Next-to-soft virtual threshold corrections in QCD

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Introduction

- In QCD improved Parton model, the inclusive cross-section factorises into perturbatively calculable coefficient functions(CFs), Δ_{ab} and non-perturbative PDF.
- The CFs near threshold region, $z \to 1$

$$\Delta_{ab} \sim a\,\delta(1-z) + b_i D_i + c_i\, L_z^i + d$$

$$D_i = \left[\frac{\ln^i(1-z)}{1-z}\right]_+$$

$$L_z^i = \ln^i(1-z)$$

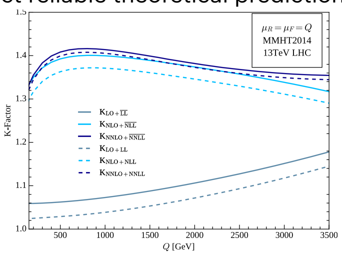
- NSV terms in $z \to 1$
- Collinear logarithmic corrections

Motivation & Objective

- NSV terms have significant contribution in cross-section

a_s^3	L_z^5	D_5	L_z^4	D_4	*Total SV*	*Total NSV*
$gg \to H$	117.95%	96.72%	103.36%	20.65%	-2.28%	25.83%
DY	8.59%	5.44%	9.82%	2.62%	0.02%	1.49%

- These logarithms spoil the perturbbativity of the series
- High-energy resummation is necessary to extract reliable theoretical predictions



Theoretical Framework

- Starting with the mass factorisation,

$$\frac{1}{z}\hat{\sigma}_{c\bar{c}}(z,\epsilon) = \sigma_0 \Gamma_{cc}(\mu_F^2, z, \epsilon) \otimes (\Delta_{c\bar{c}}(\mu_F^2, z, \epsilon) \otimes \Gamma_{\bar{c}\bar{c}}(\mu_F^2, z, \epsilon)$$

- UV finite CF for the diagonal channel is given by,

$$\Delta_{c\bar{c}}(z,\epsilon, q^2 \mu_R^2, \mu_F^2) = (\Gamma^T)^{-1}_{cc} \otimes \{ Z^2_{c,UV} |\hat{F}_c(Q^2,\epsilon)|^2 S_c(q^2, z, \epsilon)\} \otimes (\Gamma)^{-1}_{cc}$$

- Soft-Collinear operator, S_c ,

$$\ln S_c(\hat{a}_s, q^2, \mu^2, z, \epsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right) S_\epsilon^i \left\{ \Phi_{SV}^{c,(i)}(z,\epsilon), \Phi_{NSV}^{c,(i)}(z,\epsilon) \right\}$$

- Using the RG evolution equation and the energy evolution equation of S_c

$$\Phi_f^{c,(1)}(z,\epsilon) = \frac{1}{\epsilon}\mathcal{G}^c_{f,1}(z,\epsilon)\,,$$

$$\Phi_f^{c,(2)}(z,\epsilon) = \frac{1}{\epsilon^2}\Big(-\beta_0 \mathcal{G}^c_{f,1}(z,\epsilon)\Big) + \frac{1}{2\epsilon}\mathcal{G}^c_{f,2}(z,\epsilon)\,,$$

$$\Phi_f^{c,(3)}(z,\epsilon) = \frac{1}{\epsilon^3}\Big(\frac{4}{3}\beta_0^2 \mathcal{G}^c_{f,1}(z,\epsilon)\Big) + \frac{1}{2\epsilon}\Big(-\frac{1}{3}\beta_1 \mathcal{G}^c_{f,1}(z,\epsilon) - \frac{4}{3}\beta_0 \mathcal{G}^c_{f,2}(z,\epsilon)\Big) + \frac{1}{3\epsilon}\mathcal{G}^c_{f,3}(z,\epsilon)$$

