Muon g-2 Theory

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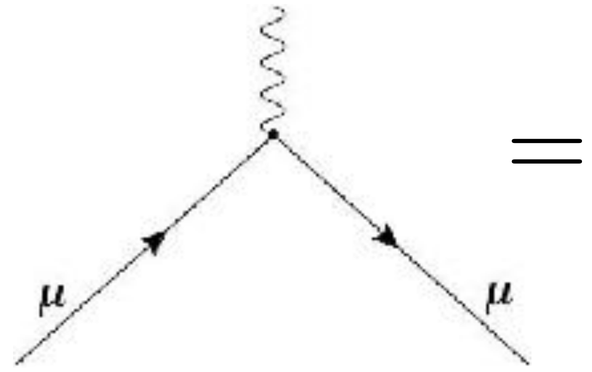
Anomalous magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\quad \vec{\mu} = g\frac{e}{2m}\vec{S}$

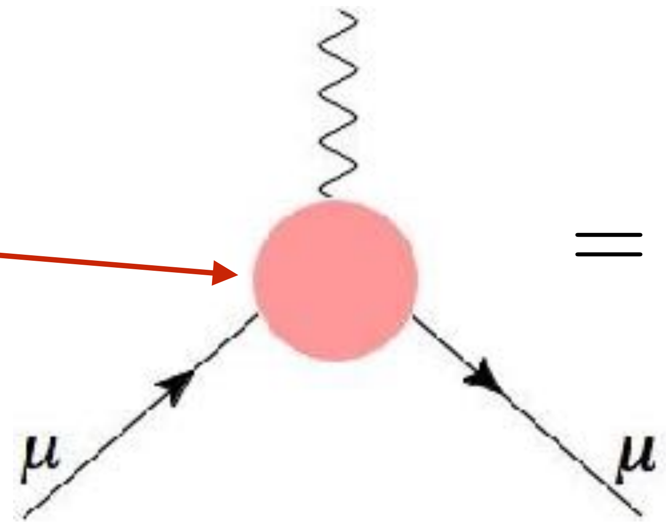
Dirac (leading order): $g = 2$



$$= (-ie)\,\bar{u}(p')\gamma^{\mu}u(p)$$

Quantum effects (loops):

All SM particles contribute



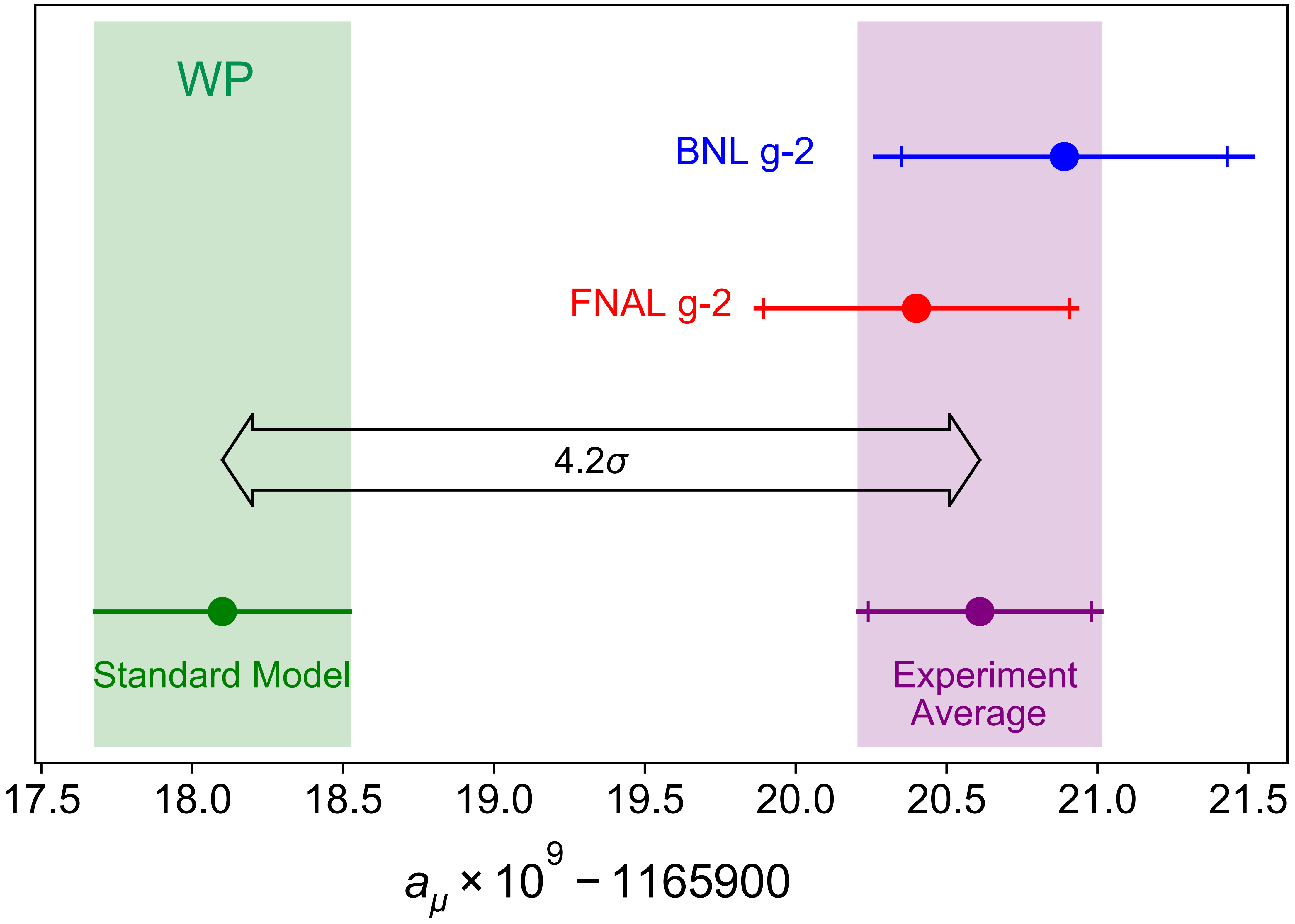
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right]u(p)$$

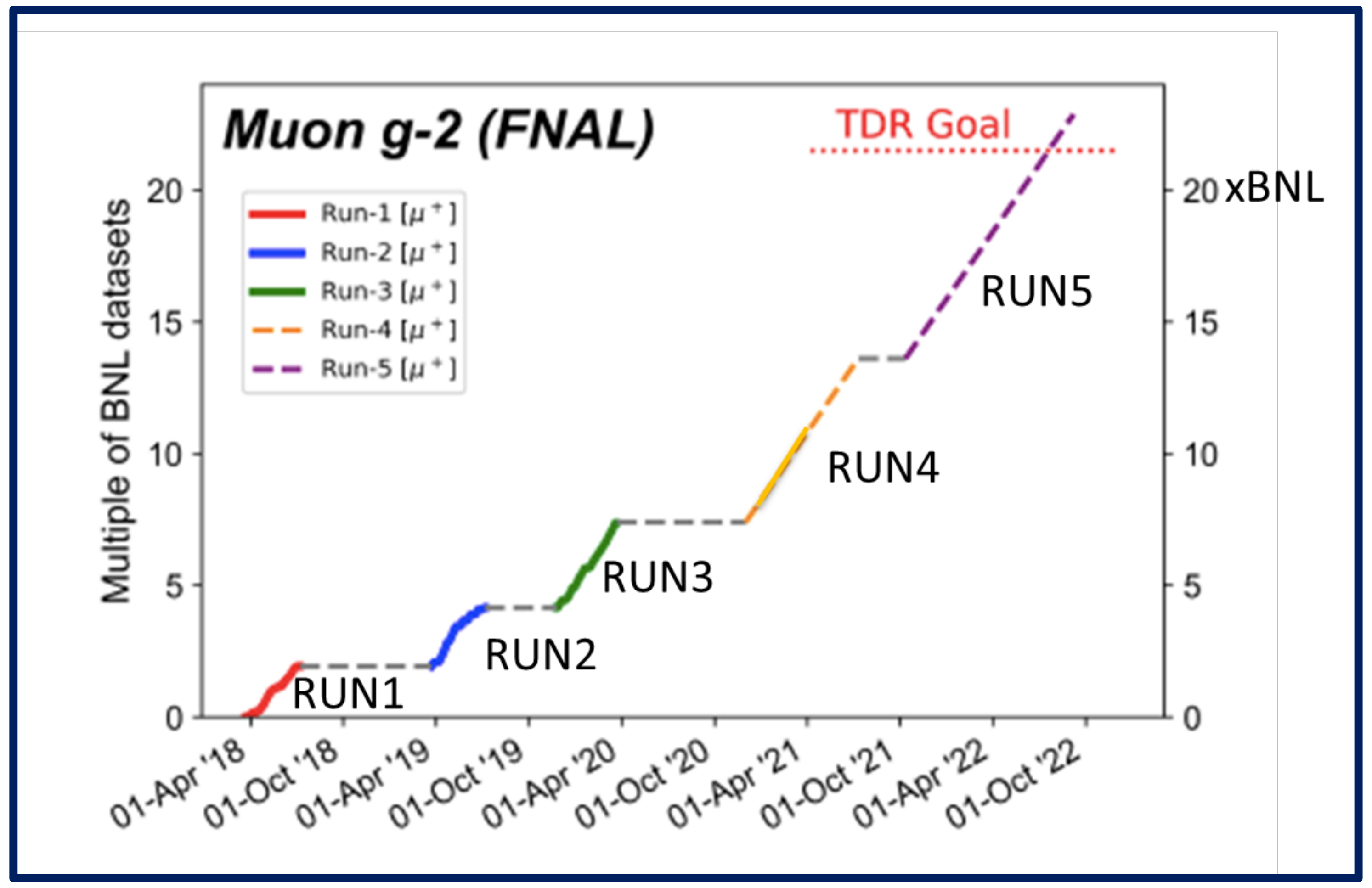
Note: $F_1(0) = 1$ and $\quad g = 2 + 2\,F_2(0)$

Anomalous magnetic moment: $\boxed{a \equiv \frac{g-2}{2} = F_2(0)}$

Muon g-2: experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021. [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Analysis of runs 2 and 3 is now underway.





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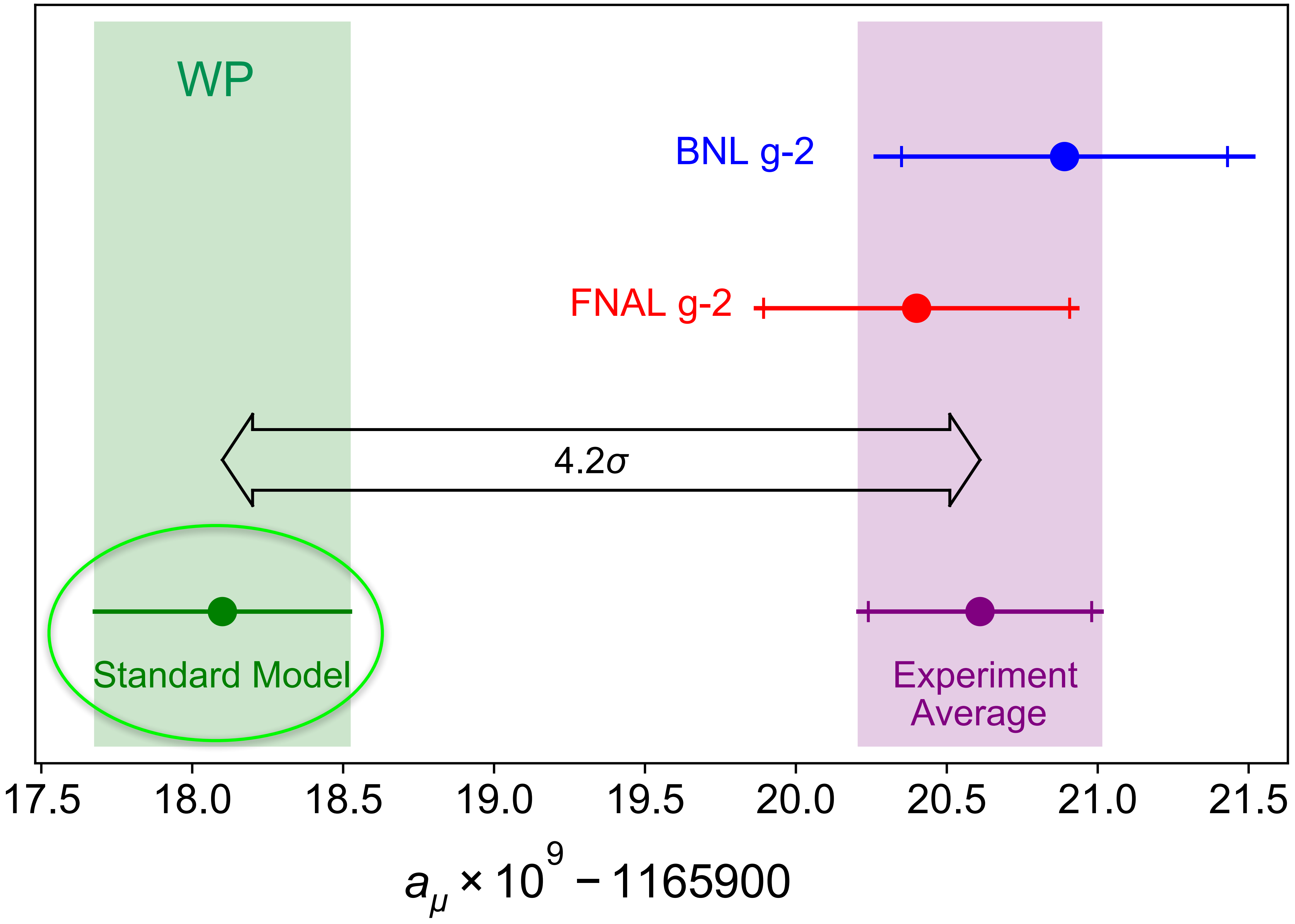
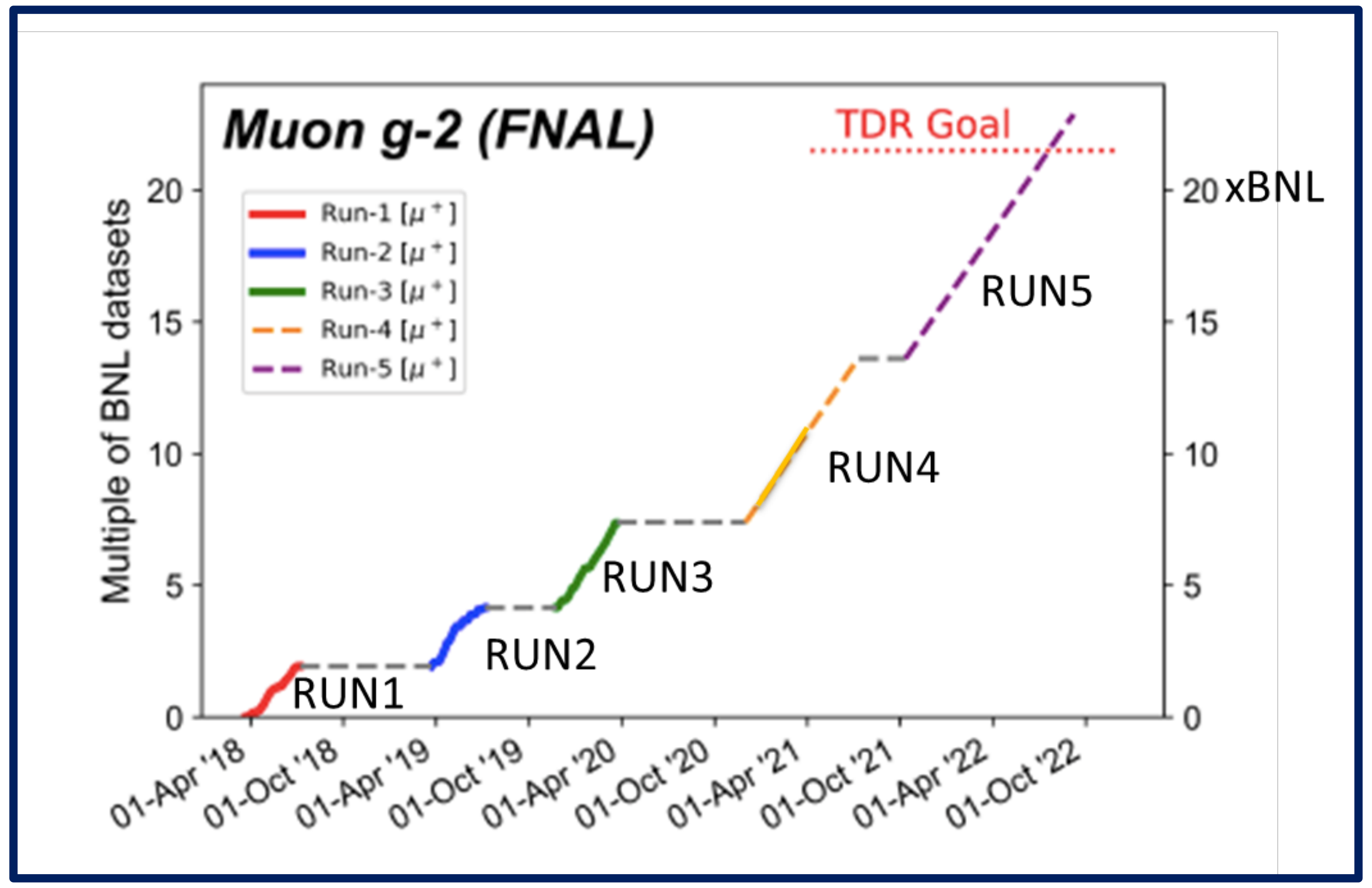




Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022

- 1st White Paper published in 2020 (132 authors, 82 institutions, 21 countries)
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- 2nd White Paper (~2023): First discussions @ KEK meeting in June 2021
  expect to develop a concrete plan (outline, authors) @ Higgs Centre workshop

Muon g-2 Theory Initiative

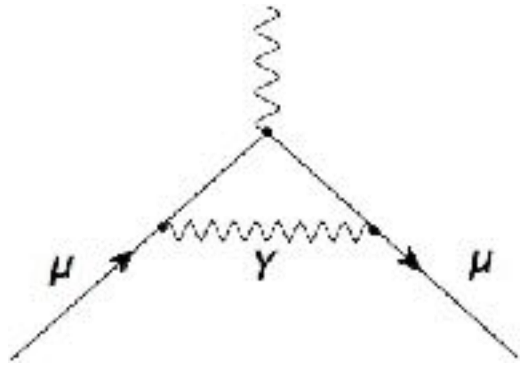
Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK) J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston) Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

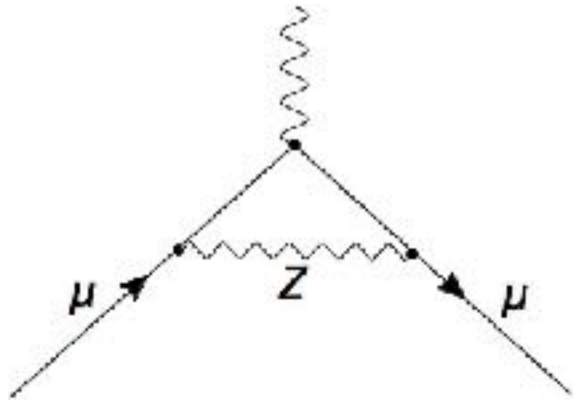
website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$\boxed{a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})}$$

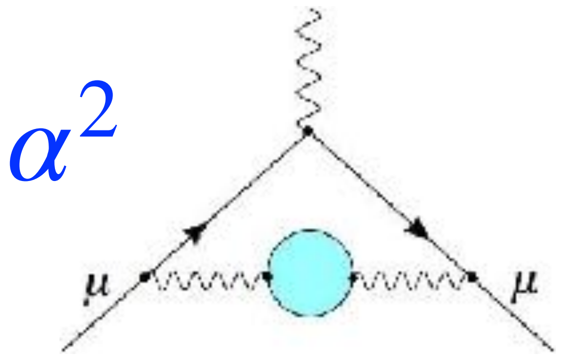
QED 

$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

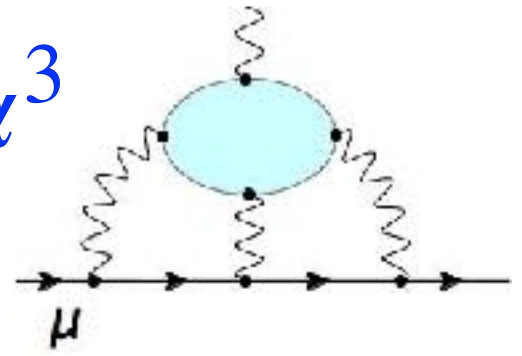
EW 

$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

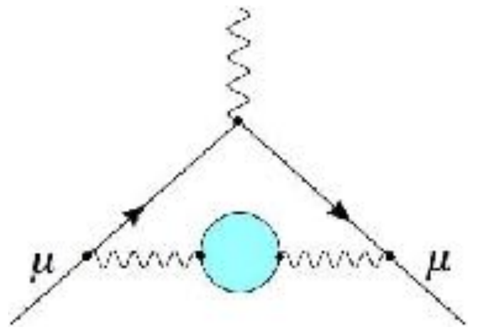
Hadronic corrections

HVP α^2 

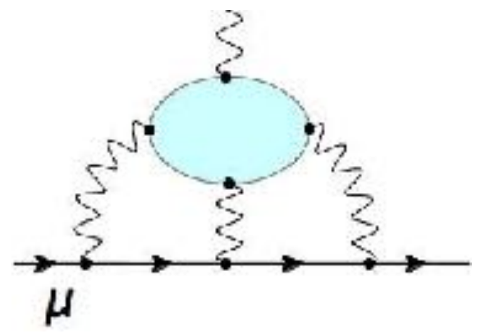
$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

HLbL α^3 

$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm



Hadronic Corrections

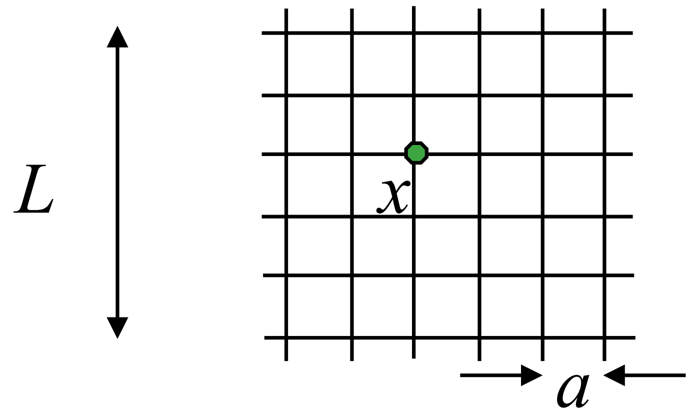


Two different, independent strategies:

1. For HVP, use dispersion relations to rewrite integral in terms of hadronic cross section:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision.
New dispersive approach developed for HLbL

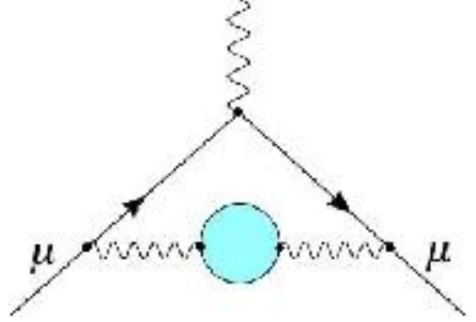
2. Direct calculation using Euclidean Lattice QCD



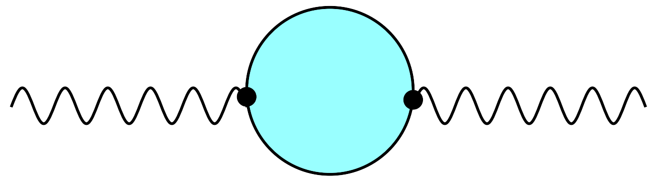
Approximations:
discrete space-time (spacing a)
finite spatial volume (L), and time extent (T)
…

Integrals are evaluated numerically using Monte Carlo methods.

