

# **Charged Current Deep Inelastic scattering at three loops**

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# Content

- Introduction to the Deep Inelastic Scattering

▲ Charged current deep-inelastic scattering at three loops.

S. Moch and M. R. arXiv:0704.1740v1 [hep-ph]

▲ Differences between CC coefficient functions, applications

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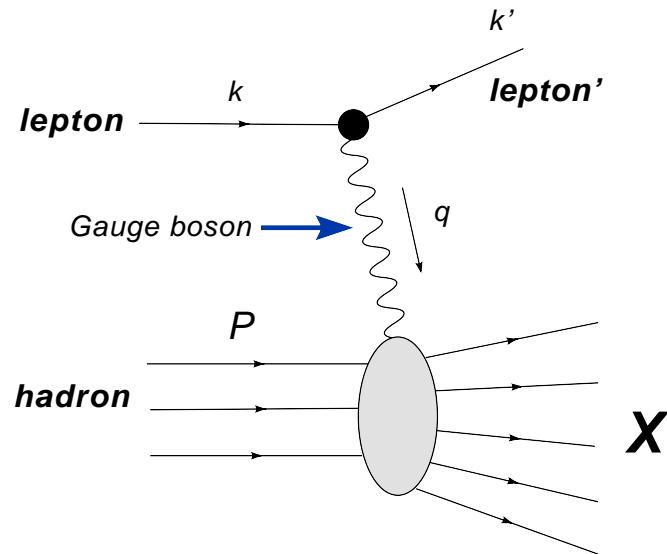
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- Deep-inelastic lepton-hadron scattering ( $e^\pm p$ ,  $e^\pm n$ ,  $\nu p$ ,  $\bar{\nu} p$ , ... - collisions)

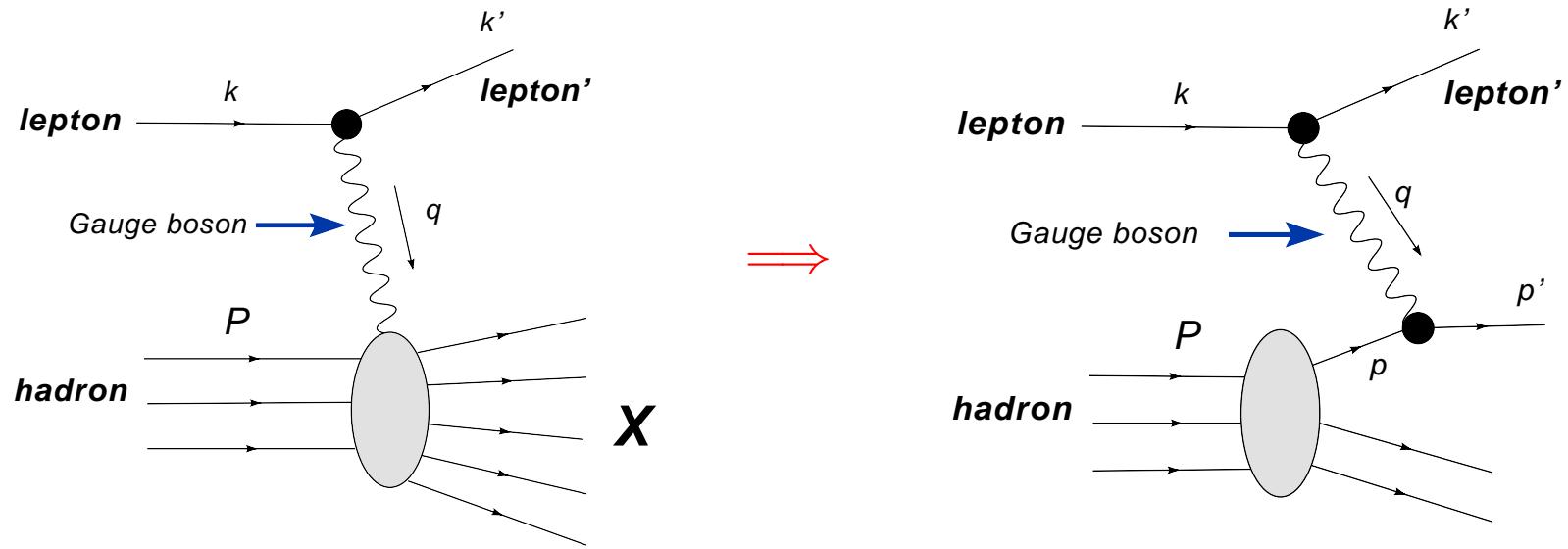
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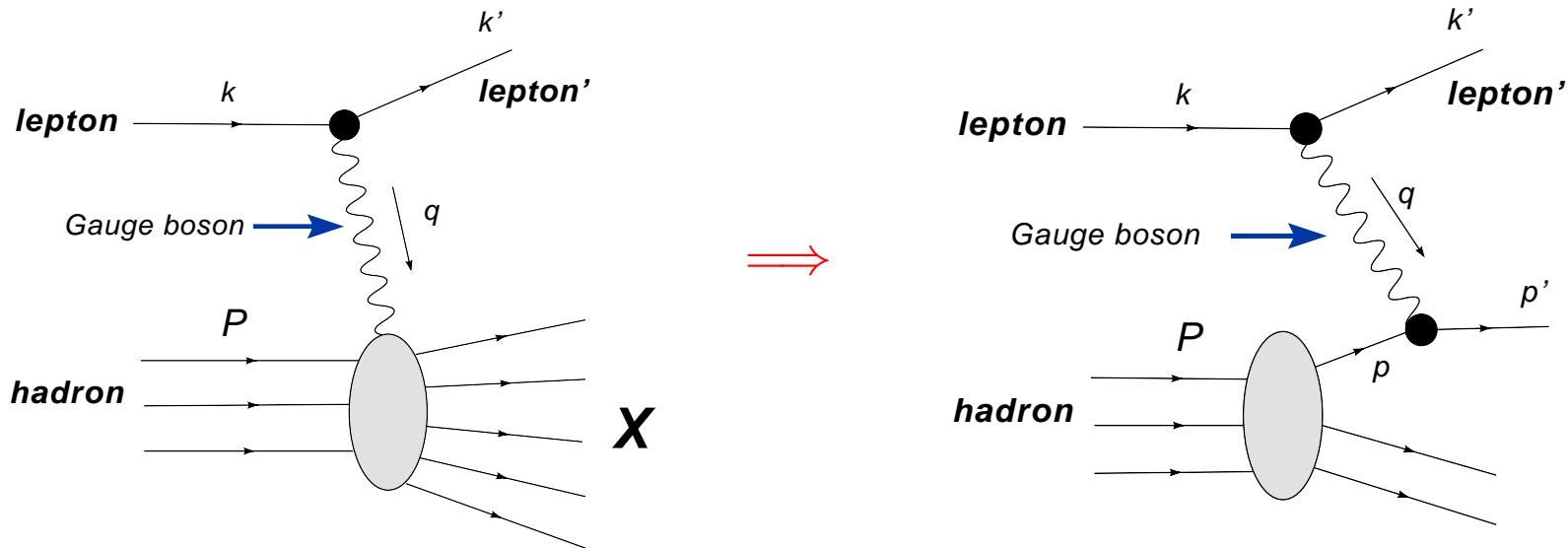
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- Gauge boson:

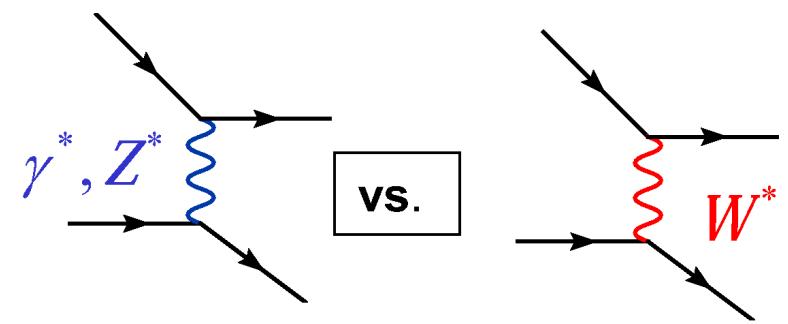
$\gamma$ ,  $Z^0$  - NC  
 $W^\pm$  - CC

## Kinematic variables

- momentum transfer  $Q^2 = -q^2 > 0$
- Bjorken variable  $x = Q^2/(2P \cdot q)$
- Inelasticity  $y = (P \cdot q)/(P \cdot k)$

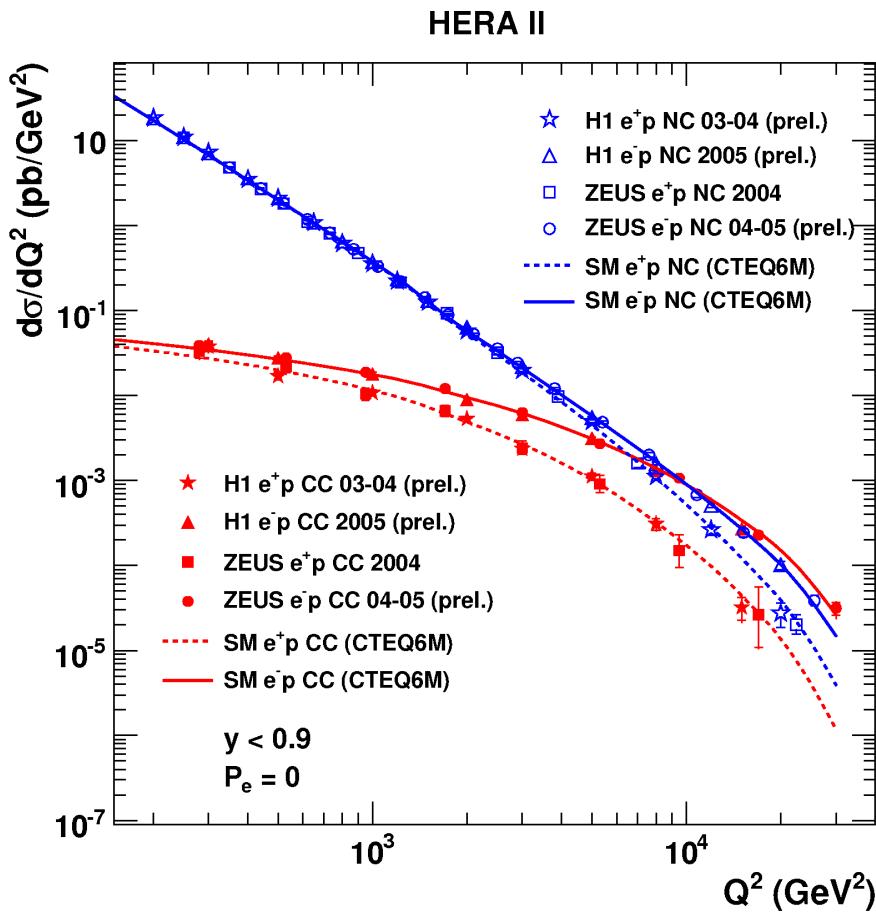
# DIS experiments

- EW unification at HERA:  
neutral vs . charged current

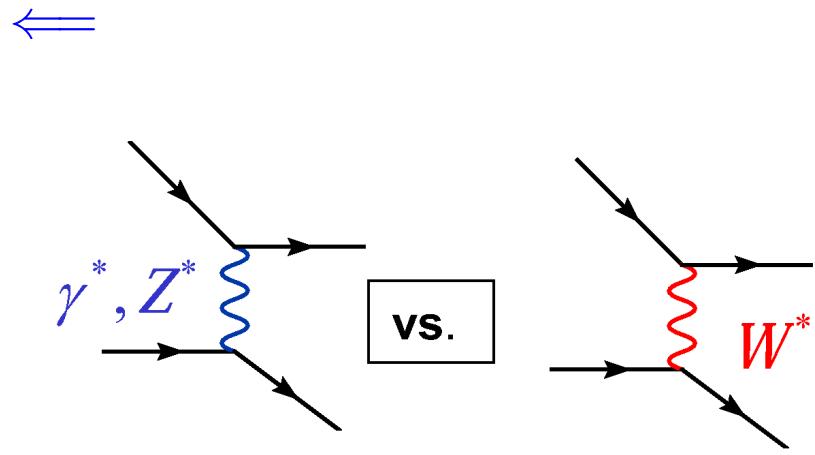


# DIS experiments

- EW unification at HERA:  
neutral vs . charged current



Charged and neutral deep inelastic scattering cross sections become comparable when  $Q^2$  reaches the electroweak scale



- Polarized charged current DIS at HERA

