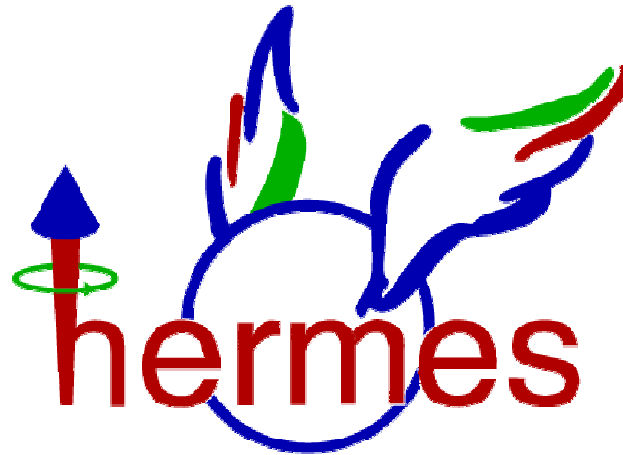
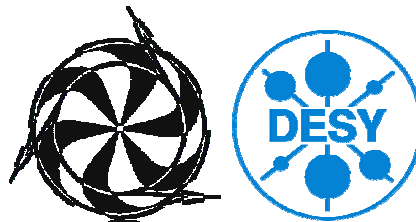


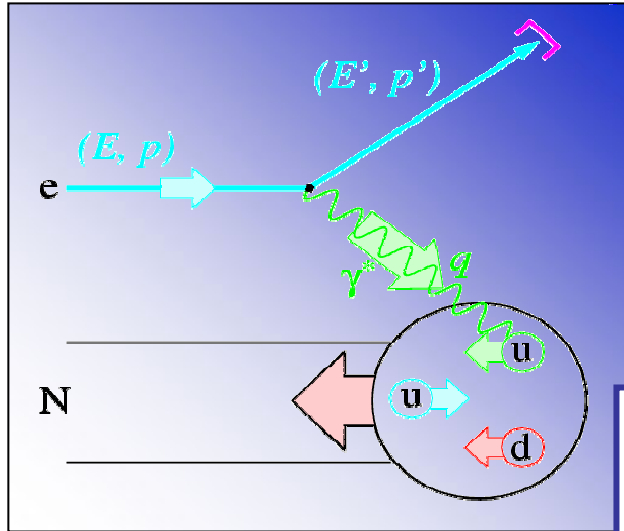
Measurement of the spin structure functions g_1^p and g_1^d at HERMES



Lara De Nardo
TRIUMF/DESY



Inclusive *Deep Inelastic* Scattering



$L_{\mu\nu}$ is exact in QED

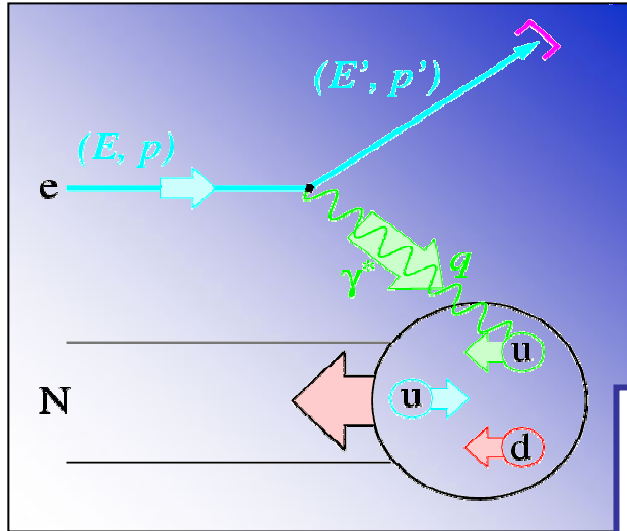
$$\frac{\partial^2 \sigma}{\partial \Omega \partial E'} \propto L_{\mu\nu} W^{\mu\nu}$$

$$g_1 \approx \langle e^2 \rangle \left[\Delta C_\Sigma \otimes \Delta \Sigma + \Delta C_G \otimes \Delta G + \Delta C_{NS} \otimes \Delta q_{NS}^{p,n} \right]$$

$$= \frac{1}{2} \langle e^2 \rangle \left[\Delta \Sigma + \Delta q_{NS}^{p,n} \right] \quad \text{in LO QCD}$$

$$W^{\mu\nu} = -g^{\mu\nu} \mathbf{F}_1 + \frac{p^\mu p^\nu}{\nu} \mathbf{F}_2 + \frac{i}{\nu} \epsilon^{\mu\nu\lambda\sigma} q^\lambda s^\sigma \mathbf{g}_1 + \frac{i}{\nu^2} \epsilon^{\mu\nu\lambda\sigma} q^\lambda (p \cdot q s^\sigma - s \cdot q p^\sigma) \mathbf{g}_2.$$

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$$\underbrace{-r_{\mu\nu} \mathbf{b}_1 + \frac{1}{6} (s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu}) \mathbf{b}_2 + \frac{1}{2} (s_{\mu\nu} - u_{\mu\nu}) \mathbf{b}_3 + \frac{1}{2} (s_{\mu\nu} - t_{\mu\nu}) \mathbf{b}_4}_{\text{spin 1}}$$

Measured Inclusive Asymmetries

$$P_{zz} = 0.83 \pm 0.03$$

$$A_{zz} \sim 0.01$$

$$\frac{b_1^d}{F_1^d} = -\frac{3}{2} A_{zz}$$

$$\sigma = \sigma_{unpol} \left[1 + P_B P_z A_{||} + \underbrace{\frac{1}{2} P_{zz} A_{zz}}_d \right]$$

