

Hadron physics : theory

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Outline:

conventional and exotic hadrons

their masses, decay widths and other properties

from theory

Hadrons

$G=\text{gluon}$, $q=\text{quark}=u,d,s,c,b$

Today we know (from exp and theory) that hadrons with the following minimal quark and gluon contents.

There may be more categories, but these are not reliably confirmed yet.

minimal quark (q) and gluon (G) contents

conventional hadrons

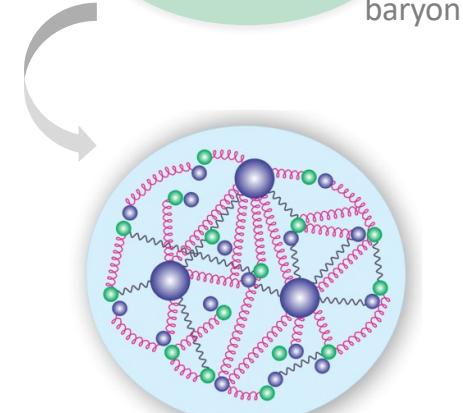
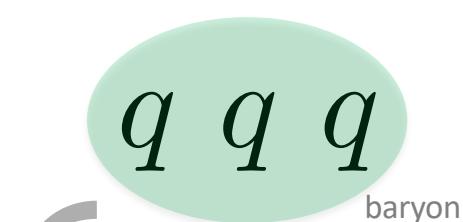
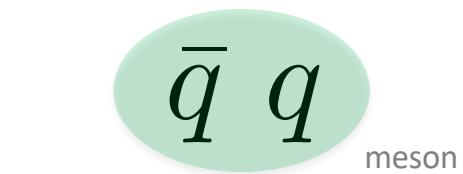
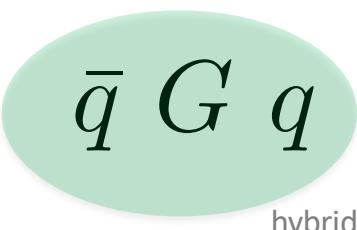
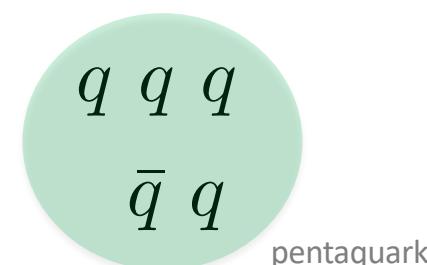
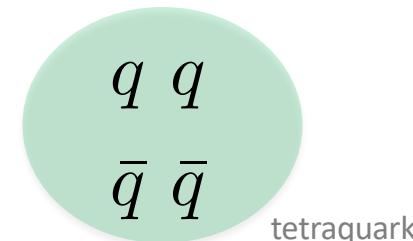


image:
<https://physicstoday.scitation.org/do/10.1063/PT.5.7167/full/>

exotic hadrons



experimentaly discovered example

$$\frac{c}{\bar{u}} \frac{c}{d} = T_{cc} \quad [\text{LHCb 2021}]$$

$$uud \bar{c} c = P_c \quad [\text{LHCb 2015}]$$

$$\bar{u} G d = \pi_1 \quad [\text{several exp.}]$$

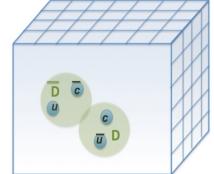
$J^{PC} = 1^{-+}$

QCD: $\mathcal{L}_{QCD} = \frac{1}{4}G_a^{\mu\nu}G_a^{\mu\nu} + \bar{q}i\gamma_\mu(\partial^\mu + ig_s G_a^\mu T^a)q - m_q\bar{q}q$ $g_s \cancel{\ll} 1$ at hadronic energy scale

Theoretical approaches: nonperturbative approach to QCD is needed to address hadrons

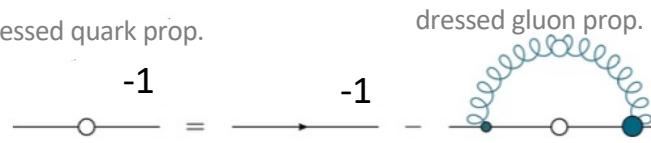
Lattice QCD $\langle C \rangle = \int D\mathbf{G} D\mathbf{q} D\bar{\mathbf{q}} C e^{-S_{QCD}^E/\hbar}$ $S_{QCD}^E = \int d^4x_E \mathcal{L}_{QCD}^E(m_q, g_s)$

often “non-precision” studies: $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

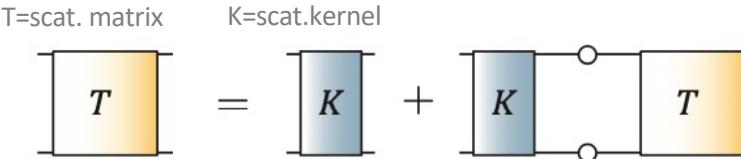


Dyson-Schwinger equations + **Bethe-Salpeter equations :** T, K computed with certain truncations

dressed quark prop. dressed gluon prop.



T=scat. matrix K=scat.kernel



Effective field theories : effective degrees of freedom + symmetries of QCD + expansion (in E, p, $1/m_Q$)

Quark Models

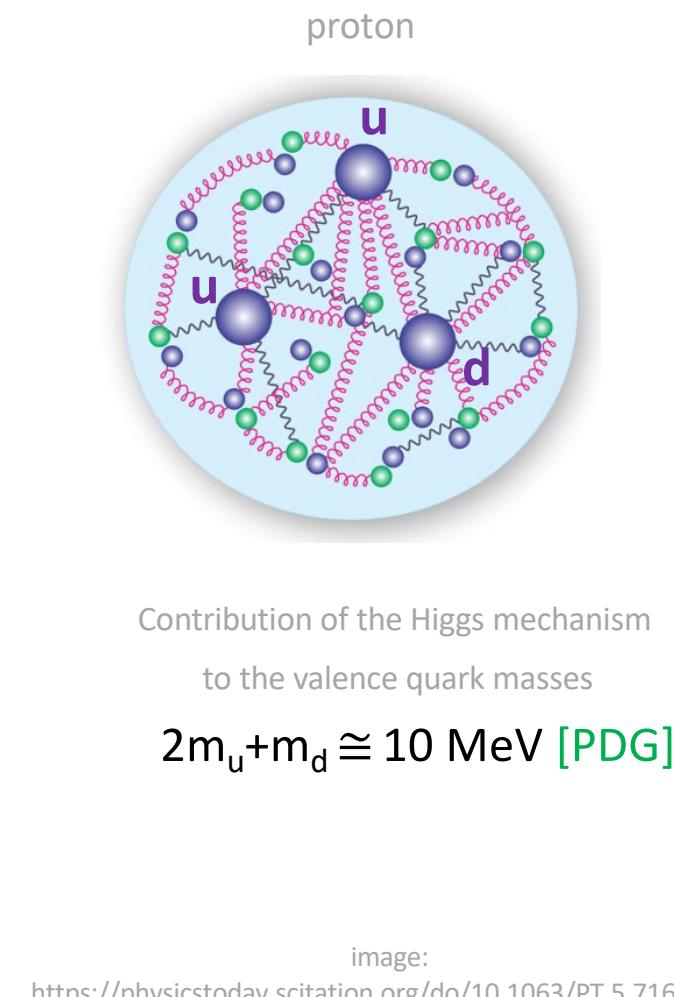
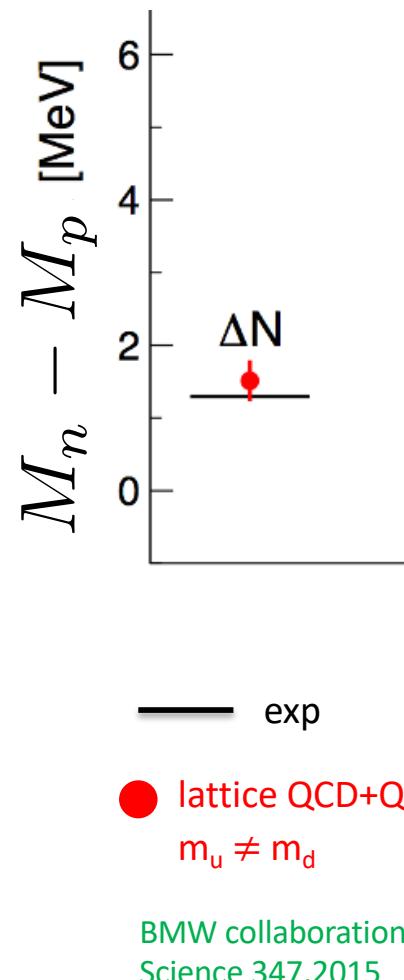
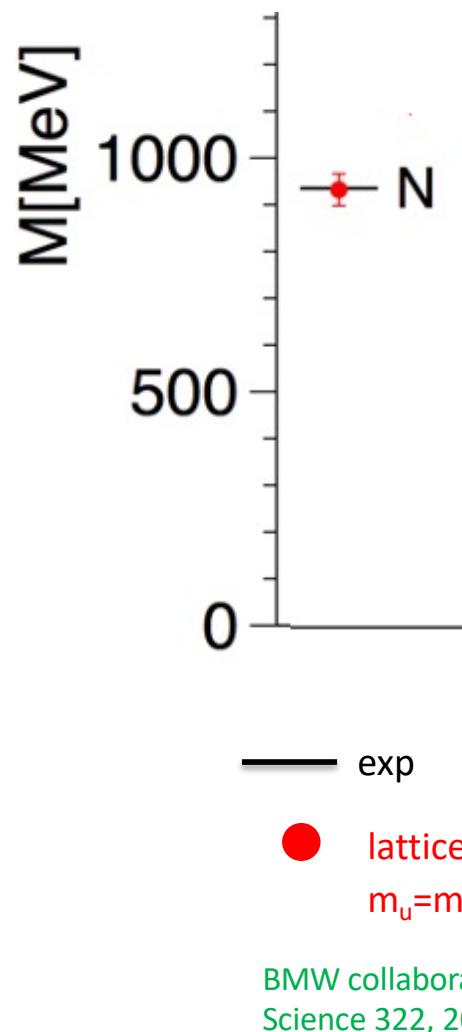
Models : effective degrees of freedom + symmetries of QCD + but not rigorous expansion (in E, p, $1/m_Q$),

Sum rules these do not extract scattering amplitudes and do not rigorously extract resonances

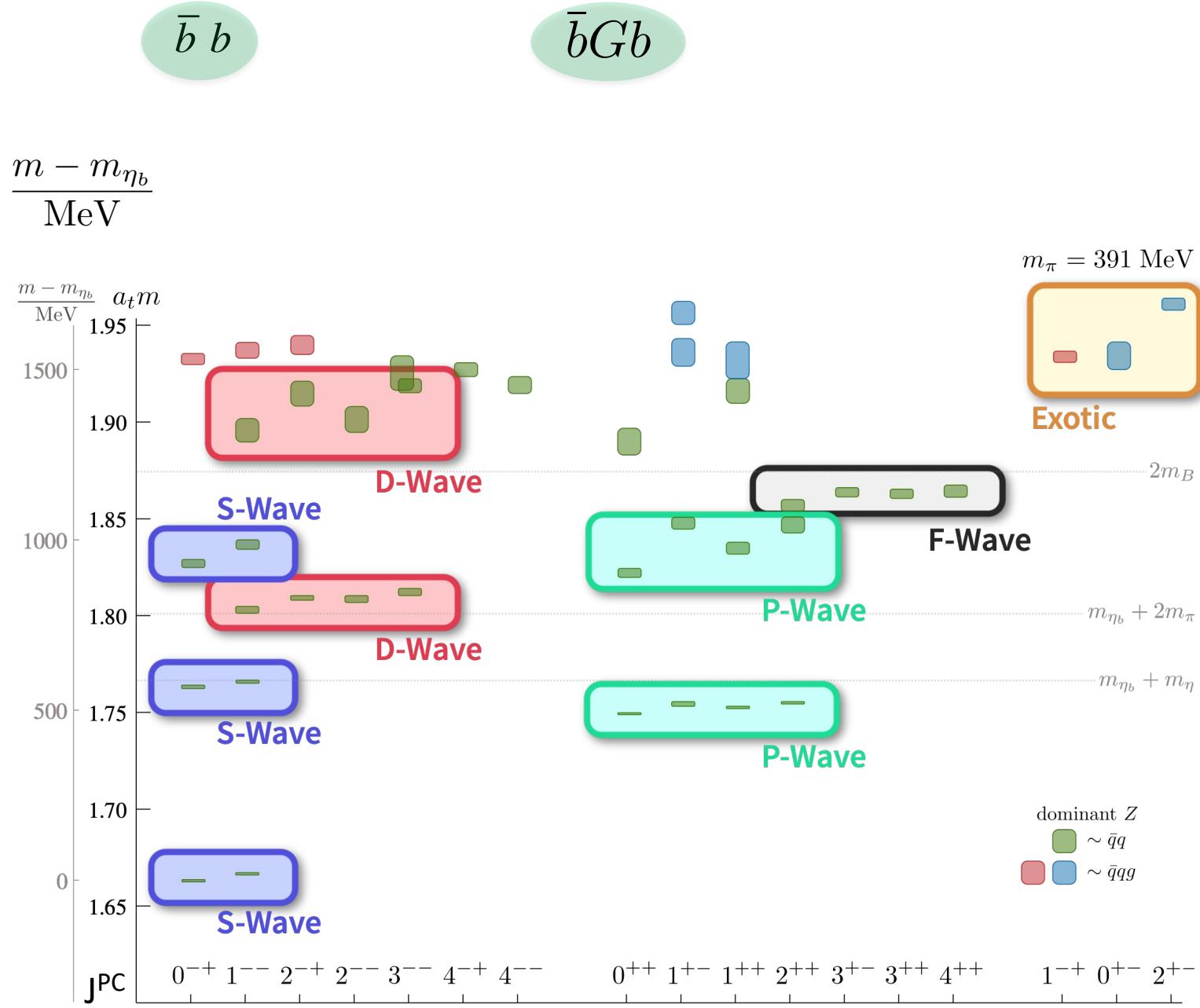
This talk:

- focus on first two ab-initio approaches where results are available: strongly stable hadrons and certain strongly decaying hadrons
- solid ab-initio results not yet available for many exotic states discovered in exp: present result from other approaches

Proton and neutron mass constitute more than 99% of the bright universe mass

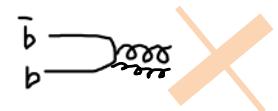


Bottomonia and bottomonium hybrids



Lattice QCD
relativistic b quarks
[Ryan & Wilson \(HadSpec\)](#)
[2008.02656, JHEP](#)

strongly stable below $\underline{B}B$ if
 $\underline{b}\underline{b}$ annihilation is omitted



$m_{\text{hybrid}} \geq 10.9 \text{ GeV}$

other predictions of hybrids:

from quenched lattice static potentials / EFT

[Brambilla et al, 1805.07713, 1908.11699, PRD](#)

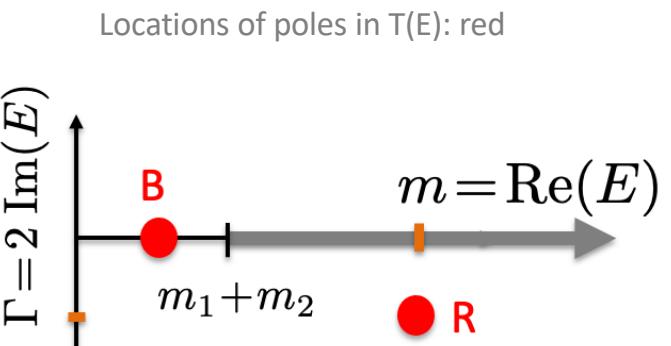
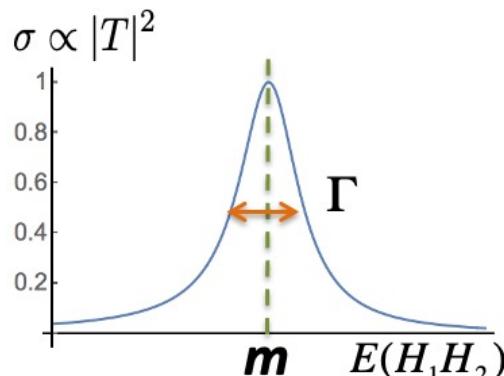
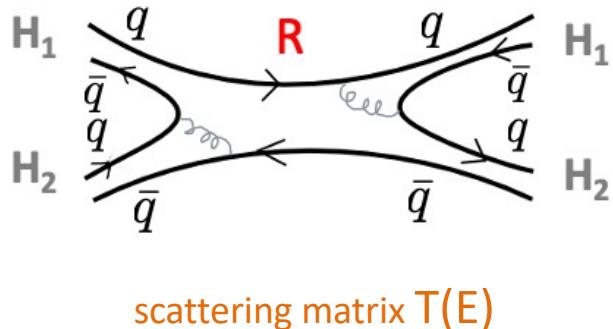
[Schlosser & Wagner, 2111.00741](#)

Strongly decaying hadronic resonances

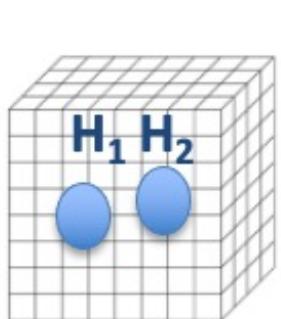


Strongly decaying resonances

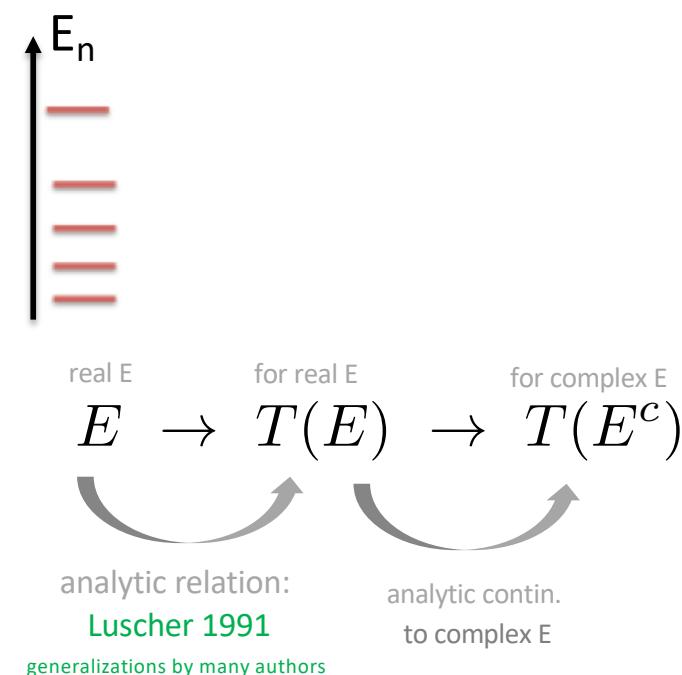
have to be extracted from the scattering of H_1H_2



Scattering matrix $T(E)$ from lattice QCD



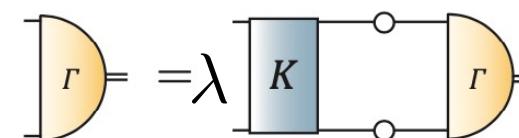
many resonances and bound states extracted in this way by now
(apologies for not covering all)



Poles in $T(E)$ from Bethe-Salpeter approach

Γ = BS amplitude of a state

K = scat. kernel

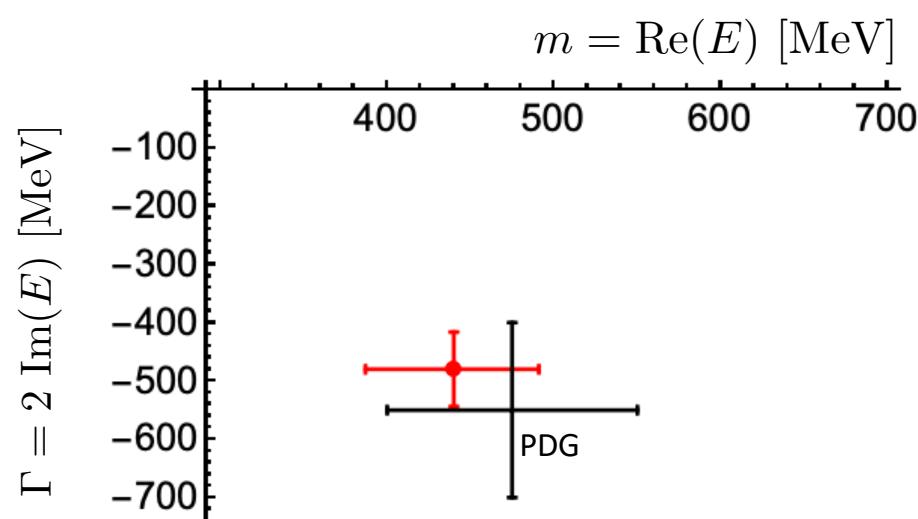


$$\Gamma = \lambda K \Gamma$$

- solve this eigenvalue equation to find the resonance pole
- in practice solutions $\lambda \neq 1$ are found for P^2 below threshold (in singularity free region) and then analytically extrapolated to find P^2 that correspond to $\lambda = 1$ above th.
- first results for resonances in QCD became available only recently

R. Williams, 1804.11161 PLB, Miramontes, Sanchis-Alepuz (2019)
Eichman, Duarte, Pena, Stadler, PRD 100 (2019), Santowsky, Eichmann, Fischer, Wallbott, Williams, 2007.06495, PRD Miramontes, Sanchis-Alepuz, Alkofer, PRD 103 (2021)

Resonance $\sigma = f_0(500)$



Lattice QCD, dynamical u&d quarks

Guo, Alexandru, Mai, Molina, Doring, 1803.02897, PRD

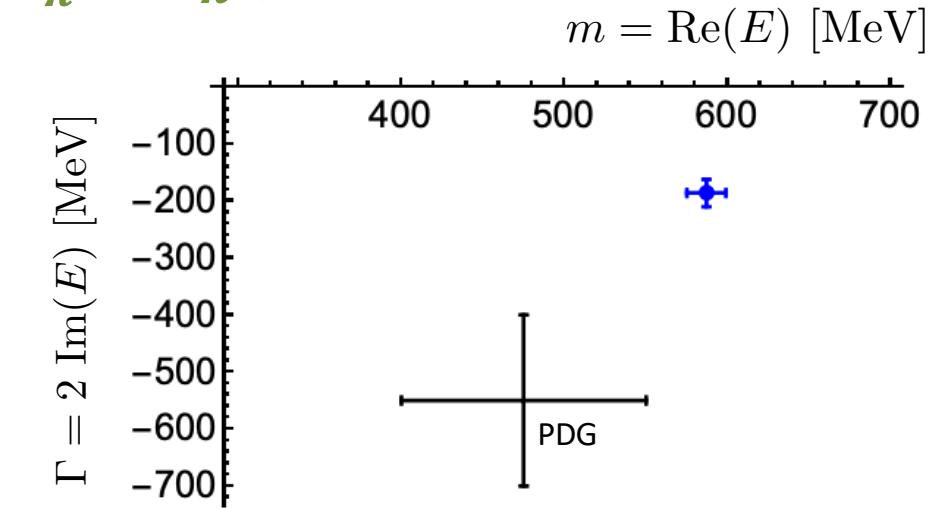
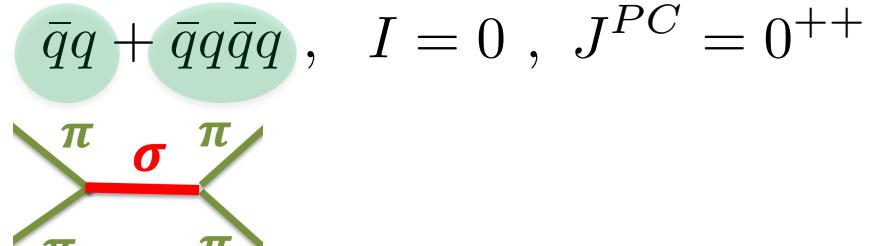
$$m_\pi = 227, 315 \text{ MeV} \rightarrow m_\pi^{phy}$$

chiral unitary extrapolation

the only other lattice result for sigma pole:

Briceno, Dudek, Edwards, Wilson, 1607.05900 , PRL

See also works by RBC/UKQCD 2103.15131 PRD, Pelaez, Rodas



Dyson-Schwinger + Bethe-Salpeter equations

Santowsky, Eichmann, Fischer, Wallbott, Williams, 2007.06495, PRD

$$K^{(4)} = \boxed{\text{blue}} + \boxed{\text{red}} - \boxed{\text{red}} + \boxed{\text{red}} \text{ (truncation)}$$

+ perm.

+ perm.

coupled qq and $qqqq$

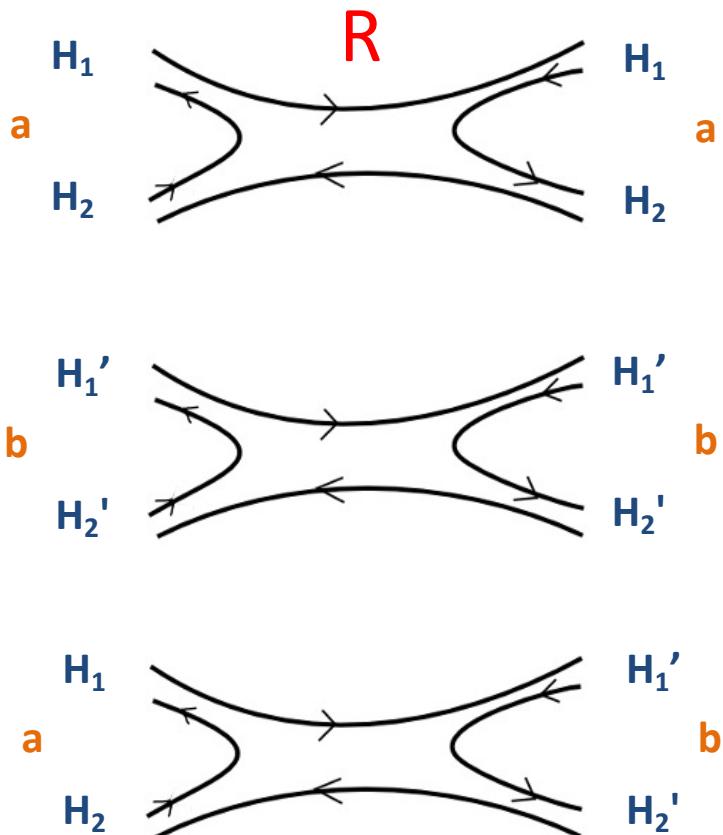
several simplifications undertaken

the width determined from qq part

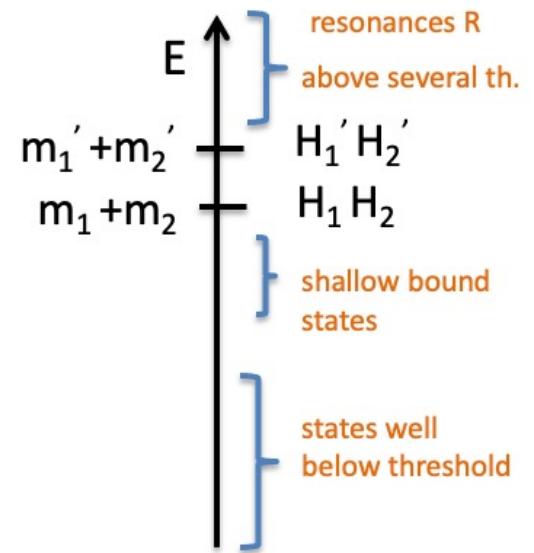
currently: only qualitative comparison to exp