### CC cross section modified by polarization:

$$\sigma_{CC}^{e^\pm p}(P_e) = (1 \pm P_e) \cdot \sigma_{CC}^{e^\pm p}(P_e = 0)$$

$$P_e = \frac{N_R - N_L}{N_R + N_L}$$

- Cross section is linearly proportional to polarization  $P_e$
- **Standard model prediction:** vanishing cross section for  $P_e = +1(-1)$  in  $e^{-(+)} \text{ scattering}$

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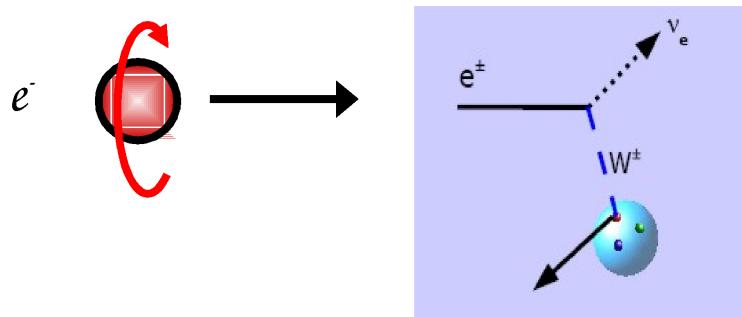
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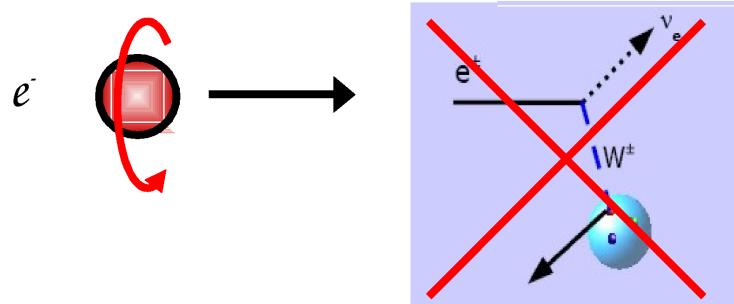
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■ *righthanded electrons do not!*



