CP violation and mixing in charm and beauty hadrons

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for the LHCb Collaboration

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**CP violation experiments**

- \(e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}\) or: \(\rightarrow \psi(3770) \rightarrow D\bar{D}\)
- Known initial state but comparatively low cross section
- \(pp \rightarrow gg \rightarrow b\bar{b}\) with general-purpose detectors
- Huge cross section but messy. Not what they’re built for.
- or dedicated detector with tuned-down pile-up: LHCb.
- Optimised vertexing, hadron PID and a software trigger
Topics and parallel sessions

- Precision measurement of $B_s$ meson oscillations
- Observation of $D^0$ meson mass difference
- Observation of direct $CP$ violation with $D^0$ mesons
- Time-dependent $CP$ violation search with $D^0$ mesons
- $CP$ violation searches with $D^{(*)}$ mesons
- $CP$ violation in charmless $B^+$ decays
- Unitarity Triangle: $\beta$
- Unitarity Triangle: $\gamma$
- $CP$ violating phase with $B_s$ mesons

Wed 11:20: Yu Zhang
“Hadronic charm decays at BESIII”

Wed 11:40: Joan Ruiz Vidal
“CPV and mixing in charm at LHCb”

Wed 10:00: Markus Reif
“Hadronic $B$ decays at Belle II”

Wed 12:00: Radek Zlebcik
“Charm and TDCPV in $B$ decays at Belle II”

Wed 10:40: Jordy Butter
“CPV and CKM measurements with beauty decays at LHCb”

Wed 12:00: Radek Zlebcik
“Charm and TDCPV in $B$ decays at Belle II”
$B_s \leftrightarrow \bar{B}_s$ mixing frequency: $\Delta m_s$

- Mixing occurs if the eigenstates of the hamiltonian are not aligned with the interactions eigenstates:
  \[
  |B_{sH}\rangle = p |B_s\rangle - q |\bar{B}_s\rangle \quad |B_{sL}\rangle = p |B_s\rangle + q |\bar{B}_s\rangle
  \]

\[
|B_s(t)\rangle = g_+ |B_s\rangle + \frac{q}{p} g_- |\bar{B}_s\rangle
\]

\[
g_\pm = \frac{1}{2} \left[ e^{-i M_L t - \Gamma_L t/2} \pm e^{-i M_H t - \Gamma_H t/2} \right]
\]
Mixing occurs if the eigenstates of the hamiltonian are not aligned with the interactions eigenstates:

\[
|B_{sH}(t)\rangle = \frac{p}{q}g_{+} \vert B_s \rangle + \frac{q}{p}g_{-} \vert \bar{B}_s \rangle
\]

\[
|B_{sL}(t)\rangle = \frac{p}{q}g_{+} \vert B_s \rangle + \frac{q}{p}g_{-} \vert \bar{B}_s \rangle
\]

\[
|B_{s}(t)\rangle = g_{+} \vert B_s \rangle + \frac{q}{p}g_{-} \vert \bar{B}_s \rangle
\]

\[
g_{\pm} = \frac{1}{2} \left[ e^{-i M_{L}t - \Gamma_{L}t/2} \pm e^{-i M_{H}t - \Gamma_{H}t/2} \right]
\]

average decay width
\[
\Gamma_s = (\Gamma_H + \Gamma_L)/2
\]

decay width difference
\[
\Delta \Gamma_s = \Gamma_H - \Gamma_L
\]

Probability to observe [un]mixed \( B_s \) meson
\[
P(t) \sim e^{-\Gamma_{s}t} \left[ \cosh \left( \frac{\Delta \Gamma_{s}t}{2} \right) + C \cdot \cos(\Delta m_{s}t) \right]
\]

Constant \( C = -1 \) (+1) for mixed (unmixed) \( B_s \) meson

\[
\Delta m_{s} = m_{H} - m_{L}
\]
Illustration: initial $\rightarrow$ mixed $\rightarrow$ mix back $\rightarrow$ mix again ... 