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations



Hadronic vacuum polarization



$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

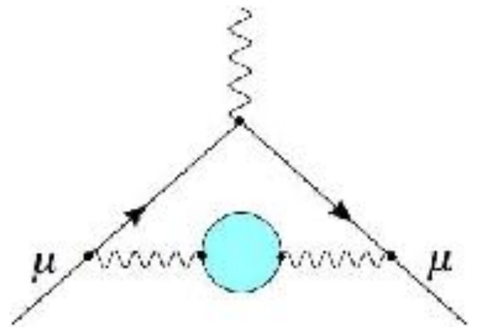
Leading order HVP correction: $$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)}$$

- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\exp}(s)}$$

Dominant contributions from low energies
$\pi^+\pi^-$ channel: 73% of total

- Use direct integration method, summing up cross sections for all possible hadronic channels up to ~ 2 GeV



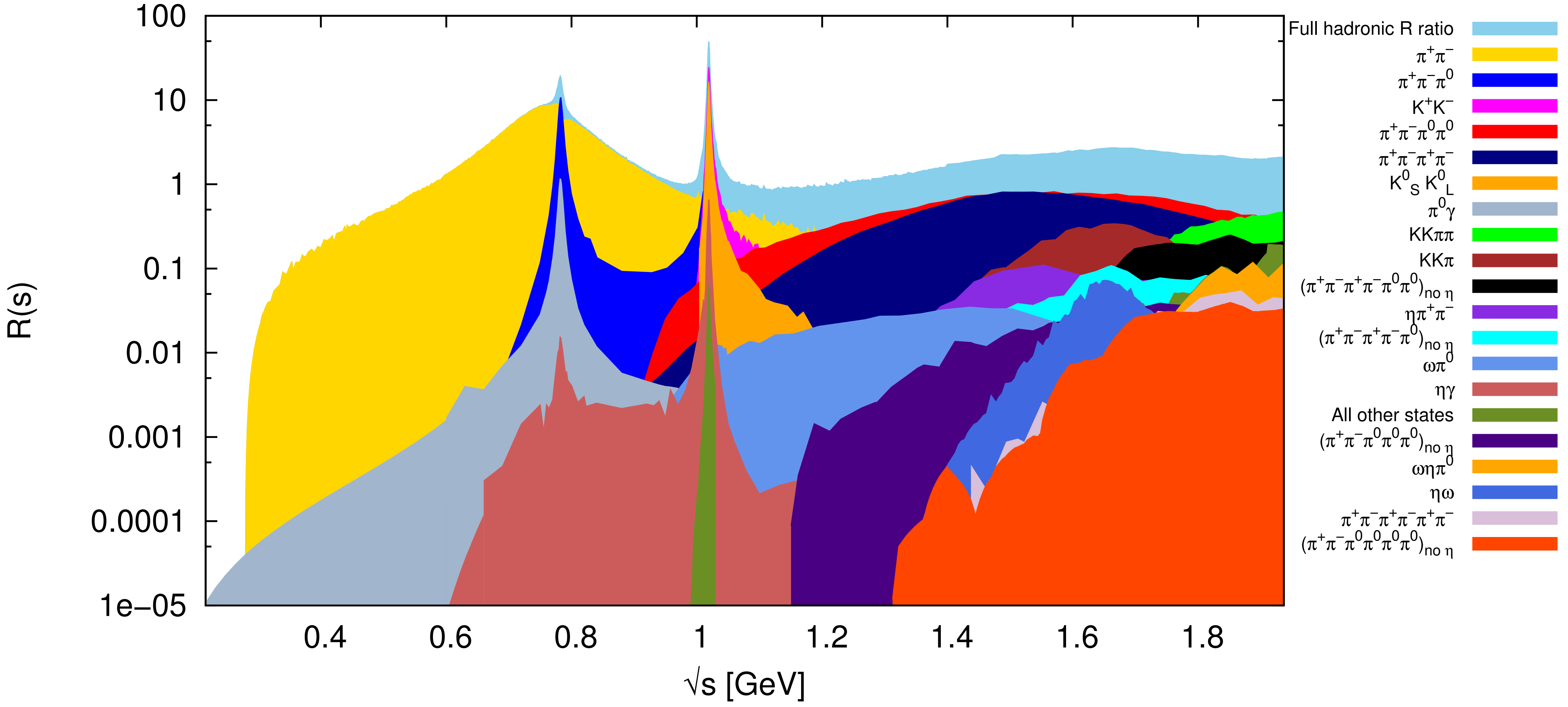
HVP: data-driven

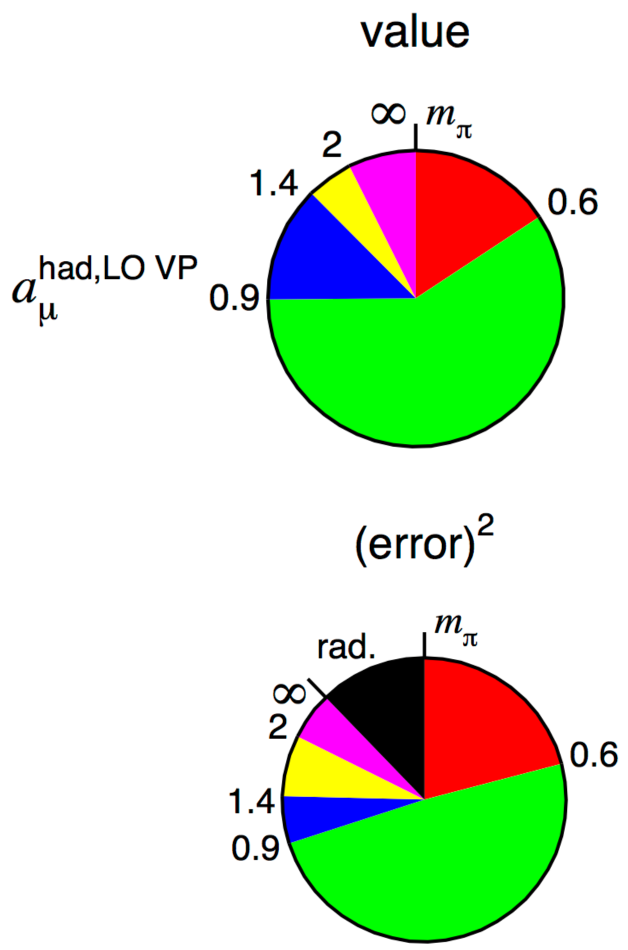
Z. Zhang for DHMZ @ INT g-2 workshop

[M. Davier et al, arXiv:1908.00921]

Channel	$a_\mu^{\text{had, LO}}\,[10^{-10}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07
$\psi(2S)$ (BW integral)	1.57 ± 0.03
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$
$R_{\text{QCD}}\,[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
$R_{\text{QCD}}\,[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04
$R_{\text{QCD}}\,[9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01
$R_{\text{QCD}}\,[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00
$R_{\text{QCD}}\,[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00
$R_{\text{QCD}}\,[> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$

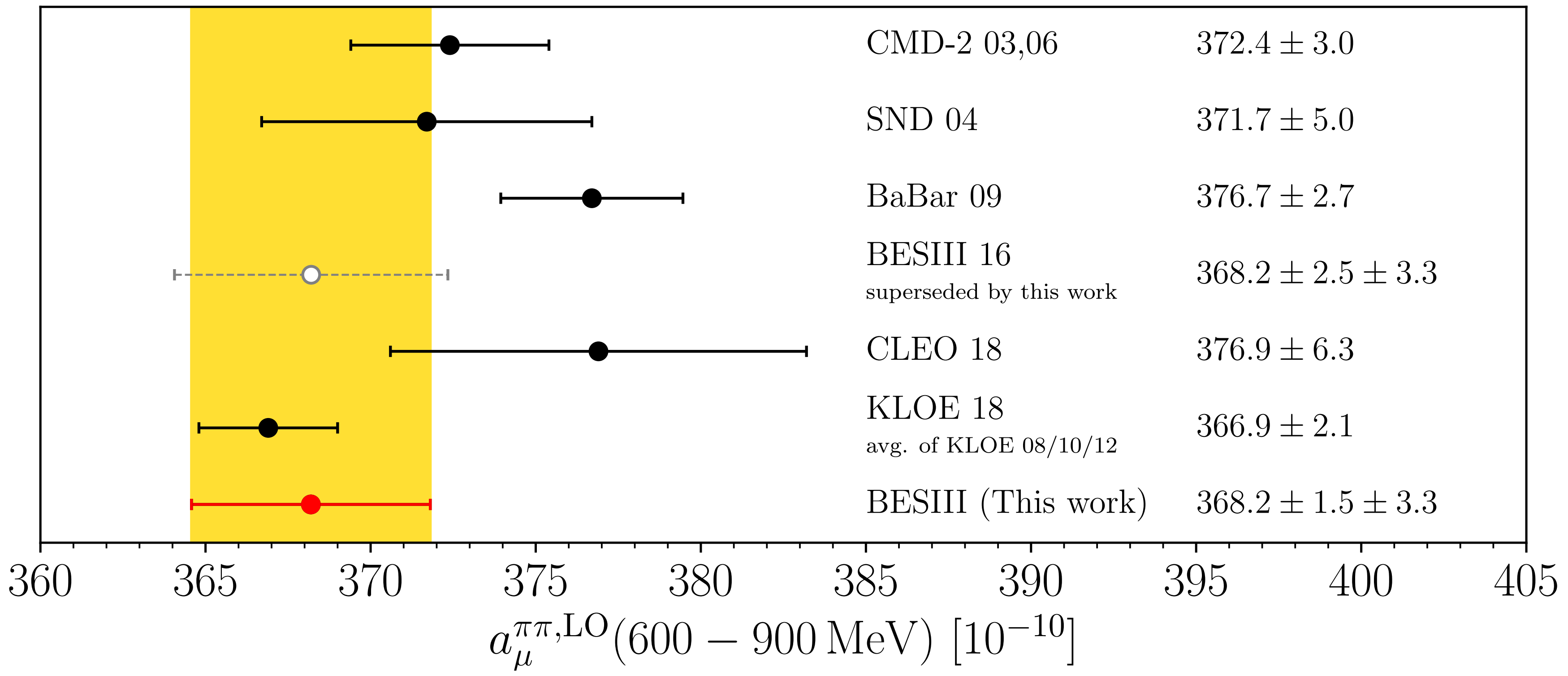
[A. Keshavarzi et al, arXiv:1802.02995]





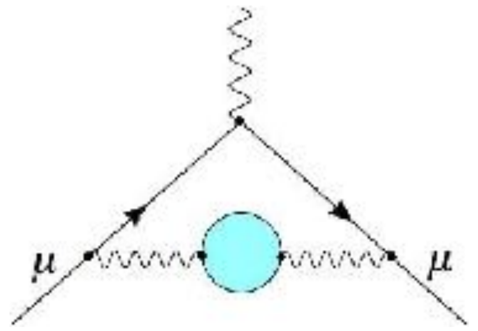
Tensions between BaBar and KLOE data sets:

[M. Ablikim et al (BES III), arXiv:2009.05011]



- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions

➠ conservative merging procedure



HVP: data-driven

In 2020 WP:

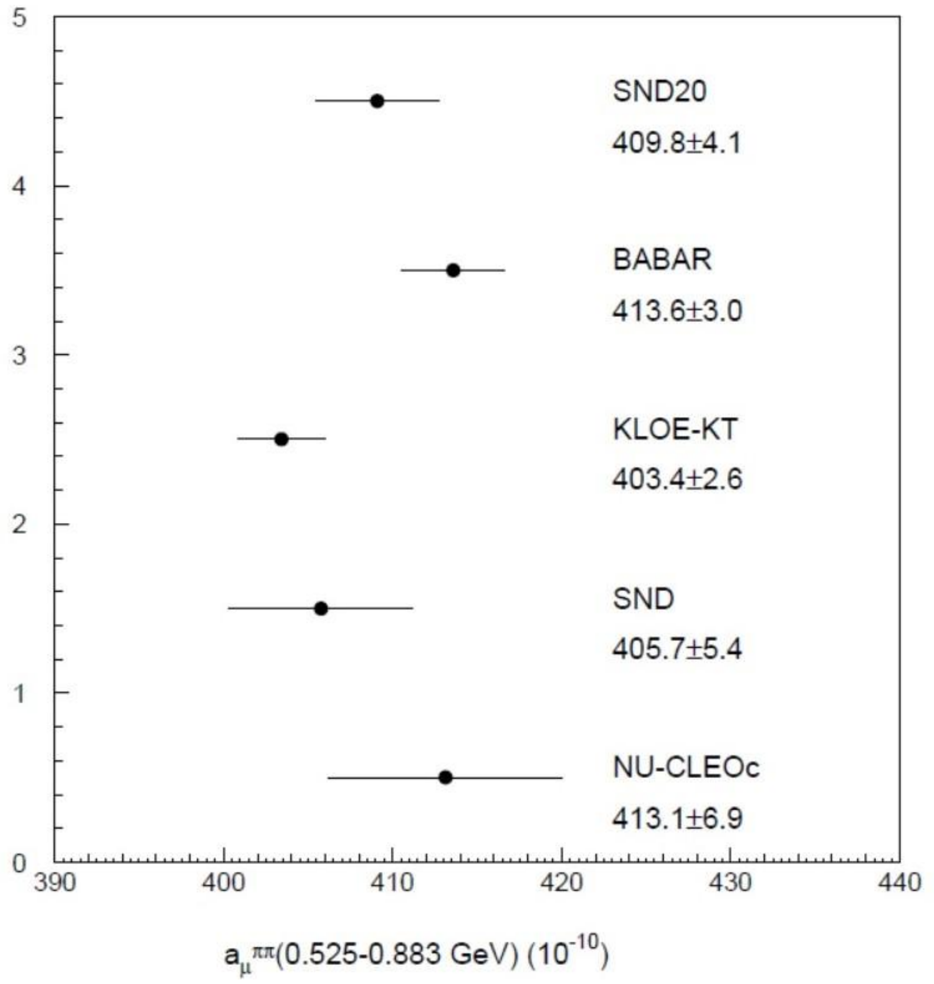
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019

$$a_\mu^{\text{HVP,LO}} = 693.1\,(2.8)_{\text{exp}}\,(0.7)_{\text{DV+pQCD}}\,(2.8)_{\text{BaBar-KLOE}} \times 10^{-10}$$

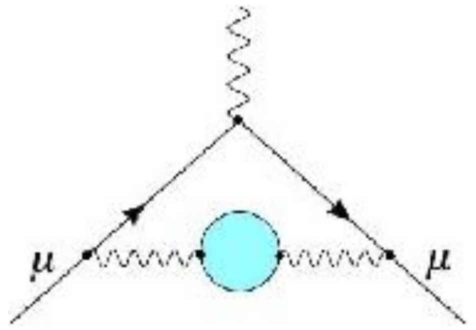
$$= 693.1\,(4.0) \times 10^{-10}$$

[M. Davier @ KEK workshop]

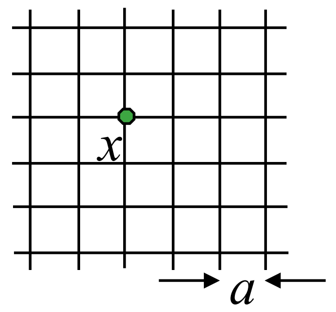


Ongoing work:

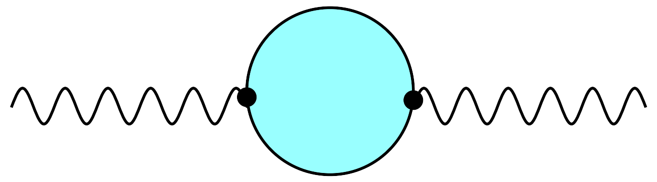
- BaBar: new analysis of large (7×) data set in $\pi\pi$ channel (1-2 years), also $\pi\pi\pi$, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- CMD-3: ongoing analyses for $\pi\pi$ and other channels
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: will have high-statistics data for low-energy cross sections.
- Some experiments performing blind analyses to resolve the tensions (esp. for $\pi\pi$ channel)
- Developing NNLO Monte Carlo generators (STRONG 2020 workshop next week https://agenda.infn.it/event/28089/)



Lattice HVP: Introduction



[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

 $\hat{\Pi}(q^2)$

Leading order HVP correction: $$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)}$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

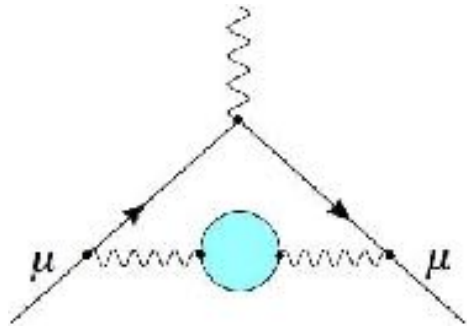
Compute correlation function: $$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$$

and $$\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$$

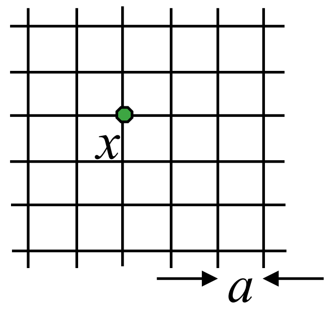
[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)}$$



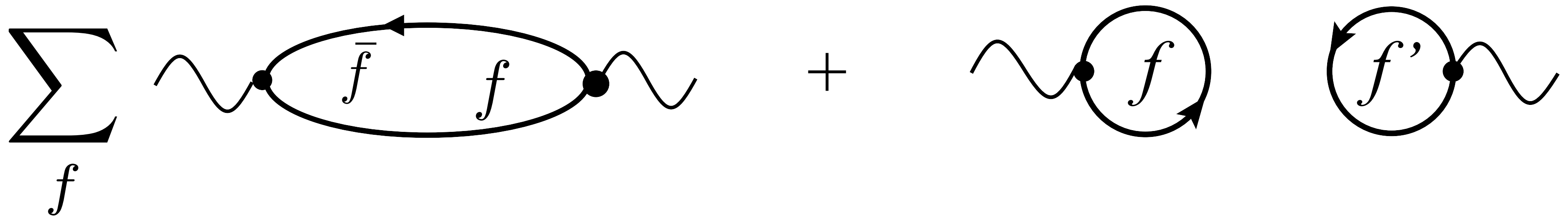
Lattice HVP: Introduction



Calculate a_μ^{HVP} in Lattice QCD:

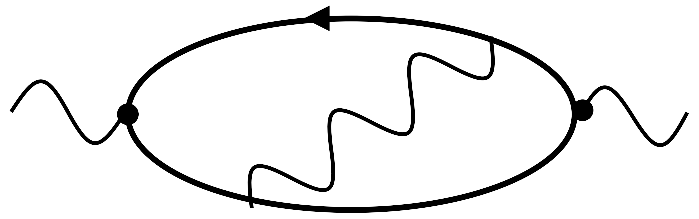
$$a_\mu^{\mathrm{HVP,LO}} = \sum_f a_{\mu,f}^{\mathrm{HVP,LO}} + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)
  Note: almost always $m_u = m_d$



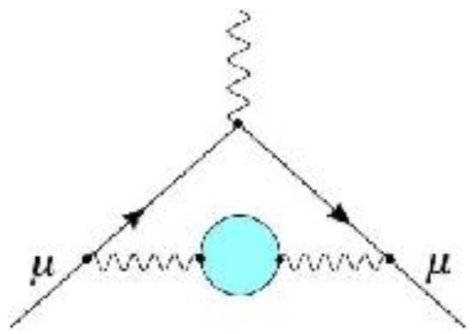
f= ud, s, c, b

- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

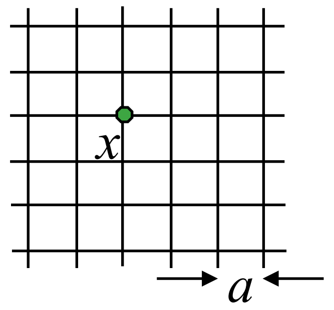


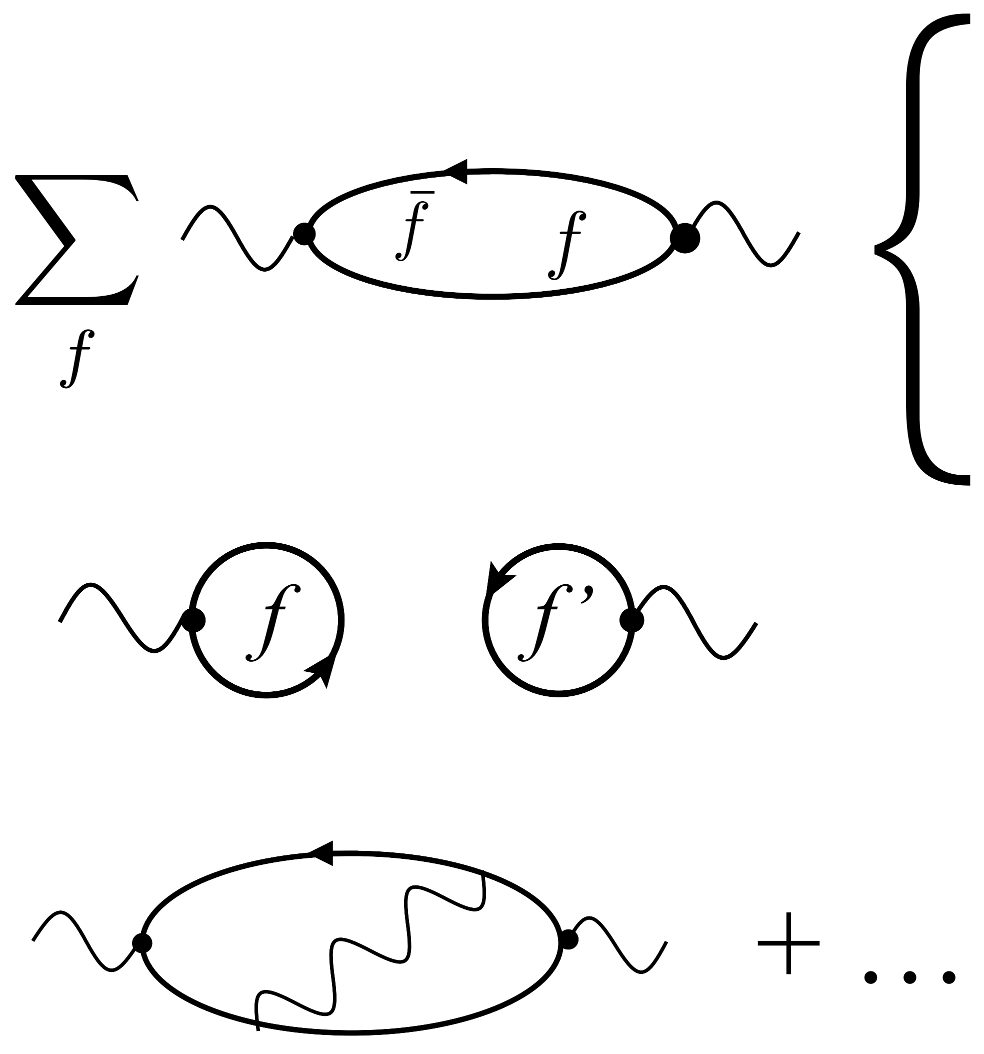
$+ \ldots$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with $m_u \neq m_d$



Lattice HVP: Introduction

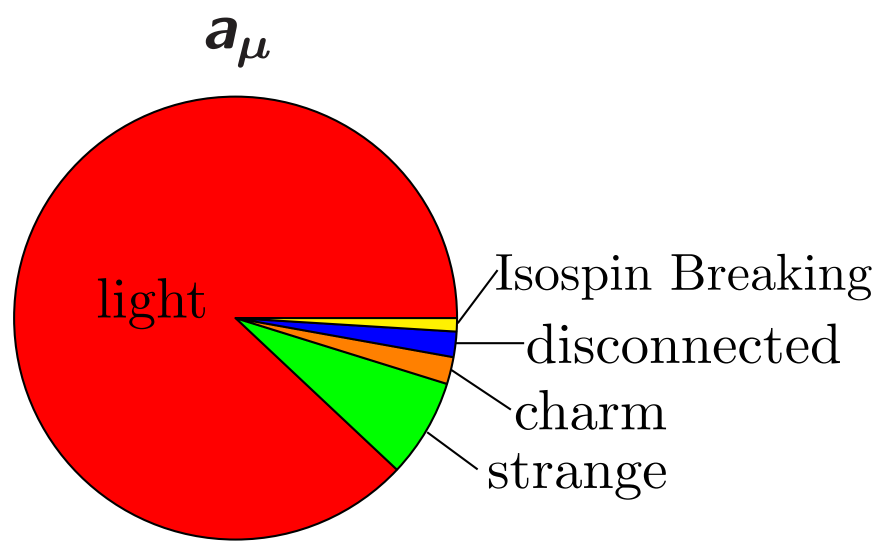


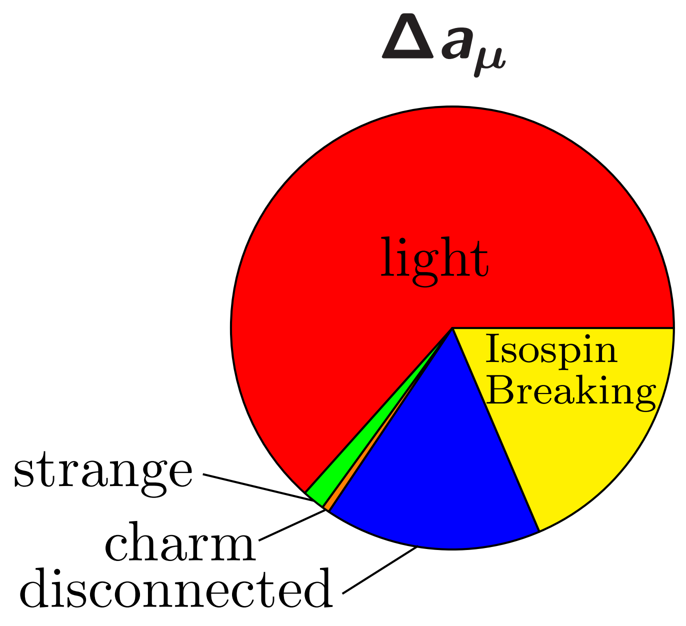


- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- s,c,b-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total

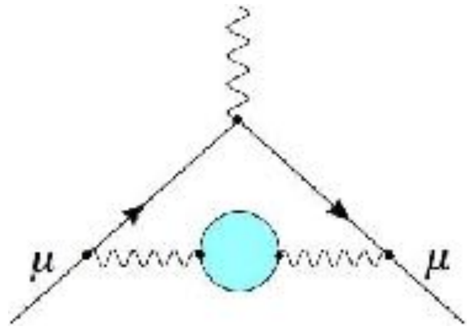
$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$

V. Gülpers @ Lattice HVP workshop





WP lattice HVP combination



HVP: Comparison

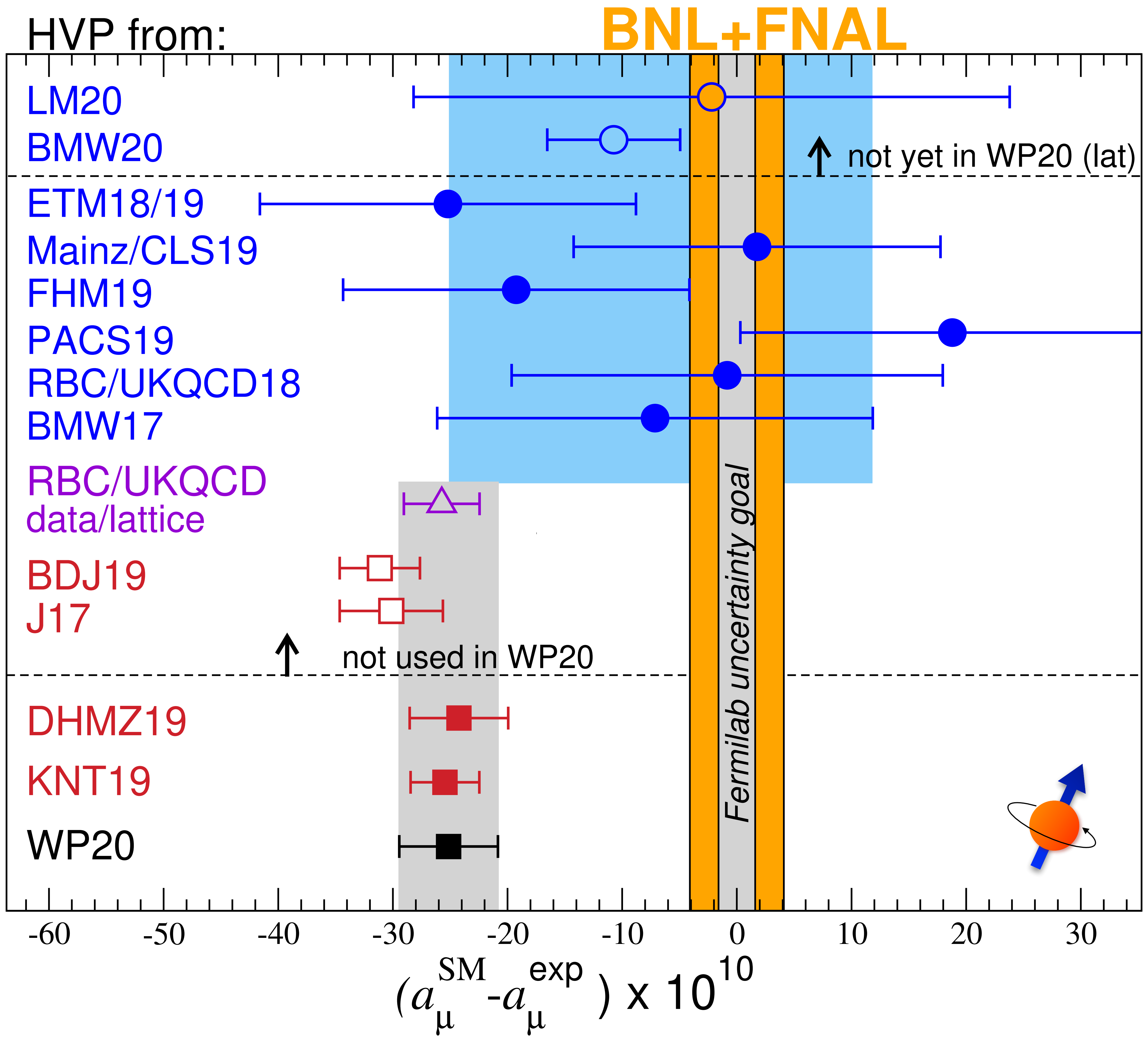
$$a_\mu^{\rm SM} \quad a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right]$$

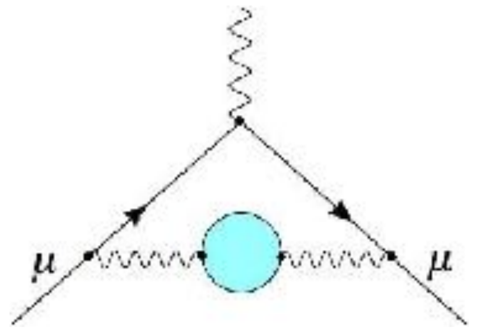
Lattice QCD + QED

hybrid: combine data & lattice

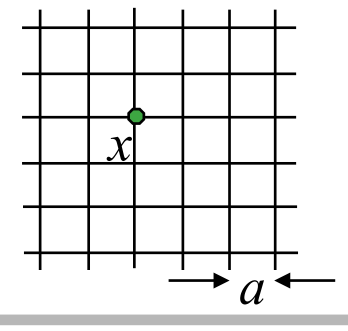
data driven

+ unitarity/analyticity constraints



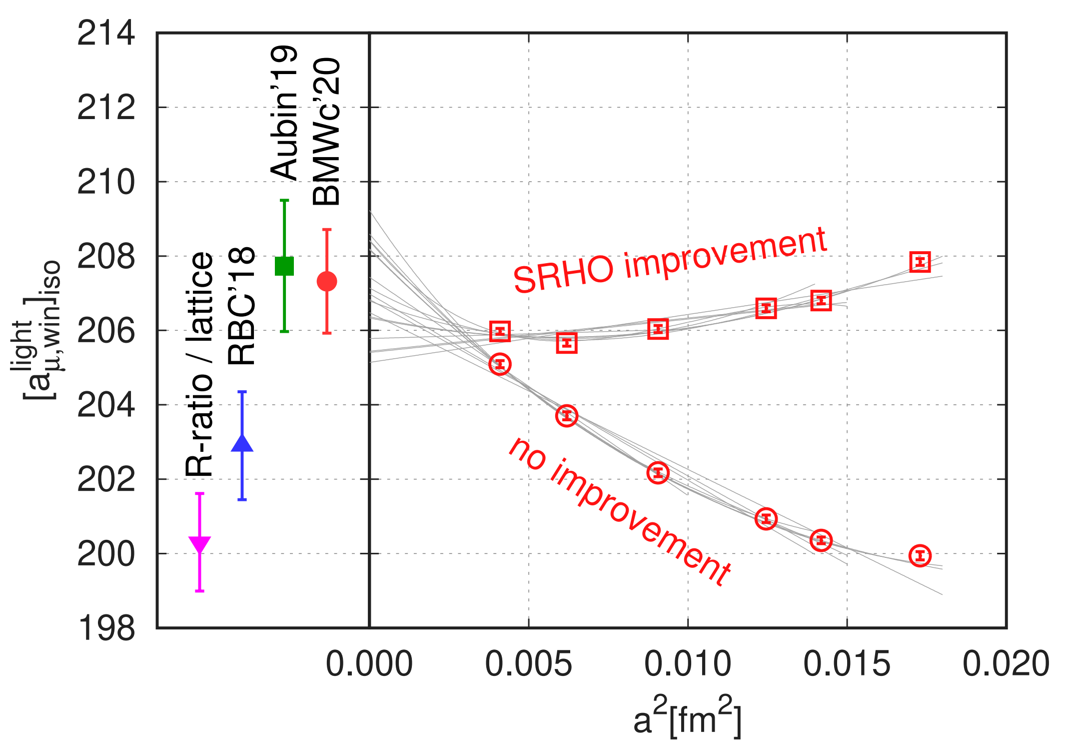


HVP: lattice



In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\rm HVP,LO} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021) first complete LQCD calculation with sub-percent error $(0.8\,\%)$ but in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window:



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

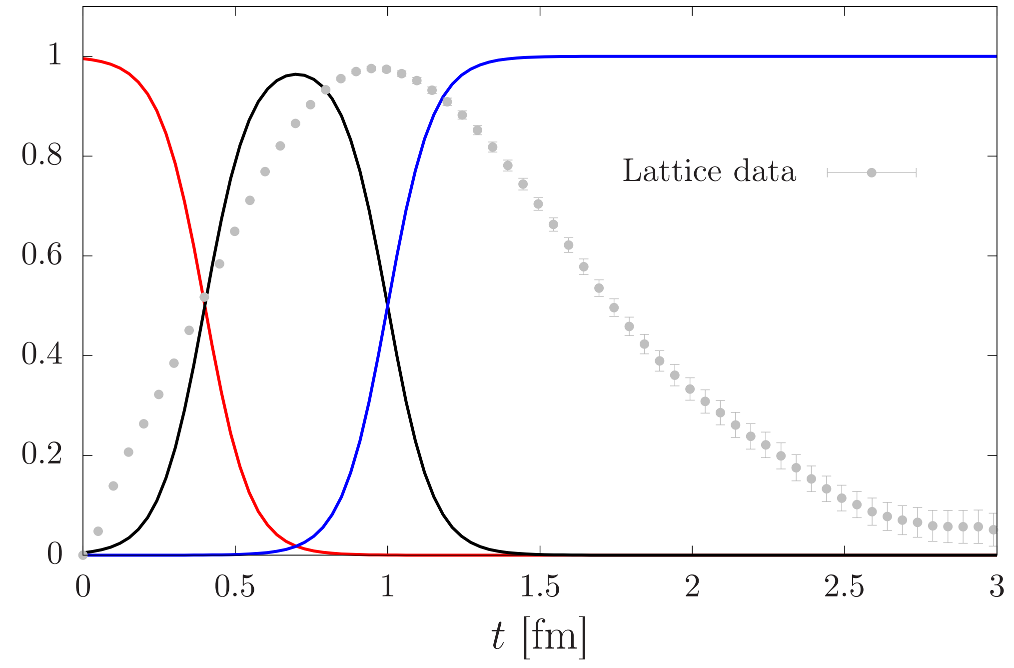
$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

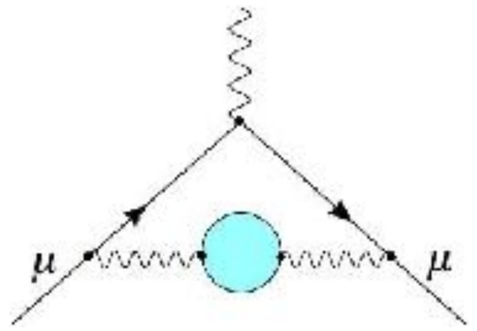
Short Distance (SD)	$t : 0 \to t_0$
Intermediate (W)	$t : t_0 \to t_1$
Long Distance (LD)	$t : t_1 \to \infty$

$t_0 = 0.4\,\text{fm}, t_1 = 1.0\,\text{fm}$

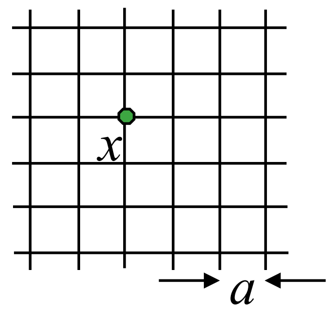
H. Wittig @ Lattice 2021



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & with disperse approach
- Internal cross check:
  Compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$



HVP: lattice



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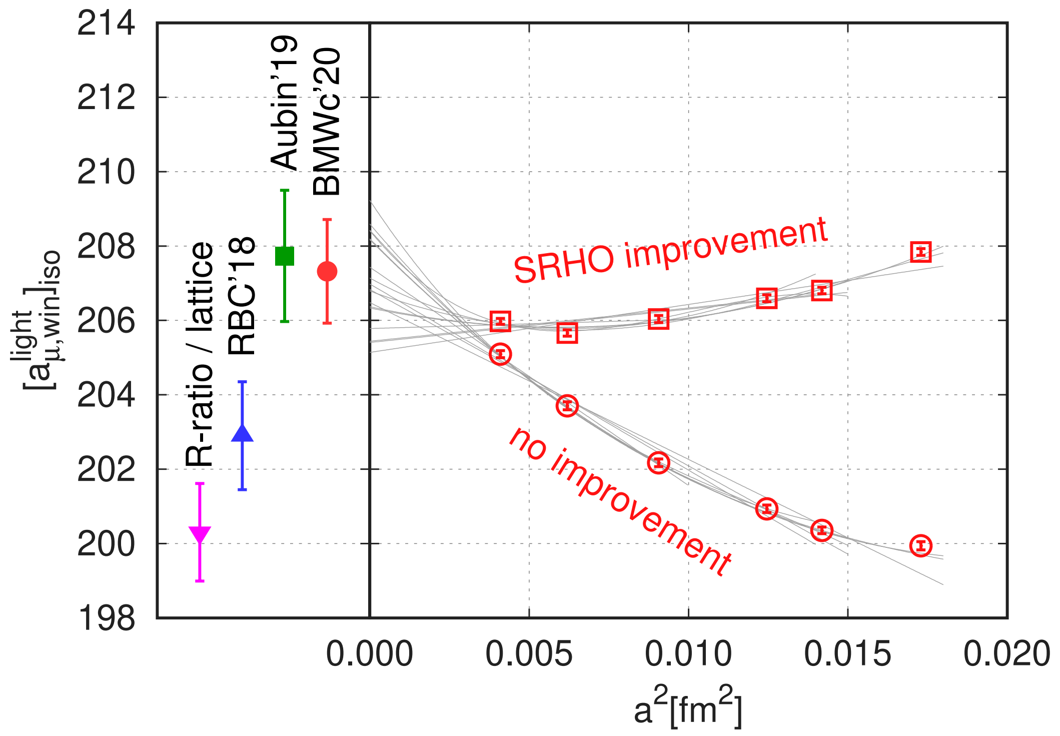