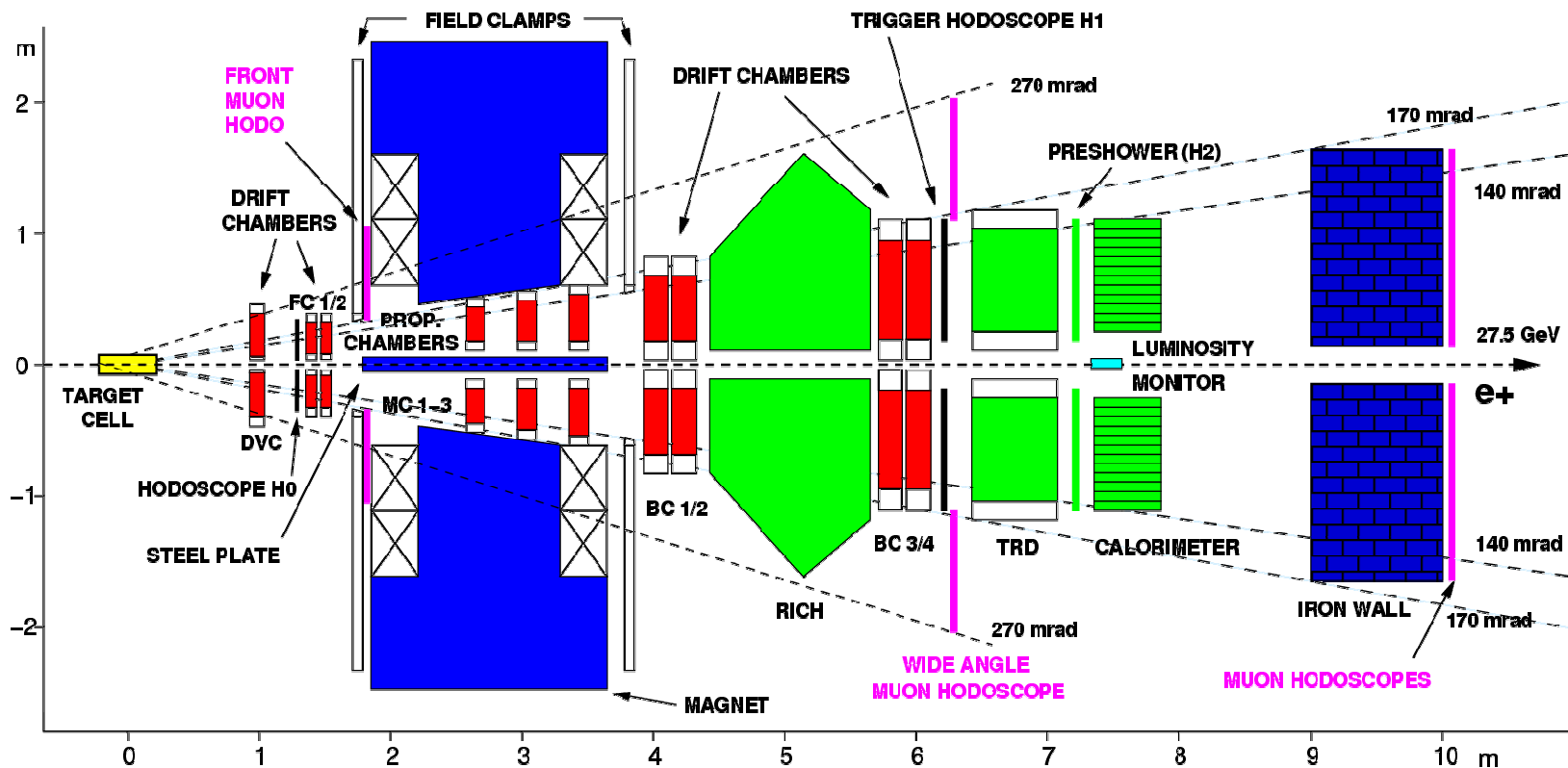
measured DIS cross section

inclusive asymmetry:

$$A_{||} = \frac{\sigma^{\leftarrow} - \sigma^{\rightarrow}}{\sigma^{\leftarrow} + \sigma^{\rightarrow}} = \frac{1}{P_B P_z} \cdot \frac{\frac{N^{\leftarrow}}{L^{\rightarrow}} - \frac{N^{\rightarrow}}{L^{\leftarrow}}}{\frac{N^{\leftarrow}}{L^{\rightarrow}} + \frac{N^{\rightarrow}}{L^{\leftarrow}}}$$

$$g_1(x, Q^2) = \underbrace{\frac{1}{1 - \frac{y}{2} - \frac{1}{4} y^2 \gamma}}_{\text{kinematic factors}} \left[\underbrace{\frac{Q^4}{8\pi\alpha^2 y}}_{\text{param.}} \underbrace{\frac{\partial^2 \sigma_{unpol}}{\partial x \partial Q^2}}_{\text{param.}} \underbrace{A_{||}(x, Q^2)}_{\text{measured}} + \underbrace{\frac{y}{2} \gamma^2}_{\text{kin. fact.}} \underbrace{g_2(x, Q^2)}_{\text{param.}} \right]$$

