Parton densities: progress and challenges

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University of Liverpool

- Introduction: partons in hard processes, LHC Higgs production
- Higher-order effects in the parton evolution, small-x logarithms
- Parton evolution in practice: evolution codes and benchmarks
- Input shapes and factorization schemes, heavy-quark densities
- Highlights of recent parton analyses, future constraints at LHC
- Instead of a summary: W-mass at the LHC

DIS 2007, Munich, 16-04-07

W.L. van Neerven, 1947 – 2007

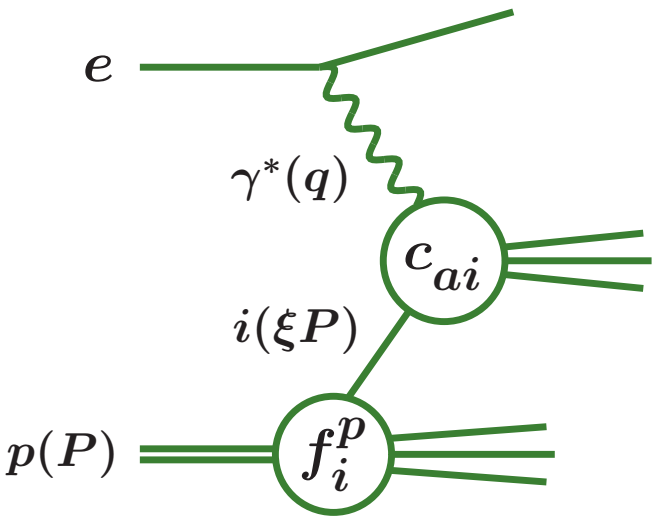
Photo courtesy of DESY

One of the greatest theorists of deep-inelastic processes

- ...
- Electro-weak corrections for LEP
- 2nd order QCD corr's for DY process
- NLO heavy-quark production in pp
- O (α_s^2) coefficient functions in DIS
- Heavy quarks in structure functions
- Polarized NLO splitting functions
- 2nd order corr's for fragmentation
- Variable-flavour partons and DIS
- QCD corr's to LHC Higgs production

Parton distributions and hard processes (I)

Example: inclusive photon-exchange deep-inelastic scattering (DIS)



Hard scale, Bjorken variable

$$Q^2 = -q^2$$
$$x = Q^2/(2P\cdot q)$$

Lowest order, quarks: $x = \xi$

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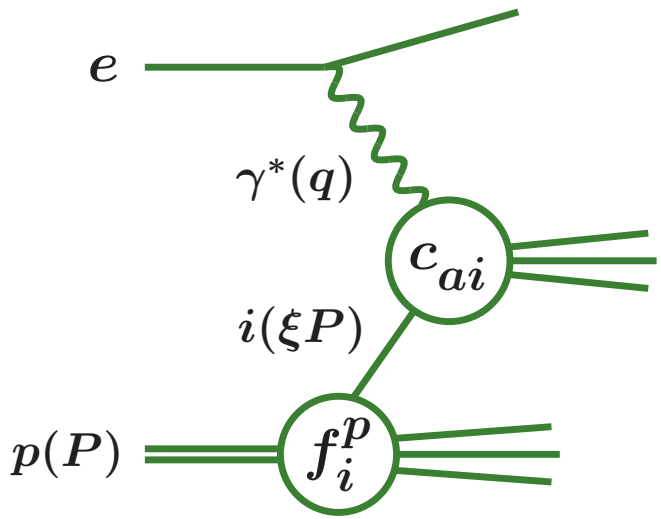

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Structure functions $F_{2,L}$ **[up to** $1/Q^2$ **terms – cut data to suppress]**

$$x^{-1}F_a^p(x,Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, c_{a,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^p(\xi,\mu^2)$$

Coefficient functions: scheme, scale $\mu = \mathcal{O}(Q)$**, Mellin convolution**

Parton distributions and hard processes (II)

Parton distributions f_i : evolution equations ($\otimes$ = Mellin conv.)

