcal{B}$

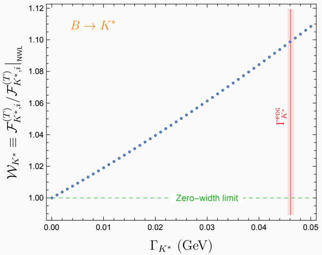
Lepton-Flavour Universality Ratios

- to LO in α_e, SM prediction differs from 1 only due to $4m_\mu^2/q^2$ factors
  - various groups agree on predictions in the SM

- radiative corrections
  - semi-analytic calculation of integrated R_K agrees with *PHOTOS*-based simulation [Bordone,Isidori,Pattori 1605.07633]
  - double-differential distribution can suffer from large correction, requires more careful treatment compatible with current best practice [Isidori,Nabeebaccus,Zwicky 2009.00929]
  - no structure-dependent studies yet for rare semileptonic decays, but important insights gained from QED factorization studies for $B_s \to \mu\mu$ and non-leptonic $B \to K\pi$ decays [Beneke,Bobeth,Szafron 1908.07011] [Beneke,Böer,Toelstede,Vos 2008.10615]

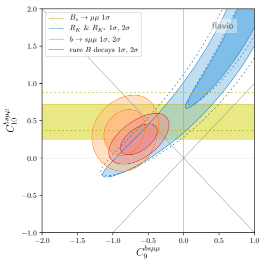
- large uncertainties, since (local) form factors cannot cancel
- largest deviations seen at small q^2 values: $1\,\text{GeV}^2 \leq q^2 \leq 6\,\text{GeV}^2$
  - current lattice QCD results limited to $q^2 \gtrsim 12\,\text{GeV}^2$
  - theory predictions at small q^2 dominated by QCD light-cone sum rules with large uncertainties
- first attempt to account for non-zero width in $K^* \to K\pi$
  - SM prediction grows by $\sim 20\%$, increasing tensions
  - effect cancels in ratios (LFU, ang. obs.)

[Descotes-Genon 1908.02267]

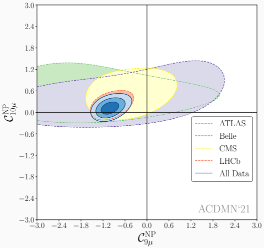


- normalization cancels hadronic form factors partially
  - theory correlations indispensable
  - using lattice QCD info if available, heavy-quark expansion if not
- major task: disentangle non-local contributions from WET coefficients C_7 & C_9
- non-local effects: using pertubative QCD at time-like momentum transfer below narrow charmonium resonances
  - a-posteriori tests seem to indicate that non-local effects are not driving the anomalies
  - nevertheless, poses presently the largest systematic uncertainty in the determination of the C_9 WET coefficient

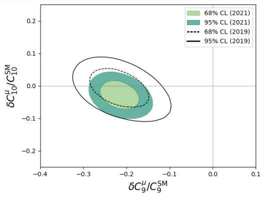
Interpretation — Weak Effective Theory



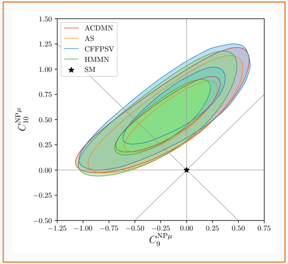
[Altmannsh.,Stangl 2103.13370]



[Alguero et al. 2104.08921]



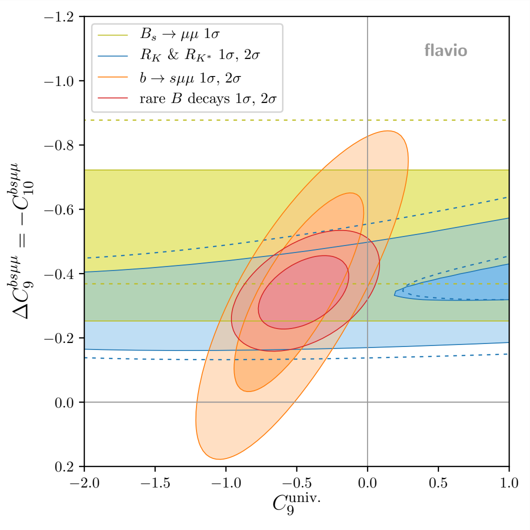
[Hurth et al. 2104.10058]



[B. Capdevila, M. Fedele, S. Neshatpour, and P. Stangl]