Resonances from coupled-channel scattering

$$R \rightarrow H_1 H_2, H'_1 H'_2, \dots$$

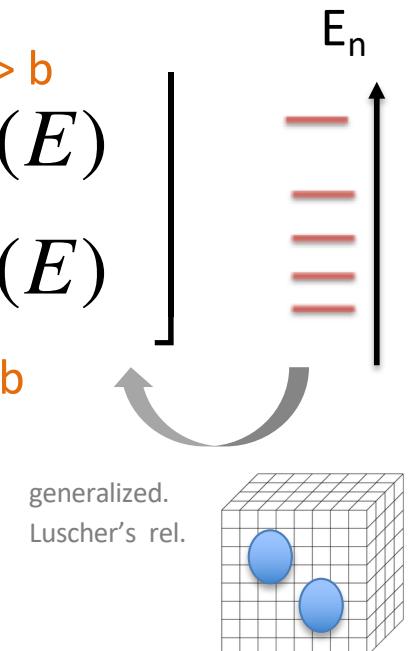


channel a : $H_1 H_2$
 channel b : $H'_1 H'_2$

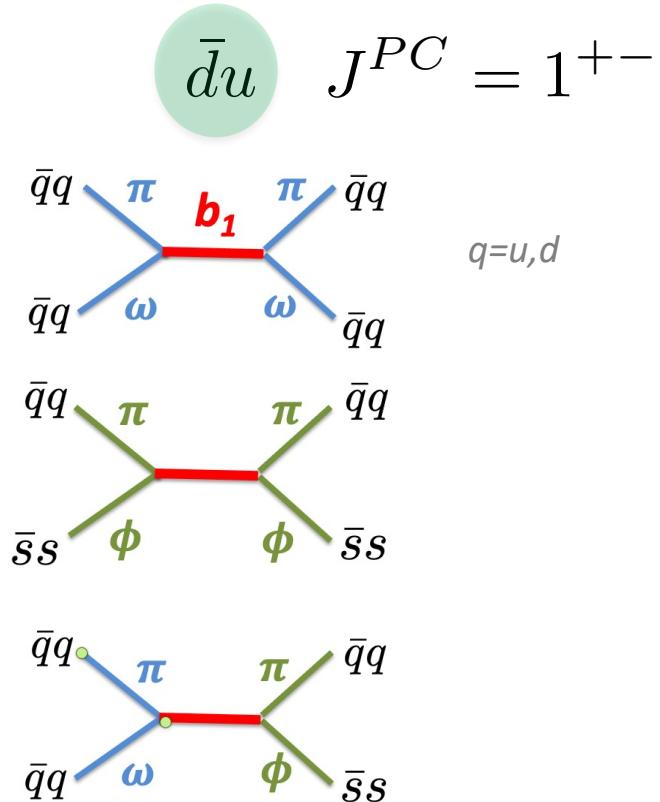


$$T(E) = \begin{bmatrix} & a \rightarrow a & a \rightarrow b \\ & T_{aa}(E) & T_{ab}(E) \\ & T_{ab}(E) & T_{bb}(E) \\ b \rightarrow a & & b \rightarrow b \end{bmatrix}$$

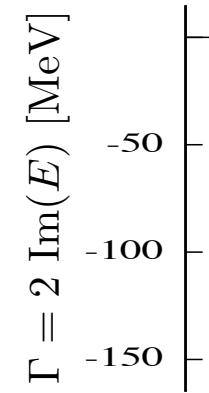
- lattice QCD studies extracted $T(E)$ for several resonances
- most results by HadSpec. coll.



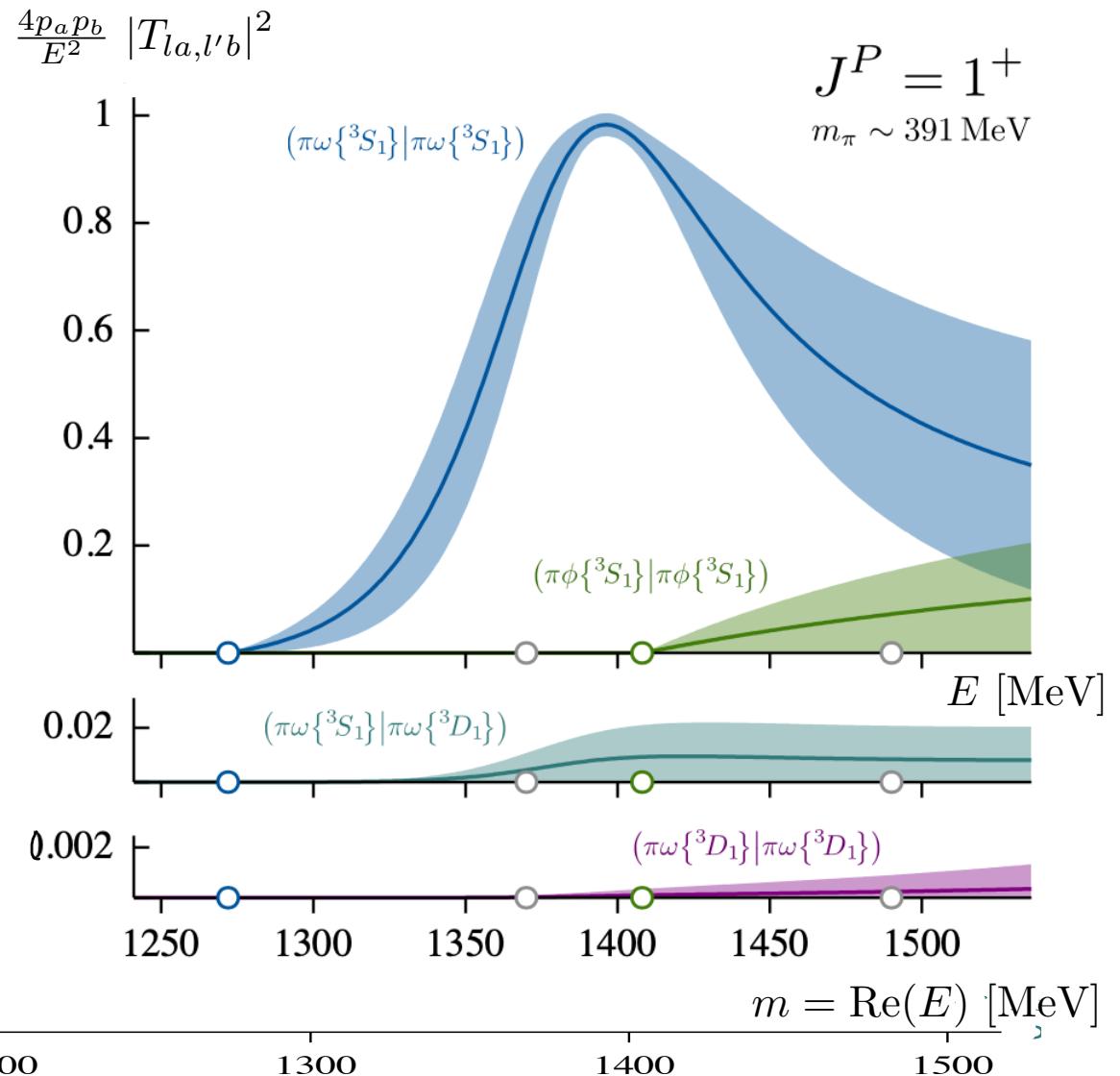
b_1 resonance from lattice



Woss et al, HadSpec,
1904.04136, PRD



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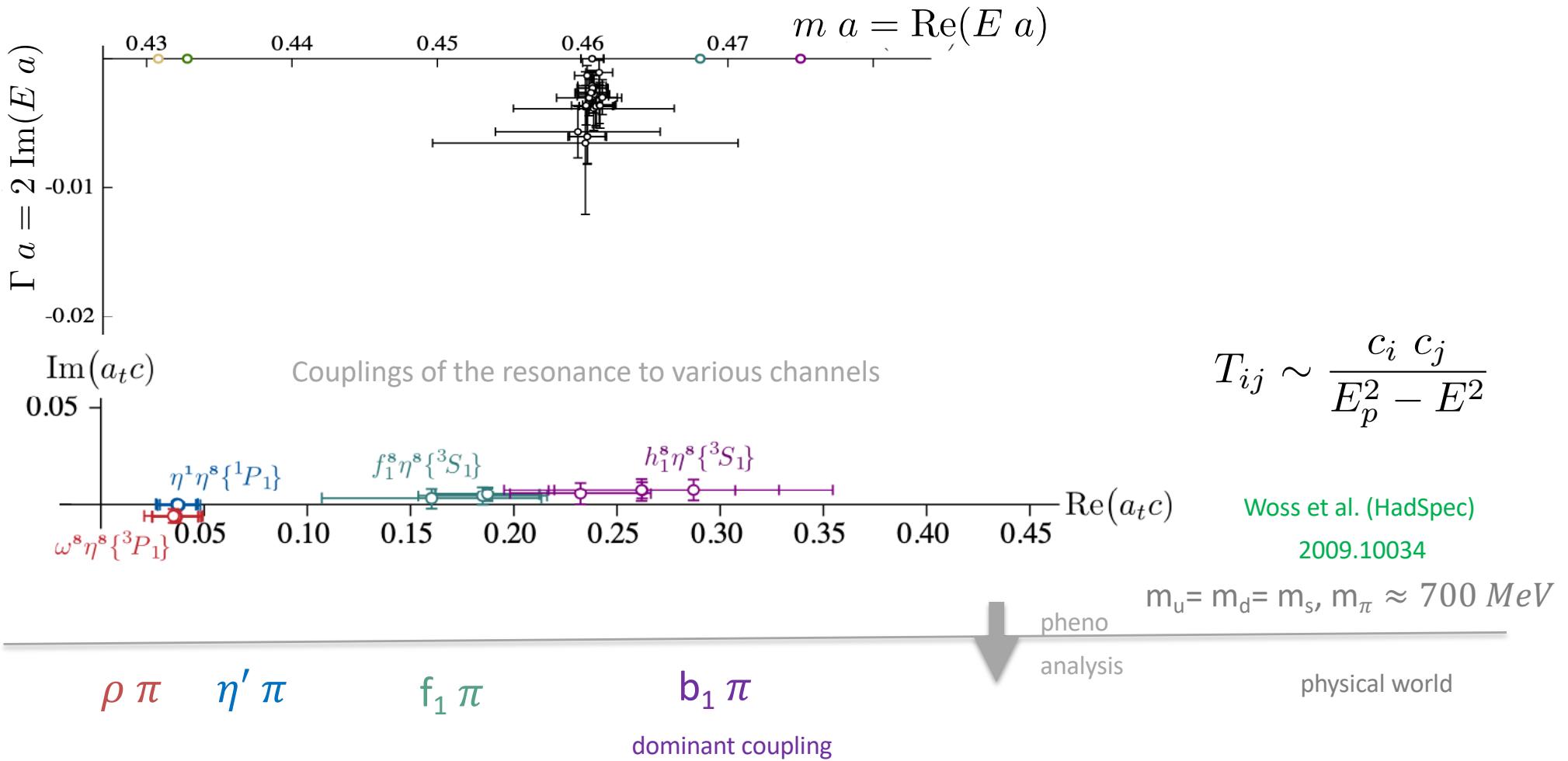
$\text{lat } (m_\pi=391 \text{ MeV})$

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light hybrid meson π_1 from lattice

$\bar{d}Gu$

$J^{PC} = 1^{-+}$



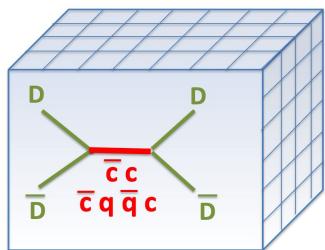
resemblance to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$

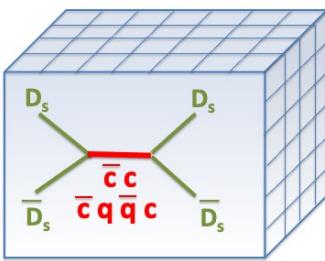
$q=u,d,s$ $I=0$



$\bar{D}_s D_s$

state

likely related to $X(3915)/\chi_{c0}(3930)$
[BaBar, LHCb 2009.00026]; explaining why
it has narrow width to $\bar{D}D$. Predicted
by Lebed, Polosa 1602.08421