Flavour tagging vital to identify initial flavour

$P(t)$

$B_s \leftrightarrow \overline{B}_s$

$B^0 \leftrightarrow \overline{B}^0$

$\overline{D}^0 \leftrightarrow D^0$

$\sim 1$ cm at LHCb

Small mixing probability
$\Delta m_s$ with $B_s \rightarrow D_s^{\pm} \pi^{\mp}$

- Identify decay flavour with Cabibbo-favoured $b \rightarrow c\pi^-$ vs. $\bar{b} \rightarrow \bar{c}\pi^+$ transition
- DCS contribution negligible
- Identify initial flavour with flavour tagging.
- Effecting performance at LHCb: 6.1% of ideal.

- An initial mass fit identifies 379$k$ signal events
- Result with $B_s \rightarrow D_s\pi$ combined with earlier result using $B_s \rightarrow D_s\pi\pi$ (JHEP 03 (2021) 137)

$$\Delta m_s = 17.7656 \pm 0.0057 \text{ ps}^{-1}$$
- Consistent with, but far more precise than, SM prediction (due to QCD calculation limitations)

\[ c.f. \sim 31\% \text{ at Belle(II)} \]
On the other hand, charm mixing is slow

- Probability of mixing too small to measure complete oscillations
- Though flavour tagging is near-perfect: $D^{*+} \rightarrow D^0\pi^+$ or $B^- \rightarrow D^0\mu^-\nu$
- Instead, look for a time-dependant change in the rate of DCS decays

$$\Gamma(D^0 \rightarrow K^+\pi^-) = |g_+(t) r_D e^{-i\delta_D} + g_-(t)|^2$$

$$R = \frac{D^0 \rightarrow K^+\pi^-}{\bar{D}^0 \rightarrow K^+\pi^-}$$
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$$

$$
e^{-\Gamma t} \left[ \frac{1}{2} r_D^2 (\cosh(y\Gamma t) + \cos(x\Gamma t)) + \frac{1}{2} (\cosh(y\Gamma t) - \cos(x\Gamma t)) + r_D (\cos \delta_D \sinh(y\Gamma t) - \sin \delta_D \sin(x\Gamma t)) \right]
$$

$$
x \equiv \Delta m/\Gamma \quad y \equiv \Delta \Gamma/2\Gamma
$$

dimensionless parameters $O(1\%)$

$$
R = \frac{D^0 \rightarrow K^+ \pi^-}{D^0 \rightarrow K^+ \pi^-} \approx r_D^2 + r_D (y \cos \delta_D - x \sin \delta_D) \left( \frac{t}{\tau} \right) + \frac{y^2 + x^2}{4} \left( \frac{t}{\tau} \right)^2
$$
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\]

Problem: need a precise \( \delta_D \) for precision \( x \) & \( y \)

\[
x \equiv \Delta m/\Gamma \quad y \equiv \Delta \Gamma/2\Gamma
\]

Dimensionless parameters O(1%)
Bin-flip method with $D \rightarrow K_S^0 \pi^+ \pi^-$

- Use charge-conjugate multi-body decay. Subdivided into bins with a variety of $\delta_D$ (called $X_b$ here)
- Measure the relative phases between bin $+b$ and $-b$, $X_b$ using quantum-correlated $D^0\bar{D}^0$ pairs in a $e^+e^- \rightarrow \psi(3770)$ dataset ($\bullet$). At LHCb, measure time-dependent “DCS” ratio $R_b$ in each bin-pair.


30 M events
Observation of the neutral charm $\Delta m$

- Formalism expanded and generalised to include the binning scheme and $CP$ violation

$$R_{bj}^\pm \approx \frac{r_b + r_b \frac{\langle t^2 \rangle_j}{4} \text{Re}(z_{CP}^2 - \Delta z^2) + \frac{\langle t^2 \rangle_j}{4} |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \langle t \rangle_j \text{Re}(X_b^*(z_{CP} \pm \Delta z))}{1 + \frac{\langle t^2 \rangle_j}{4} \text{Re}(z_{CP}^2 - \Delta z^2) + r_b \frac{\langle t^2 \rangle_j}{4} |z_{CP} \pm \Delta z|^2 + \sqrt{r_b} \langle t \rangle_j \text{Re}(X_b(z_{CP} \pm \Delta z))}.$$  