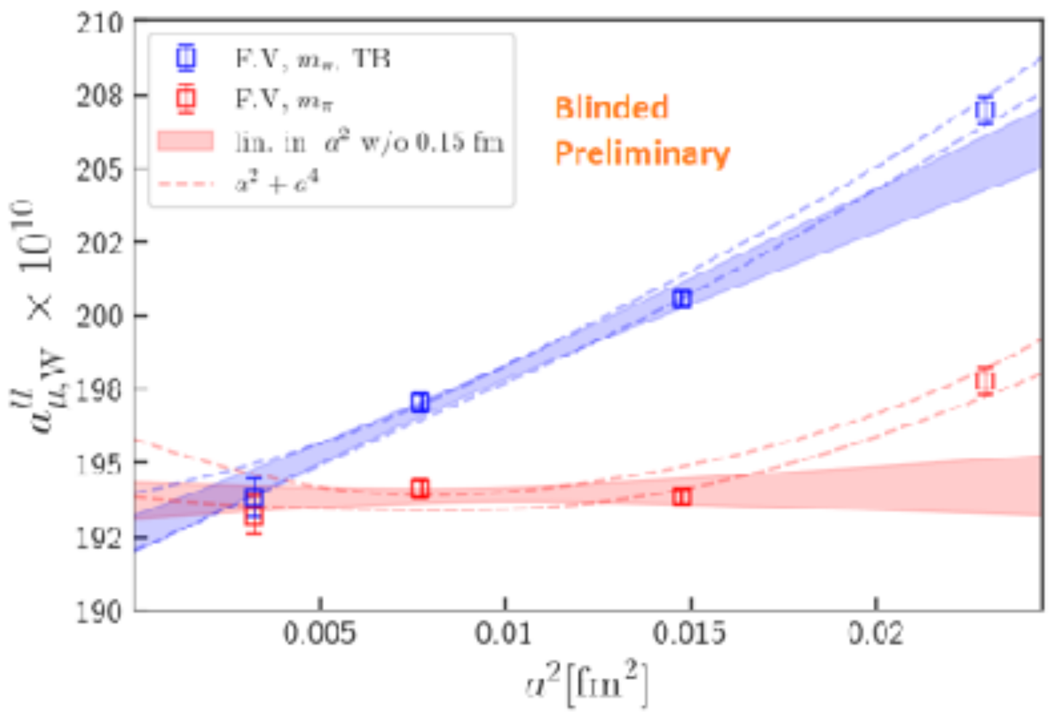
-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

Ongoing work:

- Expect new results from RBC/UKQCD, FNAL/HPQCD/MILC, ETMc, Aubin et al, …in the coming months:

S. Lahert (FNAL/HPQCD/MILC)



- blind analyses being done in FNAL/HPQCD/MILC and RBC/UKQCD
- Including $\pi\pi$ states for refined long-distance computation
  (Mainz, RBC/UKQCD, FNAL/MILC)
- Developing method average for lattice HVP — started at KEK workshop (June 2021), based on detailed comparisons
  - list of sub quantities (and their definitions)
  - common prescription for separating QCD & QED
  - quality criteria for inclusion
- Most groups plan to include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

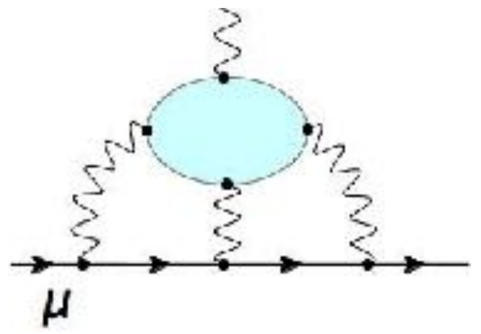
Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.

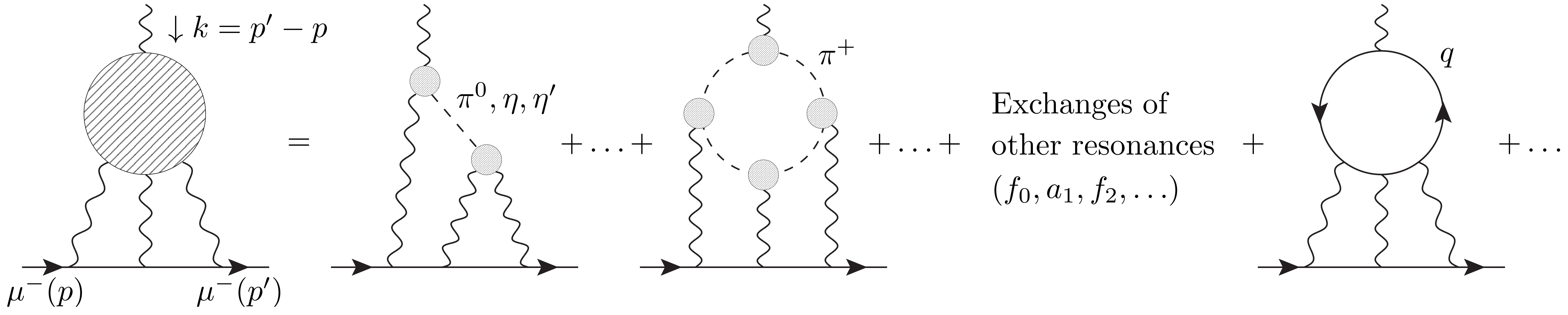
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer 2021]

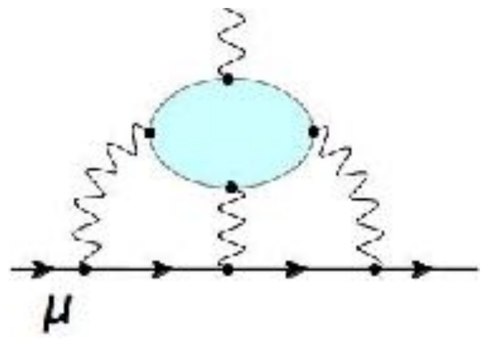


Hadronic Light-by-light

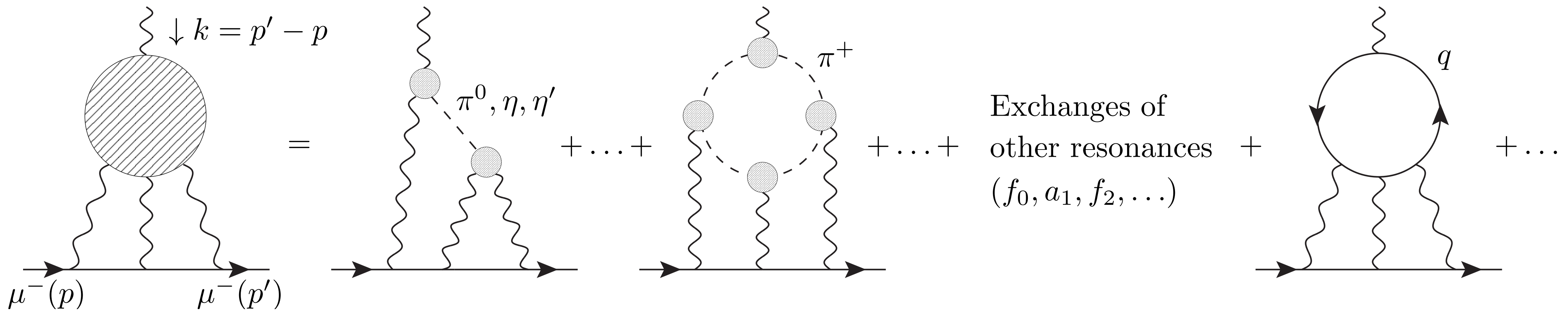
Hadronic light-by-light: Target: ≲ 10% total error



- ✦ previous estimates “Glasgow consensus” use models of QCD
- ✦ used to evaluate individual contributions to HLbL scattering tensor
- ✦ theory error not well determined and not improvable



Hadronic Light-by-light

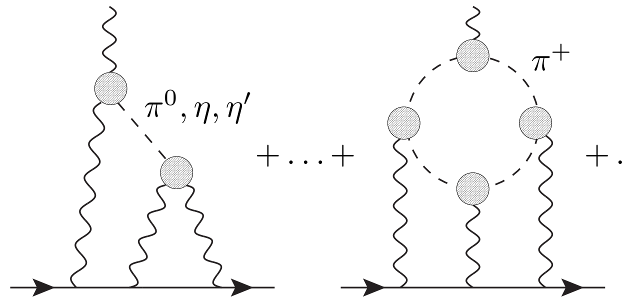


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

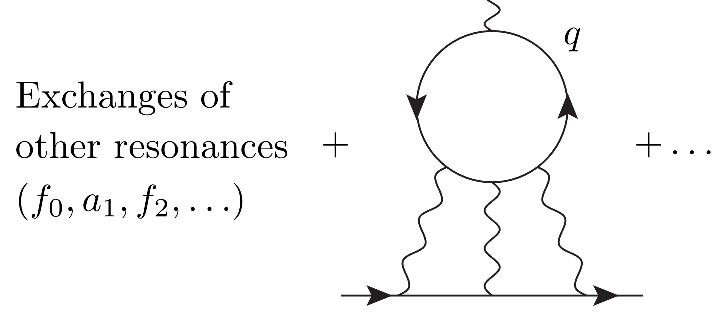
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



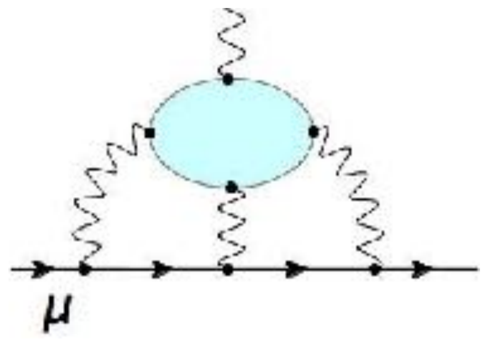
- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ η, η' pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 15\,\%$ of total):

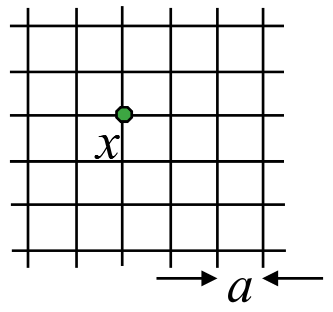


- ✦ Not yet well known
  ➠ dominant contribution to total uncertainty

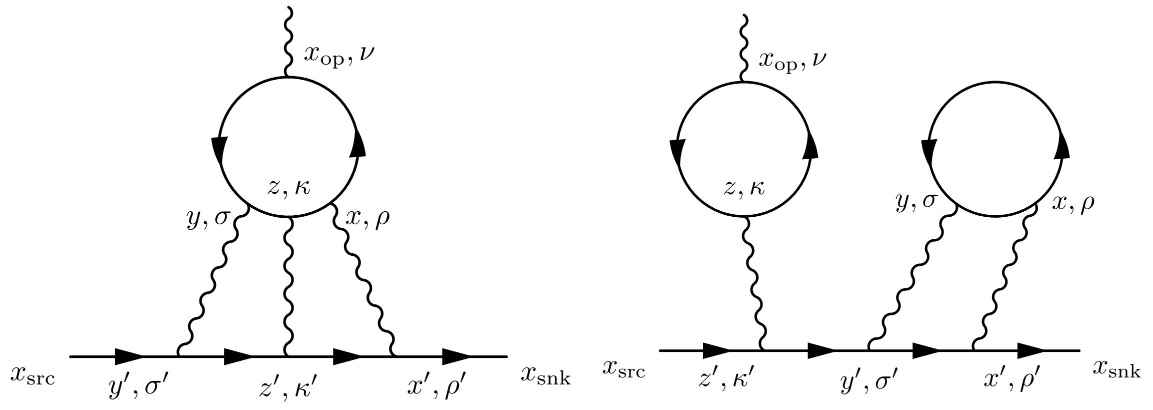
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - BESIII ramping up $\gamma^{(*)}\gamma^*$ program

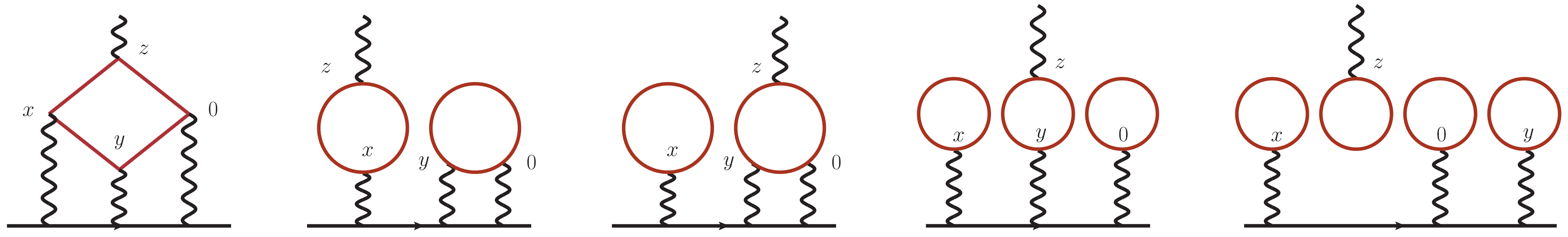


Hadronic Light-by-light



Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





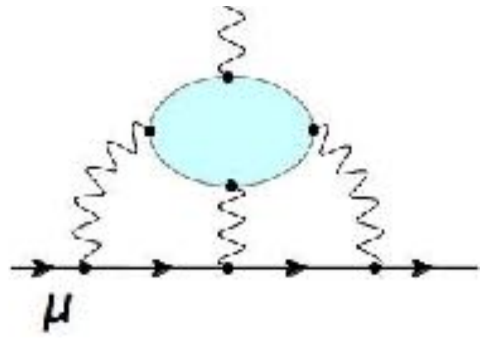
- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + QED$_\mathrm{L}$ (finite volume)
  ➠ $1/L^2$ FV effects
  stochastic evaluation of position space sums

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm},\ L \sim 4.5 - 9.3\,\mathrm{fm}$

- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum)
  ➠ $e^{-m_\pi L}$ FV effects
  semi-analytic QED kernel function

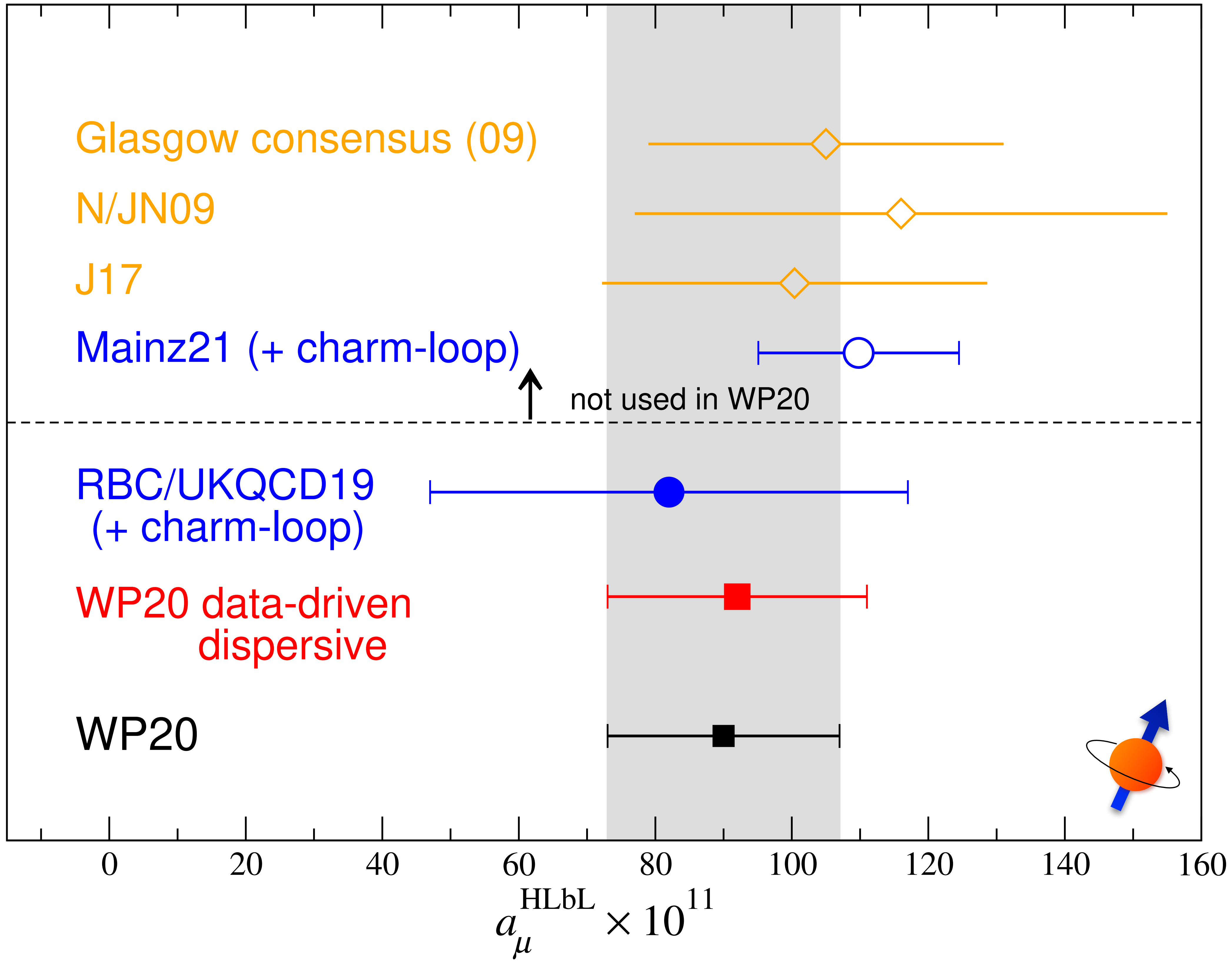
  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV},\ a \approx 0.05 - 0.1\,\mathrm{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ✦ Both groups will continue to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)



HLbL: Comparison

$a_\mu^{\rm HLbL}$



models of QCD +EFT, large N_c

Lattice QCD + QED

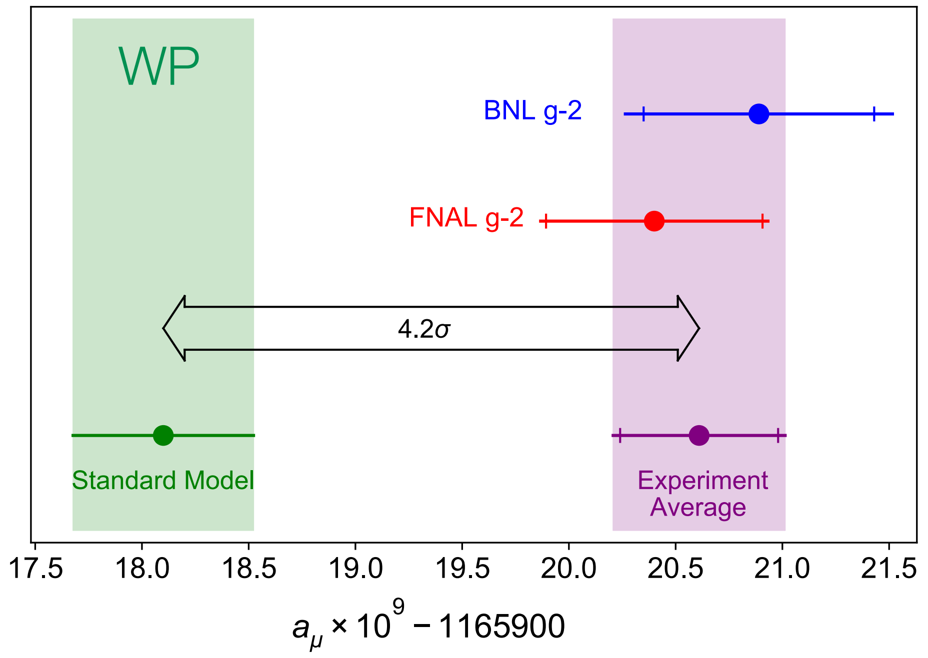
data + dispersive approach

Now well-determined in two independent approaches, systematically improvable

Timeline


Physics Reports

The anomalous magnetic moment of the muon in the Standard Model



Theory Initiative:

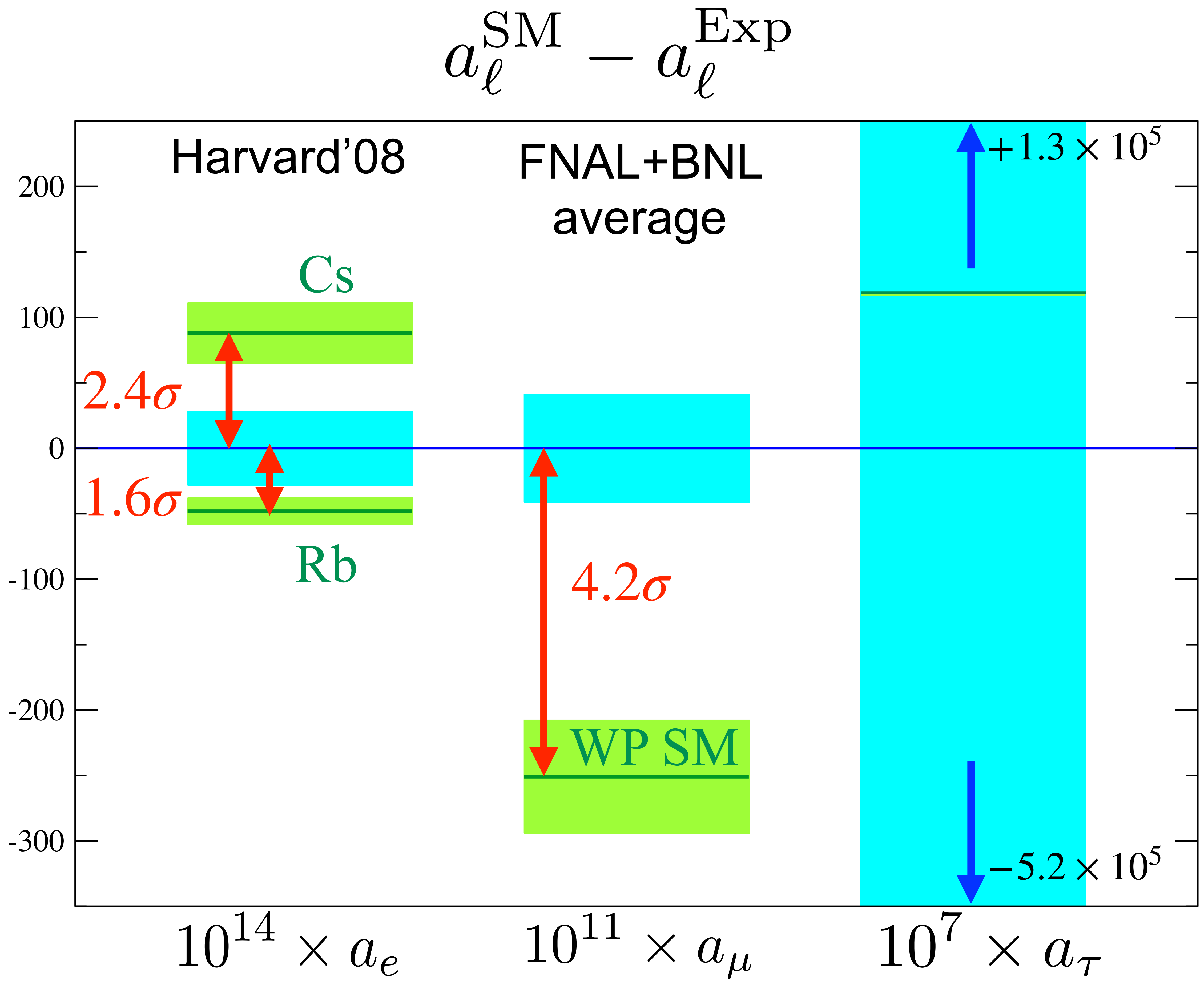
- ongoing activities: develop method average for Lattice HVP
- plan to update WP with new SM predictions (~ 2023)

★ TI workshops: Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre

Summary

- The QED and EW contributions are known very precisely
  Hadronic contributions determine the uncertainty in the SM prediction.
- dispersive HVP: ~0.6% error [0.34ppm]
  based on well-tested experimental data, will be improved with new measurements (coming soon).
- lattice HVP: first LQCD calculation with sub-percent uncertainty by BMWc
  but in tension with data-driven approach
  and mild tensions between lattice results for intermediate window
- dispersive HLBL: ~20% error [0.15ppm]
  newly developed dispersive approach with almost fully quantified errors
  systematically improvable
- lattice HLbL: two complete lattice calculations
  consistent with each other and with data-driven result
  systematically improvable

Lepton moments summary



Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4\times10^4$$

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Beyond the SM possibilities

a_μ is loop-induced, conserves CP & Flavor, flips chirality.

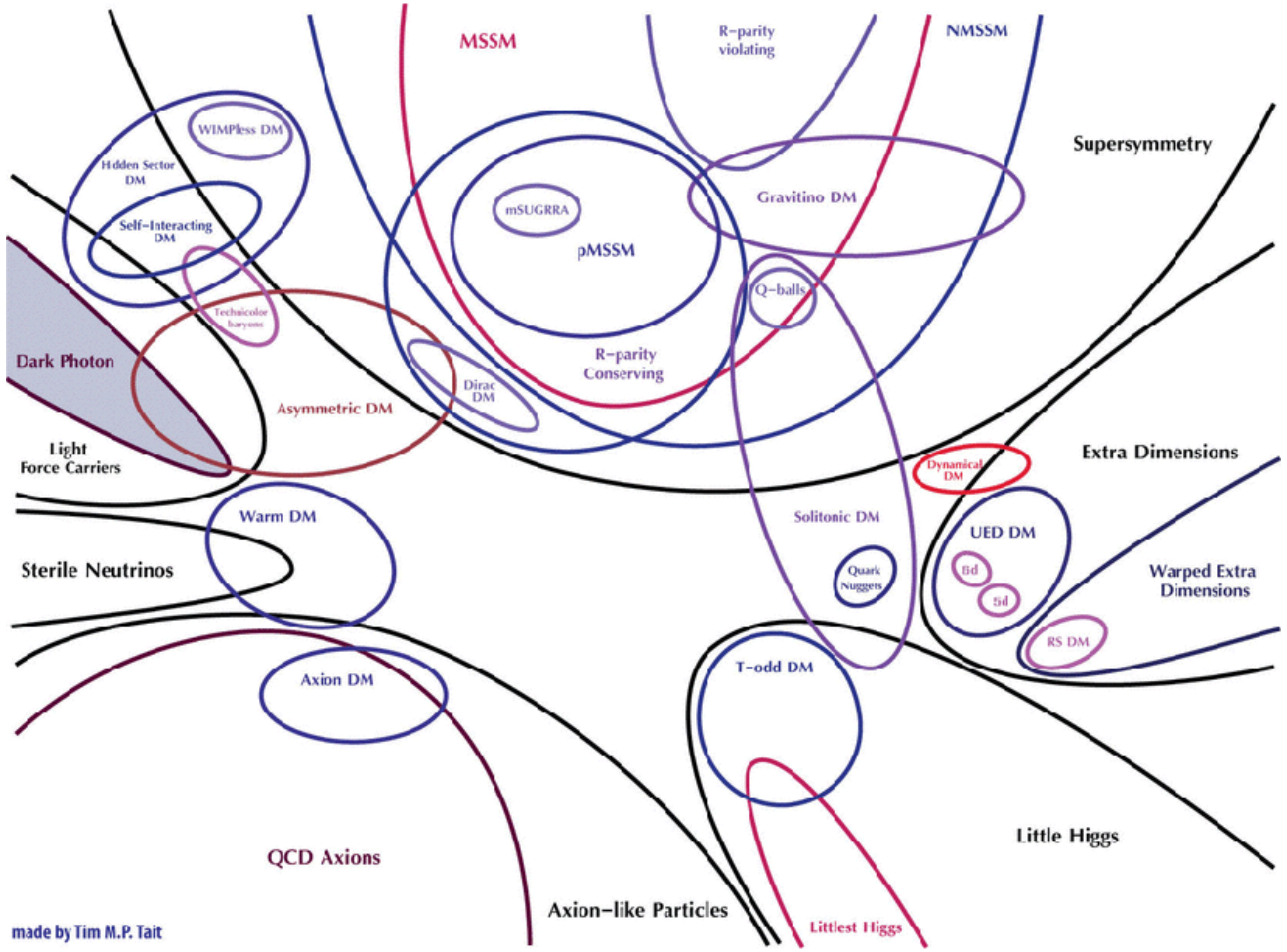
The difference between Exp-SM is large:

$$\Delta a_\mu = 251(59) \times 10^{-11} \;>\; a_\mu(\mathrm{EW})$$

Generically expect:

$$a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$$

Can still be accommodated by many BSM theories, with constraints.

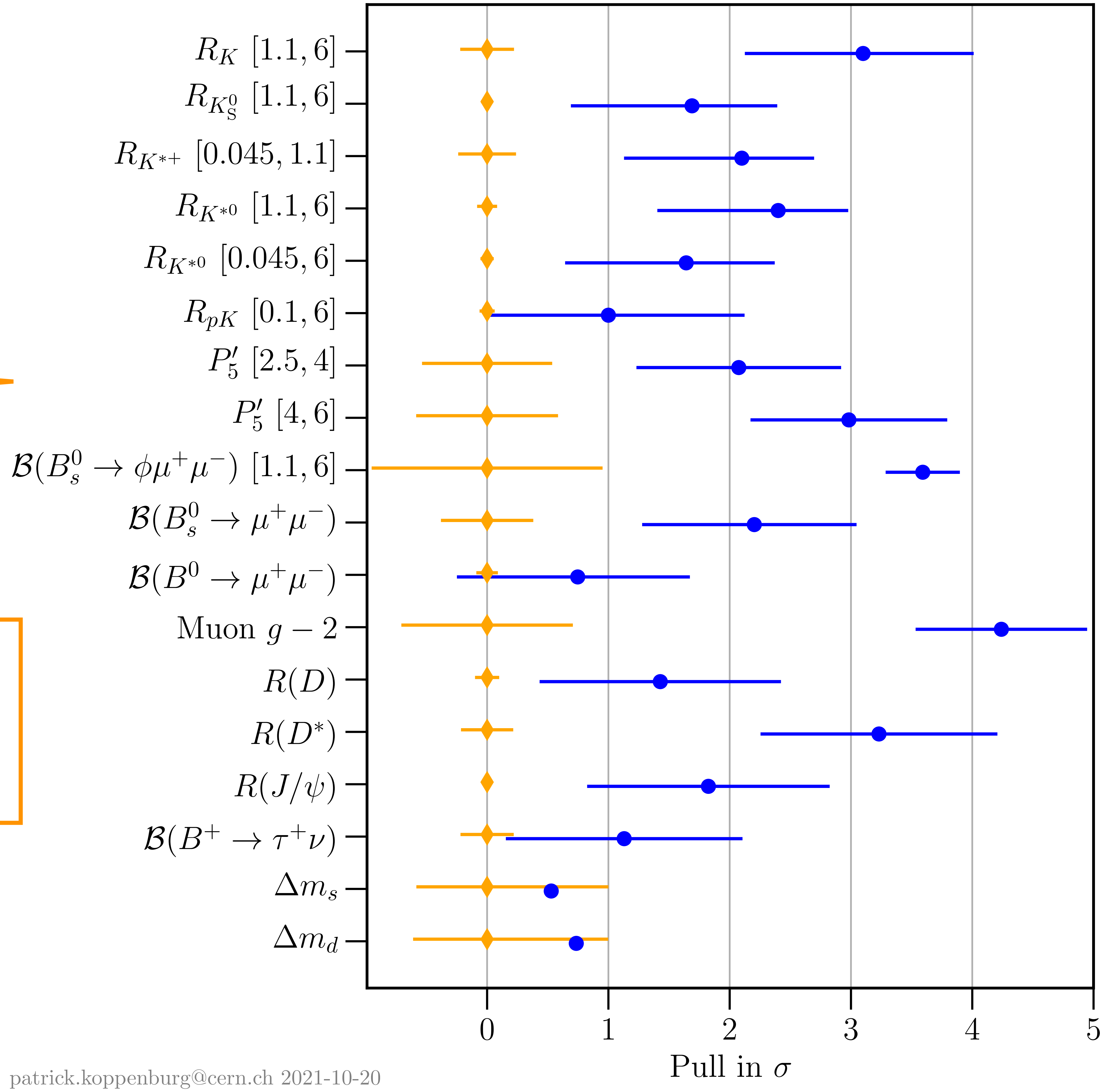


Flavor Anomalies

Patrick Koppenburg [http://www.scholarpedia.org/article/Rare_decays_of_b_hadrons]

Cherry-picked selection

theoretical SM predictions



experimental measurements

LQCD uncertainties are being improved for many flavor quantities ….

Experimental errors are also improving:
Belle II, LHCb,
Fermilab muon g-2, ….
and new experiments will add more information.

Outlook

- Theory Initiative:
  - WP update ~2023 will include any new available results and a method average for lattice HVP, HLbL
  - Concrete plans for writing WP update (outline, authors,…) @ next workshop
- Programs and plans in place to improve the hadronic contributions in both approaches
  - data-driven HVP: new, precise experimental measurements to resolve the tension between BaBar and KLOE
  - lattice HVP: new, precise results from several lattice groups coming soon, needed to scrutinize BMW result
  - disp. HLbL: quantify sub leading contributions (SDCs, axial vectors)
  - Lattice HLbL: refine direct calculations. Also efforts to compute pseudo scalar pole contributions
- If tensions between data-driven HVP and lattice HVP are resolved, SM predictions will likely reach desired precision
- Beyond 2025: MUonE (space-like momentum measurement of $\Delta\alpha$) will provide more information/cross checks.

➠ Next workshop of the Muon g-2 Theory Initiative: 5-9 Sep 2022

Fermilab

UNIVERSITY of WASHINGTON

INSTITUTE for NUCLEAR THEORY

U.S. DEPARTMENT OF ENERGY

Office of Science

ILLINOIS

KEK

NEC

Fermi Research Alliance LLC

Thank you!

UNIVERSITIES RESEARCH ASSOCIATION

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HIM HELMHOLTZ Helmholtz-Institut Mainz

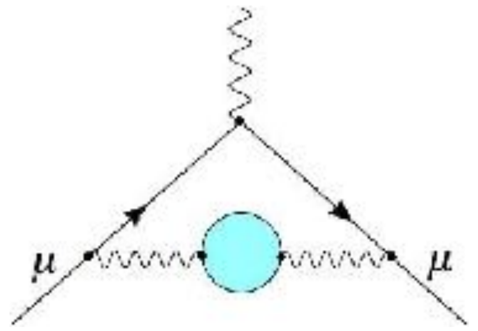
SFB 1044 THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

PRISMA

UCONN DEPARTMENT OF PHYSICS

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Appendix



HVP: data-driven

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

[B. Malaescu @ INT g-2 workshop]

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

Include constraints using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, Colangelo et al, arXiv:1810.00007; HHKS19, Hoferichter et al, arXiv:1907.01556]

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(2.8)_{\text{exp}}\,(2.8)_{\text{sys}}\,(0.7)_{\text{DV+pQCD}} \times 10^{-10} = 693.1\,(4.0) \times 10^{-10}$$

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

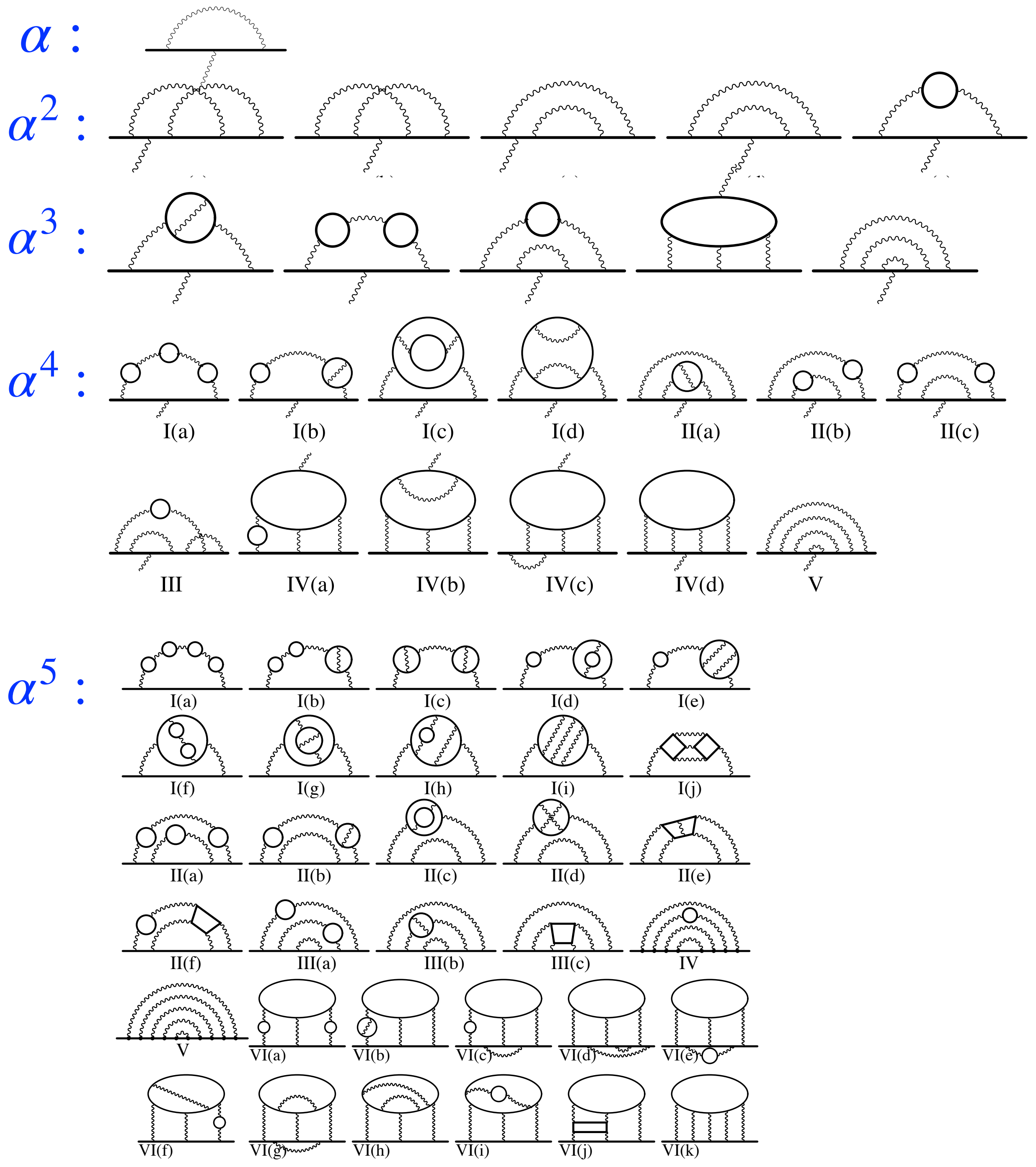
$$a_\mu(\text{QED}) = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0} \left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\text{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]