The HERMES Spectrometer



Reconstruction: $\delta p/p < 2\%$, $\delta\theta < 1$ mrad

Internal Gas Target: unpol: H_2 , D_2 , He, N, Ne, Kr, Xe, He, H , D , H

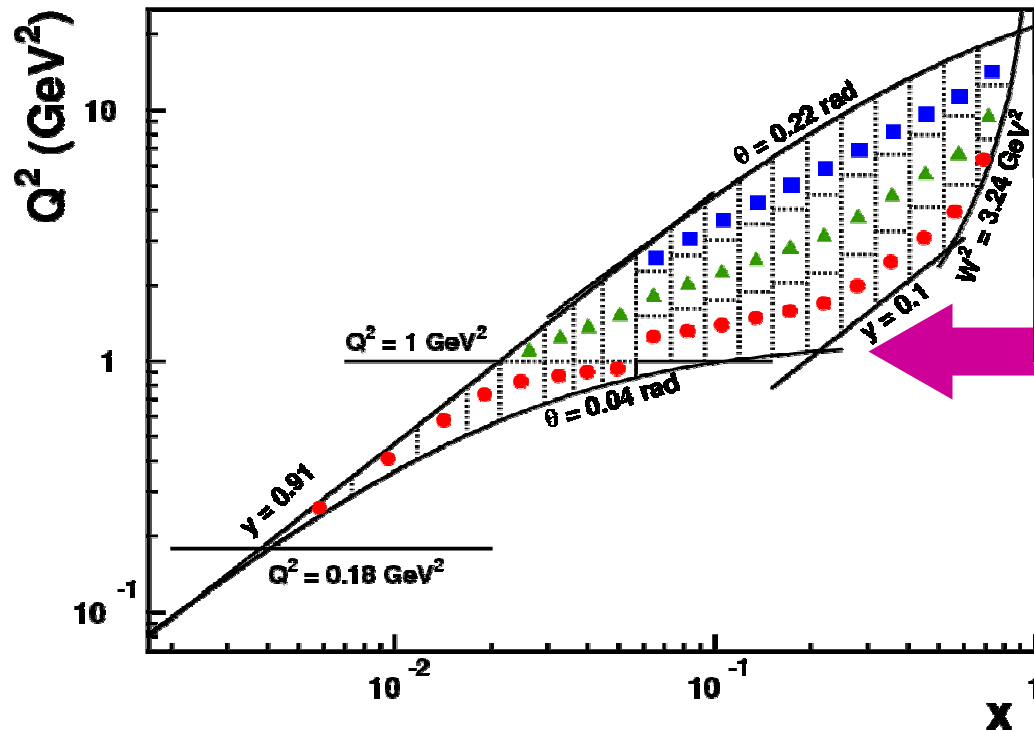
Particle ID: TRD, Preshower, Calorimeter --- 1997: Cherenkov, 1998: RICH

The data

Target	Year	Luminosity (pb^{-1})	#	P_{target} (%)
H	1996	12.6	670,000	75.9 ± 3.2
H	1997	37.3	2,800,000	85.1 ± 3.2
D	2000	138.7	10,900,000	85.1 ± 3.2 (+) 84.0 ± 3.1 (-)

$$P_{\text{beam}} \sim (53 \pm 1.8)\%$$

$$P_{\text{beam}} \sim (53 \pm 1.0)\%$$

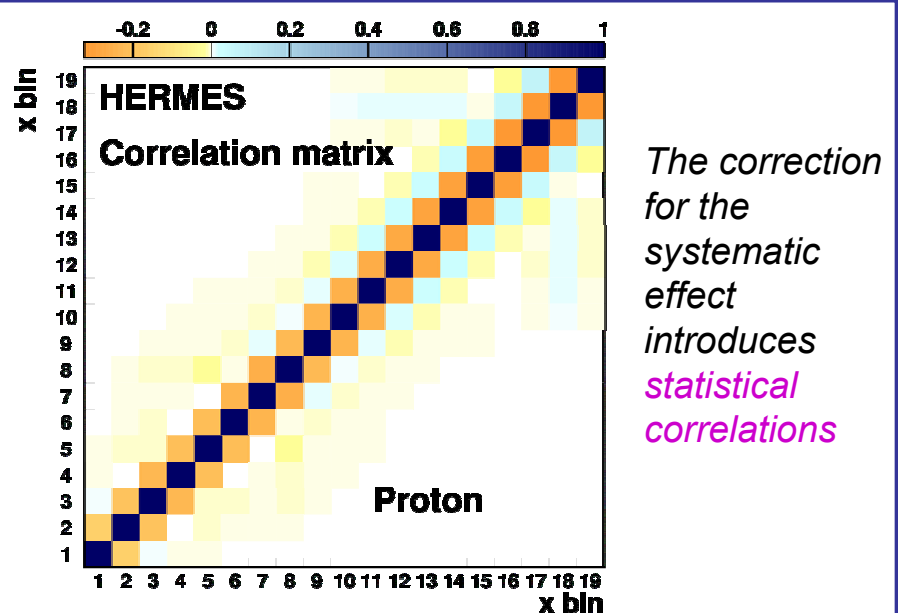
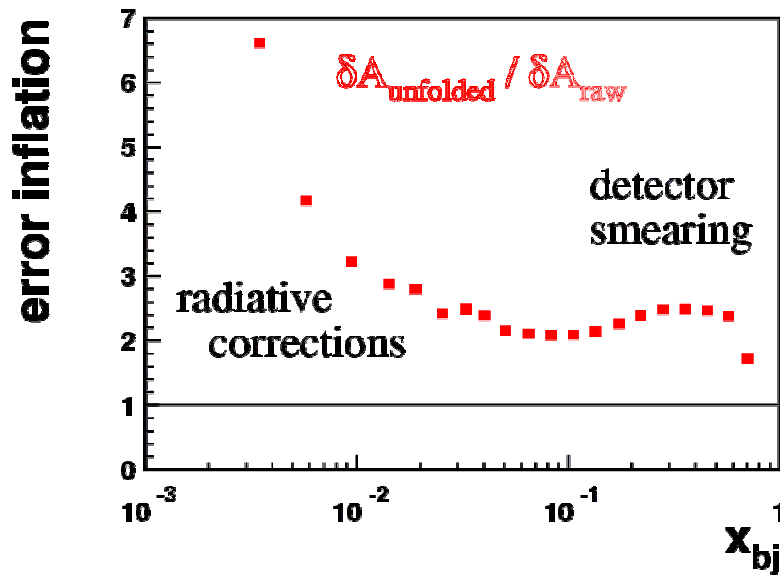
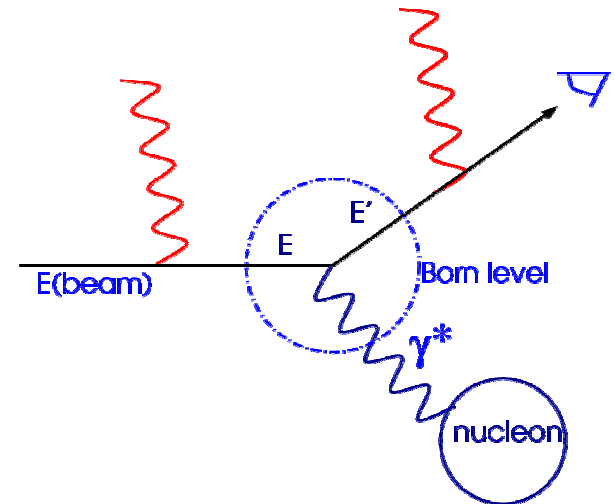