for Dalitz bin $b$ and decay-time bin $j$

$$z_{CP} \pm \Delta z \equiv -(q/p)^\pm 1 (y + ix) \quad x \equiv \Delta m / \Gamma \quad y \equiv \Delta \Gamma / 2 \Gamma$$

dimensionless parameters $O(1\%)$
Observation of the neutral charm $\Delta m$

- Formalism expanded and generalised to include the binning scheme and CP violation

$$R_{bj}^\pm \approx \frac{r_b + r_b \langle t^2 \rangle_j}{4} \frac{\text{Re}(z_{CP}^2 - \Delta z^2)}{\text{Re}(z_{CP}^2 - \Delta z^2) + r_b \langle t^2 \rangle_j} |z_{CP} + \Delta z|^2 \sqrt{r_b \langle t \rangle_j} \text{Re}[X_b^*(z_{CP} + \Delta z)] \ .$$

for Dalitz bin $b$ and decay-time bin $j$

$$z_{CP}^{\pm \Delta z} \equiv -(q/p)^{\pm 1} (y + ix) \quad x \equiv \Delta m/\Gamma \quad y \equiv \Delta \Gamma/2\Gamma$$

dimensionless parameters $O(1\%)$

$r_b = R_{bj}$ at $t = 0$

$X_b = \exp(i\delta_p(b))$ where $\delta_p(b)$ is the strong-phase difference between $+b$ and $-b$

$> 7\sigma$

$x = (3.98_{-0.54}^{+0.56}) \times 10^{-3},$

$y = (4.6_{-1.5}^{+1.3}) \times 10^{-3},$

$|q/p| = 0.996 \pm 0.052,$

$\text{arg}(q/p) = 0.056_{-0.051}^{+0.047}.$
New global combination

\[ x = (0.409 \pm 0.049) \times 10^{-3} \]
\[ y = (0.615 \pm 0.056) \times 10^{-3} \]
\[ |\frac{q}{p}| - 1 = -0.005 \pm 0.016 \]
\[ \arg(q/p) = (-2.5 \pm 1.2)^\circ \]

No-CPV in charm mixing now excluded at 1.6\(\sigma\)
Time dependant CPV in charm

- Lifetime asymmetry of $D^0$ and $\bar{D}^0$ to a CP eigenstate, $f = K^+K^-, \pi^+\pi^-$ expected to be $\mathcal{O}(10^{-5})$

$$A_{CP}(f, t) \equiv \frac{\Gamma(D^0 \to f, t) - \Gamma(\bar{D}^0 \to f, t)}{\Gamma(D^0 \to f, t) + \Gamma(\bar{D}^0 \to f, t)} \approx a^d_f + \Delta Y_f \frac{t}{\tau_{D^0}}$$

slope measurement only

$$\Delta Y_f \approx -\frac{\hat{\Gamma}_{D^0 \to f} - \hat{\Gamma}_{\bar{D}^0 \to f}}{\hat{\Gamma}_{D^0 \to f} + \hat{\Gamma}_{\bar{D}^0 \to f}}$$

Asymmetry of the effective decay widths
Time dependent CPV in charm

- Lifetime asymmetry of $D^0$ and $\bar{D}^0$ to a CP eigenstate, $f = K^+K^-$, $\pi^+\pi^-$ expected to be $\mathcal{O}(10^{-5})$

\[
A_{CP}(f, t) \equiv \frac{\Gamma(D^0 \to f, t) - \Gamma(\bar{D}^0 \to f, t)}{\Gamma(D^0 \to f, t) + \Gamma(\bar{D}^0 \to f, t)} \approx a_f^d + \Delta Y_f \frac{t}{\tau_{D^0}}
\]

\[
\Delta Y_f \approx -\frac{\hat{\Gamma}_{D^0 \to f} - \hat{\Gamma}_{\bar{D}^0 \to f}}{\hat{\Gamma}_{D^0 \to f} + \hat{\Gamma}_{\bar{D}^0 \to f}}
\]

Asymmetry of the effective decay widths

This measurement

\[
\Delta Y = \frac{1}{2} \left[ \left( |p| \left| \frac{A_f}{A_f^*} \right| + |q| \left| \frac{A_f}{A_f^*} \right| \right) x \sin \phi_y - \left( |p| \left| \frac{A_f}{A_f^*} \right| - |q| \left| \frac{A_f}{A_f^*} \right| \right) y \cos \phi_y \right]
\]
Observation of a direct $CP$ violating effect in charm