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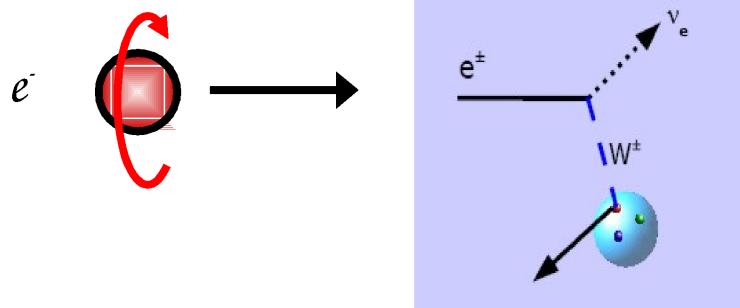
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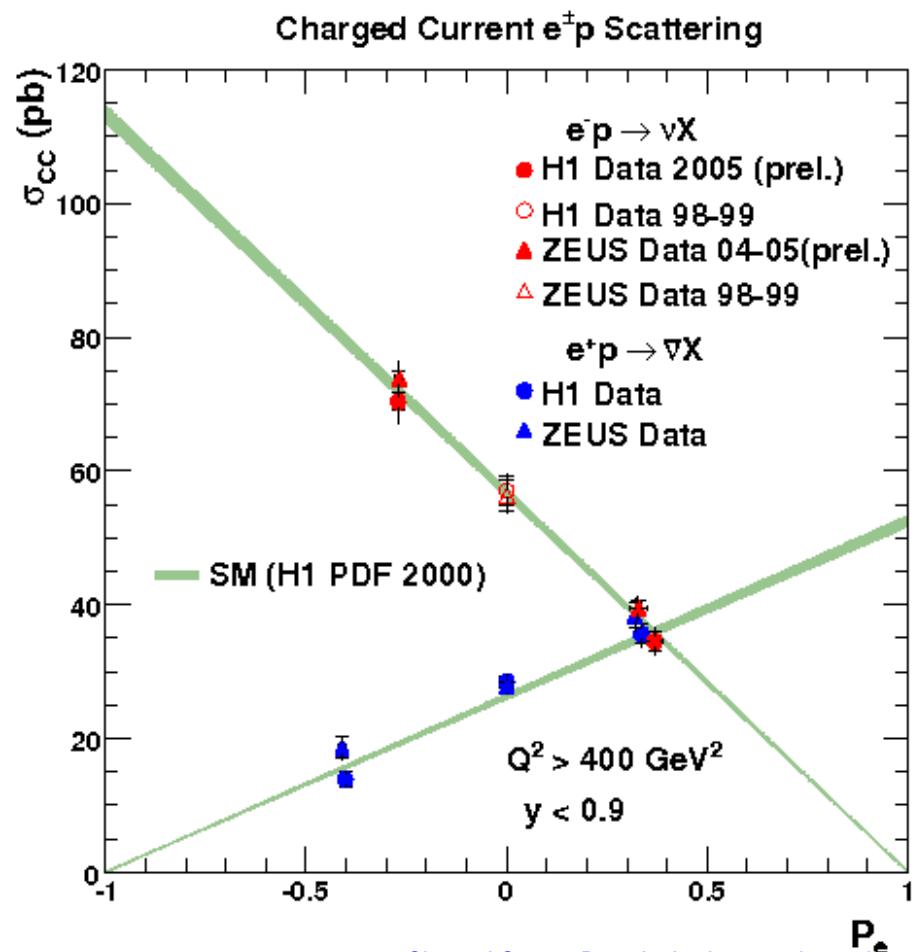
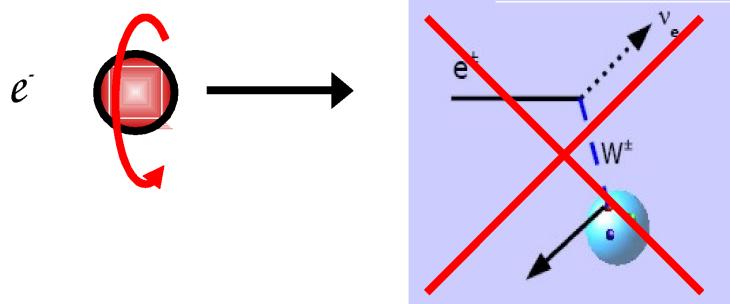
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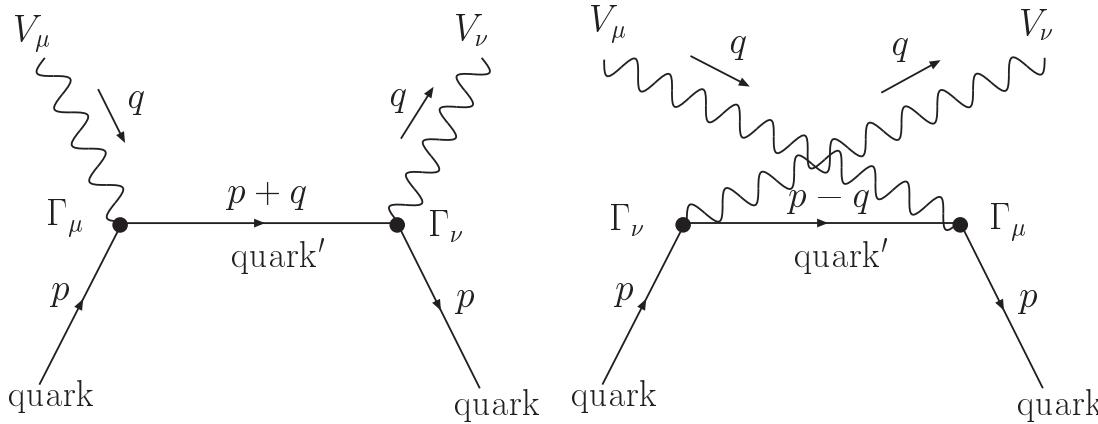