Full analysis published in
Phys.Rev. **D75** (2007)012007

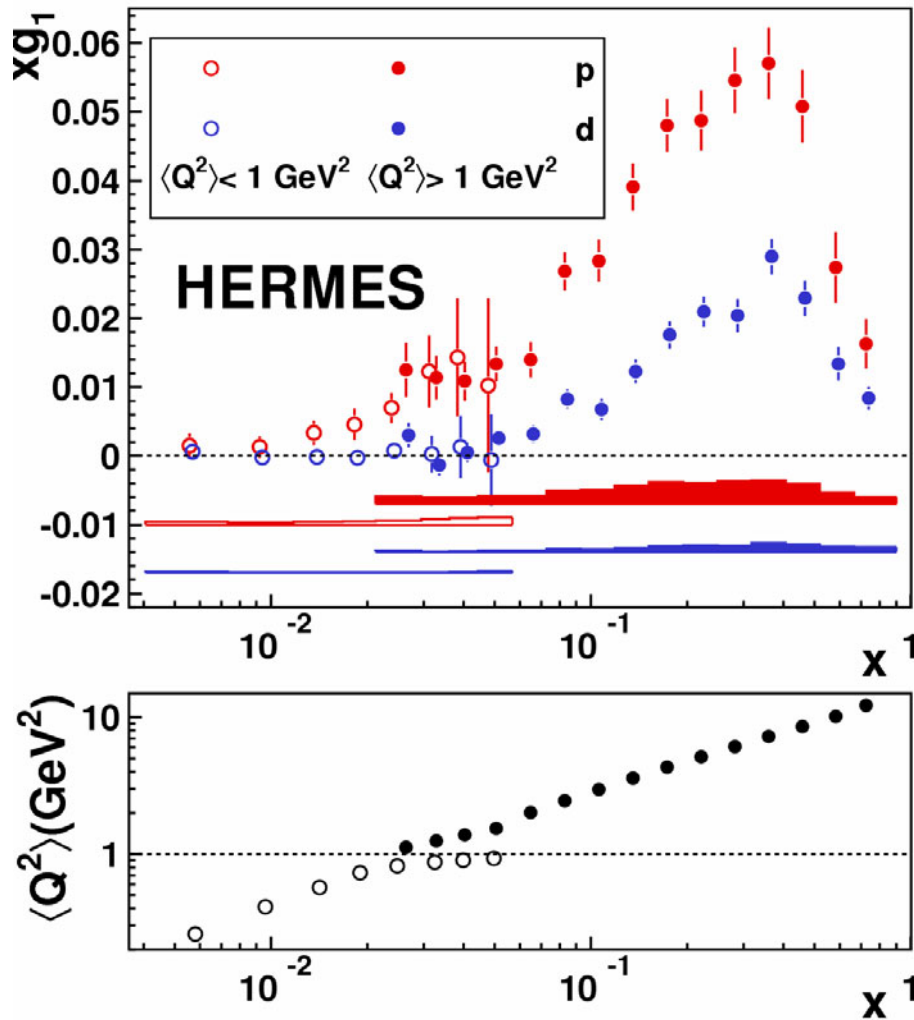
The cut at $Q^2=1\text{GeV}^2$
identifies the DIS region

Unfolding of radiative corrections

- Measured events have to be corrected for:
 - ➔ Background tail (radiation from (quasi)-elastic)
 - ➔ Radiation from DIS and detector smearing
- The smearing of events is simulated through a MonteCarlo which includes a **full detector description** and a **model for the cross-section**
- The approach is **independent** on the model for the asymmetry in the measured region