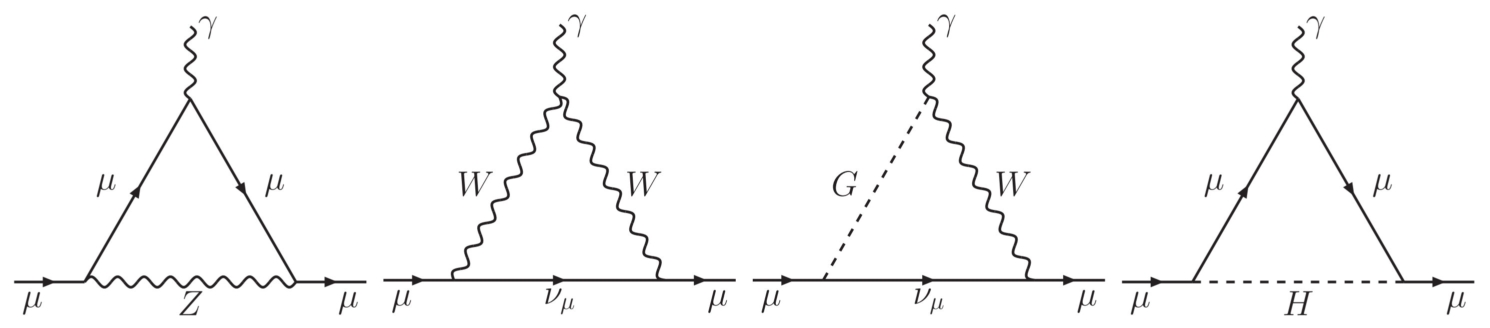


Muon g-2: SM contributions

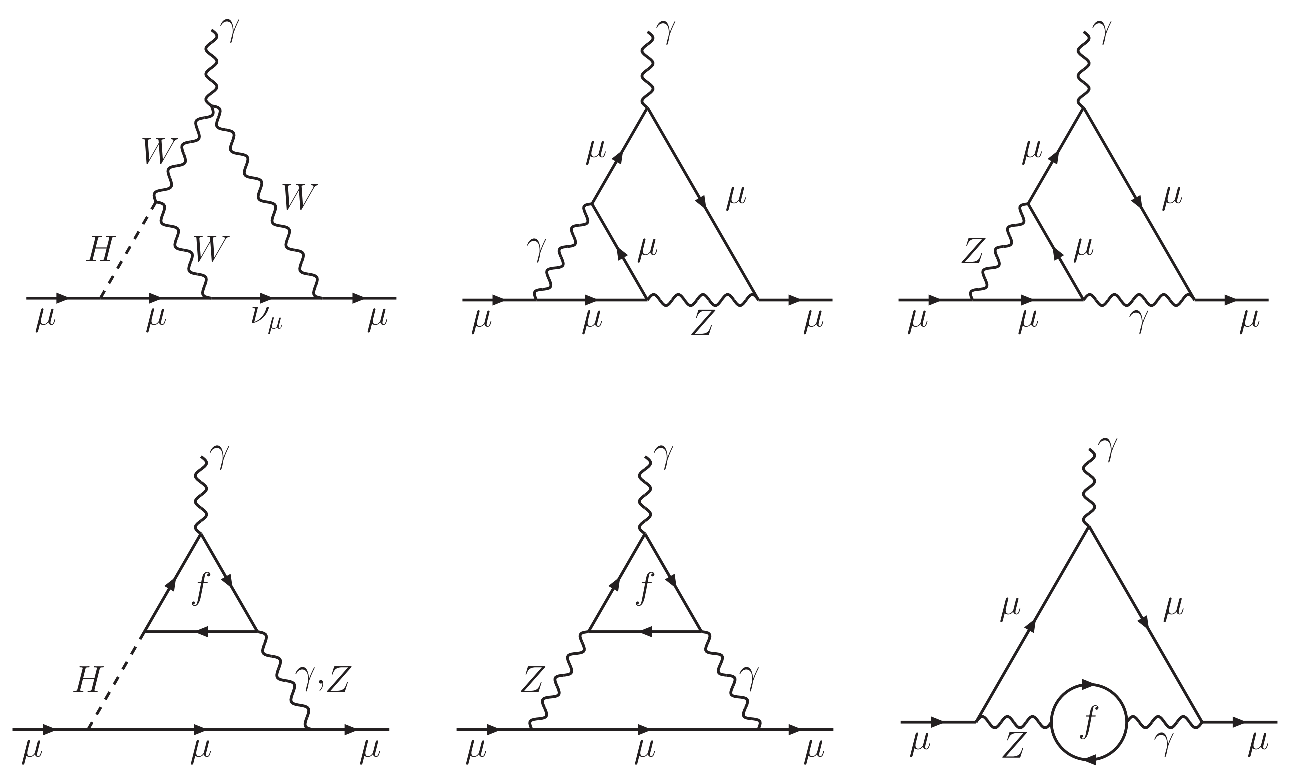
$$\boxed{a_\mu = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{EW}) + a_\mu(\mathrm{hadronic})}$$

Electroweak
(contributions from W,Z,H,.. bosons)

1-loop



2-loop



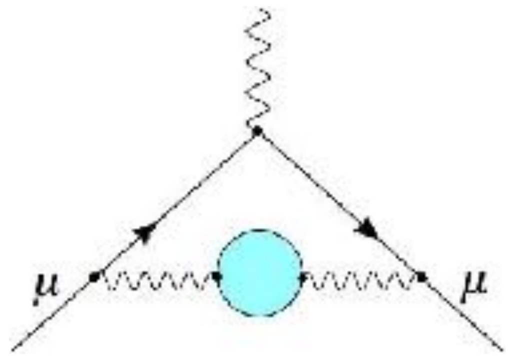
$$\boxed{a_\mu(\mathrm{EW}) = 153.6\,(1.0) \times 10^{-11}}$$

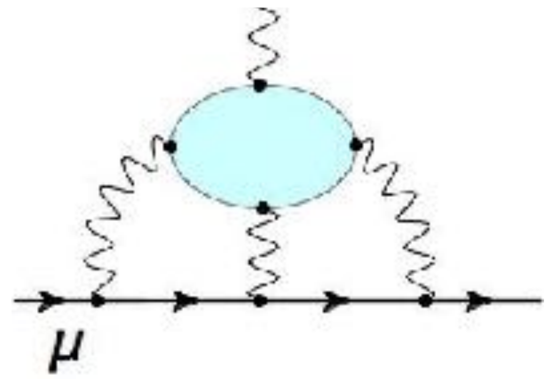
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

$$\boxed{a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})}$$

leading hadronic

α^2 

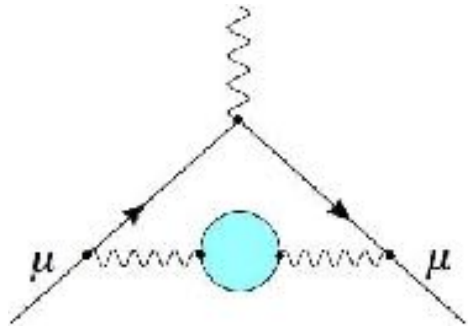
α^3 

✦ The hadronic contributions are written as:

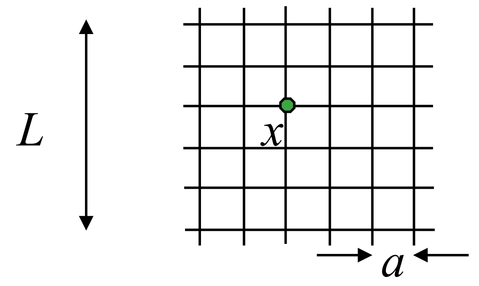
$$a_\ell(\text{hadronic}) = a_\ell^{\text{HVP, LO}} + a_\ell^{\text{HVP, NLO}} + a_\ell^{\text{HVP, NNLO}} + \ldots$$
$$+ a_\ell^{\text{HLbL}} + a_\ell^{\text{HLbL, NLO}} + \ldots$$

α^2 α^3 α^4

$\sim \quad 10^{-7}$



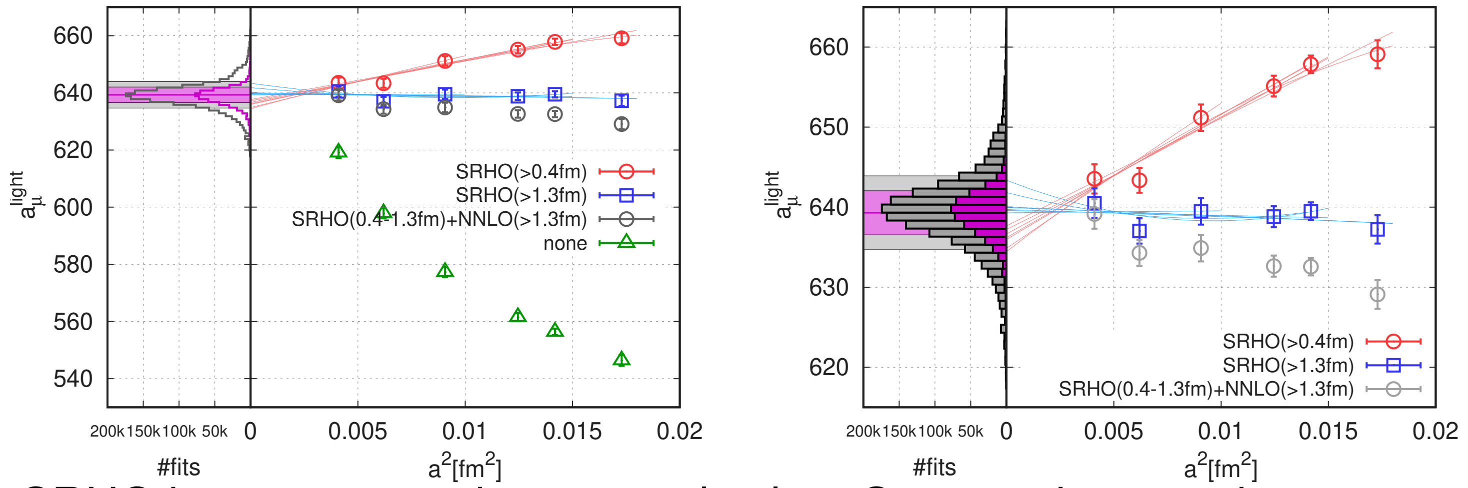
Lattice HVP: Continuum extrapolation



Kalman Szabo (BMWc) @ Lattice 2021

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\mathrm{SRHO}}(a) + a_\mu^{\mathrm{RHO}}$
- reduces lattice artefact, also makes a^2 dependence linear



SRHO improvement gives central value. Systematic errors by:

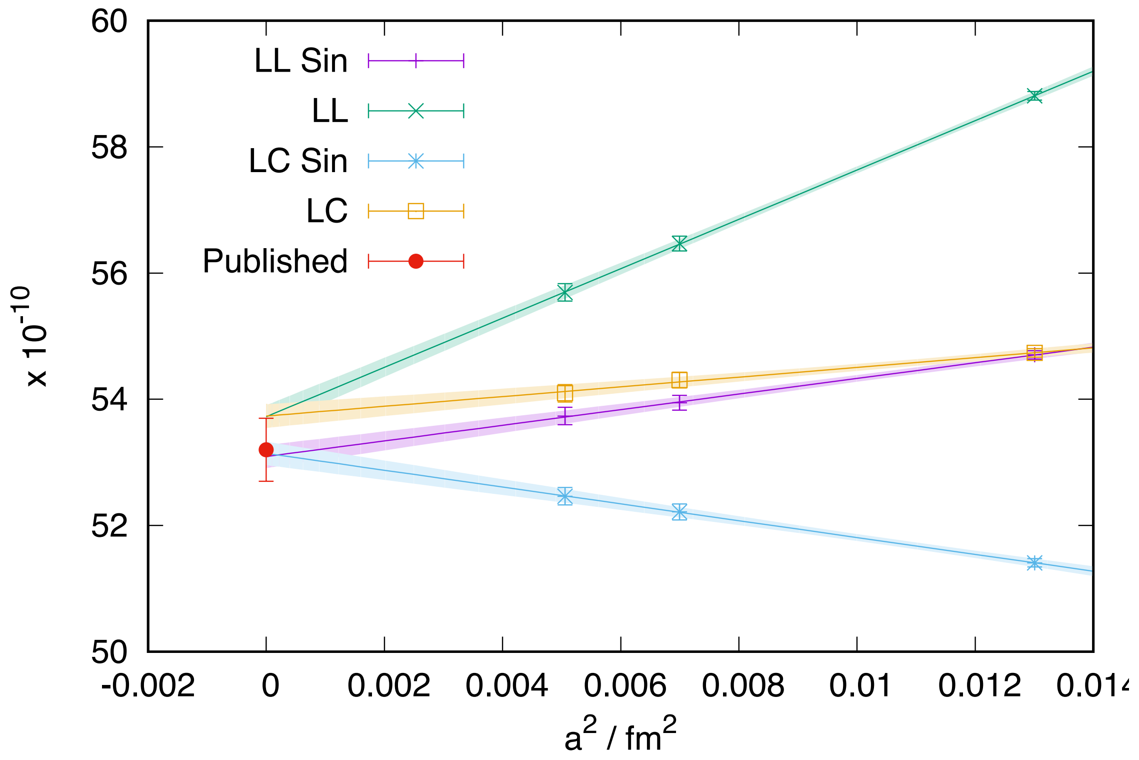
1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

- Large taste-breaking effects with BMW set-up
  ➠ uncorrected data not easily fit to power series, i.e.

1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\frac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\frac{1}{a})\right]^2$

Christoph Lehner (RBC/UKQCD) @ Lattice 2021

- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):

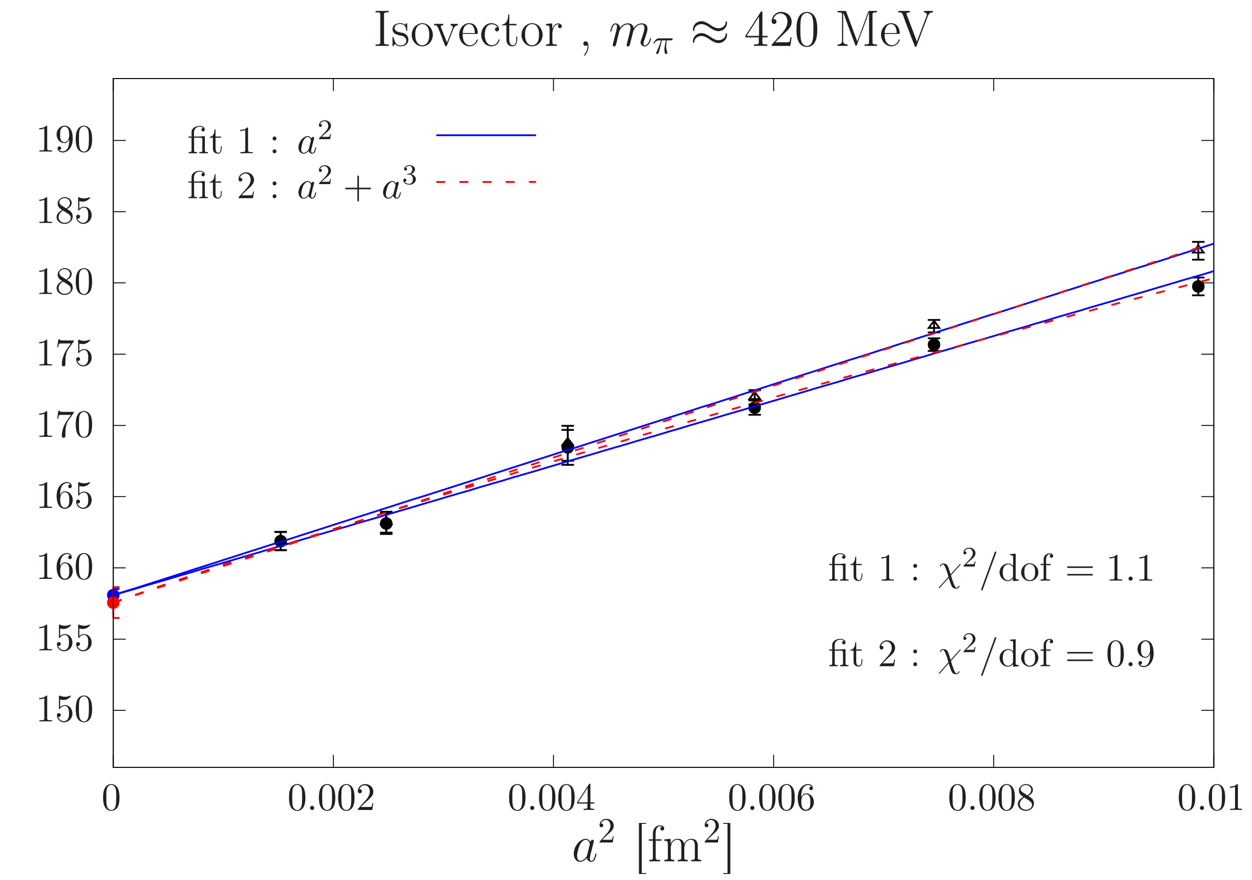


- For light quark use new 96I ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

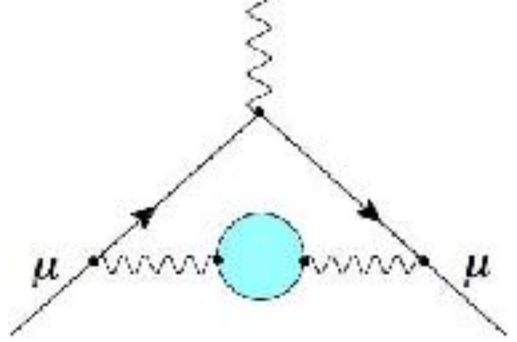
- RBC:
  Currently adding add a third lattice spacing

- Fermilab-HPQCD-MILC:
  planning to add a 5th lattice spacing (0.042 fm).

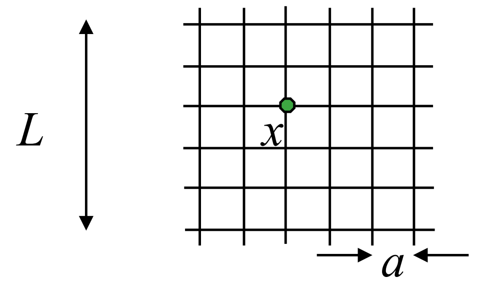
Hartmut Wittig (Mainz) @ Lattice 2021



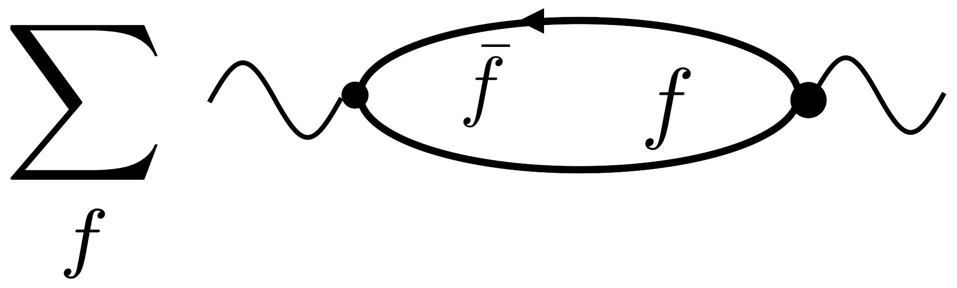
- Mainz and ETMc perform combined chair and continuum extrapolaiton