- several groups carrying out fits, with varying assumptions and datasets
  - stunning agreement between results of four of the major fitting groups when considering common subset of data
- scenario dependent tensions
  - tension $> 5\,\sigma$ for all-operator fits to all data [all semi-leptonic ops]
  - tension $\gtrsim 4\,\sigma$ for fits to “clean” subset of data

Universal vs Non-Universal BSM Contributions



[Altmannshofer,Stangl 2103.13370]

- ▸ several groups investigate both LFU and LFUV contrib.
- + tension larger than in μ-only assumption!
- – LFU part sensitive to non-local form factors
- ▸ accurate interpretation requires accurate predictions of non-local form factors

LFU observables:

- SM prediction very clean; e.m. radiative contributions seem under control
- for confirmation, measurements independent of LHCb seem mandatory looking at Belle (II), ATLAS, and CMS

What's Next?

LFU observables:

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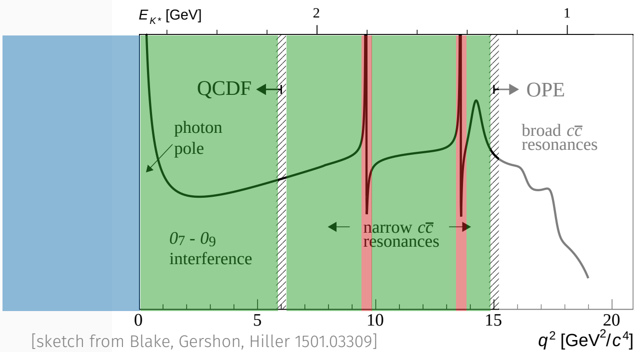
non-LFU observables:

- overwhelming number of measurements are of non-LFU observables
  - large variety of q^2 bins
  - across LHC experiments and BaBar/Belle
- branching ratios & angular observables require further theory improvements
  - theory uncertainties currently limiting factor in fit significances!

parametrize non-local effects [Bobeth et al. 1707.07305; Gubernari et al. 2011.09813]



[sketch from Blake, Gershon, Hiller 1501.03309]

- predict non-local form factors in timelike region
- extrapolate to spacelike region
- account for experimental measurements of non-leptonic decays
- global fit based on recent parametrization in preparation [Gubernari,Reboud,DvD,Virto w.i.p.]

Conclusion

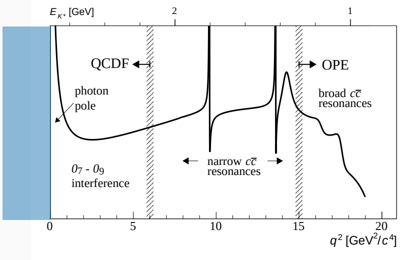
Summary

- $b \to c\tau^-\overline{\nu}$ anomalies seem stable
  - recent lattice QCD analyses (HPQCD, FNAL/MILC) pave road toward high-precision theory-only predictions for $\overline{B} \to D^*\tau^-\overline{\nu}$
  - looking forward to complementary measurements by LHC experiments and Belle II
- longstanding $b \to s\mu^+\mu^-$ anomalies make us #cautiouslyexcited
  - significances of the $b \to s\mu^+\mu^-$ anomalies have been increasing with growing data sets
  - LFU observables are limited by experimental data
  - non-LFU observables are limited by theory
    - non-local form factors single-largest systematic theory uncertainty

Backup Slides

Compute Light-Cone OPE

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$

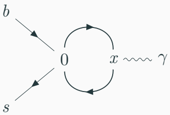


- expansion in operators w/ light-like sep. $x^2 \simeq 0$

  [Khodjamirian, Mannel, Pivovarov, Wang 2010]

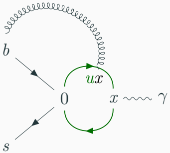
- employing light-cone expansion of charm propagator

  [Balitsky, Braun 1989]



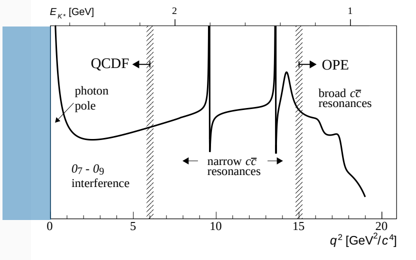
$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{coeff \#1}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$+ \,(\text{coeff \#2}) \times [\,\bar{s}_L \gamma^\alpha (i n_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,]$$