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

Initial conditions incalculable in pQCD. Lattice: low moments only

$\Rightarrow$ **predictions: fits of suitable reference processes, universality**

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Expansion in α_s: splitting functions P, coefficient functions c_a

$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \ldots$$

$$c_a = \underbrace{\alpha_s^{n_a}\left[c_a^{(0)}\right.}_{} + \alpha_s c_a^{(1)} + \alpha_s^2 c_a^{(2)} + \ldots \left.\right]$$

LO: approximate shape, rough estimate of rate

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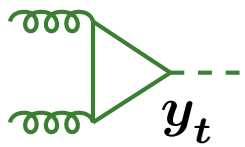
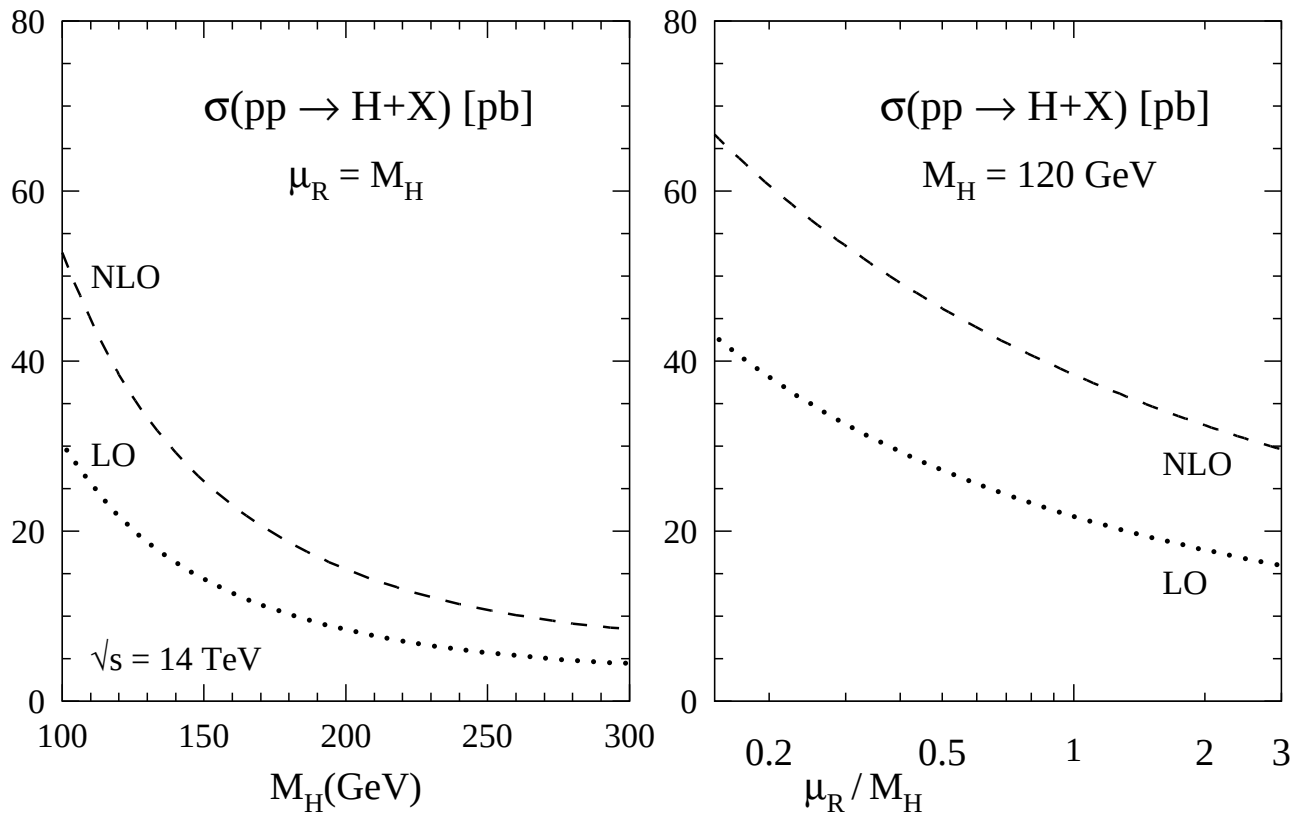
NLO: first real prediction of size of cross sections

NNLO, $P^{(2)}$, $c_a^{(2)}$: first serious error estimate of pQCD predictions

Higgs boson production at the LHC (I)

Dominant channel: $gg \to H + X$ via top-quark loop





Spira et al. (95)