The SV and NSV parts of S_c have the functional form,

$\mathcal{G}^c_{SV,1}(z,\epsilon) = \frac{2A_1}{1-z} + \epsilon \frac{\mathcal{G}^{c,(1)}_{SV,1}}{1-z} + \mathcal{O}(\epsilon^2)$	$\mathcal{G}^c_{NSV,1}(z,\epsilon) = 2D_1 + 2C_1 \ln(1-z) + \epsilon \mathcal{G}^{c,(1)}_{NSV,1}(z) + \mathcal{O}(\epsilon^2)$
$\mathcal{G}^c_{SV,2}(z,\epsilon) = \frac{2A_2}{1-z} - 2\beta_0 \frac{\mathcal{G}^{c,(1)}_{SV,1}}{1-z} + \mathcal{O}(\epsilon)$	$\mathcal{G}^c_{NSV,2}(z,\epsilon) = 2D_2 + 2C_2 \ln(1-z) - 2\beta_0 \mathcal{G}^{c,(1)}_{NSV,1}(z) + \mathcal{O}(\epsilon)$

The master formula for $\Delta_{c\bar{c}}$,

$$\ln \Delta_c(q^2, \mu_R^2, \mu_F^2, z, \epsilon) = \left(\ln\Big(Z_{UV,c}(\hat{a}_s, \mu^2, \mu_R^2, \epsilon)\Big)^2 + \ln\Big|\hat{F}_c(\hat{a}_s, \mu^2, Q^2, \epsilon)\Big|^2\right)\delta(1-z) + \ln S_c(\hat{a}_s, \mu^2, q^2, z, \epsilon) - 2\mathscr{C}\ln\Gamma_{cc}(\hat{a}_s, \mu^2, \mu_F^2, z, \epsilon)$$

NSV Resummation

- To study all order behaviour resummation is done in kinematical region $z \to 1$
- The modified resummed expression after including NSV terms is given by,

$$\Delta_{c,N}(q^2, \mu_R^2, \mu_F^2) = \Big(\sum_{i=0}^{\infty} a_s^i(\mu_R^2) \tilde{g}_{0,i}(q^2, \mu_R^2, \mu_F^2)\Big) \exp\Big(\Psi^c_{SV,N}(q^2, \mu_F^2) + \Psi^c_{NSV,N}(q^2, \mu_F^2)\Big)$$

where,

$$\Psi^c_{\rm SV,N} = g_1^c(\omega) \ln N + \sum_{i=0}^{\infty} a_s^i(\mu_R^2) g^c_{i+2}(\omega)$$

and,

$$\Psi^c_{NSV,N} = \frac{1}{N}\Big(\sum_{i=0}^{\infty} a_s^i(\mu_R^2) h_i^c(\omega, N)\Big)$$

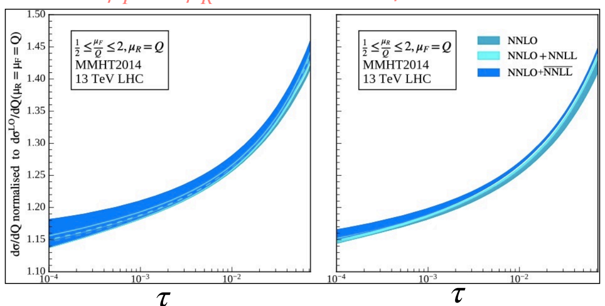
$$h_0^c(\omega, N) = h_{00}^c(\omega) + h_{01}^c(\omega) \ln N\,,\; h_i^c(\omega, N) = \sum_{k=0}^{i} h_{ik}^c(\omega)\;\; \ln^k(N)$$

Phenomenology

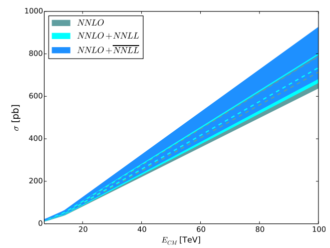
The resummed result at a given accuracy, say N^nLL is computed by taking the difference between the resummed result and the same truncated upto order a_s^n,

$$\sigma_N^{\rm N^nLO+N^nLL} = \sigma_N^{\rm N^nLO} + \sigma^{(0)} \sum_{ab\in\{q,\bar{q}\}} \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} (\tau)^{-N} \delta_{a\bar{b}} f_{a,N}(\mu_F^2) f_{b,N}(\mu_F^2) \times \left(\Delta_{q,N}\Big|_{\rm N^nLL} - \Delta_{q,N}\Big|_{tr\rm N^nLO}\right).$$