Compute Light-Cone OPE

$$4m_c^2 - q^2 \gg \Lambda^2_{\text{hadr.}}$$



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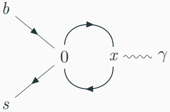
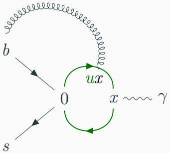




$$\Rightarrow \mathcal{H}_\lambda = \text{coeff \#1} \times \mathcal{F}_\lambda + \mathcal{H}_\lambda^{\text{spect.}}$$
$$+ \text{coeff \#2} \times \tilde{\mathcal{V}}_\lambda$$

- leading part identical to QCD fact. results [Beneke, Feldmann, Seidel '01&'04]
- subleading matrix element $\tilde{\mathcal{V}}_\lambda$ can be inferred from B-LCSRs

  [Khodjamirian, Mannel, Pivovarov, Wang '10; Gubernari, DvD, Virto '21]

Compute Soft gluon matrix elements

matrix elements of a single operator appearing at subleading power in the LCOPE

$$\tilde{\mathcal{V}}_\lambda \sim \langle M|\, \bar{s}(0)\gamma^\rho P_L G^{\alpha\beta}(-un^\mu)b(0)\,|\bar{B}\rangle$$

for $B \to K^{(*)}$ and $B_s \to \phi$ transitions

- matrix element has been prev. calculated in light-cone sum rules

  [Khodjamirian, Mannel, Pivovarov, Wang '10]

- physical picture provides that the soft gluon field originates from the $\bar{B}$ meson
  - analytical results independent of two-particle $b\bar{q}$ Fock state inside the $\bar{B}$
  - expressions start with three-particle $b\bar{q}G$ Fock state, and their light-cone distribution amplitudes (LCDAs)

$$\Phi(t, u) \sim \langle 0|\, \bar{q}(x)\, G^{\mu\nu}(ux)\Gamma h_v^b(0)\,|\bar{B}(vM_B)\rangle \qquad x^\mu = tn^\mu$$

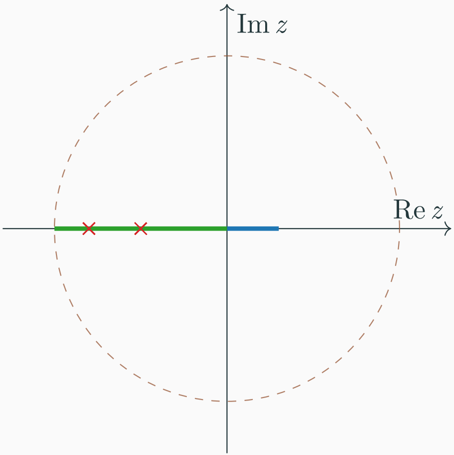
  - original results lacking four out of eight three-particle LCDAs [Gubernari,DvD,Virto '20]

Compute Soft gluon matrix elements

- we calculate the soft-gluon contributions $\tilde{V}_\lambda$ to the full set of $B \to V$ and $B \to P$ nonlocal form factors using light-cone sum rules [Gubernari,DvD,Virto '20]
  - analytic results for restricted set of LCDAs in full agreement with KMPW2010 [Khodjamirian, Mannel, Pivovarov, Wang '10]
  - result of restricted set fails to reproduce duality thresholds obtained from local form factor sum rules [Gubernari, Kokulu, DvD '18]
  - cross check: our results reproduce the (local) duality thresholds!
  - our numerical results differ significantly from KMPW2010
    - reduction by factor ~ 100, differences well understood!
    - reduction by ~ 10 from update inputs, and ~ 10 from cancellations due to new terms
  - conclusion: soft-gluon contributions are not numerically relevant for $q^2 < 0$

Extrapolate Parametrisation of the nonlocal form factors

- map q^2 to new variable z that develops branch cut at $q^2 = 4M_D^2$ [Bobeth, Chrzaszcz, DvD, Virto '17]
  - branch cut is mapped onto unit circle in z
- data and theory live inside the unit circle
  - real-valued $q^2 \leq 4M_D^2$ is mapped to real-valued z
- expand in z
  + resonances J/ψ, $\psi(2S)$ can be included (poles/Blaschke factors)
  + easy to use in a fit to theory and data
  + compatible with analyticity
  - expansion coefficients unbounded!