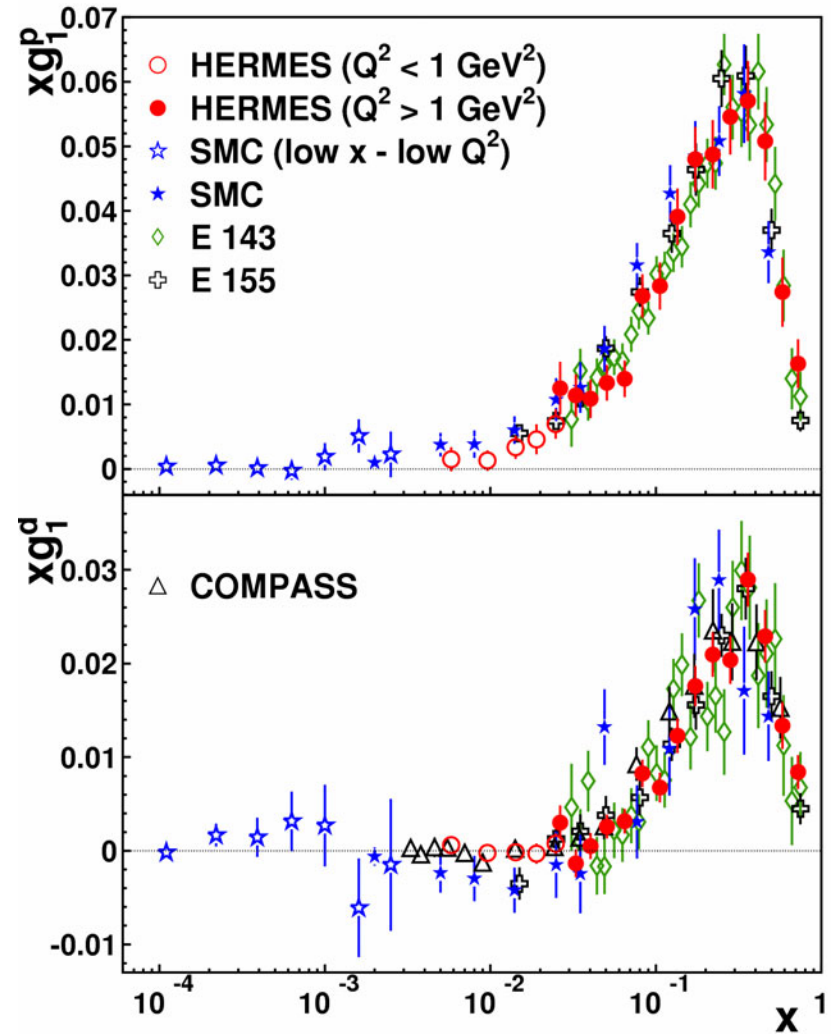
g_1 results



■ Statistical uncertainties are *diagonal* elements of covariance matrix

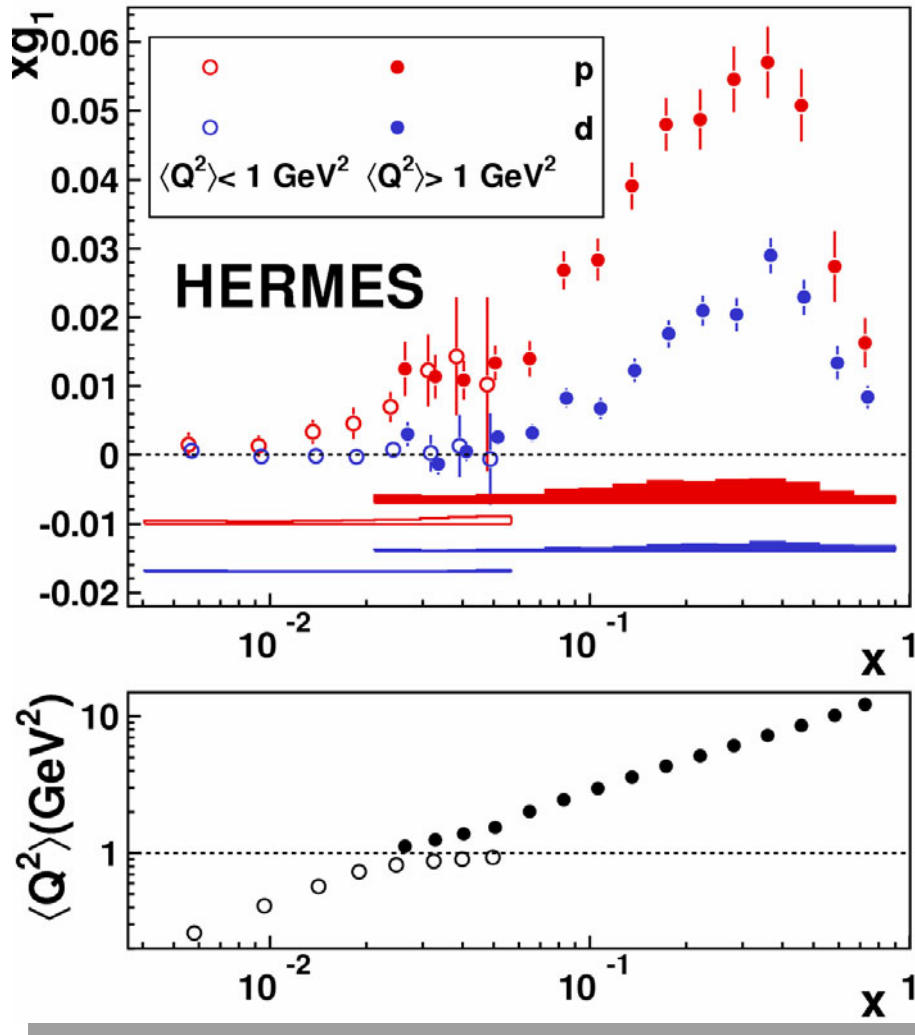
■ Systematic unc. are dominated by *target and beam* polarization

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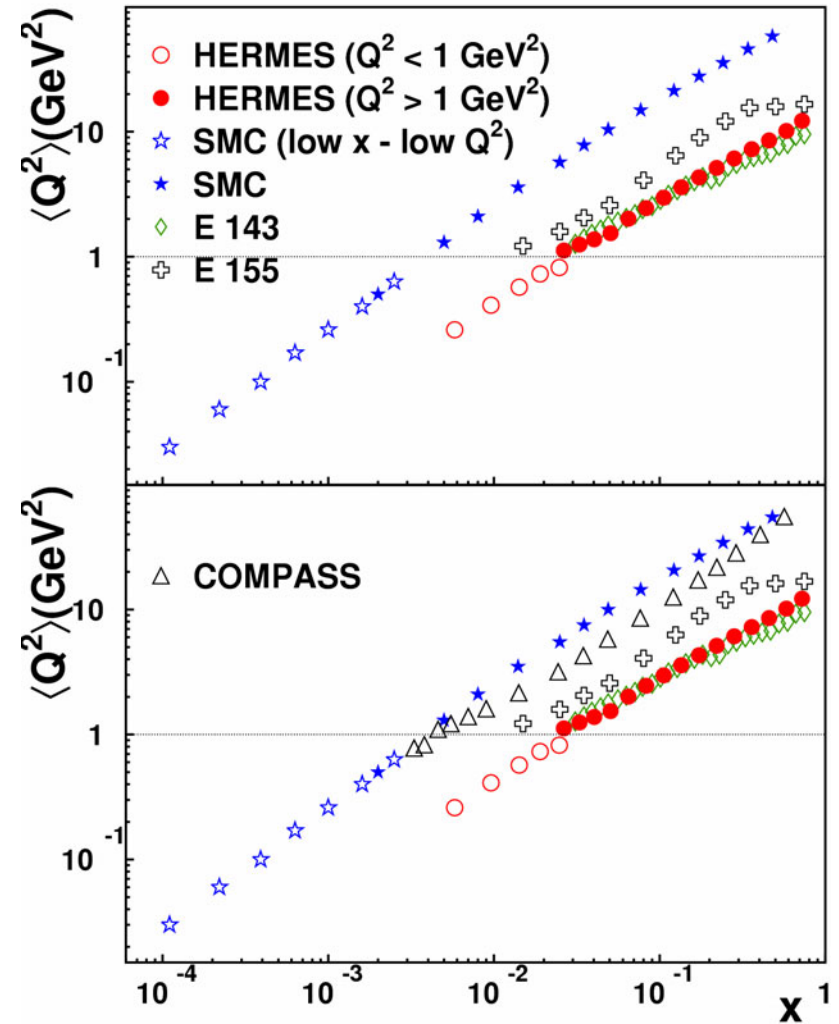
DIS 2007