- Achieved using a difference method to cancel common systematics
  \[
  \Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \\
  \approx \Delta a_{CP}^{dir} + \frac{\Delta(t)}{\tau_{D^0}} \Delta Y \\
  \text{dominates} \quad \approx +3 \times 10^{-5}, \text{ with earlier } \Delta Y \text{ result}
  \]

- Using both $D^{*+} \to D^0 \pi^+$ (top) and $B^- \to D^0 \mu^- \nu$ self-tagging sources of $D^0$ decays,
  \[
  \Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4}
  \]
  \[5.3\sigma \text{ from zero}\]

To be determined if $KK$ or $\pi\pi$ or neither are consistent with zero.
Other direct CPV searches with charm

- Belle dataset (0.98 ab\(^{-1}\)) used to improve branching fractions and search for CP asymmetries

With \(D^0\) mesons,

\[ D^0 \rightarrow \pi^+ \pi^- \eta \text{ shown. (} Q \equiv \Delta m \)\]

- \(A_{CP}(D^0 \rightarrow \pi^+ \pi^- \eta) = [0.9 \pm 1.2 \text{ (stat)} \pm 0.5 \text{ (syst)}] \text{%} \)
- \(A_{CP}(D^0 \rightarrow K^+ K^- \eta) = [-1.4 \pm 3.3 \text{ (stat)} \pm 1.1 \text{ (syst)}] \text{%} \)
- \(A_{CP}(D^0 \rightarrow \phi \eta) = [-1.9 \pm 4.4 \text{ (stat)} \pm 0.6 \text{ (syst)}] \text{%} \)

- **13k events**
- **1462 events**
- **728 events**

With \(D_s^+\) mesons,

\[ D_s^+ \rightarrow \pi^+ \eta \text{ shown.} \]

- \(A_{CP}(D_s^+ \rightarrow K^+ \pi^0) = 0.064 \pm 0.044 \pm 0.011 \)
- \(A_{CP}(D_s^+ \rightarrow K^+ \eta) = 0.021 \pm 0.021 \pm 0.004 \)
- \(A_{CP}(D_s^+ \rightarrow \pi^+ \eta) = 0.002 \pm 0.003 \pm 0.003 \)

- **12k events**
- **14k events**
- **223k events**

All consistent with zero
The observation of Charge-Parity (CP) violation with $D^+ \rightarrow \pi^+\pi^0$ asymmetries is expected to be of the order $10^{-3}$.

- Final state should be symmetric in flavour.
- So must be a $I = 2$ as $I_3 = 1$.
- Thus $\Delta I = \frac{3}{2}$. ($\Delta I = \frac{1}{2}$ in $W^+$ exchange)
- $\Delta I = 1$ in gluon line would be a flavour change, so second diagram forbidden.
- So one set of CKM factors contributes.
- Thus: zero CP violation.

- Very challenging reconstruction for LHCb. Trick: use $\pi^0 \rightarrow \gamma e^+e^-$, including conversions

Belle: 6.6k events (0.92 ab$^{-1}$)

LHCb 26k events (6 fb$^{-1}$)


LHCb: JHEP 06 (2021) 019
$B \rightarrow K\pi$

- Long-standing puzzle. Assuming isospin symmetry of the spectator quark, expect $I_{K\pi} = 0$.

$$I_{K\pi} = A_{K^+\pi^-} + A_{K^0\pi^+} \frac{B(K^0\pi^+)}{B(K^+\pi^-)} \tau_{B^0} - 2A_{K^+\pi^0} \frac{B(K^+\pi^-)}{B(K^+\pi^-)} \tau_{B^0} - 2A_{K^0\pi^0} \frac{B(K^0\pi^0)}{B(K^+\pi^-)}$$

- LHCb now provide new information with 16.7k events. $A_{CP}(B^+ \rightarrow K^+\pi^0) = 0.025 \pm 0.015 \pm 0.006 \pm 0.003$