Extrapolate New parametrisation w/ dispersive bound

matrix elements $\mathcal{H}$ arise from nonlocal operator [Gubernari,DvD,Virto '20]

$$\mathcal{O}^\mu(Q;x) \sim \int d^4y\, e^{iQ\cdot y}\, T\{J^\mu_{\text{em}}(x+y), [C_1 O_1 + C_2 O_2](x)\}$$

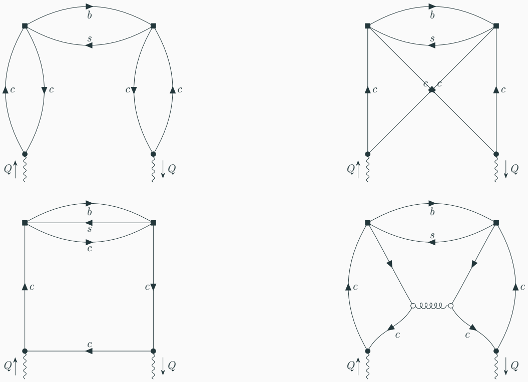
construct four-point operator to derive a dispersive bound

- define matrix element of "square" operator

$$\left[\frac{Q^\mu Q^\nu}{Q^2} - g^{\mu\nu}\right]\Pi(Q^2) \equiv \int d^4x\, e^{iQ\cdot x}\, \langle 0|\, T\{\mathcal{O}^\mu(Q;x)\,\mathcal{O}^{\dagger,\nu}(Q;0)\}\,|0\rangle$$

- $\Pi(Q^2)$ has two types of discontinuities
  - from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
  - from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

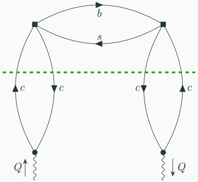
Extrapolate Cuts of Π

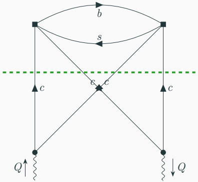


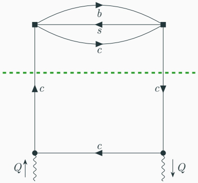
diagrams start at three loops

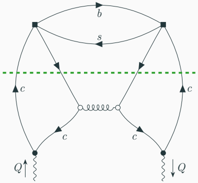
- diagrams to LO in α_s: top, and bottom left
- one diagram to NLO in α_s (bottom right), for illustration only

Extrapolate Cuts of Π









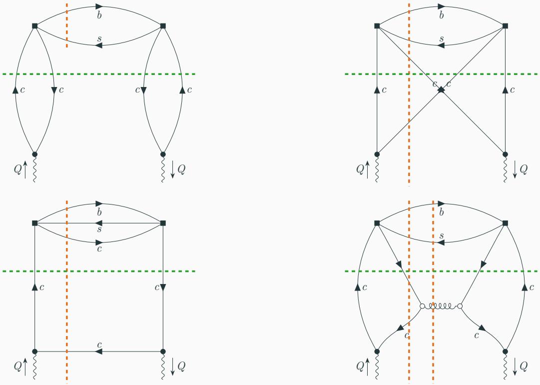
diagrams start at three loops

- diagrams to LO in α_s: top, and bottom left
- one diagram to NLO in α_s (bottom right), for illustration only

discontinuities

- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)

Extrapolate Cuts of Π



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discontinuities

- from intermediate unflavoured states ($c\bar{c}$, $c\bar{c}c\bar{c}$, ...)
- from intermediate $b\bar{s}$-flavoured states ($b\bar{s}$, $b\bar{s}g$, $b\bar{s}c\bar{c}$, ...)

Extrapolate Dispersion relation for Π

dispersive representation of the $b\bar{s}$ contribution to derivative of Π

$$\chi(Q^2) \equiv \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \Pi(Q^2) = \frac{1}{2!}\left[\frac{d}{dQ^2}\right]^2 \frac{1}{2i\pi} \int_{(m_b+m_s)^2}^{\infty} ds\, \frac{\mathrm{Disc}_{b\bar{s}}\,\Pi(s)}{s-Q^2} > 0$$

- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be computed in the local OPE
  $\rightarrow \chi^{\text{OPE}}(Q^2)$
- $\mathrm{Disc}_{b\bar{s}}\,\Pi$ can be expressed in terms of the nonlocal form factors $|\mathcal{H}_\lambda|^2$
  $\rightarrow \chi^{\text{had}}(Q^2)$