g_1 results



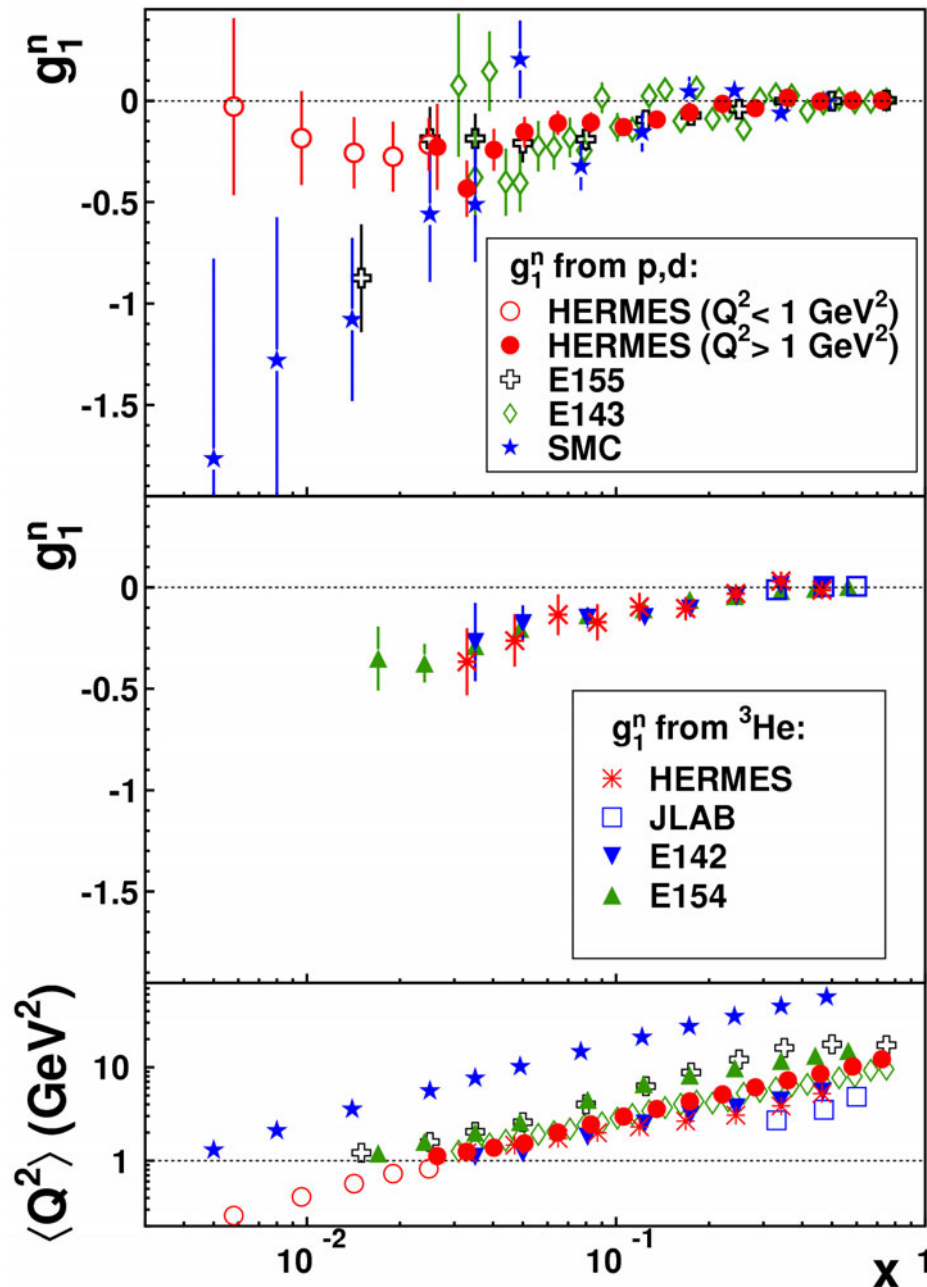
■ Statistical uncertainties are *diagonal* elements of covariance matrix
 ■ Systematic unc. are dominated by *target and beam* polarization

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DIS 2007

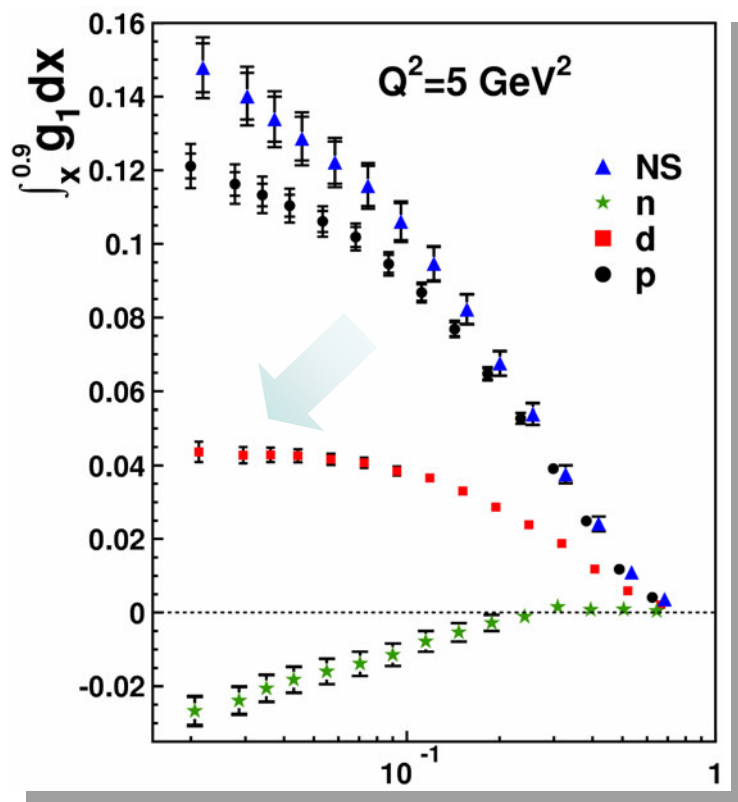
Neutron results



$$g_1^n = \frac{2}{1 - \frac{3}{2}\omega_D} \cdot g_1^d - g_1^p$$

- g_1^n negative everywhere except at very high- x
- Low- Q^2 data tends to zero at low- x
 - ◆ Does not support earlier conjecture of strong decrease for $x \rightarrow 0$