μ_F and μ_R scale uncertainty for Drell-Yan



E_{CM} Variation for $gg \to H$



$\sqrt{s}$	NNLO (pb/GeV)	NNLO+NNLL$_{SV}$ (pb/GeV)	NNLO+NNLL$_{SV+NSV}$ (pb/GeV)
7 TeV	$14.5570^{+1.60}_{-1.54}$	$15.2911^{+1.22}_{-1.36}$	$16.6944^{+2.44}_{-1.80}$
8 TeV	$18.5517^{+2.03}_{-1.95}$	$19.4548^{+1.55}_{-1.73}$	$21.2064^{+3.15}_{-2.31}$
13 TeV	$42.3392^{+4.53}_{-4.41}$	$44.1706^{+3.58}_{-3.97}$	$47.8966^{+7.47}_{-5.38}$
14 TeV	$47.6973^{+5.09}_{-4.96}$	$49.7258^{+4.04}_{-4.47}$	$53.8807^{+8.46}_{-6.09}$
100 TeV	$714.3105^{+74.13}_{-74.05}$	$735.0223^{+63.55}_{-69.22}$	$784.0407^{+139.60}_{-99.05}$

Results

Using 3-loop results, predictions for the first three logs till 7-loop,

$$\begin{aligned}\Delta_g^{NSV} = {}& a_s\Delta_g^{NSV(1)} + a_s^2\Delta_g^{NSV(2)} + a_s^3\Delta_g^{NSV(3)} \\ &+ a_s^4\Big[\Big\{-\frac{4096}{3}C_A^4\Big\}L_z^7 + \Big\{\frac{98560}{9}C_A^4 - \frac{7168}{9}n_f C_A^3\Big\}L_z^6 + \Big\{\Big(-\frac{298240}{9} + 23552\zeta_2\Big)C_A^4 \\ &+ \frac{174208}{27}n_f C_A^3 - \frac{4096}{27}n_f^2 C_A^2\Big\}L_z^5 + \mathcal{O}(L_z^4)\Big] + a_s^5\Big[\Big\{-\frac{8192}{3}C_A^5\Big\}L_z^9 + \Big\{\frac{96256}{3}C_A^5 \\ &- \frac{8192}{3}C_A^4 n_f\Big\}L_z^8 + \Big\{\Big(-\frac{12283904}{81} + \frac{262144}{3}\zeta_2\Big)C_A^5 + \frac{2569216}{81}C_A^4 n_f - \frac{81920}{81}n_f^2 C_A^3\Big\}L_z^7 \\ &+ \mathcal{O}(L_z^6)\Big] + a_s^6\Big[\Big\{-\frac{65536}{15}C_A^6\Big\}L_z^{11} + \Big\{\frac{9490432}{135}C_A^6 - \frac{180224}{27}C_A^5 n_f\Big\}L_z^{10} + \Big\{\Big(\frac{671744}{3}\zeta_2 \\ &- \frac{4261888}{9}\Big)C_A^6 + \frac{8493056}{81}C_A^5 n_f - \frac{327680}{81}n_f^2 C_A^4\Big\}L_z^9 + \mathcal{O}(L_z^8)\Big] \\ &+ a_s^7\Big[\Big\{-\frac{262144}{45}C_A^7\Big\}L_z^{13} + \Big\{\frac{3309568}{27}C_A^7 - \frac{1703936}{135}C_A^6 n_f\Big\}L_z^{12} + \Big\{\Big(-\frac{449429504}{405} \\ &+ \frac{1310720}{3}\zeta_2\Big)C_A^7 + \frac{11583488}{45}C_A^6 n_f - \frac{917504}{81}n_f^2 C_A^5\Big\}L_z^{11} + \mathcal{O}(L_z^{10})\Big] + \mathcal{O}(a_s^8)\,.\end{aligned}$$

Conclusion

- Using IR factorisation and UV renormalisation group invariance, we provide an all order perturbative result in strong coupling constant.
- We present an integral representation which resum both SV and NSV logarithms to all orders.
- The inclusion of NSV terms enhances the precision as can be seen in the plots.

References

[1] A.H.Ajjath, Pooja Mukherjee, V. Ravindran, Aparna Sankar and Surabhi Tiwari: On next to soft corrections to DIS and SIA processes, *JHEP* 04 (2021) 131 [2007.12214]

[2] A.H.Ajjath, Pooja Mukherjee and V. Ravindran: On next to soft corrections to Drell-Yan and Higgs Boson productions, arXiv: 2006.06726

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