- First measurement of $A_{K^0\pi^0}$ by Belle II

- Belle II predict $\sigma(A_{K^0\pi^0})$ reach 0.025 with 50 ab$^{-1}$

Progress likely to be dominated by Belle II


Belle II ($K^+\pi^0$): [https://arxiv.org/abs/2105.04111](https://arxiv.org/abs/2105.04111)

Belle II ($K^0\pi^0$): [https://arxiv.org/abs/2104.14871](https://arxiv.org/abs/2104.14871)
Unitarity triangle, $\beta$, $\gamma$

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
1 - \lambda^2/2 - \lambda^4/8 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \lambda^2/2 - \lambda^4/8(1 + 4A^2) & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + \frac{\lambda^4}{2}A(1 - 2(\rho + i\eta)) & 1 - \lambda^4/2A^2
\end{pmatrix} + O(\lambda^5)
\]

Forming inner product of 1st and 3rd columns gives the UT

\[
(1 - \frac{\lambda^2}{2})(\rho, \eta)
\]

Length constrained by measurements of semileptonic $b \to u$ decays and QCD calculations

\[
y = \arg\left(-\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}\right) \quad \beta = \arg\left(-\frac{V_{cb}^* V_{cd}}{V_{tb}^* V_{td}}\right)
\]

Similarly (but smaller):

\[
\beta_s = \arg\left(-\frac{V_{cb}^* V_{cs}}{V_{tb}^* V_{ts}}\right)
\]

Little progress on $\alpha$ in the last decade but Belle II will lead here with $B \to \pi\pi & \rho\rho$
**sin 2β**: the CP-violating phase of $B^0$ mixing

- Flagship result of the B-factories. LHCb has become, and should remain, competitive.

\[
A_{[c\bar{c}]K^0_S}(t) = \frac{\Gamma(B^0(t) \to [c\bar{c}]K^0_S) - \Gamma(B^0(t) \to [c\bar{c}]K^0_S)}{\Gamma(B^0(t) \to [c\bar{c}]K^0_S) + \Gamma(B^0(t) \to [c\bar{c}]K^0_S)}
\]

\[
= \frac{S \sin(\Delta m t) - C \cos(\Delta m t)}{\cosh(\Delta \Gamma t/2) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2)} \approx S \sin(\Delta m t) - C \cos(\Delta m t)
\]

$C_{SM} \approx 0$

$S_{SM} = \sin 2\beta$

(The amplitude of the sine wave after correction for mis-tagging)

\[
C(B^0 \to [c\bar{c}]K^0_S) = -0.017 \pm 0.029
\]

\[
S(B^0 \to [c\bar{c}]K^0_S) = 0.760 \pm 0.034
\]
### sin(2\beta) ≡ sin(2\phi_1)

**HFLAV Moriond 2018 PRELIMINARY**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reference</th>
<th>Value (2\beta)</th>
<th>Error (2\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>PRD 79 (2009) 072009</td>
<td>0.69 ± 0.03 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>BaBar ( \chi_c \to K )</td>
<td>PRD 80 (2009) 112001</td>
<td>0.69 ± 0.04 ± 0.07</td>
<td></td>
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<tr>
<td>BaBar J/\psi (hadronic) ( K_S )</td>
<td>PRD 69 (2004) 052001</td>
<td>1.56 ± 0.21 ± 0.42</td>
<td></td>
</tr>
<tr>
<td>Belle</td>
<td>PRL 108 (2012) 171802</td>
<td>0.67 ± 0.02 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>ALEPH</td>
<td>PLB 492, 259 (2000)</td>
<td>0.84 ± 0.16 +0.82</td>
<td>-0.14 ± 0.04</td>
</tr>
<tr>
<td>OPAL</td>
<td>EPJ C5, 379 (1998)</td>
<td>3.20 ± 0.50</td>
<td>+1.80</td>
</tr>
<tr>
<td>CDF</td>
<td>PRD 61, 07200 (2000)</td>
<td>0.79 ± 0.41</td>
<td>+0.41</td>
</tr>
<tr>
<td>LHCb</td>
<td>JHEP 11 (2017) 170</td>
<td>0.76 ± 0.03</td>
<td></td>
</tr>
<tr>
<td>Belle5S</td>
<td>PRL 108 (2012) 171801</td>
<td>0.57 ± 0.06</td>
<td>+0.56</td>
</tr>
<tr>
<td>Average</td>
<td>HFLAV</td>
<td>0.70 ± 0.02</td>
<td></td>
</tr>
</tbody>
</table>