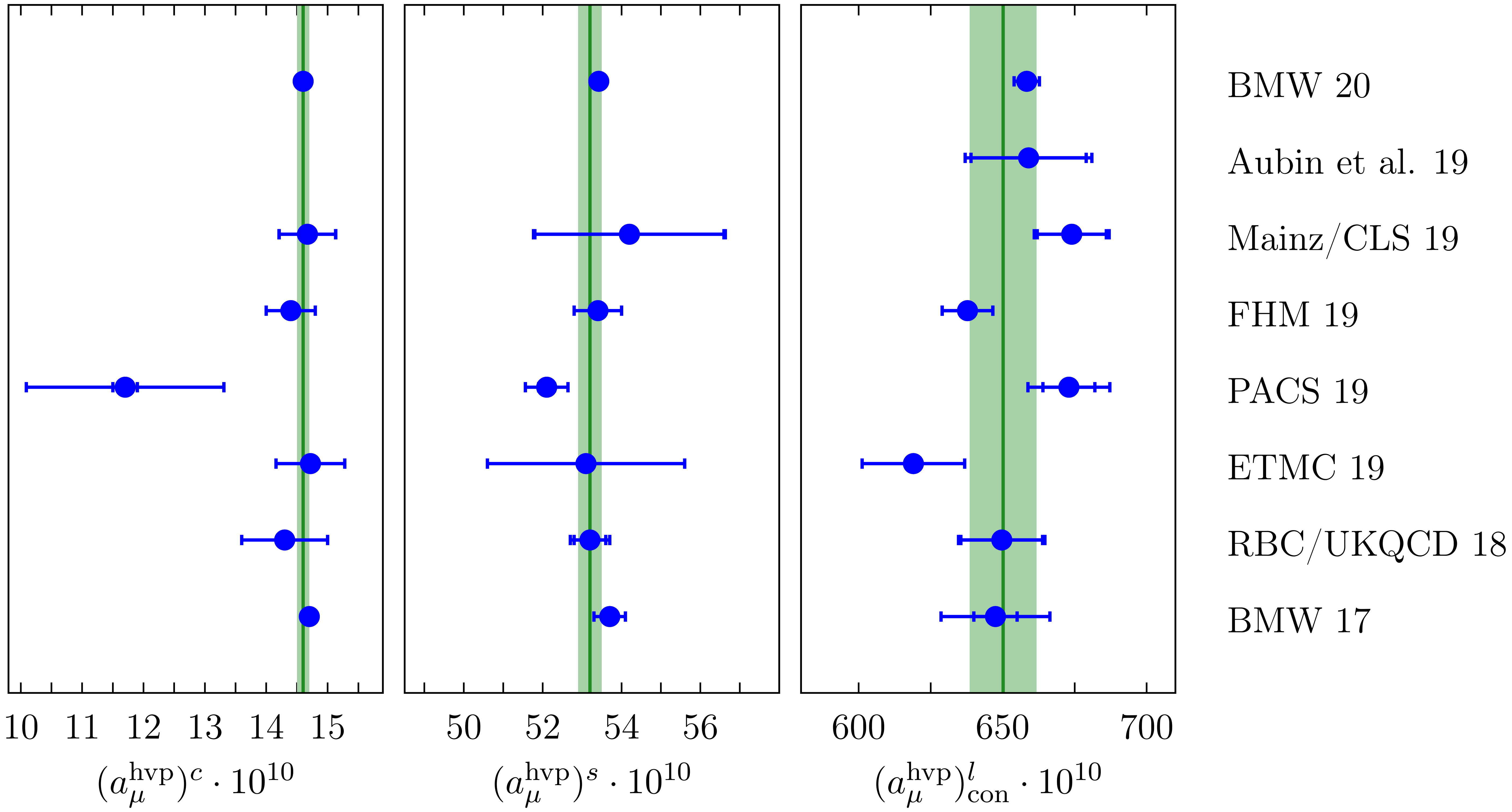


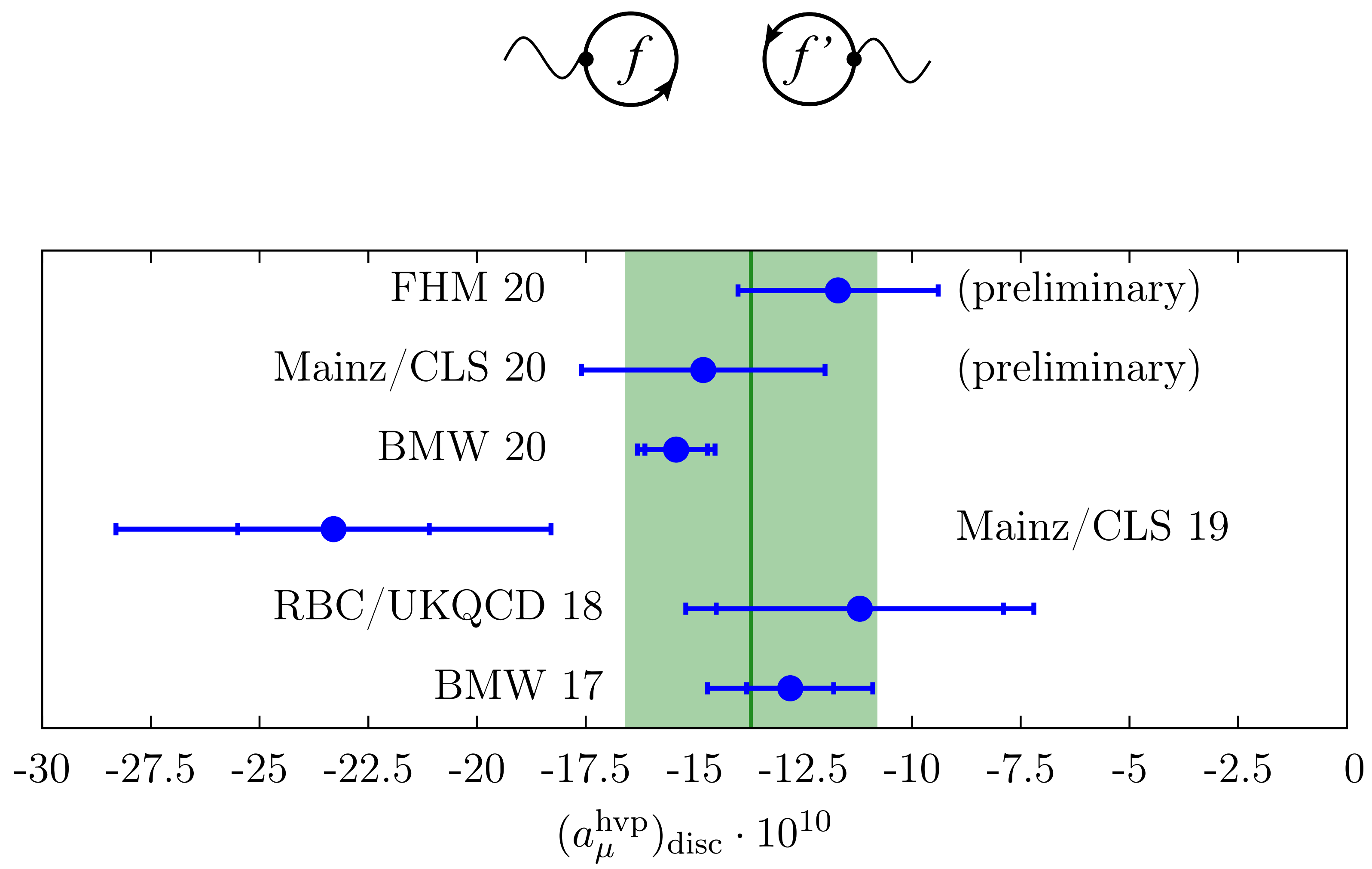
Lattice HVP: c, s, ℓ connected

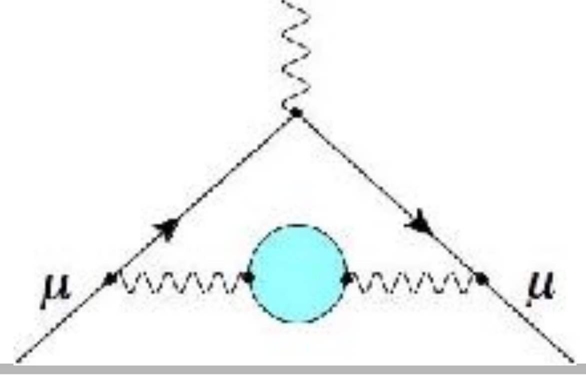


H. Wittig @ Lattice HVP workshop

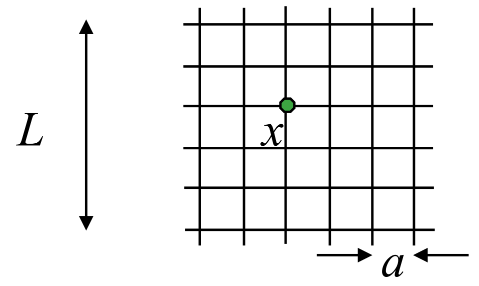






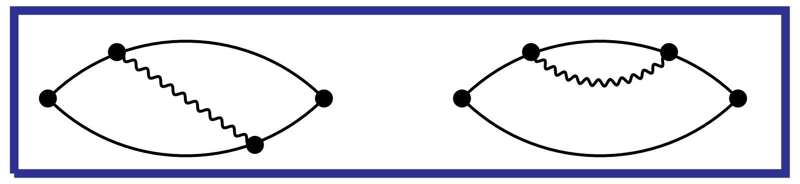


Lattice HVP: Isospin corrections

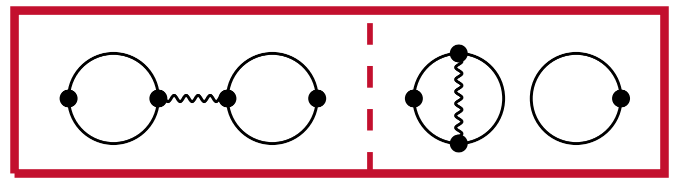


V. Gülpers @ Lattice HVP workshop

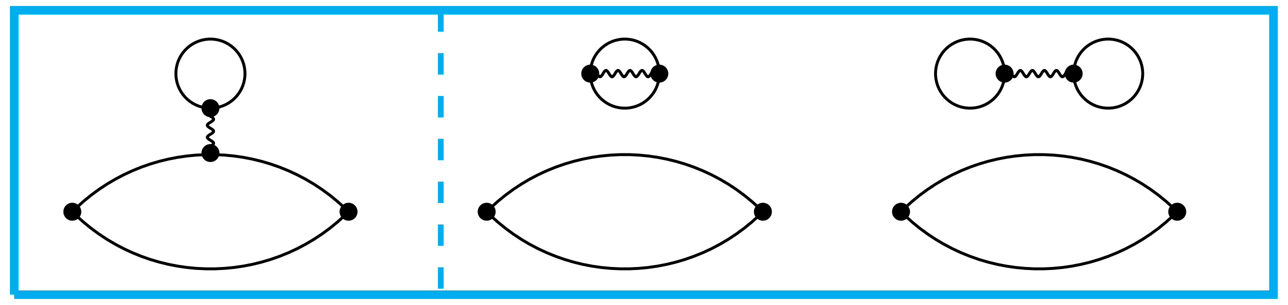
Overview of published results - contributions to $a_\mu \times 10^{10}$



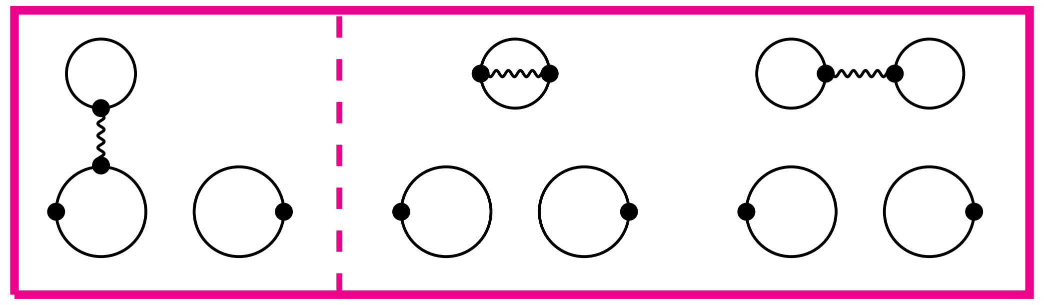
BMW $-1.27(40)(33)$
RBC/UKQCD $5.9(5.7)(1.7)$
ETM $1.1(1.0)$



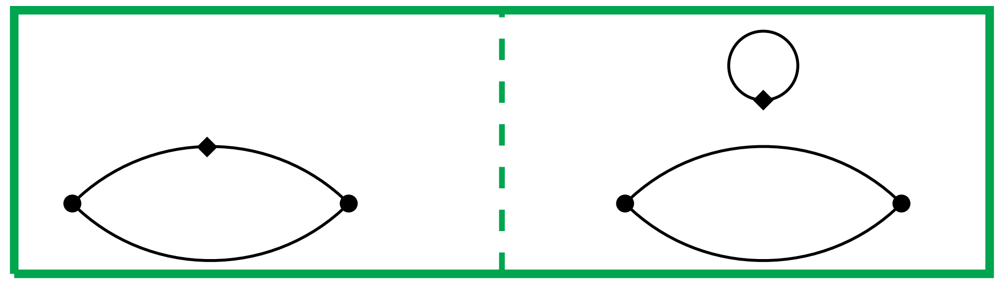
$-0.55(15)(11)$ BMW
$-6.9(2.1)(2.0)$ RBC/UKQCD



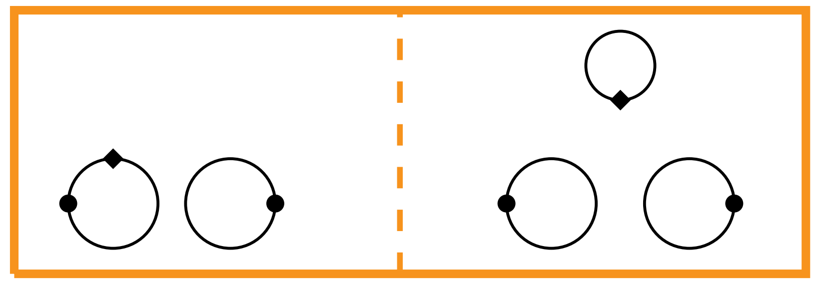
$-0.0095(86)(99)$ $0.42(20)(19)$ BMW



$0.011(24)(14)$ $-0.047(33)(23)$ BMW



$6.59(63)(53)$ BMW
$10.6(4.3)(6.8)$ RBC/UKQCD
$6.0(2.3)$ ETM
$7.7(3.7)$ $9.0(2.3)$ FHM
$9.0(0.8)(1.2)$ LM

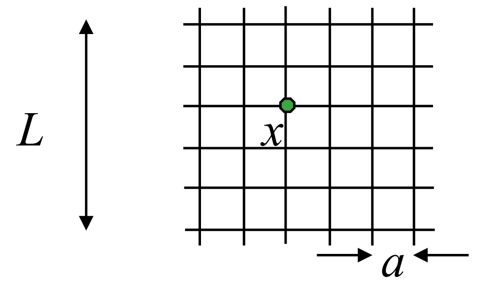


$-4.63(54)(69)$ BMW

BMW [arXiv:2002.12347]
RBC/UKQCD [Phys.Rev.Lett. 121 (2018) 2, 022003]
ETM [Phys. Rev. D 99, 114502 (2019)]
FHM [Phys.Rev.Lett. 120 (2018) 15, 152001]
LM [Phys.Rev.D 101 (2020) 074515]

- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions:
  $\delta a_\mu^{\mathrm{IB}} \lesssim 1\,\%$
- Ongoing efforts by ETMC, Mainz, FNAL-HPQCD-MILC, RBC/UKQCD.

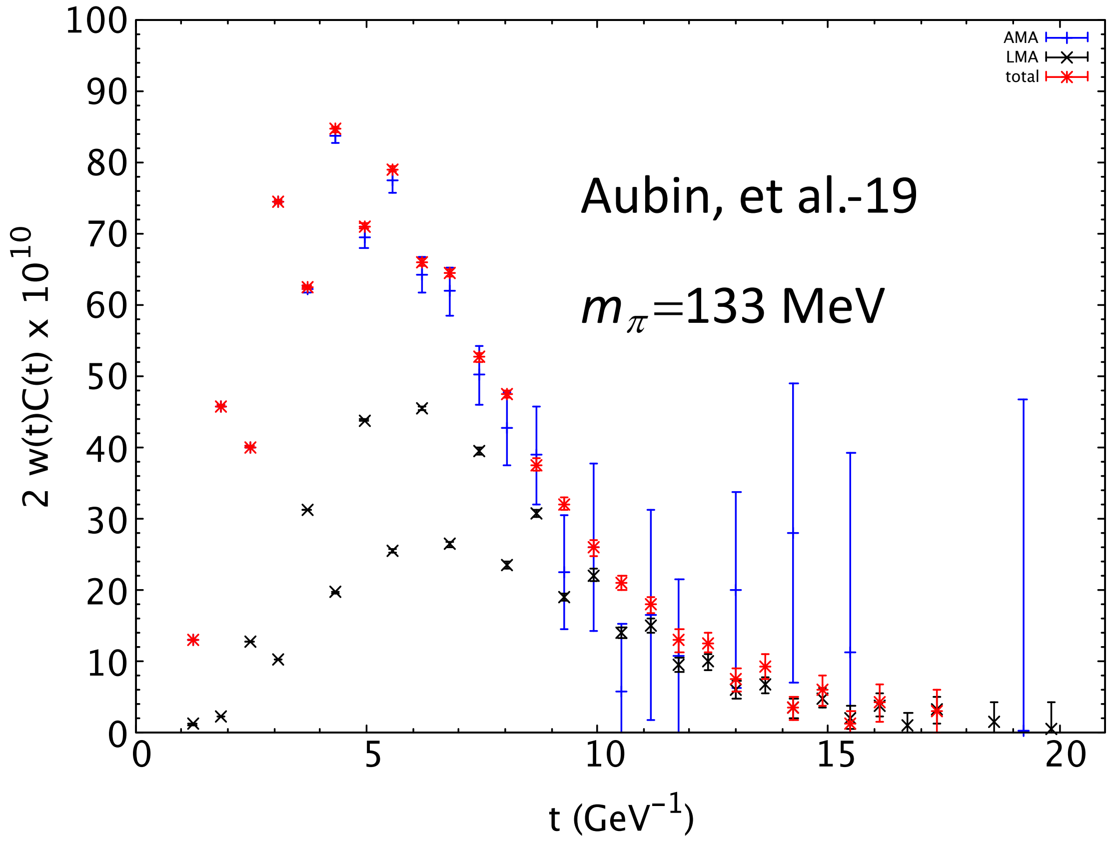
Long-distance tail



$$G(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t)\, j_i(0,0) \rangle$$

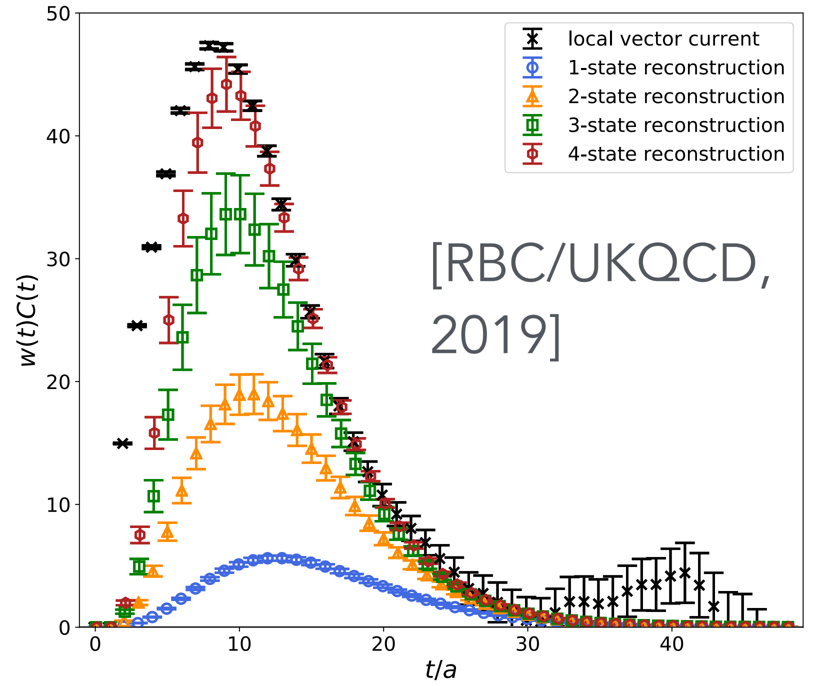
- Use noise reduction methods (AMA, LMA,...):

  Aubin et al, RBC/UKQCD, BMWc, Mainz, …



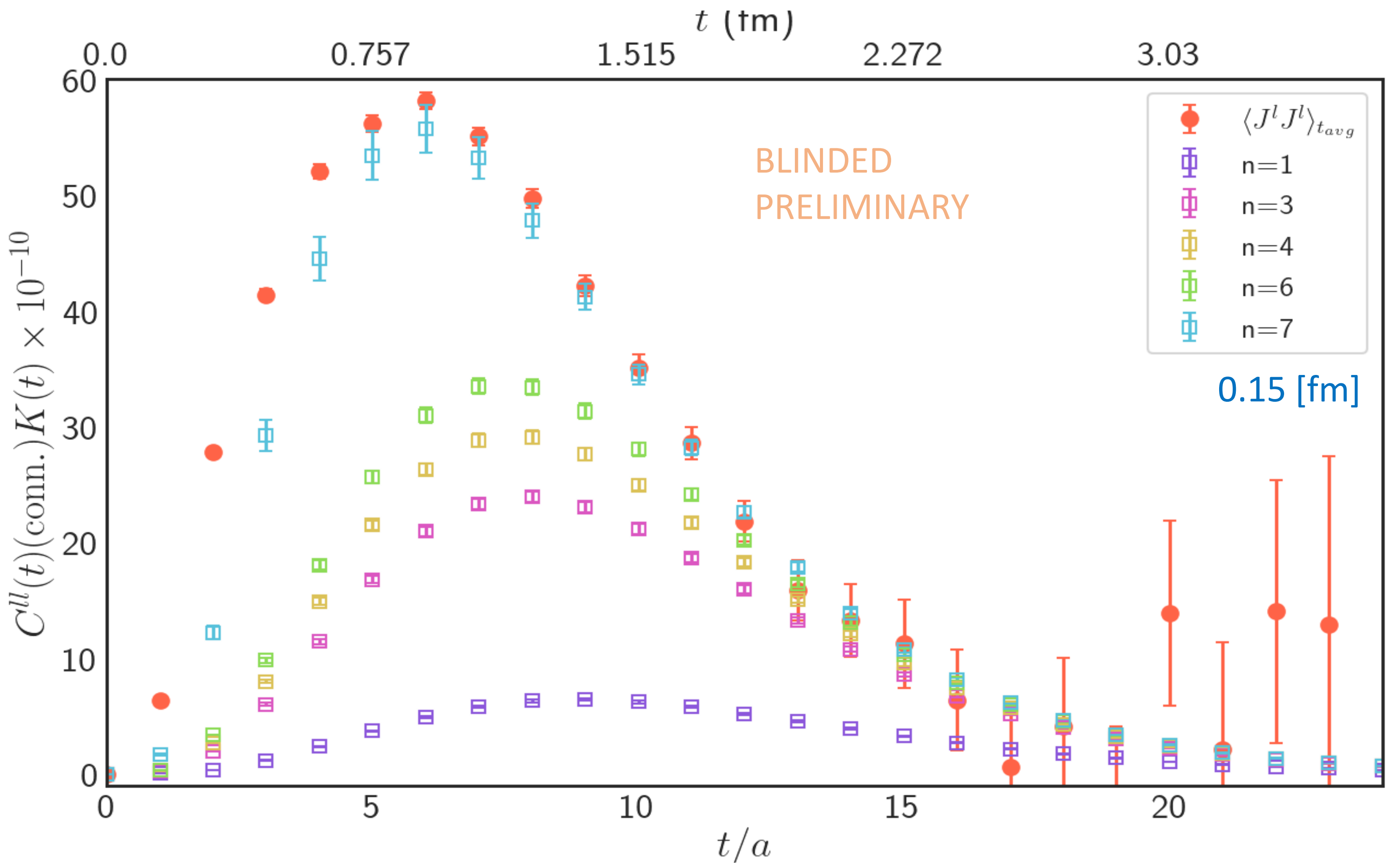
- Spectral reconstruction (RBC/UKQCD, Mainz):
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$