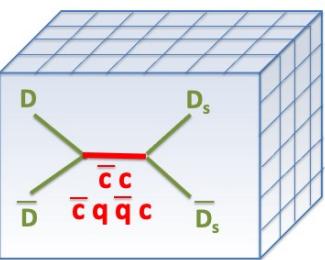


$\bar{D}D$

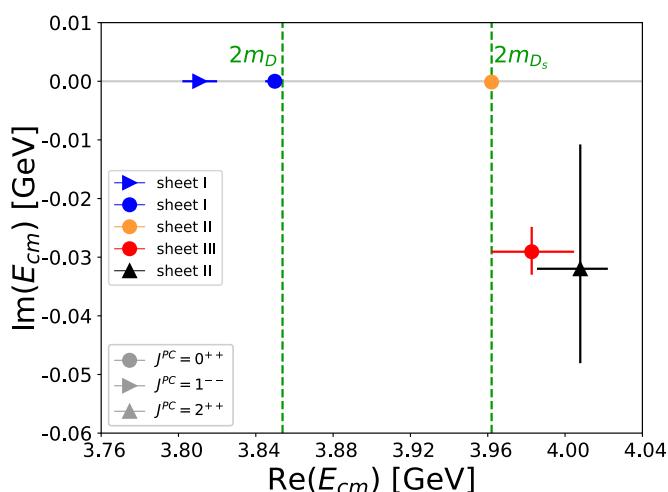
state

predicted in models [Oset et al,
0612179 PRD, Hildago Duque et al
1305.4487, Baru et al 1605.09649 PLB]

seen in dispersive analysis of exp.
data [Deineka, Danilkin et al 2111.15033]



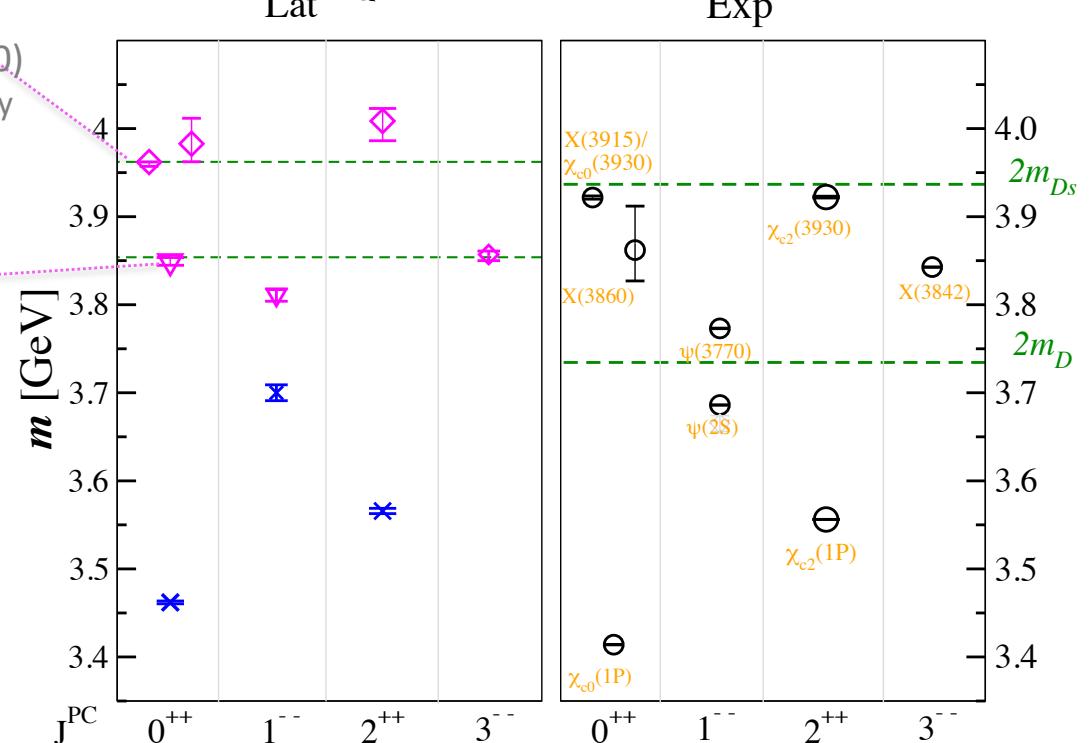
+ expected conventional charmonia



$m_\pi \simeq 280$ MeV

Lat

Exp



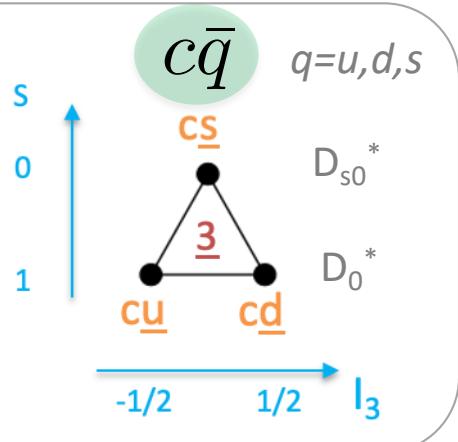
S.P., Collins, Padmanath, Mohler, Piemonte

2011.02542 JHEP, 1905.03506 PRD, 2111.02934

Scalar heavy-light mesons

$$J^P = 0^+$$

Conventional quark model



New paradigm

Du et al, 1712.07957, PRD

Albaladejo et al, 1610.06727, PLB

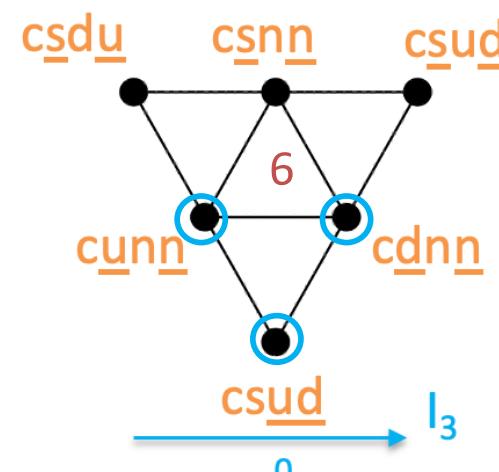
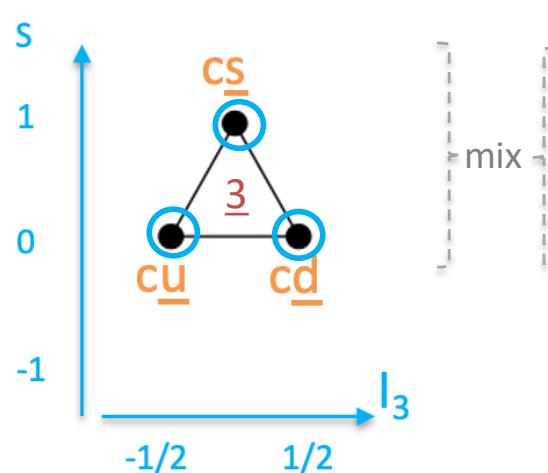
Lutz et al (2003), 0307133, PLB

Beveren, Rupp; Dmitrasinovic

lat: 2.1-2.2 GeV (pole)
PDG: 2.3 GeV (BW)
(see backup)

$$c\bar{q} + c\bar{q} q\bar{q} \quad q=u,d,s \quad n=u,d$$

$$\underline{3} \otimes \underline{8} = \underline{3} \oplus \underline{6} \oplus \underline{15} \quad \text{SU}(3)_F$$



Scattering on the lattice

S=1 Mohler et al, 1308.3175, PRL

Lang et al, 1403.8103, PRD

RQCD, 1706.01247, PRD

HadSpec 2008.06432, JHEP

S=0 Mohler et al. 1208.4059, PRD
(see backup)

HadSpec, 1607.07093, JHEP

HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

Doubly heavy tetraquarks

$$QQ' \bar{q}\bar{q}'$$

$Q=c,b$ $q=u,d,s$

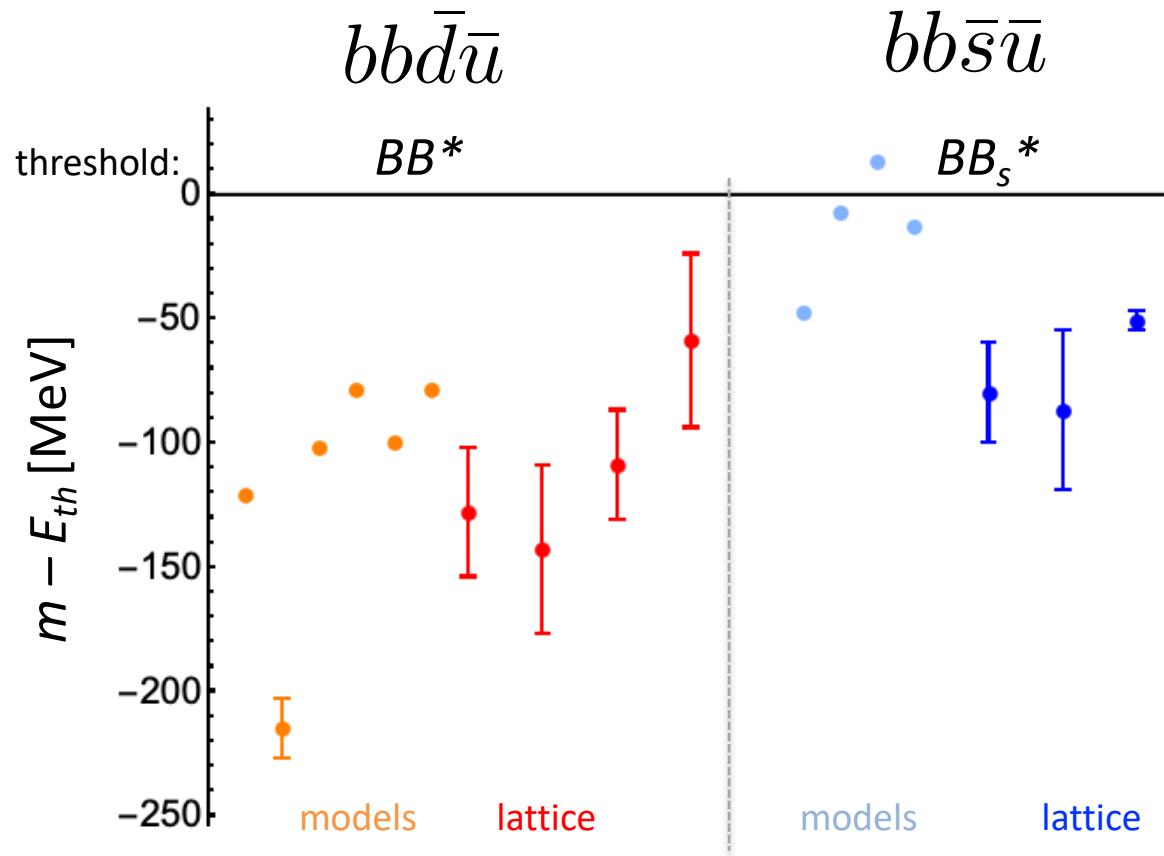
Doubly bottom tetraquarks

not found in exp, difficult to find

$bb\bar{d}\bar{u}$

$bb\bar{s}\bar{u}$

$I=0, J^P=1^+$



references from left to right

models (many more references):

Eichten and Quigg (2017) 1707.09575 PRL

Karliner and Rosner (2017) 1707.07666 PRL

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

Janc and Rosina (2004) hep-ph/0405208

lattice: most updated results

Leskovec, Meinel, Pflaumer, Wagner (2019) 1904.04197

Junnarkar, Mathur, Padmananth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

Bicudo, Wagner et al. 1612.02758 static potentials

models (many more references)

Eichten and Quigg (2017) 1707.09575 PRL

Parket al. (2018) 1809.05257

Ebert et al. (2007) 0706.3853

Silvestre-Brac and Semay (1993)

lattice: most updated results

Pflaumer, Leskovec, Meinel, Wagner (2021) 2108.10704

Junnarkar, Mathur, Padmananth (2018) 1810.12285

Frances, Colquhoun, Hudspith, Maltman (2021) preliminary

lattice	$m_{u/d}$	a [fm]
Leskovec , Pflaumer et al	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.08-0.11
Junnarkar et al.	$m_{u/d} \rightarrow m_{u/d}^{phy}$	$a \rightarrow 0$
Francis et al	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.09
Bicudo et al.	$m_{u/d} \rightarrow m_{u/d}^{phy}$	0.08

earlier results of Frances et al in 1810.10550, 1607.05214 PRL

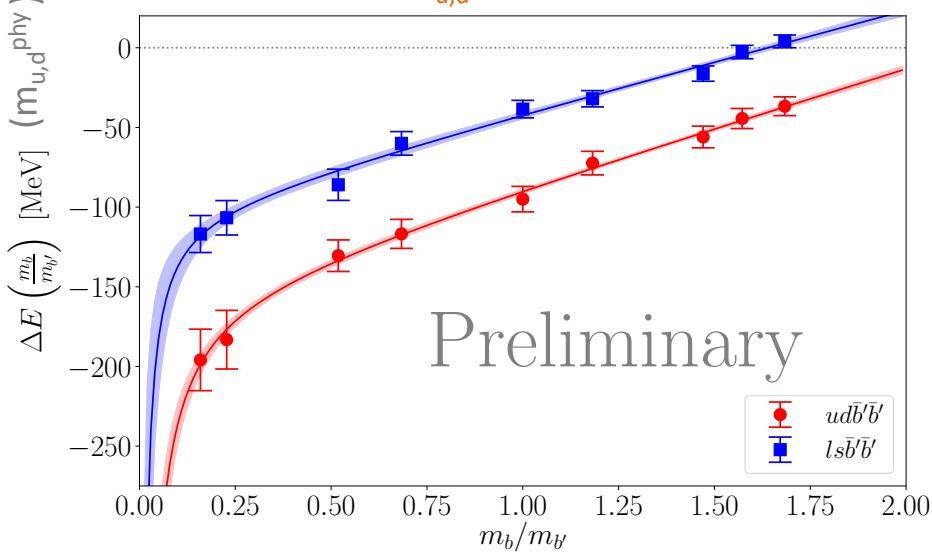
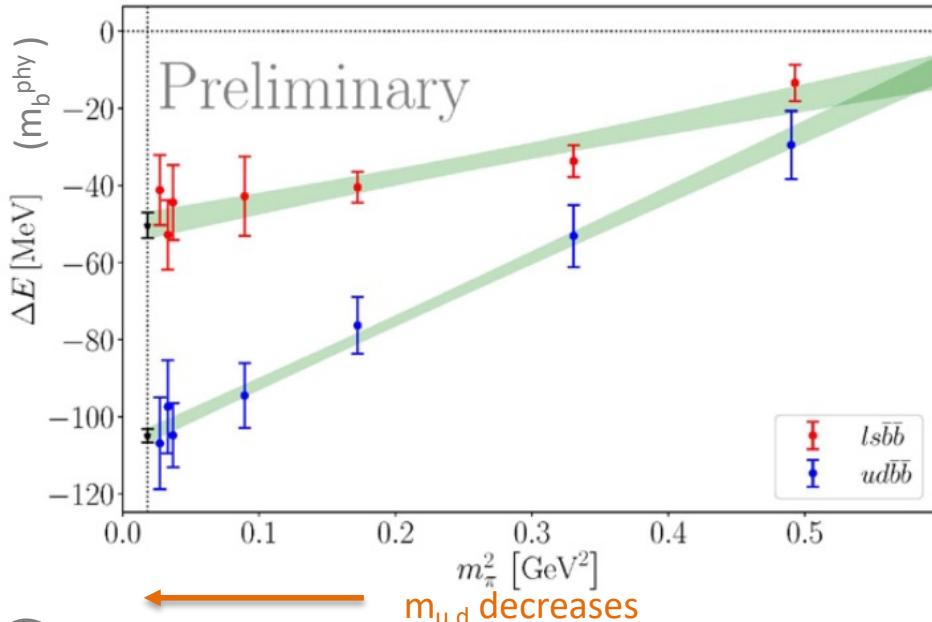
Doubly bottom tetraquarks