■ right handed electrons do not!



# Calculation

- Leading order diagrams at parton level
  - Vector and axial-vector interaction  $a\gamma^\mu + b\gamma^\mu\gamma^5$



- Mellin moments with definite symmetry properties
  - process dependent distinction even/odd  $N$  (from OPE)

$$F_i(N, Q^2) = \int_0^1 dx x^{N-2} F_i(x, Q^2), \quad i = 2, L$$

$$F_3(N, Q^2) = \int_0^1 dx x^{N-1} F_3(x, Q^2)$$

## Known

- NC (exchange via  $\gamma$  gauge boson)  $\longrightarrow F_2^{eP}$
- CC (exchange via  $W^\pm$  gauge boson)  $\longrightarrow F_2^{\nu p + \bar{\nu} p}, F_3^{\nu p + \bar{\nu} p}$

even  $N$  for  $F_2$ , odd  $N$  for  $F_3$

- NLO Bardeen, Buras, Duke, Muta '78
- N<sup>2</sup>LO Zijlstra, van Neerven '92
- N<sup>3</sup>LO Moch, Vermaseren, Vogt '05/'06

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## Unknown

- NC  $\gamma - Z$  interference at N<sup>3</sup>LO still missing
  - CC (exchange via  $W^\pm$  gauge boson)  $\longrightarrow F_2^{\nu p - \bar{\nu} p}, F_3^{\nu p - \bar{\nu} p}$
- odd  $N$  for  $F_2$ , even  $N$  for  $F_3$
- order N<sup>3</sup>LO *already known* Moch, M. R. ‘07  
best use: difference “even-odd” Moch, M. R. and Vogt. ‘07

### The calculation

- Big number of diagrams  $\Rightarrow$  need of automatization  
e.g. DIS structure functions  $F_{2,L}^{\nu p \pm \bar{\nu} p}$  - 1076 diagrams,  $F_3^{\nu p \pm \bar{\nu} p}$  - 1314 diagrams up to 3 loops