#### Should remain the most precise UT angle throughout the BelleII/LHCb era.
\[ \phi_s = -2\beta_s : \text{the CP-violating phase of } B_s^0 \text{ mixing} \]

- \( \beta_s \) is analogous to \( \beta \) but is much smaller in the SM due to smallness of the complex phase in \( V_{ts} \)
- LHCb/ATLAS/CMS use time-dependent analysis of \( B_s \to J/\psi \left( K^+K^- \right) \phi \) (LHCb also \( B_s \to J/\psi \pi^+\pi^- \))

\[ \begin{align*}
\cos(\phi_s) &= 0.99987 \\
\sin(\phi_s) &= 0.01900 \\
\cos(2\beta_s) &= 0.99995 \\
\sin(2\beta_s) &= 0.01900 \\
\end{align*} \]

HFLAV: \( \phi_s = -0.050 \pm 0.019 \)

SM: \(-2\beta_s = -0.037 \pm 0.001\)
Unitarity angle $\gamma$: ADS/GLW

- The CP-violating phase in tree-level $b \rightarrow u \oplus b \rightarrow c$ transitions is $\gamma$. Negligible penguin/ theory error

  \[
  B^- \begin{cases} 
  b \\ W^- \\ \bar{u} \\
  \end{cases} \rightarrow \bar{D}^0 \\
  \]

  \[
  B^- \begin{cases} 
  \bar{u} \\ W^- \\ s \\
  \end{cases} \rightarrow K^- \\
  \]

  \[
  B^- \begin{cases} 
  b \\ W^- \\ \bar{u} \\
  \end{cases} \rightarrow K^- \\
  \]

  \[
  B^- \begin{cases} 
  b \\ \bar{u} \\ \bar{u} \\
  \end{cases} \rightarrow D^0 \\
  \]

  \[
  \gamma = \arg \left( -\frac{V^*_{ub} V_{ud}}{V^*_{cb} V_{cd}} \right) \\
  \]

- Gives rise to large, direct asymmetries

  \[
  \Gamma(B^{\mp} \rightarrow [\{f\}]_{D} h^\mp) \propto (r_d^f)^2 + (r_b^X)^2 + 2r_d^f r_b^X \cos(\delta_b^X + \delta_d^f \mp \gamma) \\
  \]

- $\gamma$ not measured directly, but must be inferred along with other nuisance parameters, $r_B, \delta_B, \delta_D, r_D$
Unitarity angle $\gamma : K_S^0 h^+h^-$

Recent work at BESIII (🪐) has ensured $c_i, s_i$ systematics remain ‘small’ for LHCb throughout 2020s

$N^+_{i+} = h_{B^+} \left[ F_{-i} + \left( (x^+_{DK})^2 + (y^+_{DK})^2 \right) F_{+i} + 2\sqrt{F_{+i}F_{-i}} \left( x^+_{DK} c_{+i} - y^+_{DK} s_{+i} \right) \right]$

$N^-_{i+} = h_{B^+} \left[ F_{+i} + \left( (x^-_{DK})^2 + (y^-_{DK})^2 \right) F_{-i} + 2\sqrt{F_{-i}F_{+i}} \left( x^-_{DK} c_{+i} + y^-_{DK} s_{+i} \right) \right]$

$N^+_{i-} = h_{B^-} \left[ F_{+i} + \left( (x^+_{DK})^2 + (y^+_{DK})^2 \right) F_{-i} + 2\sqrt{F_{-i}F_{+i}} \left( x^+_{DK} c_{-i} + y^+_{DK} s_{-i} \right) \right]$

$N^-_{i-} = h_{B^-} \left[ F_{-i} + \left( (x^-_{DK})^2 + (y^-_{DK})^2 \right) F_{+i} + 2\sqrt{F_{+i}F_{-i}} \left( x^-_{DK} c_{-i} - y^-_{DK} s_{-i} \right) \right]$