$b\bar{b}\bar{d}\bar{u}$

$b\bar{b}\bar{s}\bar{u}$

$I=0, J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



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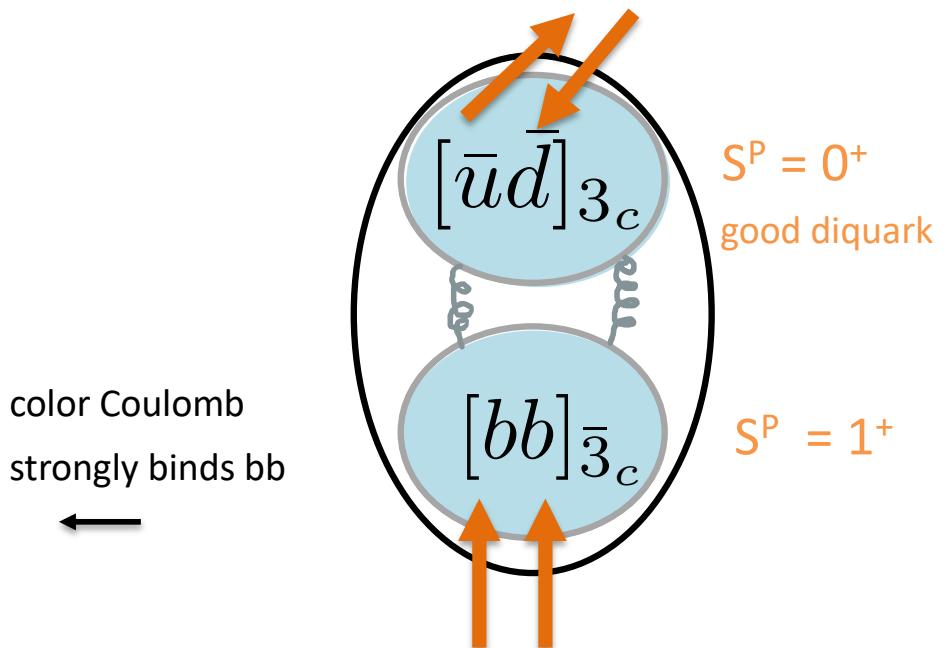
Hadron physics: theory

preliminary lattice results of Frances, Colquhoun, Hudspith, Maltman (2021)

@Lattice2021, Hadron2021

supports internal structure below

supported also by almost all model studies
Karliner and Rosner (2017), Janc and Rosina (2004), ...



Other $QQ'\bar{q}\bar{q}'$ and J^P

No other state threo. found significantly (>40 MeV) below threshold

Two that could be near threshold

$bcd\bar{u}$	Padmanath, Mathur 2111.01147	lattice energy extracted,
$ccd\bar{u}$	Junnarkar, Mathur, Pad. 1810.12285	pole not established

States near or above threshold have to be identified as poles in
scattering $T(E)$: more challenging than to establish a state well below th.

Final conclusion not yet reached.

Other $QQ'\bar{q}\bar{q}'$ and J^P

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Two that could be near threshold

$bcd\bar{u}$

Padmanath, Mathur 2111.01147

$ccd\bar{u}$

Junnarkar, Mathur, Pad. 1810.12285

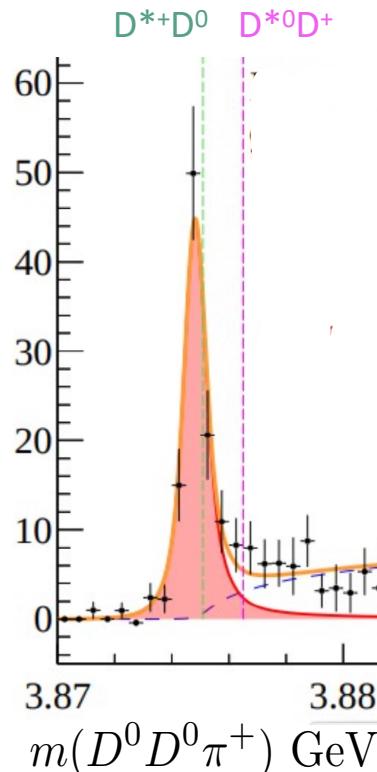
lattice energy extracted,

pole not established

States near or above threshold have to be identified as poles in
scattering T(E): more challenging than to establish a state well below th.
Final conclusion not yet reached.

Doubly charm tetraquark T_{cc}

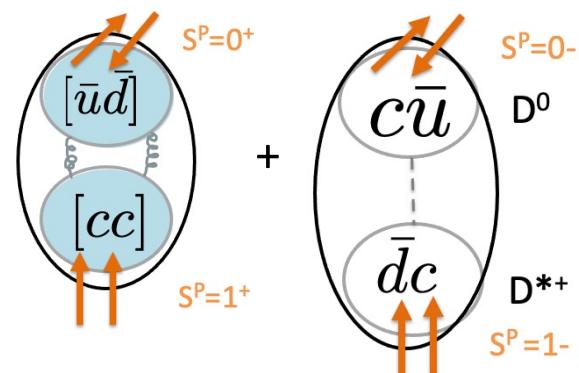
$cc\bar{d}\bar{u}$



$$\delta m = m - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{pole} = -0.36 \pm 0.04 \text{ MeV}$$

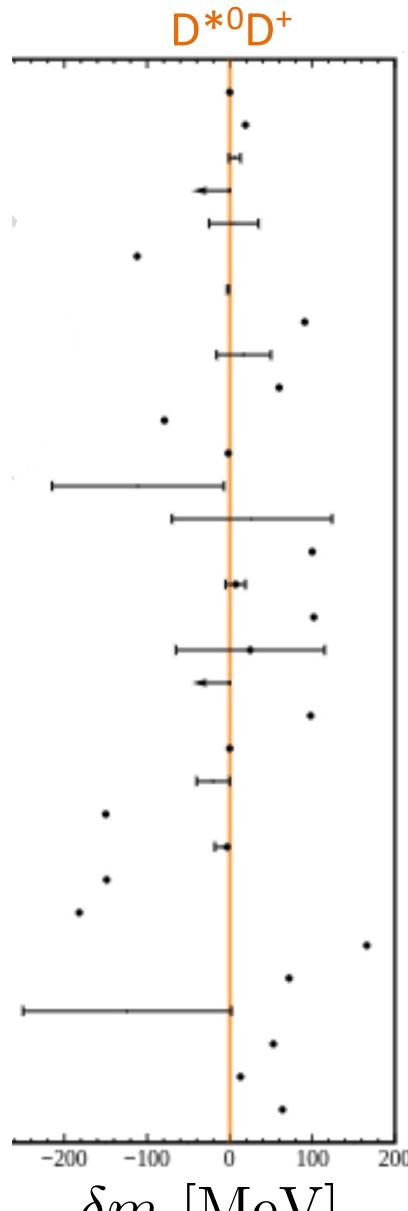
LHCb 2109.01038, 2109.01056



likely dominant

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δm [MeV]

references at the back

Theoretical predictions for T_{cc} mass ($I=0, J^P=1^+$)

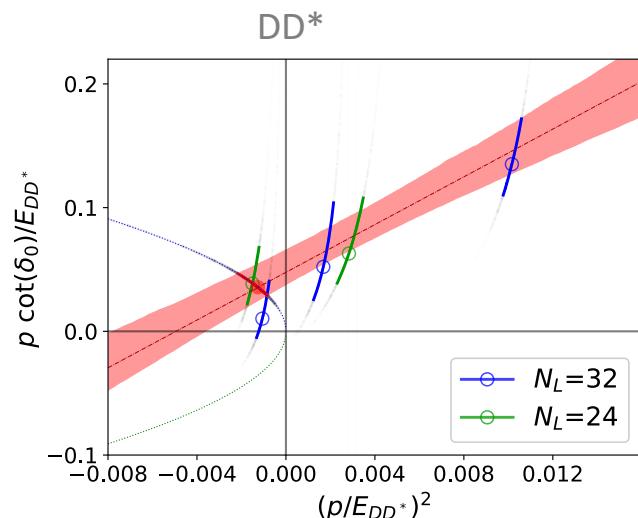
J. Carlson <i>et al.</i>	1987
B. Silvestre-Brac and C. Semay	1993
C. Semay and B. Silvestre-Brac	1994
S. Pepin <i>et al.</i>	1996
B. A. Gelman and S. Nussinov	2003
J. Vijande <i>et al.</i>	2003
D. Janc and M. Rosina	2004
F. Navarra <i>et al.</i>	2007
J. Vijande <i>et al.</i>	2007
D. Ebert <i>et al.</i>	2007
S. H. Lee and S. Yasui	2009
Y. Yang <i>et al.</i>	2009
G.-Q. Feng <i>et al.</i>	2013
Y. Ikeda <i>et al.</i>	2013
S.-Q. Luo <i>et al.</i>	2017
M. Karliner and J. Rosner	2017
E. J. Eichten and C. Quigg	2017
Z. G. Wang	2017
G. K. C. Cheung <i>et al.</i>	2017
W. Park <i>et al.</i>	2018
A. Francis <i>et al.</i>	2018
P. Junnarkar <i>et al.</i>	2018
C. Deng <i>et al.</i>	2018
M.-Z. Liu <i>et al.</i>	2019
G. Yang <i>et al.</i>	2019
Y. Tan <i>et al.</i>	2020
Q.-F. Lü <i>et al.</i>	2020
E. Braaten <i>et al.</i>	2020
D. Gao <i>et al.</i>	2020
J.-B. Cheng <i>et al.</i>	2020
S. Noh <i>et al.</i>	2021
R. N. Faustov <i>et al.</i>	2021

Doubly charm tetraquark

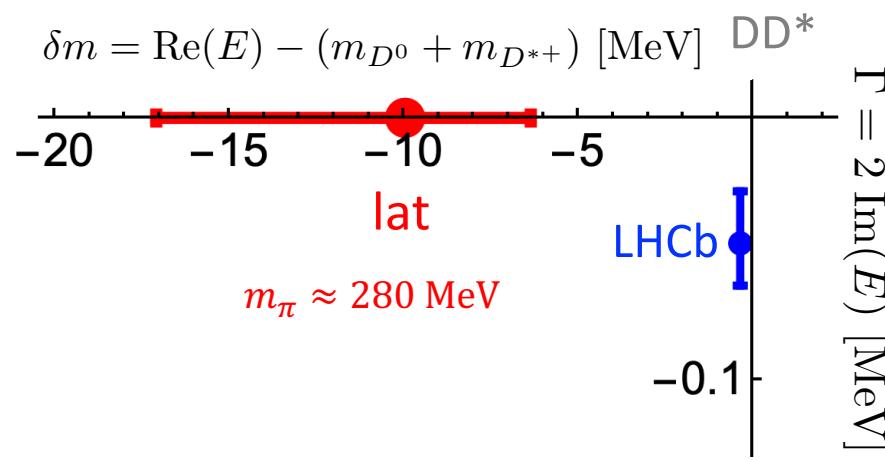
The only lattice study that extracts pole in $T(E)$

[Padmanath, S.P., S. Collins, preliminary 2022], $m_\pi \approx 280$ MeV, CLS ens.

more details at the back $E_{DD^*} \equiv m_D + m_{D^*}$ Preliminary



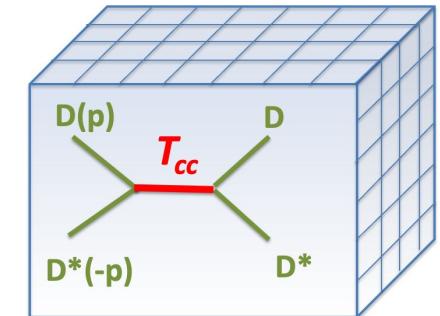
• Pole in $T \propto (p \cot \delta_0 - ip)^{-1}$



Sasa Prelovsek

$I=0, J^P=1^+$

$cc\bar{d}\bar{u}$



LHCb: T_{cc} : bound state $p = i |p|$ in the limit of stable D^*

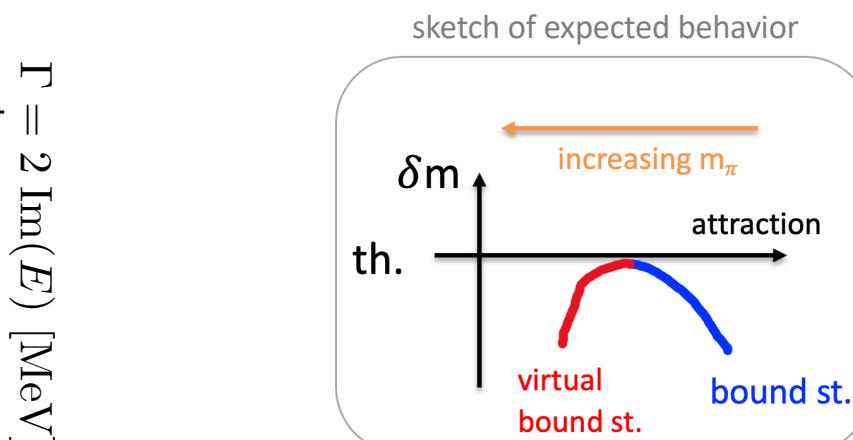
T_{cc} : resonance - decays to $DD\pi$, D^* strongly decays

lat: T_{cc} : virtual bound state $p = -i |p|$ for this m_π

$$E_B = -9.9^{+3.6}_{-7.1} \text{ MeV}$$

D^* strongly stable for this m_π

For $m_\pi > m_\pi^{\text{phy}}$ the attraction is smaller and the LHCb bound state with $\delta m=0.4$ MeV is expected to become a virtual bound state. The lattice result confirms this.