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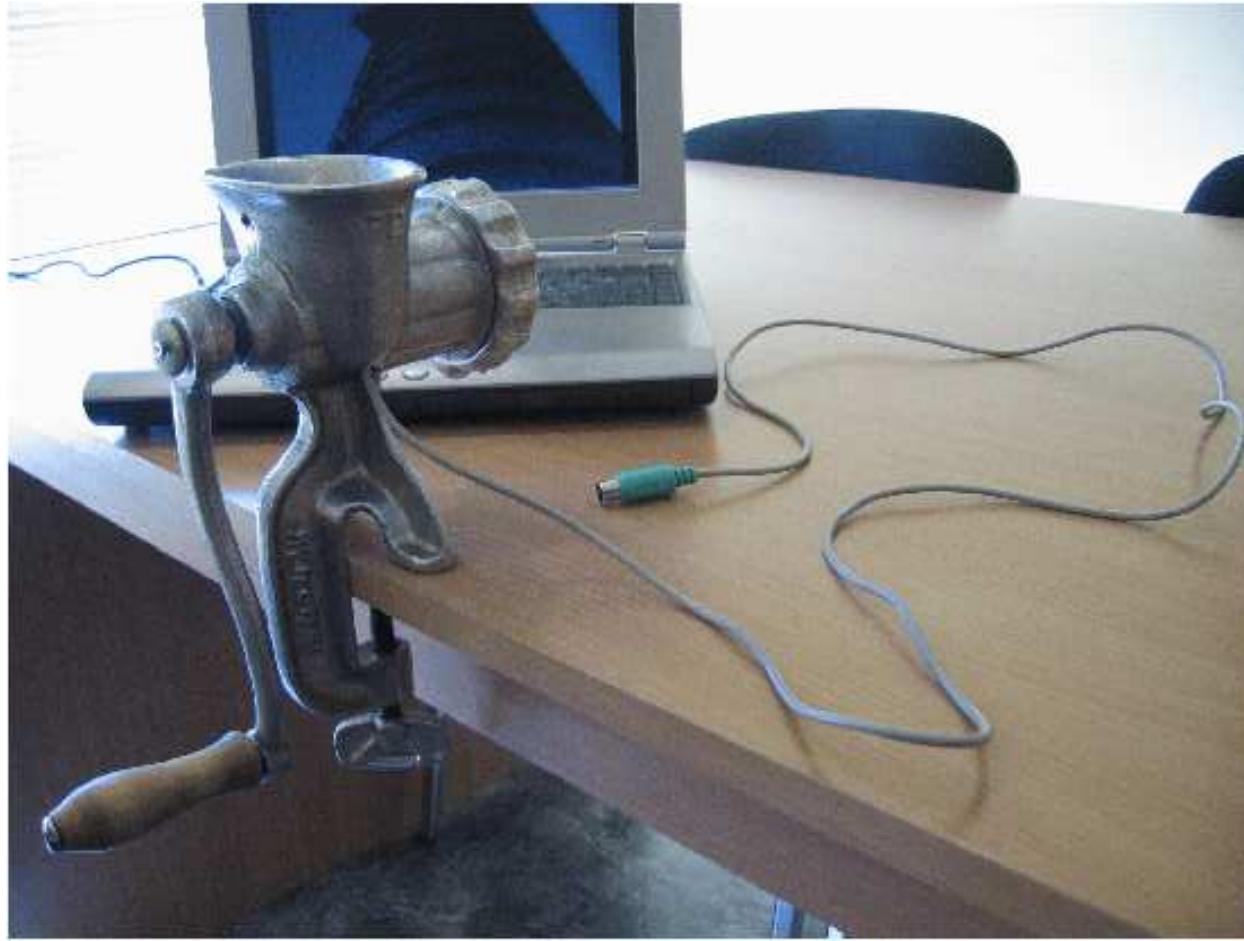
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Calculation of diagrams  $\mapsto$

- MINCER in FORM Larin, Tkachev, Vermaseren '91

What does MINCER do?

## MINCER minces integrals



$$\begin{aligned}
C_{3,10}^{\text{ns}} = & 1 + a_s C_F \frac{1953379}{138600} + a_s^2 C_F n_f \left( -\frac{537659500957277}{15975002736000} \right) + a_s^2 C_F {}^2 \left( \frac{597399446375524589}{14760902528064000} \right. \\
& \left. + \frac{7202}{105} \zeta_3 \right) + a_s^2 C_A C_F \left( \frac{5832602058122267}{29045459520000} - \frac{99886}{1155} \zeta_3 \right) \\
& + a_s^3 C_F n_f {}^2 \left( \frac{51339756673194617191}{996360920644320000} + \frac{48220}{18711} \zeta_3 \right) \\
& + a_s^3 C_F {}^2 n_f \left( -\frac{125483817946055121351353}{209235793335307200000} - \frac{59829376}{3274425} \zeta_3 + \frac{24110}{693} \zeta_4 \right) \\
& + a_s^3 C_F {}^3 \left( -\frac{744474223606695878525401307}{7088908678200207936000000} + \frac{28630985464358}{24960941775} \zeta_3 \right. \\
& \left. + \frac{151796299}{8004150} \zeta_4 - \frac{53708}{99} \zeta_5 \right) \\
& + a_s^3 C_A C_F n_f \left( -\frac{185221350045507487753}{226445663782800000} + \frac{8071097}{39690} \zeta_3 - \frac{24110}{693} \zeta_4 \right) \\
& + a_s^3 C_A C_F {}^2 \left( \frac{19770078729338607732075449}{8369431733412288000000} - \frac{619383700181}{5546875950} \zeta_3 \right. \\
& \left. - \frac{151796299}{5336100} \zeta_4 - \frac{37322}{99} \zeta_5 \right) \\
& + a_s^3 C_A {}^2 C_F \left( \frac{93798719639056648125143}{36231306205248000000} - \frac{43202630363}{20582100} \zeta_3 \right. \\
& \left. + \frac{151796299}{16008300} \zeta_4 + \frac{195422}{231} \zeta_5 \right).
\end{aligned}$$