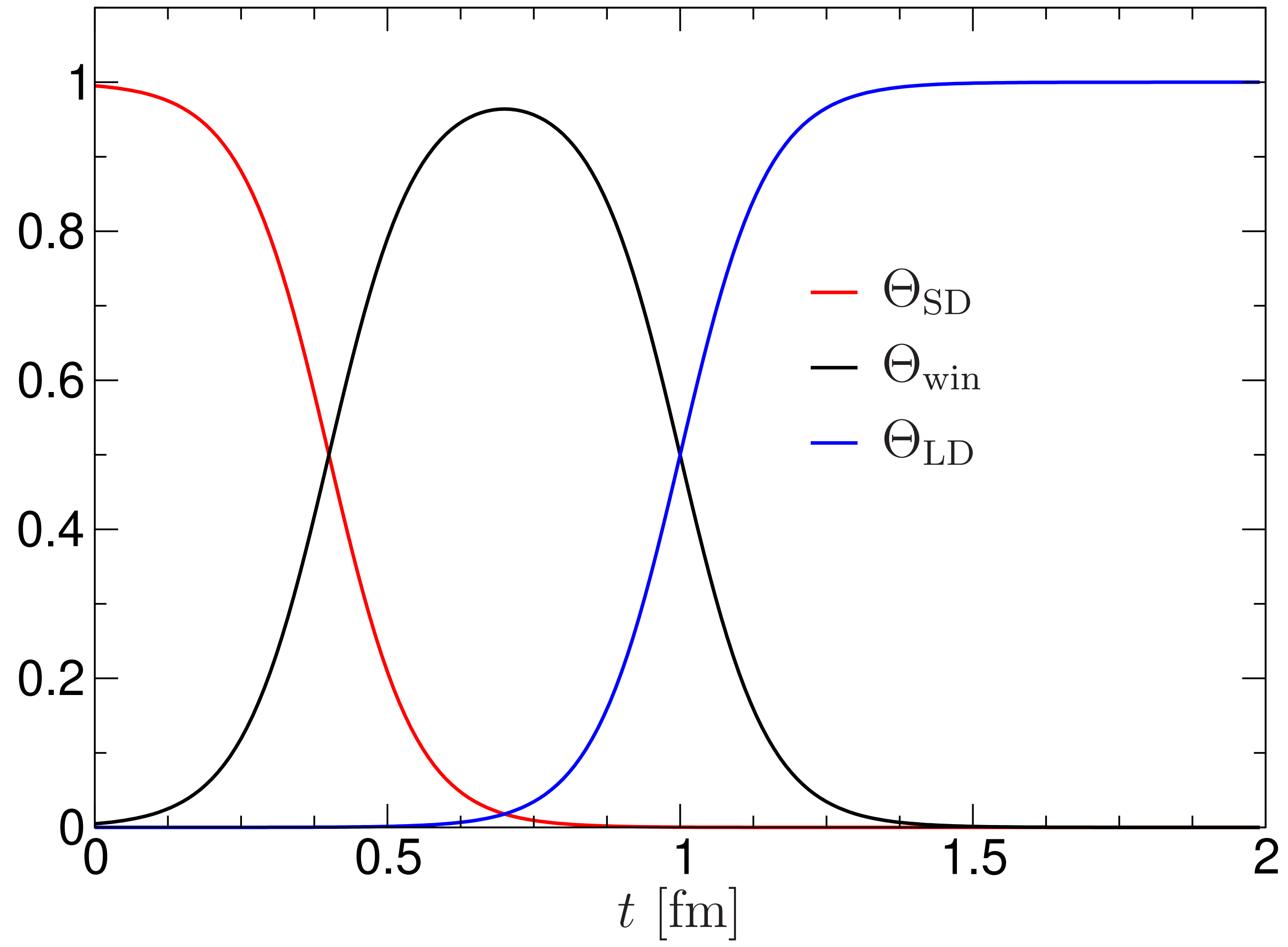
Shaun Lahert (FNAL-HPQCD-MILC) @ Lattice 2021

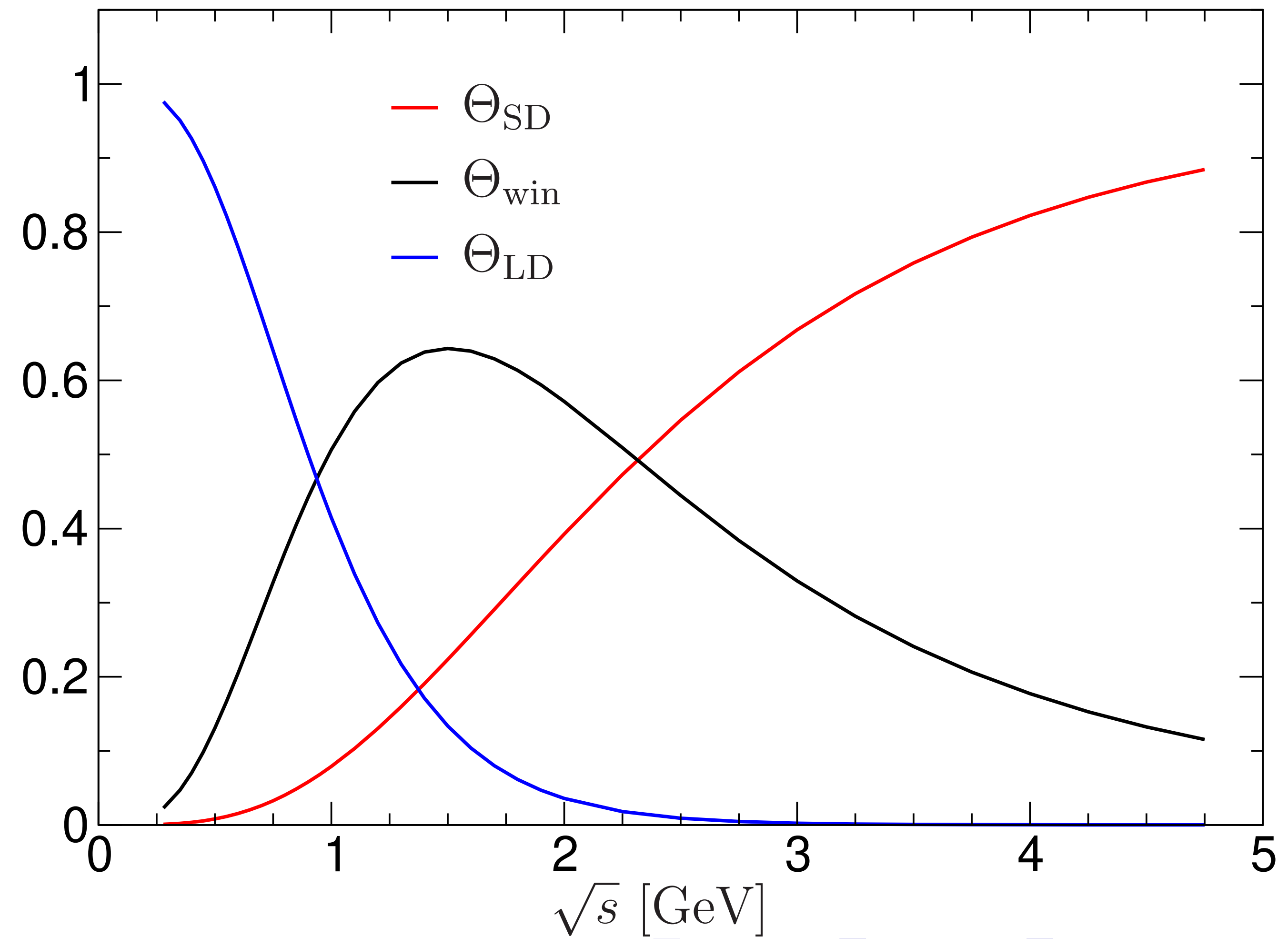
- First calculation with staggered multi-pion operators



Windows: Euclidean time vs $\sqrt{s}$

Martin Hoferichter @ Lattice HVP workshop





intermediate window	SD	percentage captured of $\pi\pi$ channel $\leq$ 1 GeV $[t_0, t_1]$ intermediate	LD
[0.4, 1.0] fm	3	28	69
[1.0, 2.0] fm	31	51	18
[1.0, 2.5] fm	31	61	9
[1.0, 3.0] fm	31	65	4

SD: $[0, t_0]$
LD: $[t_1, \infty]$
int.: $[t_0, t_1]$

For intermediate window:
~30% from $\sigma(\pi\pi) \lesssim 1\,\mathrm{GeV}$

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.

- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.

- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer 2021]

Martin Hoferichter @ Lattice HVP workshop

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

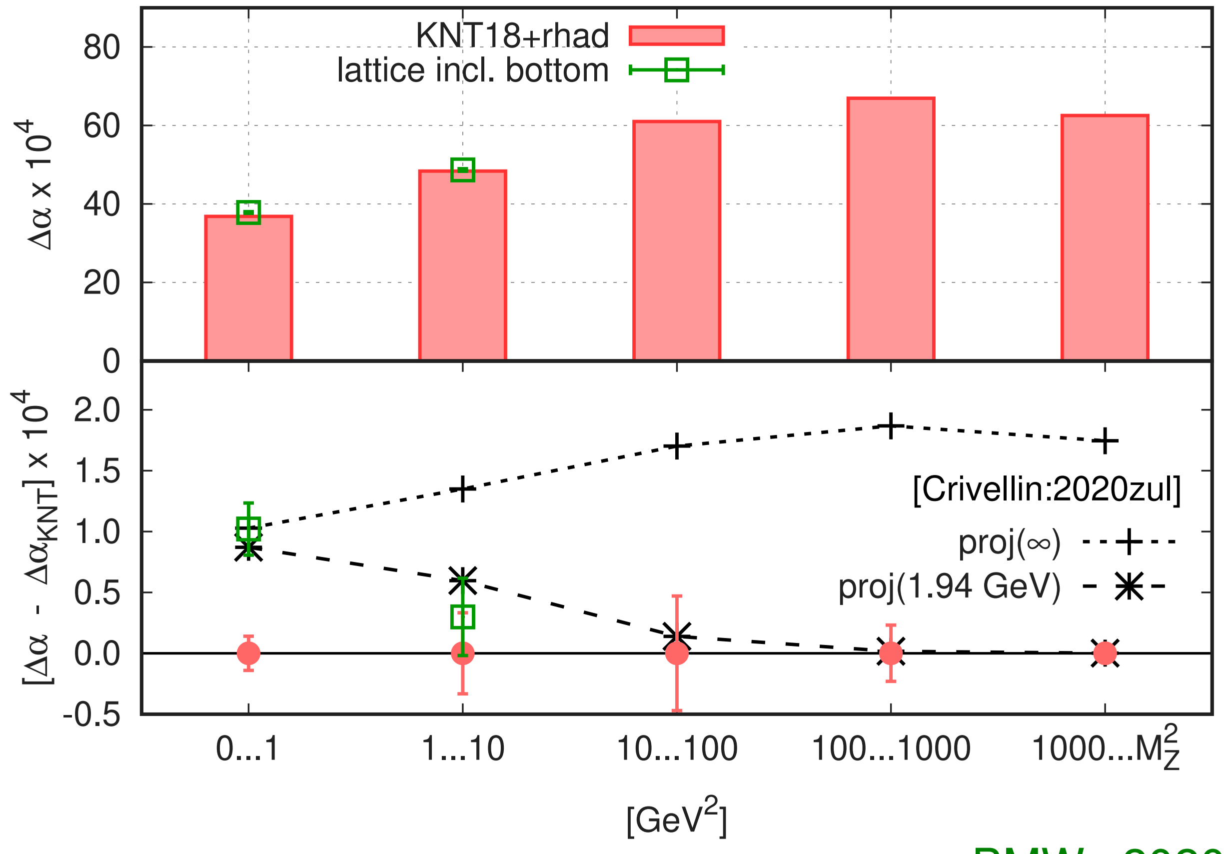
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.

- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
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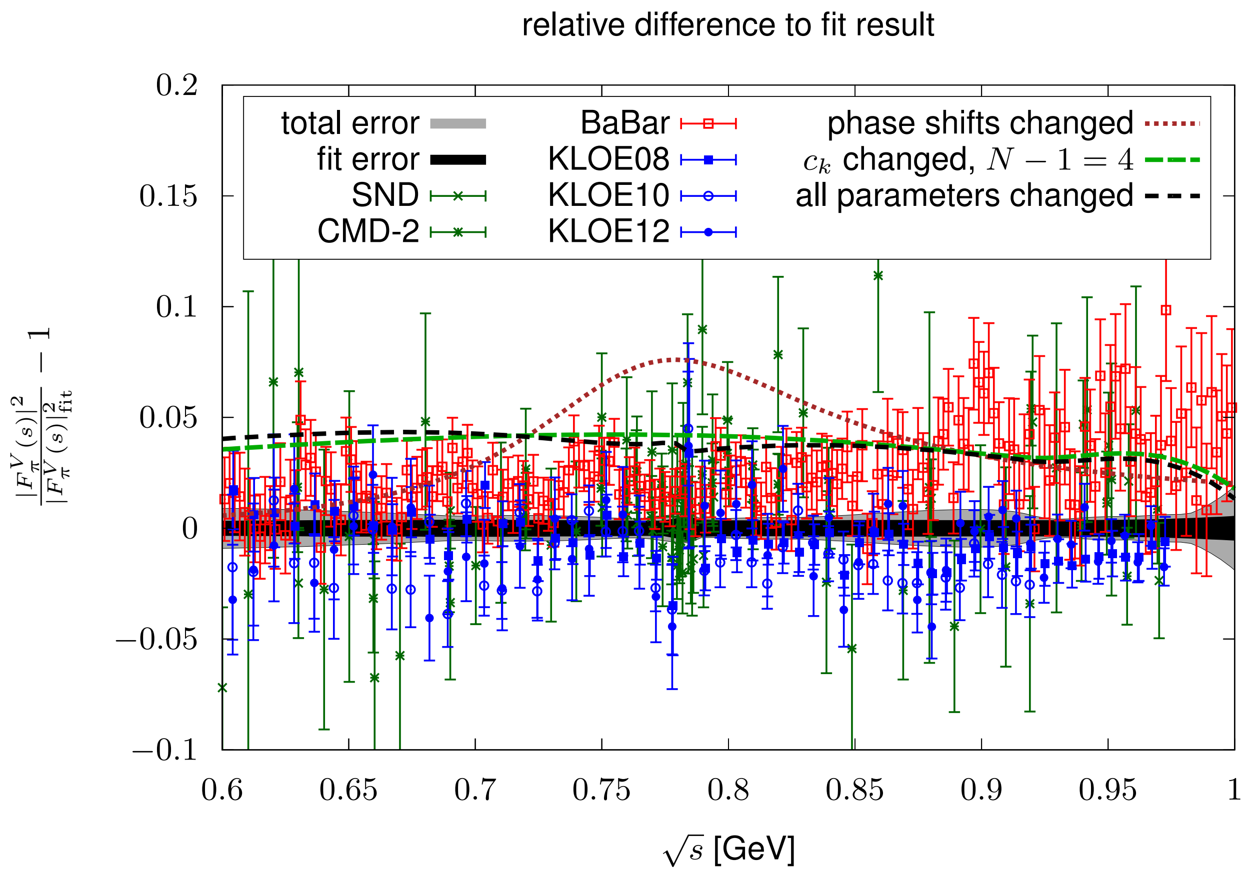
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer 2021]

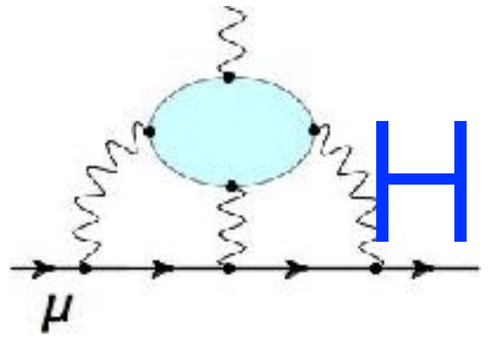
Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP   arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result: $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  $\to$ arXiv:2006.05431 [hep-ph]

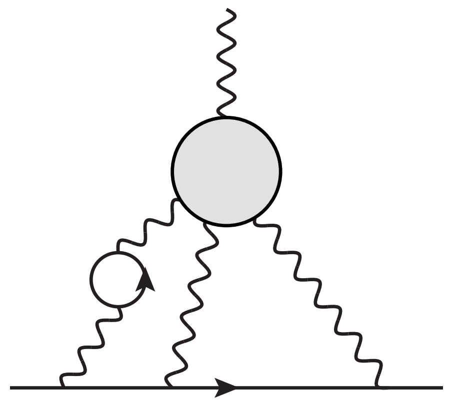




HLbL: dispersive

Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	}
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



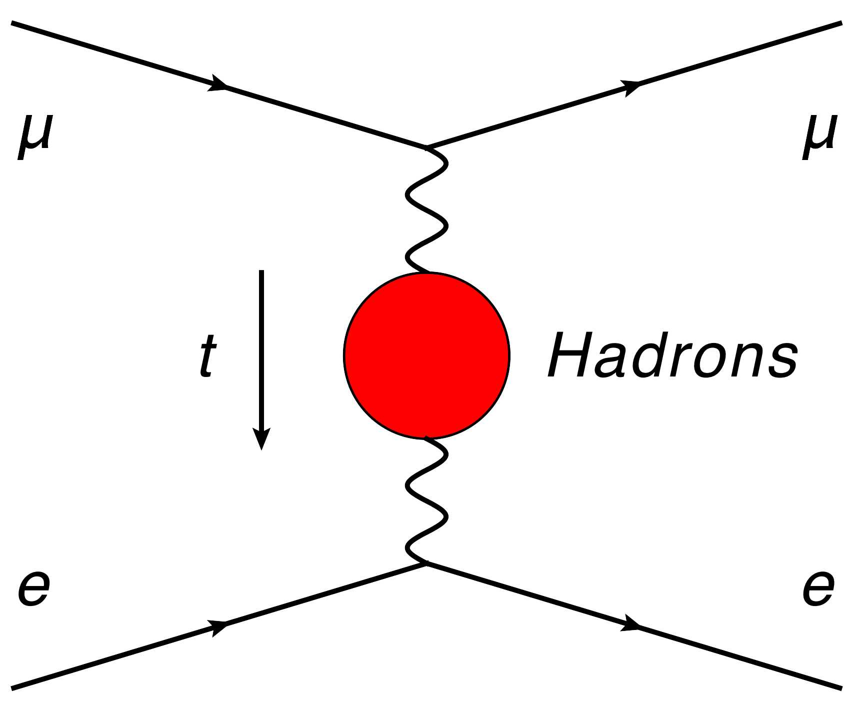
NLO HLbL contribution:

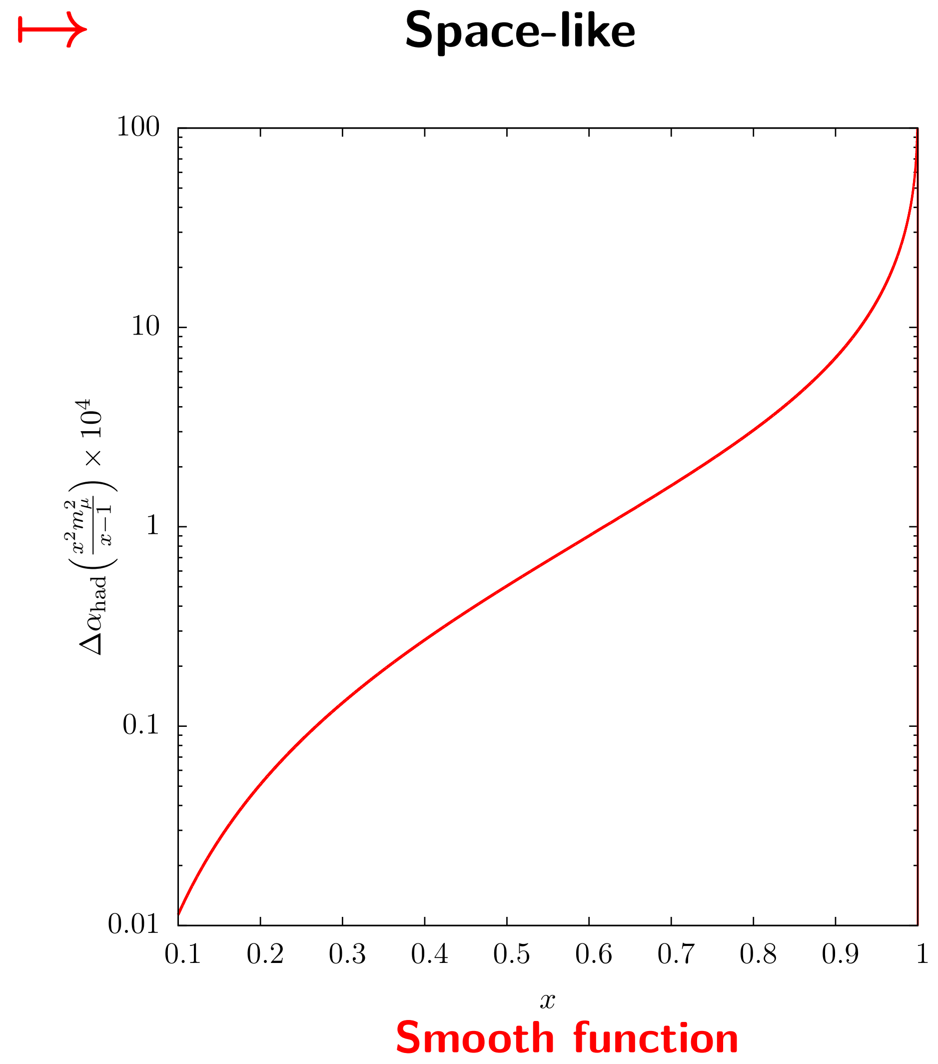
$$a_\mu^{\mathrm{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

Hadronic vacuum polarization

μ-e elastic scattering to measure a_μ^{HVP}

LOI June 2019 [P. Banerjeei et al, arXiv:2004.13663, Eur.Phys.J.C 80 (2020)]





CC, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- pilot run in 2021
- full apparatus in 2023-2024