- global quark hadron duality suggests that $\chi^{\text{OPE}}(Q^2) = \chi^{\text{had}}(Q^2)$
- parametrize $\mathcal{H}_\lambda \propto \sum_n \alpha_{\lambda,n} f_n$ with orthonormal functions f_n

$$\Rightarrow \text{dispersive bound:} \quad \chi^{\text{OPE}} \geq \sum_n |\alpha_{\lambda,n}|^2$$

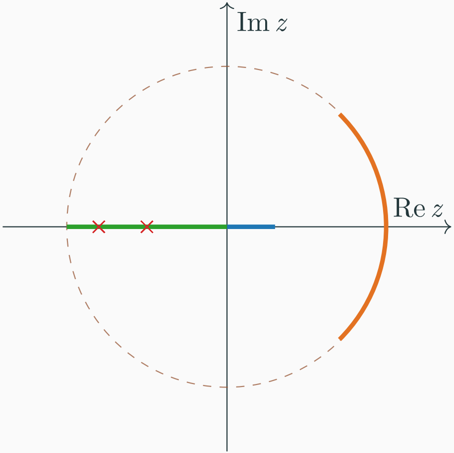
- *first application* of such a bound to nonlocal form factors
- technically more challenging than for local form factors

Extrapolate New parametrisation w/ dispersive bounds

- expand in z
  - $f_n(z)$ orthogonal on arc
  - \+ accounting for behaviour on arc produces dispersive bound on each parameter ✓

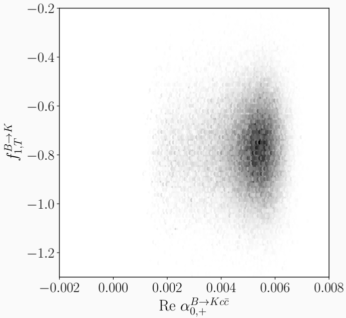
[Gubernari, DvD, Virto '20]

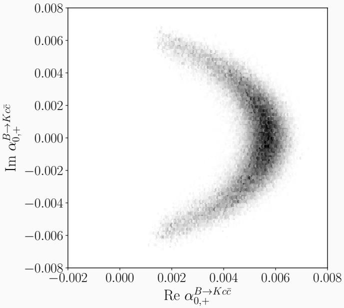
- turns so far hardly quantifiable systematic theory uncertainties into parametric uncertainties
- currently being implemented in EOS
  - open source software at *github.com/eos/eos*
  - available from PyPI for easy dissemination to both theory + experimental colleagues



Preliminary Results

- "first stage" simultaneous fit of parameters of local and non-local form factors to theory inputs + $B_{(s)} \to \{K, K^*, \phi\} J/\psi$ [Gubernari, Reboud, DvD, Virto (to appear)]





- N.B.: non-local parameters are complex numbers
- cartesian parametrisation leads to non-gaussian posterior

Preliminary Results

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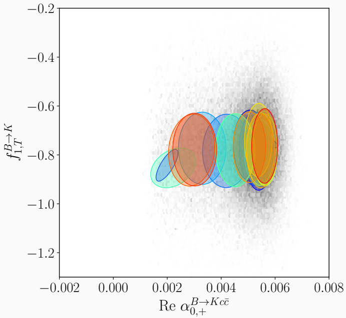
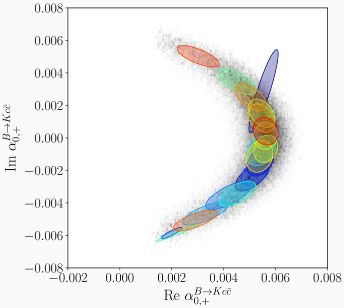
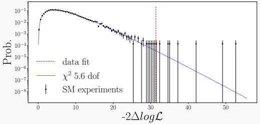




- N.B.: non-local parameters are complex numbers
- cartesian parametrisation leads to non-gaussian posterior
- successfully described by gaussian mixture density
- investigating polar parametrisation

- we plan to publish the mixture density in digital form, including a test statistic to determine a goodness of fit in BSM studies

Interpretation — Look Elsewhere Effect



[Lancierini et al. 2104.05631]

how to determine a global significance?

- fitting a few-operator scenario is not a suitable way to establish significance of a tension
  - not invariant under reparametrization
- accounting for all operators similar to Look-Elsewhere Effect [Lancierini et al. 2104.05631]
- recent conservative analysis of subset of the available data yields global significance of $3.9\,\sigma$, despite large “trial factors“
  - n.b.: should probably be interepreted as a lower bound on the global significance