# Checks

- Known Mellin moments for  $F_{2,L}^{\nu P + \bar{\nu} P}$  (even) and  $F_3^{\nu P + \bar{\nu} P}$  (odd) recalculated

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$$i \frac{-g^{\mu\nu} + (1 - \xi)q^\mu q^\nu}{q^2}$$

- Adler sum rule for DIS structure functions  $\longrightarrow C_{2,1}^{\text{ns}} = 1$

$$\int_0^1 \frac{dx}{x} (F_2^{\nu P}(x, Q^2) - F_2^{\nu N}(x, Q^2)) = 2$$

- measures isospin of the nucleon in the quark-parton model
- neither perturbative nor non-perturbative corrections in QCD

# Applications

- Gottfried sum rule (charged lepton( $l$ )-proton( $P$ ) or neutron( $N$ ) DIS)

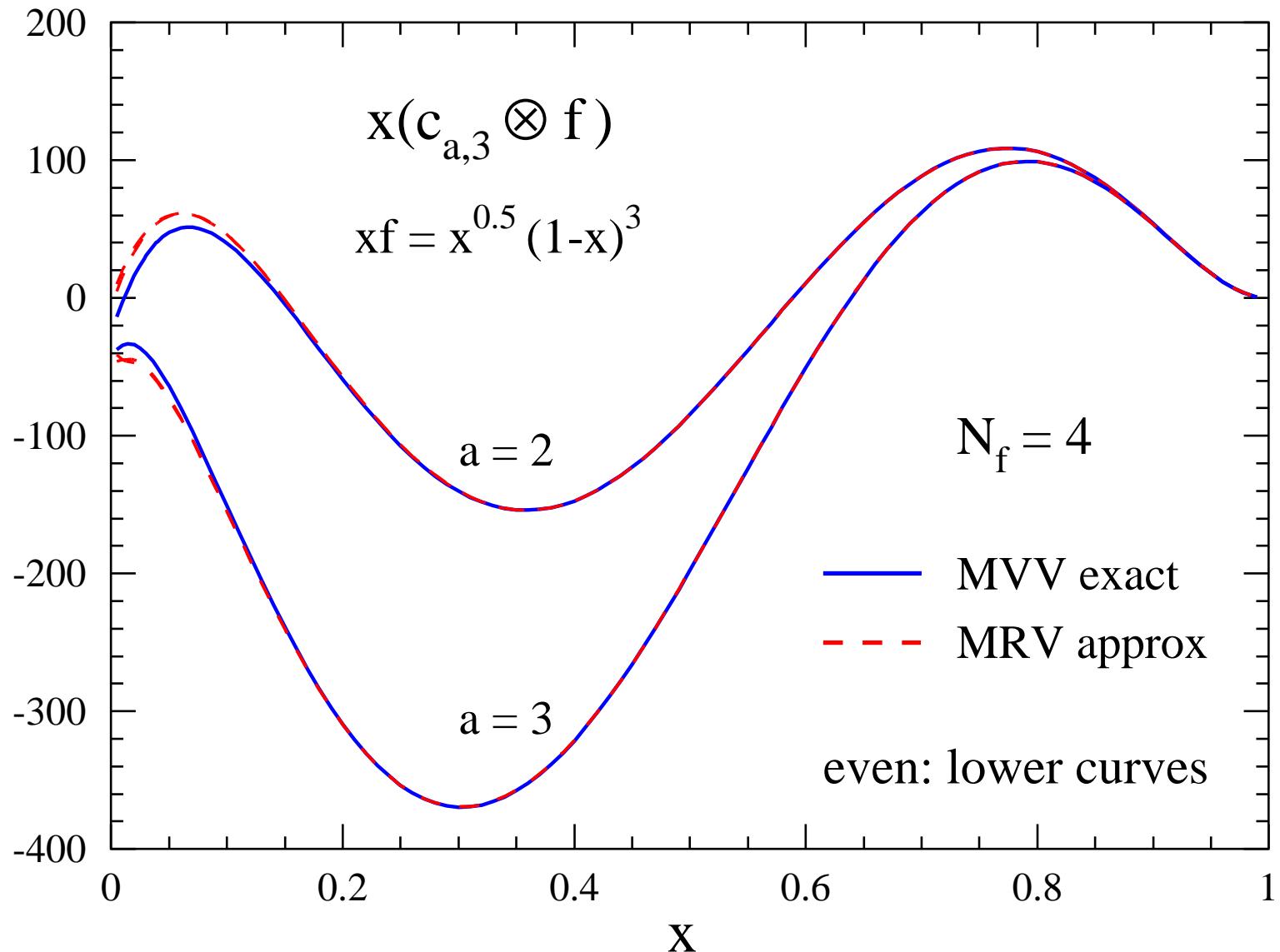
$$\int_0^1 \frac{dx}{x} (F_2^{lP}(x, Q^2) - F_2^{lN}(x, Q^2))$$