Hadron physics: theory

Exotic hadrons with Q and Q $Q=u,b$

Hadrons

$$\bar{Q}Q\bar{q}q, \bar{Q}Qqqq, \bar{Q}Q\bar{Q}Q$$

$Q=c,b$ $q=u,d,s$

$$\bar{b}b\bar{d}u$$

$$\bar{c}cuud$$

$$\bar{c}c\bar{c}c$$

$$Z_b$$

[Belle 2011]

$$P_c$$

[LHCb 2015]

$$X(6900)$$

[LHCb 2020]

GRRR!

challenging for ab-initio study
due to many decay channels:

$$\bar{Q}Q\bar{q}q \rightarrow \bar{Q}q + \bar{q}Q, \quad \bar{Q}Q + \bar{q}q$$

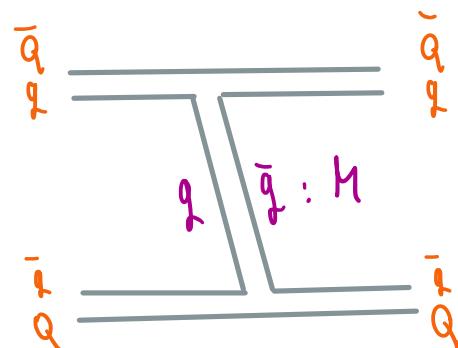
$$\bar{Q}Qqqq \rightarrow \bar{Q}q + Qqq, \quad \bar{Q}Q + qqq$$

Only partial conclusions are available
from ab-initio approaches
[reviewed e.g. in S.P. 2001.01767]
I'll discuss other approaches
(also due to lack of time).

hadronic molecule

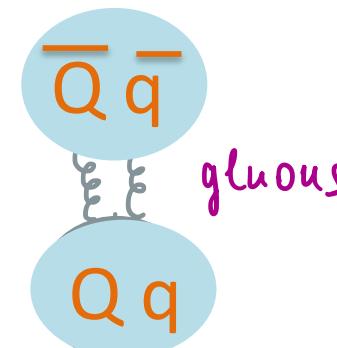
vs.

diquark antidiquark



quark spins correlated within

mesons



diquarks

Hadronic molecules

$(\bar{Q}q)(\bar{q}Q)$, $(\bar{Q}q)(Qqq)$

virtues:

- in line with most of exotics listed by Tamponi
- near thresholds
- interaction depends on isospin

Br to $\bar{Q}q + \bar{q}Q$: large

Br to $\bar{Q}Q + \bar{q}q$: small

- prevents too quick $\bar{Q}Q + \bar{q}q$

mesons exchanged (M):

- $P = \pi, \dots$ $L \propto H \partial \pi \gamma_5 \bar{H}$

$V = \rho, \dots$

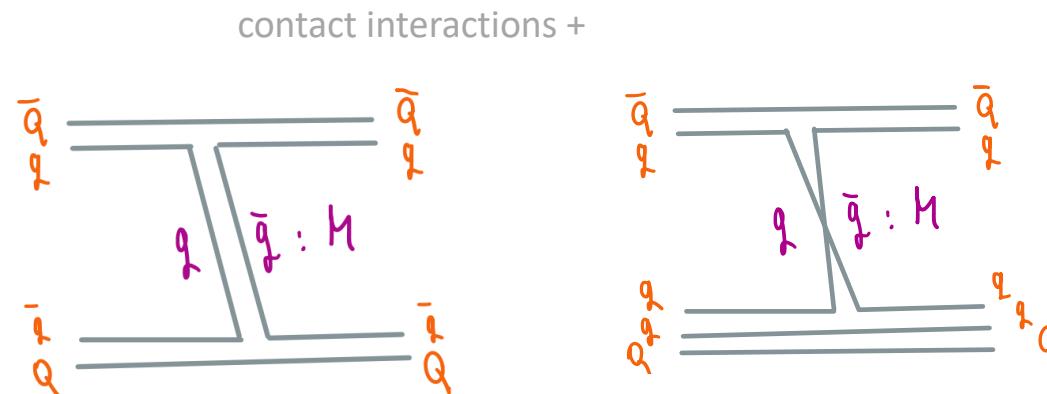
many many studies : Albaladejo, Baru, Guo, Hanhart, Oset, Meissner, Molina, Nieves et al; review 1705.00141

not natural candidates for molecules:

$bb\bar{u}\bar{d}$, $X(6900) = \bar{c}c\bar{c}\bar{c}$

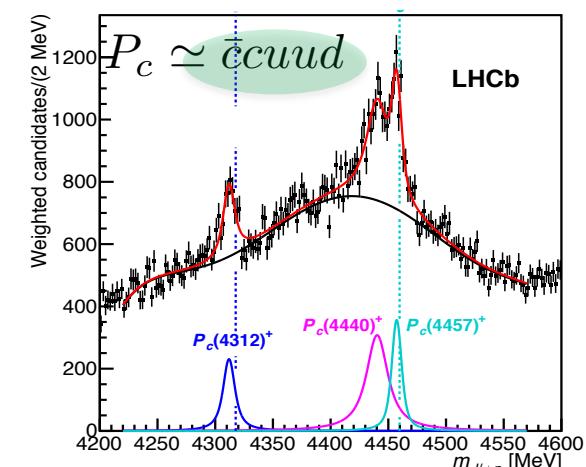
albeit see: Baru, Guo, Hanhart et al:

2107.03946, 2009.07795 PRL

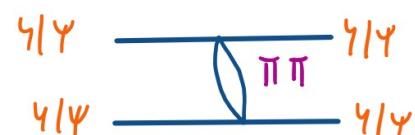


$$\Sigma_c(\frac{1}{2}^+) \bar{D}(0^-) \rightarrow J^P = \frac{1}{2}^- \quad \Sigma_c(\frac{1}{2}^+) \bar{D}^*(1^-) \rightarrow J^P = \frac{1}{2}^-, \frac{3}{2}^-$$

$\Sigma_c \bar{D}$ $\Sigma_c \bar{D}^*$



Molina, Oset, et al. 1007.0573, PRL



Hadron physics: theory

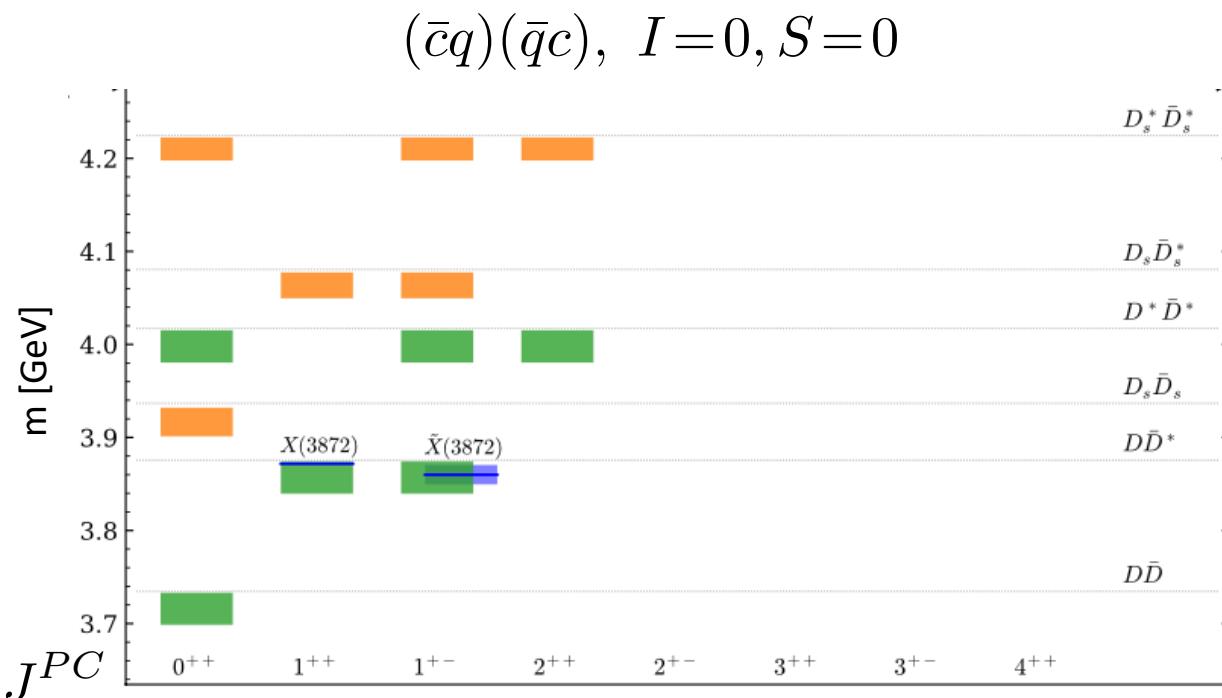
Sasa Prelovsek

A survey of molecules

$(\bar{c}q)(\bar{q}c)$, $(\bar{c}q)(cqg)$, $(\bar{q}c)(\bar{q}c)$, $(\bar{q}c)(cqg)$

229 molecules 124 molecules

q=u,d,s



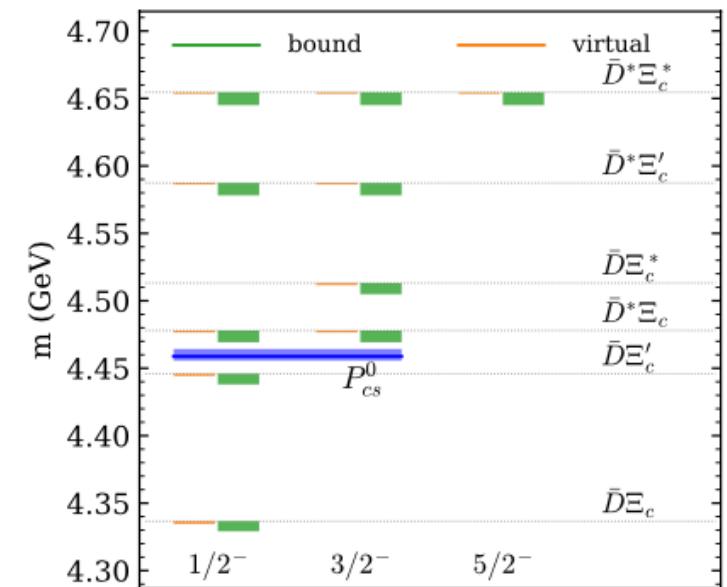
- systematic spectrum based on a single-model
- exchange of V mesons
- exchange of P omitted: subdominant near th. L $\propto H \partial\pi \gamma_5 \bar{H}$
- symmetries: ChPT, hidden local symmetry, heavy quark spin, SU(3)_{fl}
- no coupled-channel effects, no mixing with cc
- can provide only qualitative features