Integrals



Saturation in the deuteron integral is assumed

Use only deuteron data!

from hyperon beta decay
 $(a_8 = 0.586 \pm 0.031)$

$$a_0 = \frac{1}{\Delta C_S} \left[\frac{9\Gamma_1^d}{\left(1 - \frac{3}{2}\omega_D\right)} - \frac{1}{4} a_8 \Delta C_{NS} \right]$$

theory (under ΔC_S)

$\omega_D = 0.05 \pm 0.05$ (under ω_D)

theory (under ΔC_{NS})

$$\begin{aligned} \Delta u + \Delta \bar{u} &= \frac{1}{6} [2a_0 + a_8 + 3a_3] \\ \Delta d + \Delta \bar{d} &= \frac{1}{6} [2a_0 + a_8 - 3a_3] \\ \Delta s + \Delta \bar{s} &= \frac{1}{3} [a_0 - a_8] \end{aligned}$$

from neutron beta decay
 $a_3 = 1.269 \pm 0.003$

DIS 2007

	central value	uncertainties		
		theor.	exp.	evol.
a_0	0.330	0.011	0.025	0.028
$\Delta u + \Delta \bar{u}$	0.842	0.004	0.008	0.009
$\Delta d + \Delta \bar{d}$	-0.427	0.004	0.008	0.009
$\Delta s + \Delta \bar{s}$	-0.085	0.013	0.008	0.009

$Q^2 = 5 \text{ GeV}^2$, NNLO in $\overline{\text{MS}}$ scheme

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Comparisons

Exp.	Q_0^2 (GeV ²)	x range	type	Integral				
				value	stat.	syst.	param.	evol.
E143 HERMES	5	0.03 - 0.8	p	0.117 0.115	0.003 0.002	0.007 0.006	0.003 0.003	- 0.004
SMC (*) HERMES	10	0.021-0.7	p	0.120 0.119	0.005 0.003	0.007 0.007	0.003 0.003	0.002 0.005
EMC (*) HERMES	10.7	0.021-0.7	p	0.110 0.119	0.011 0.003	0.019 0.007	- 0.003	- 0.005
E155 (*) HERMES	5	0.021-0.9	p	0.124 0.121	0.002 0.002	0.009 0.007	0.003 0.003	0.005 0.005
E143 HERMES	5	0.03 - 0.8	d	0.043 0.042	0.003 0.001	0.003 0.002	- 0.001	- 0.002
SMC (*) HERMES	10	0.021-0.7	d	0.042 0.043	0.005 0.001	0.004 0.002	0.001 0.001	0.001 0.002
E155 (*) HERMES	5	0.021-0.9	d	0.043 0.044	0.002 0.001	0.003 0.002	0.003 0.001	0.003 0.003
E142 HERMES	2	0.03-0.6	n (³ He) n (p,d)	-0.028 -0.025	0.006 0.003	0.006 0.007	- 0.002	- 0.001
E154 (*) HERMES	2	0.021-0.7	n (³ He) n (p,d)	-0.032 -0.027	0.003 0.004	0.005 0.008	0.003 0.003	0.003 0.002
HERMES HERMES	2.5	0.023-0.6	n (³ He) n (p,d)	-0.034 -0.027	0.013 0.003	0.005 0.007	- 0.003	- 0.001
HERMES/ SIDIS HERMES	2.5	0.023-0.6	NS	0.147 0.138	0.008 0.005	0.019 0.013	- 0.005	- 0.003

- *Integrals provide a fair comparison of the accuracies of various experiments (all correlations taken into account)*
- *Proton data comparable with SLAC and CERN expts.*
- *Deuteron data is the most precise so far*

(*): integrals re-calculated in a smaller x range, to match HERMES

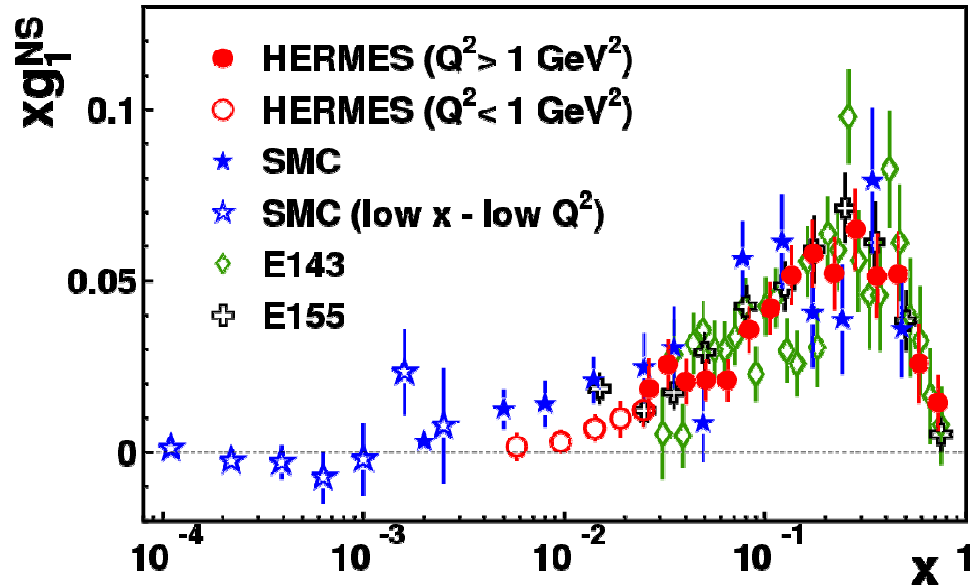
Conclusions

- **HERMES has measured g_1 for proton and deuteron for $0.0041 < x < 0.9$ and $0.18 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$**
 - **Measured results are *correlated* no longer systematically but statistically**
 - **Integrals provide a fair comparison for the statistical accuracy of various experiments:**
 - **Proton data precision is *comparable* with CERN and SLAC**
 - **Deuteron data is the *most precise* so far**
 - **The deuteron integral is observed to saturate**
 - **$a_0 = 0.330 \pm 0.011(\text{theor}) \pm 0.025(\text{exp}) \pm 0.028(\text{evol})$ at 5 GeV^2**
- agreement with COMPASS data:**
 $a_0(\text{COMPASS}) = 0.35 \pm 0.03(\text{stat}) \pm 0.05(\text{syst})$ at 3 GeV^2