- Conjecture: difference of non-singlet coefficient functions for even and odd Mellin moments **subleading** in color  $[C_F - C_A/2] \simeq 1/N_c$   
Broadhurst, Kataev, Maxwell ‘04

- $\delta C_{i,n}^{\text{ns}} = C_{i,n}^{\nu P + \bar{\nu} P} - C_{i,n}^{\nu P - \bar{\nu} P}$  with color coefficient  $[C_F - C_A/2]$
- e.g.

$$\begin{aligned}
\delta C_{2,3}^{\text{ns}} = & +a_s^2 C_F [C_F - C_A/2] \left( -\frac{4285}{96} - 122\zeta_3 + \frac{671}{9}\zeta_2 + \frac{128}{5}\zeta_2^2 \right) \\
& + a_s^3 C_F [C_F - C_A/2]^2 \left( \frac{1805677051}{466560} - \frac{2648}{9}\zeta_5 + \frac{10093427}{810}\zeta_3 - \frac{1472}{3}\zeta_3^2 \right. \\
& \quad \left. - \frac{7787113}{1944}\zeta_2 + \frac{55336}{9}\zeta_2\zeta_3 - \frac{378838}{45}\zeta_2^2 - \frac{8992}{63}\zeta_2^3 \right) \\
& + a_s^3 C_F^2 [C_F - C_A/2] \left( -\frac{5165481803}{1399680} + \frac{40648}{9}\zeta_5 - \frac{9321697}{810}\zeta_3 + \frac{1456}{3}\zeta_3^2 \right. \\
& \quad \left. + \frac{8046059}{1944}\zeta_2 - 4984\zeta_2\zeta_3 + \frac{798328}{135}\zeta_2^2 - \frac{56432}{315}\zeta_2^3 \right) \\
& + a_s^3 n_f C_F [C_F - C_A/2] \left( \frac{20396669}{116640} - \frac{1792}{9}\zeta_5 + \frac{405586}{405}\zeta_3 - \frac{139573}{486}\zeta_2 \right. \\
& \quad \left. + \frac{1408}{9}\zeta_2\zeta_3 - \frac{50392}{135}\zeta_2^2 \right).
\end{aligned}$$

## $\alpha_s^3$ part of structure functions



## ● NuTeV experiment - Paschos-Wolfenstein relation

- Exact relation for massless quarks and isospin zero target

Paschos,Wolfenstein'73, Llewelin Smith'83

$$R^- = \frac{\sigma(\nu_\mu N \rightarrow \nu_\mu X) - \sigma(\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)} = \frac{1}{2} - \sin^2 \theta_W$$

- measurement of  $\sin^2 \theta_W$  NuTeV '01 with large deviations from Standard model expectations

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- QCD corrections to the Paschos-Wolfenstein relation
  - second moments of valence PDFs  $q^- = \int dx x(q - \bar{q})$
  - expansion in isoscalar combination  $u^- + d^-$   
Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; Moch, McFarland '03

$$R^- = \frac{1}{2} - \sin^2 \theta_W + \left[ 1 - \frac{7}{3} \sin^2 \theta_W + \frac{8\alpha_s}{9\pi} \{ 1 + \alpha_s 1.689 + \alpha_s^2 (3.661 \pm 0.002) \} \left( \frac{1}{2} - \sin^2 \theta_W \right) \right] \times \left( \frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-} \right)$$

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- main uncertainties in  $s^-$

Martin, Roberts, Stirling, Thorne '04; Lai, Nadolsky, Pumplin, Stump, Tung, Yuan '07

- QCD corrections under control Moch, M. R., Vogt '07

# Summary

- New results for fixed Mellin moments at order  $\alpha_s^3$

$F_{2,L}^{\nu p - \bar{\nu} p}$  (odd) and  $F_3^{\nu p - \bar{\nu} p}$  (even)

- and differences “even-odd”

▲ interesting: OPE based moments

[ $F_{2,L}^{\nu p - \bar{\nu} p} = 1, 3, 5, \dots$ ;  $F_3^{\nu p - \bar{\nu} p} = 2, 4, 6, \dots$  ]:

weight  $w$  of zeta functions up to  $2l - 1$  ( $l$  - number of loops)

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- approximations in  $x$ -space for “even-odd” differences available

⇒ sufficient for HERA-CC,  $\nu$  - DIS (e.g. Alekhin makes use of it)