— bound — virtual

$p = i |p|$ $p = -i |p|$

$$m_B = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} < m_1 + m_2$$

$(\bar{c}u)(cds), I=0, S=1$

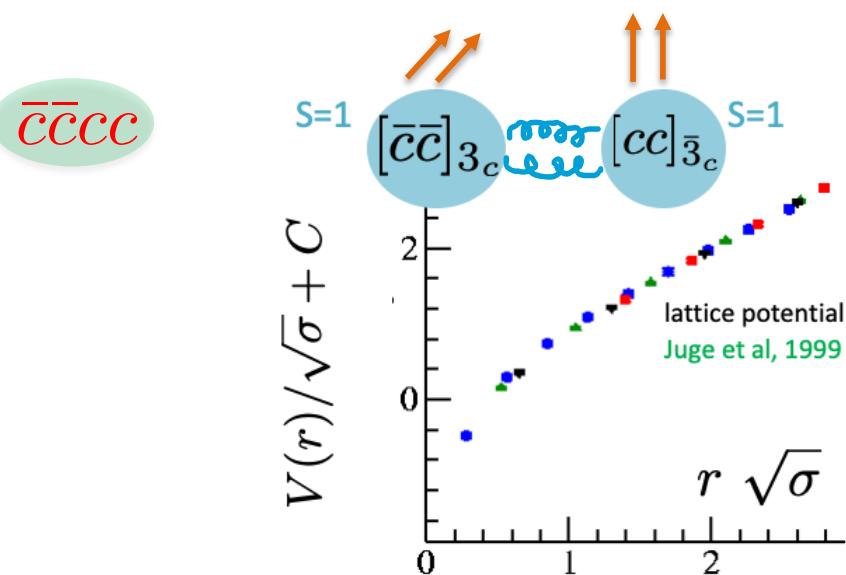


Dong, F.-K. Guo, Zou, 2101.01021, 2108.02673

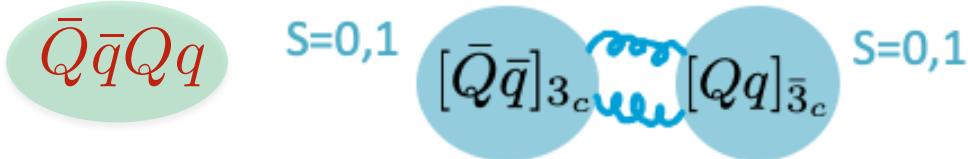


predictions for two values of cut-off

Hadrons from diquarks



Lattice QCD: no indication for strongly stable

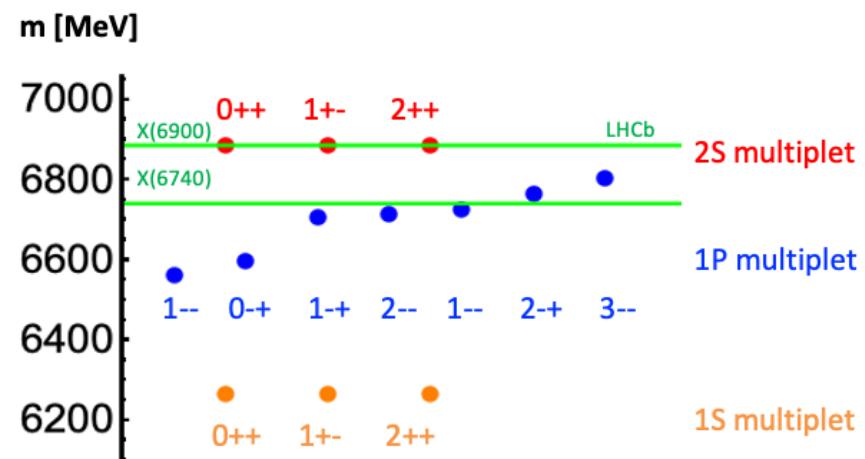


Explains several observed features.

LHCb: 2006.16957

Giron, Lebed, 2008.01631 PRD X(6900) used to fix diquark mass

Bedolla, J. Ferretti, Roberts, Santopinto 1911.00960, EPJC



[Hughes, Eichten, Davies, HPQCD, 1710.03236, PRD]

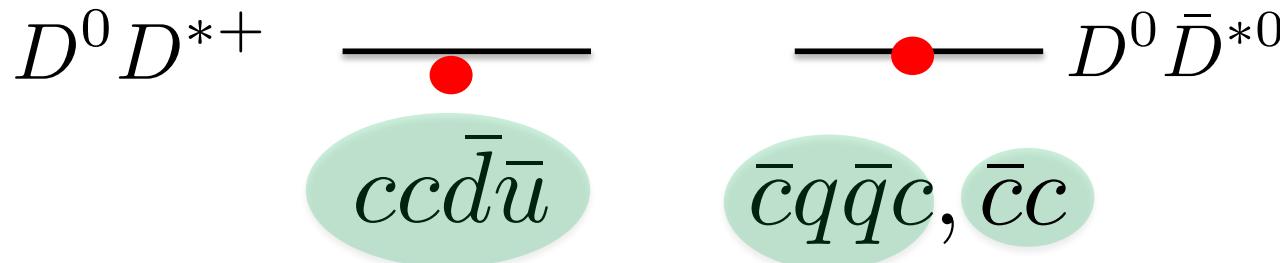
many works by Jaffe, Maiani, Polosa, Riquer, Piccinini, Pilloni, Esposito, Lebed, Giron, Ferretti, Santopinto, ..

Drawbacks:

- independent of isospin: same spectra for I=0 and 1
- sizable width to $(\bar{Q}Q)(\bar{q}q)$
- ground state $J^P = 0+$ (experimental states have $J^P = 1+$)
maybe 0+ is too broad to be observed

Partial solutions to all these aspects were proposed

A puzzle comparing T_{cc} and $X(3872)$



Why both reside within 1 MeV of threshold in exp ? There are many differences ...

Similarities:

- $J^P=1^+$, $I=0$
- in molecular picture: attraction via light vector exchange [e.g. Guo et al. 2101.01021, 2108.02673]

Differences:

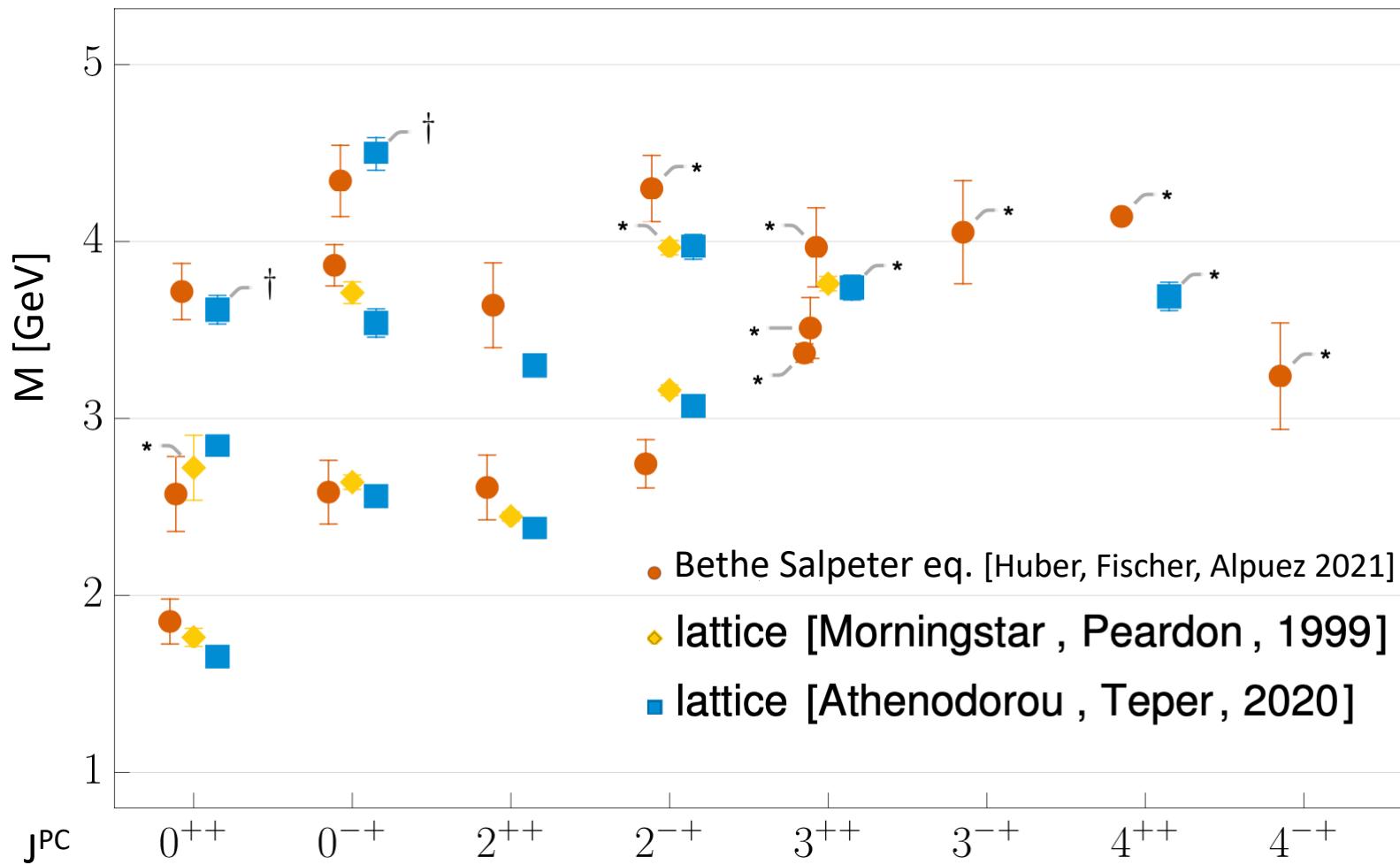
- in molecular picture: attraction from one-pion exchange for $X(3872)$ [Tornquist 1994]
slight attraction from one-pion exchange for T_{cc} [eg. Du, Guo, Hanhart, 2110.13765]
- presence of Fock component \underline{cc} for $X(3872)$ [e.g. Padmanath, Lang, SP 1503.03257, PRD]
- presence of Fock component $\underline{[cc][ud]}$ for T_{cc}

Glueballs from QCD without sea-quark effects

gluons



omitted to prevent glueballs from strong decay to mesons



2110.09180, EPJC

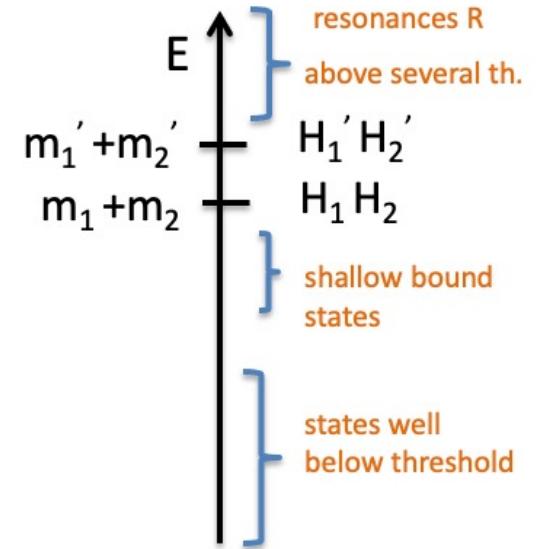
2007.06422, JHEP

Conclusions

Compliments to experiments for GREAT results !!

Status on hadron spectrum from Lattice QCD :

- **strongly stable** : “straightforward, done”
- **strongly decaying to 1 channel** “mostly done”
- **2-3 channels** “challenging, some of them studied”
- **> 3 channels** “very challenging, mostly unexplored”
- $P_c, Z_c, Z_b, X(6900), \dots$ decay via many channels: reason why lattice has only partial conclusions on those
- $R \rightarrow H_1 H_2 H_3$ “very difficult” [formalism by Sharpe, Hansen, Briceno, Doring, Mai, Rusetsky, Lopez et. al]
 $a_1 \rightarrow \pi\pi\pi$ [GWU, 2107.03973], $\pi\pi\pi$ [several groups], KKK [GWU]
- why $m_{u,d} > m_{u,d}^{\text{phy}}$? Smaller number of decay channels, smaller statistical errors on E and T(E)



Status on hadron spectrum from Bethe-Saptelet + Dyson-Schwinger equations :

strongly stable : “mostly done”

strongly decaying to 1 channel : “few resonances done over past few years”

> 2 channels: “not done yet”

Exotic hadrons: one picture can not explain all exotic hadron states; for each exotic hadron there is at least one viable picture

Theory predicts many conventional and exotic hadrons yet to be discovered

Thanks to friends and colleagues

M. Padmanath, Marek Karliner , Feng-Kun Guo, Richard Lebed, Anthony Frances,
Sara Collins, Mark Wagner, Luka Leskovec, Elena Sanotpinto, Christian Fischer,
Sinead Ryan, Michael Doring