$x^\pm_{DK} \equiv r^\pm_{DK} \cos(\delta^\pm_{DK} \pm \gamma)$ and $y^\pm_{DK} \equiv r^\pm_{DK} \sin(\delta^\pm_{DK} \pm \gamma)$.
Unitarity angle $\gamma : K_S^0 h^+ h^-$

- Recent work at BESIII (⊙) has ensured $c_i, s_i$ systematics remain ‘small’ for LHCb throughout 2020s

- First major result on $\gamma$ using Belle + BelleII data

$$\phi_3 = (78.4 \pm 11.4 \pm 0.5 \pm 1.0)^\circ$$

expect ~1° with 50 ab$^{-1}$

$$\gamma = (68.7^{+5.2}_{-5.1})^\circ$$
Unitarity angle $\gamma$ extracted in combination

- All $B \to DX$ results are combined with time-dependent charm results

<table>
<thead>
<tr>
<th>$B$ decay</th>
<th>$D$ decay</th>
<th>Ref.</th>
<th>Dataset</th>
<th>Status since Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^+ \to D^+ h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[27]</td>
<td>Run 1&amp;2</td>
<td>Updated</td>
</tr>
<tr>
<td>$B^+ \to D^0 h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[28]</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>$B^+ \to D h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[29]</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>$B^+ \to Dh^+$</td>
<td>$D \to K^0 h^+ h^-$</td>
<td>[30]</td>
<td>Run 1&amp;2</td>
<td>Updated</td>
</tr>
<tr>
<td>$B^+ \to D^+ h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[31]</td>
<td>Run &amp;2(*)</td>
<td>As before</td>
</tr>
<tr>
<td>$B^+ \to D^0 h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[32]</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>$B^0 \to D^+ h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[33]</td>
<td>Run &amp;2(*)</td>
<td>New</td>
</tr>
<tr>
<td>$B^0 \to D^0 h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[34]</td>
<td>Run</td>
<td>As before</td>
</tr>
<tr>
<td>$B^0 \to D h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[35]</td>
<td>Run 1&amp;2</td>
<td>Updated</td>
</tr>
<tr>
<td>$B^0 \to D h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[36]</td>
<td>Run 1</td>
<td>As before</td>
</tr>
<tr>
<td>$B^0 \to D^0 h^+$</td>
<td>$D \to h^+ h^-$</td>
<td>[37]</td>
<td>Run 1&amp;2</td>
<td>New</td>
</tr>
<tr>
<td>$D^0 \to h^+ h^-$</td>
<td>[38-40]</td>
<td>Run 1&amp;2</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to h^+ h^-$</td>
<td>[41]</td>
<td>Run 1</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to h^+ h^-$</td>
<td>[42-45]</td>
<td>Run 1&amp;2</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to h^+ h^-$</td>
<td>[46]</td>
<td>Run 1</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to K^+ h^-$</td>
<td>[47]</td>
<td>Run 1&amp;2</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to K^+ h^-$</td>
<td>[48]</td>
<td>Run 1</td>
<td>New</td>
<td></td>
</tr>
<tr>
<td>$D^0 \to K^0 h^+ h^-$</td>
<td>[49,50]</td>
<td>Run 1&amp;2</td>
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</tr>
<tr>
<td>$D^0 \to K^0 h^+ h^-$</td>
<td>[51]</td>
<td>Run 1</td>
<td>New</td>
<td></td>
</tr>
</tbody>
</table>

$\gamma = (65.4^{+3.8}_{-4.2})^\circ$

CKMfitter expectation

$(65.7^{0.9}_{-2.7})^\circ$

UTFit expectation

$(65.8 \pm 2.2)^\circ$

$x = (0.400^{+0.052}_{-0.053})\%$

$y = (0.630^{+0.033}_{-0.030})\%$
Conclusion

- LHCb is delivering a wealth of measurements on heavy-flavour CP violation and mixing
- All major results are compatible with the SM expectation
- But the search goes on. LHCb upgrade will provide a factor 5-10 more statistics
- And BelleII aims to improve B-factory statistics by a factor ~50
- No time to be playing football in the park.