EXTRA SLIDES

Bjorken Sum Rule



$$g_1^{NS} \equiv g_1^p - g_1^n = 2 \left[g_1^p - \frac{g_1^d}{1 - \frac{3}{2}\omega_D} \right]$$

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \frac{1}{6} a_3 \Delta C_{NS}(\alpha_s(Q^2))$$

Assuming the **validity of the BSR**, and the **saturation of the deuteron integral**, we can estimate the proton integral in the unmeasured low-x region:

	BJS	Estimated $\Gamma_1^p - \Gamma_1^n(meas.)$
LO	0.2116 ± 0.0005	$0.0316 \pm 0.0008 \pm 0.0025 \pm 0.0079 \pm 0.0025$
NLO	0.1923 ± 0.0009	$0.0219 \pm 0.0008 \pm 0.0025 \pm 0.0079 \pm 0.0025$
NNLO	0.1856 ± 0.0015	$0.0186 \pm 0.0009 \pm 0.0025 \pm 0.0079 \pm 0.0025$
NNNLO	0.1821 ± 0.0019	$0.0169 \pm 0.0013 \pm 0.0025 \pm 0.0079 \pm 0.0025$

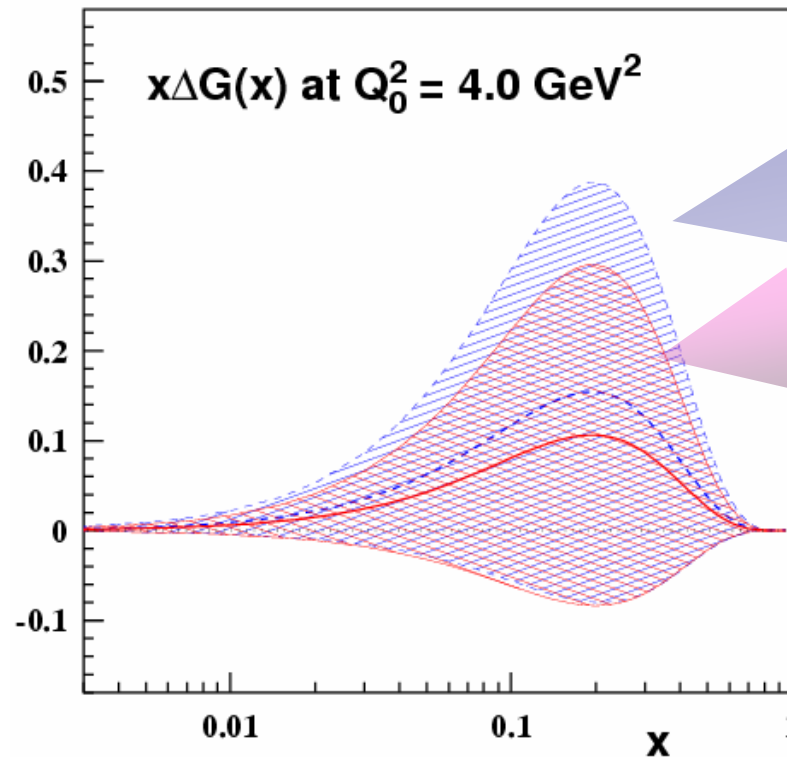
a_3, α_s, ω_D

proton
integral

deuteron
integral

Q^2 evolution

QCD fits to g_1 world data



(test done with BB code,
Nucl.Phys.**B636**(2002)225)

Without HERMES g_1^d


With HERMES g_1^d

- As effect of the inclusion of the HERMES data, the gluon moment goes from 0.32 ± 0.47 to 0.22 ± 0.39 .
- The effect on the other parton distributions is much less visible
- $\Delta\Sigma = 0.22 \pm 0.11 \pm 0.05(\text{exp}) \pm 0.06(\text{theo})$

More on Integrals

Use same code as BB fit

(Blumlein&Boettcher, Nucl.Phys.B**636**(2002)225)



$$g_1(x_i, Q_i^2) = g_1(x_i, Q_0^2) + \left[g_1^{fit}(x_i, Q_i^2) - g_1^{fit}(x_i, Q_0^2) \right] \text{ Evolution to a common } Q^2=Q_0^2$$

$$\Gamma_1(Q_0^2) = \sum_i \frac{g_1(\langle x \rangle_i, Q_0^2)}{g_1^{fit}(\langle x \rangle_i, Q_0^2)} \int_{x_i}^{x_{i+1}} dx g_1^{fit}(x, Q_0^2) \quad \text{Integral}$$

$$\sigma^2 = \sum_{ij} \left[\int_{x_i}^{x_{i+1}} dx g_1^{fit}(x, Q_0^2) \right] \cdot \left[\int_{x_j}^{x_{j+1}} dx g_1^{fit}(x, Q_0^2) \right] \cdot \frac{\text{cov}(g_1)_{ij}}{g_1^{fit}(\langle x \rangle_i, Q_0^2) g_1^{fit}(\langle x \rangle_j, Q_0^2)}$$

Statistical uncertainty on the integral

The integrals at $Q^2=5 \text{ GeV}^2$

	$\int_{0.021}^{0.9} dx g_1$	uncertainties			
		stat.	syst.	par.	evol.
$Q^2=5 \text{ GeV}^2$					
p	0.1211	0.0025	0.0068	0.0028	0.0050
d	0.0436	0.0012	0.0018	0.0008	0.0026
n	-0.0268	0.0035	0.0079	0.0031	0.0018
NS	0.1479	0.0055	0.0142	0.0055	0.0049