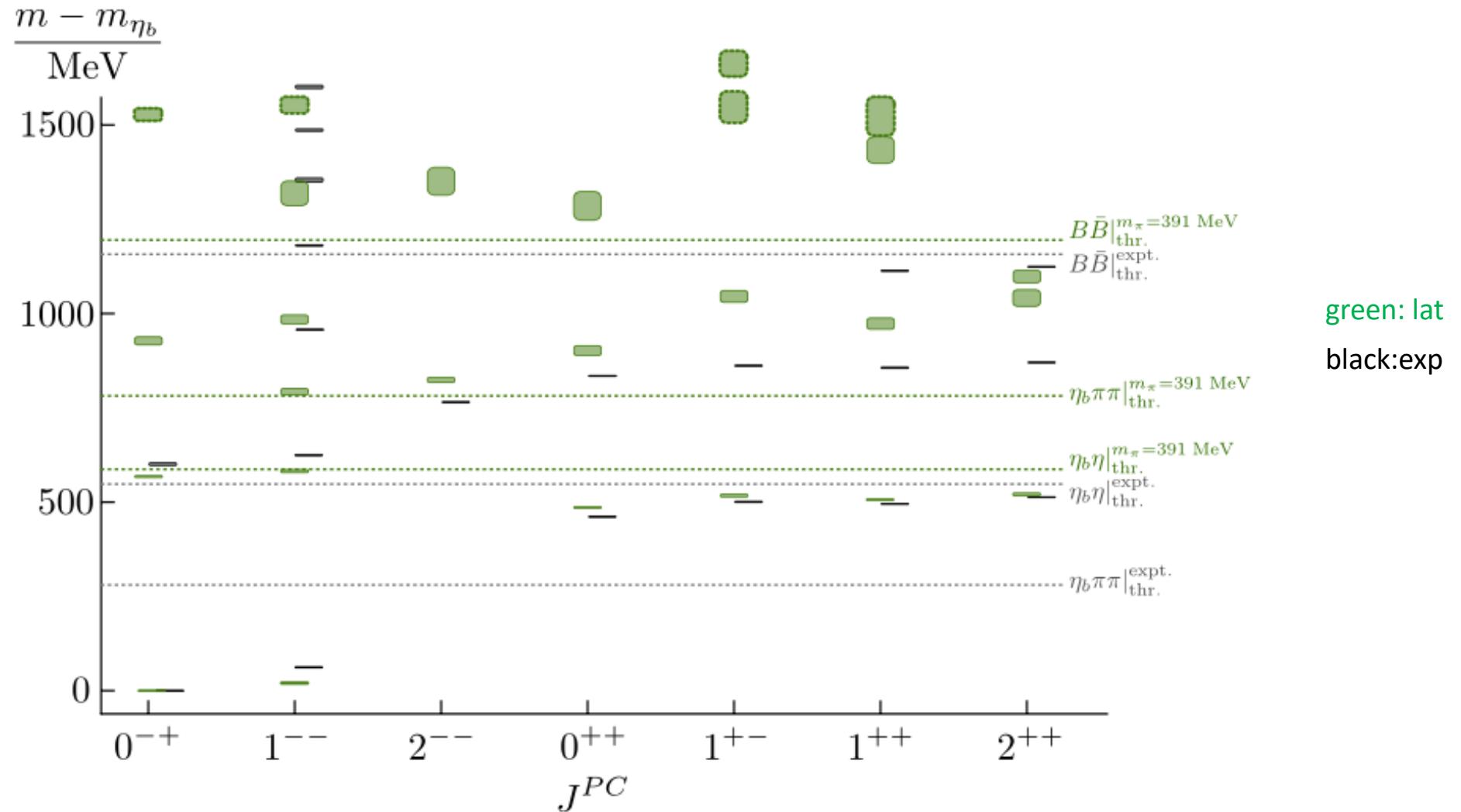
Appologies

... many interesting and relevant results could not be presented

backup

bottomonia

Ryan & Wilson (HadSpec)
2008.02656, JHEP



Sigma resonance with Bethe-Salpeter approach

$$T^{(4)} = \bar{\Psi} \cdots \Psi \Rightarrow \Psi = K^{(4)} \cdots \Psi$$

$$K^{(4)} = \text{[Diagram] } + \text{ perm.} - \text{ [Diagram]} + \text{ perm.} \text{ truncations}$$

Santowsky, Eichmann, Fischer, Wallbott, Williams,
2007.06495, PRD

$$\Psi = \text{[Diagram]} \Phi_1 + \text{[Diagram]} \Phi_2 + \text{[Diagram]} \Phi_3$$

$$\Phi_1 = \text{[Diagram]} \Phi_2 + \text{[Diagram]} \Phi_3$$

$$\begin{aligned} \text{pion} \cdots \Phi_1 &= \text{[Diagram]} \Phi_2 + \text{[Diagram]} \Phi_3 + \text{[Diagram]} \Gamma^* \\ [\bar{q}q] \cdots \Phi_3 &= \text{[Diagram]} \Phi_1 + \text{[Diagram]} \Phi_2 + \text{[Diagram]} \Gamma^* \\ \Gamma^* &= K^{(2)} \Gamma^* + K^{(2)} \Phi_1 + K^{(2)} \Phi_3 \end{aligned}$$

Scalar D_0^* resonance

Puzzle: why strange and non-strange scalars mesons almost degenerate
 $m[D_0^*(2300)] \approx m[D_{s0}^*(2317)]$

Mass of the lowest scalar charmed resonance (it is broad due to decay to $D\pi$)

Mass from BW fit [PDG]

$D_0^*(2300)$ MASS	2343 ± 10 MeV ($S = 1.5$)
--------------------	---------------------------------

$D_0^*(2300)$ WIDTH	229 ± 16 MeV
---------------------	------------------

Mass from pole (lattice [2102.04973](#), [1208.4059](#) and effective field theories) give lower value
 $m=2.1-2.2$ GeV. This makes $D0^*$ in more natural partner of $D_{s0}^*(2317)$

Comment on the $D0^*$ mass from the lattice simulation [2102.04973](#)

The BW fit at real energies rendered $m \approx 2.32$ GeV, while the corresponding pole in the complex plan based on the same lattice data is at $m \approx 2.12$ GeV.

It is not surprising that mass from BW fit and the pole differ so much in this case, since this resonance is broad and not very high above the threshold

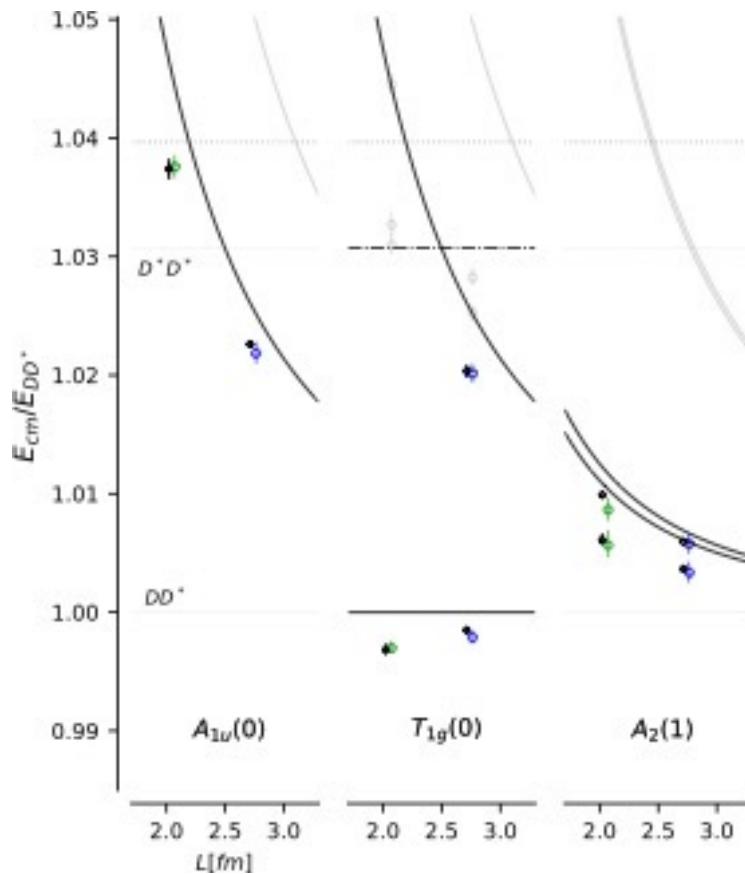
Lattice study of a doubly charm tetraquark

[Padmanath, S.P., S. Collins, preliminary 2022] $E_{DD^*} \equiv m_D + m_{D^*}$

m_π [MeV]	m_K [MeV]	m_D [MeV]	m_{D^*} [MeV]	Preliminary
280(3)	467(2)	1927(2)	2050(2)	

CLS ens: $m_{u,d} > m_{u,d}^{phy}$, $m_s < m_s^{phy}$

$$P_{\text{tot}} = 0 \quad P_{\text{tot}} = 0 \quad P_{\text{tot}} = 1 \frac{2\pi}{L}$$



Lattice eigen-energies used in fit of $T(E)$ using $L=0,1$: blue, green circles

Analytic energies based on extracted $T(E)$: black circles

Non-interacting energies: black lines

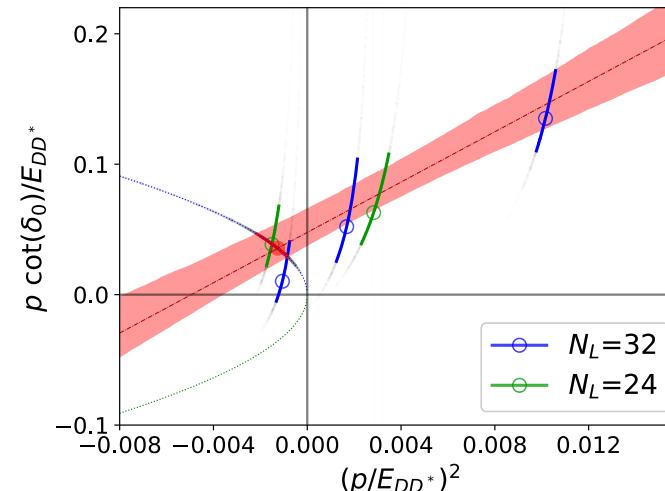
Lattice energies not used in the fit of $T(E)$: gray circles



$I=0, J^P=1^+$

$$\mathcal{O} = \sum_i D(\vec{p}_{1i}) D^*(\vec{p}_{2i})$$

Results for partial wave $L=0$



$$T \propto (p \cot \delta_0 - ip)^{-1}$$

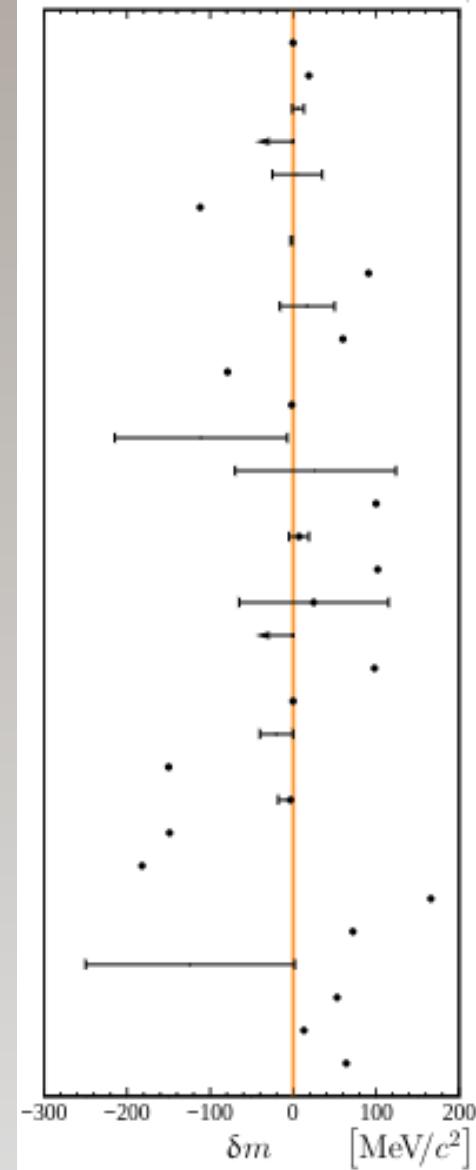
$$p \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

$$a_0 = 1.04 \pm 0.29 \text{ fm}, \quad r_0 = 0.96 \pm 0.19 \text{ fm}$$

$$E_B = -9.9 {}^{+3.6}_{-7.1} \text{ MeV}$$

Theory predictions

Reference		Year	$\delta'm$ [MeV/c ²]
J. Carlson, L. Heller and J. A. Tjon	36	1987	~ 0
B. Silvestre-Brac and C. Semay	37	1993	+19
C. Semay and B. Silvestre-Brac	38	1994	[-1, +13]
S. Pepin, F. Stancu, M. Genovese and J. M. Richard	39	1996	< 0
B. A. Gelman and S. Nussinov	40	2002	[-25, +35]
J. Vijande, F. Fernandez, A. Valcarce, A. and B. Silvestre-Brac	41	2003	-112
D. Janc and M. Rosina	42	2004	[-3, -1]
F. Navarra, M. Nielsen and S. H. Lee	43	2007	+91
J. Vijande, E. Weissman, A. Valcarce	44	2007	[-16, +50]
D. Ebert, R. N. Faustov, V. O. Galkin and W. Lucha	45	2007	+60
S. H. Lee and S. Yasui	46	2009	-79
Y. Yang, C. Deng, J. Ping and T. Goldman	47	2009	-1.8
G.-Q. Feng, X.-H. Guo and B.-S. Zou	48	2013	-215
Y. Ikeda, B. Charron, S. Aoki, T. Doi, T. Hatsuda, T. Inoue, N. Ishii, K. Murano, H. Nemura and K. Sasaki	49	2013	[-70, +124]
S.-Q. Luo, K. Chen, X. Liu, Y.-R. Liu and S.-L. Zhu	50	2017	+100
M. Karliner and J. Rosner	51	2017	7 ± 12 → 1
E. J. Eichten and C. Quigg	52	2017	+102
Z. G. Wang	53	2017	+25 ± 90
G. K. C. Cheung, C. E. Thomas, J. J. Dudek and R. G. Edwards	54	2017	≤ 0
W. Park, S. Noh and S. H. Lee	55	2018	+98
A. Francis, R. J. Hudspith, R. Lewis and K. Maltman	56	2018	~ 0
P. Junnarkar, N. Mathur and M. Padmanath	57	2018	[-40, 0]
C. Deng, H. Chen and J. Ping	58	2018	-150
M.-Z. Liu, T.-W. Wu, V. Pavon Valderrama, J.-J. Xie and L.-S. Geng	59	2019	-3 ⁺⁴ ₋₁₅
G. Yang, J. Ping and J. Segovia	60	2019	-149
Y. Tan, W. Lu and J. Ping	61	2020	-182
Q.-F. Lü, D.-Y. Chen and Y.-B. Dong	62	2020	+166
E. Braaten, L.-P. He and A. Mohapatra	63	2020	+72
D. Gao, D. Jia, Y.-J. Sun, Z. Zhang, W.-N. Liu and Q. Mei	64	2020	[-250, +2]
J.-B. Cheng, S.-Y. Li, Y.-R. Liu, Z.-G. Si, T. Yao	65	2020	+53
S. Noh, W. Park and S. H. Lee	66	2021	+13
R. N. Faustov, V. O. Galkin and E. M. Savchenko	67	2021	+64



Refs. for theory predictions

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Charmonium resonances from lattice

$$\Gamma \equiv g^2 p_D^{2l+1} / m^2$$

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