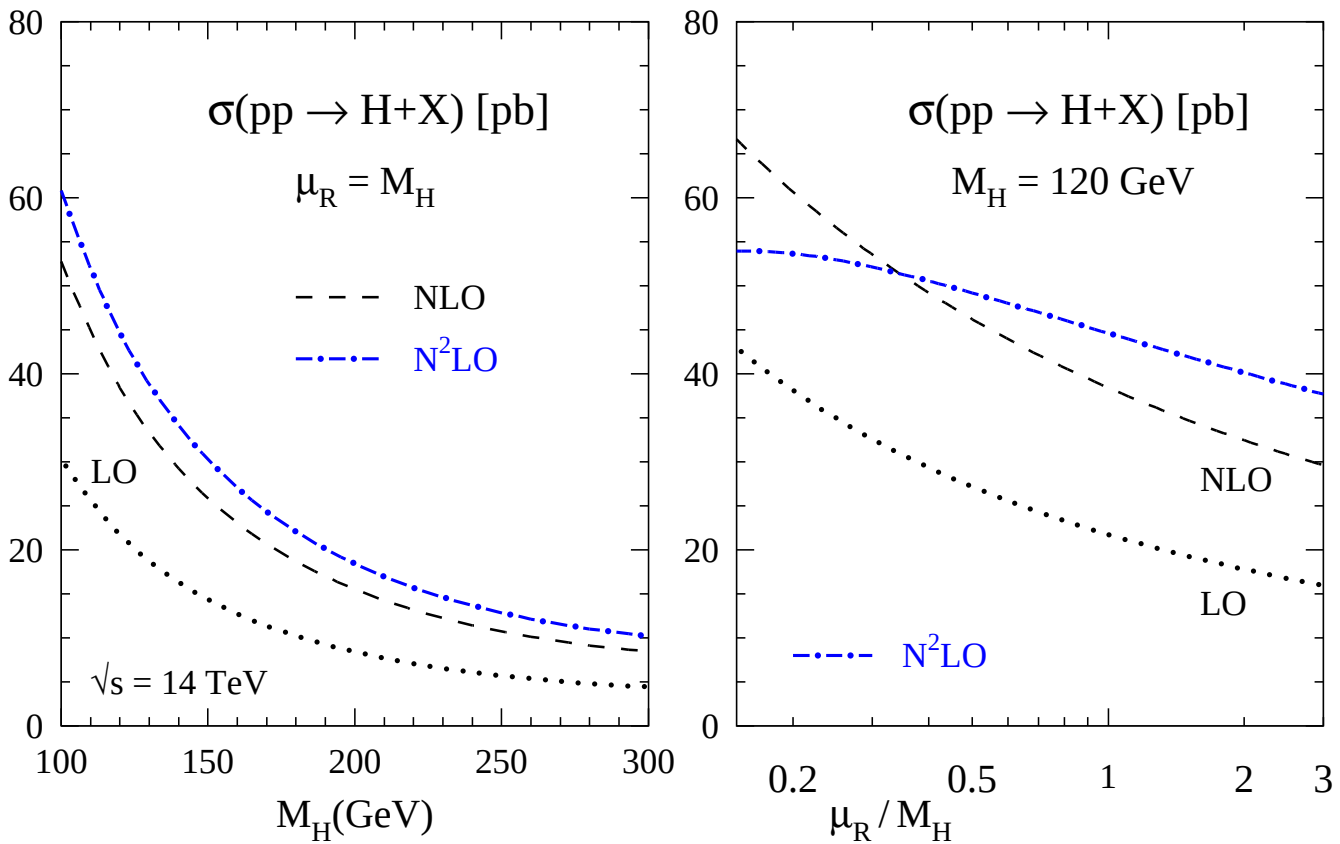
Error guess: apparent convergence, variation of scale μ (how far?)

Next-to-leading order (NLO) insufficient for quantitative prediction

Higgs boson production at the LHC (II)

$\hat{\sigma}_{\mathrm{NNLO}}$: Harlander, Kilgore (02); Anastasiou, Melnikov (02, 05 [σ_{diff}])

Partons including NNLO: MRST (plot), Alekhin.



Higher-order uncertainties: $\sim 15\%$ at NNLO

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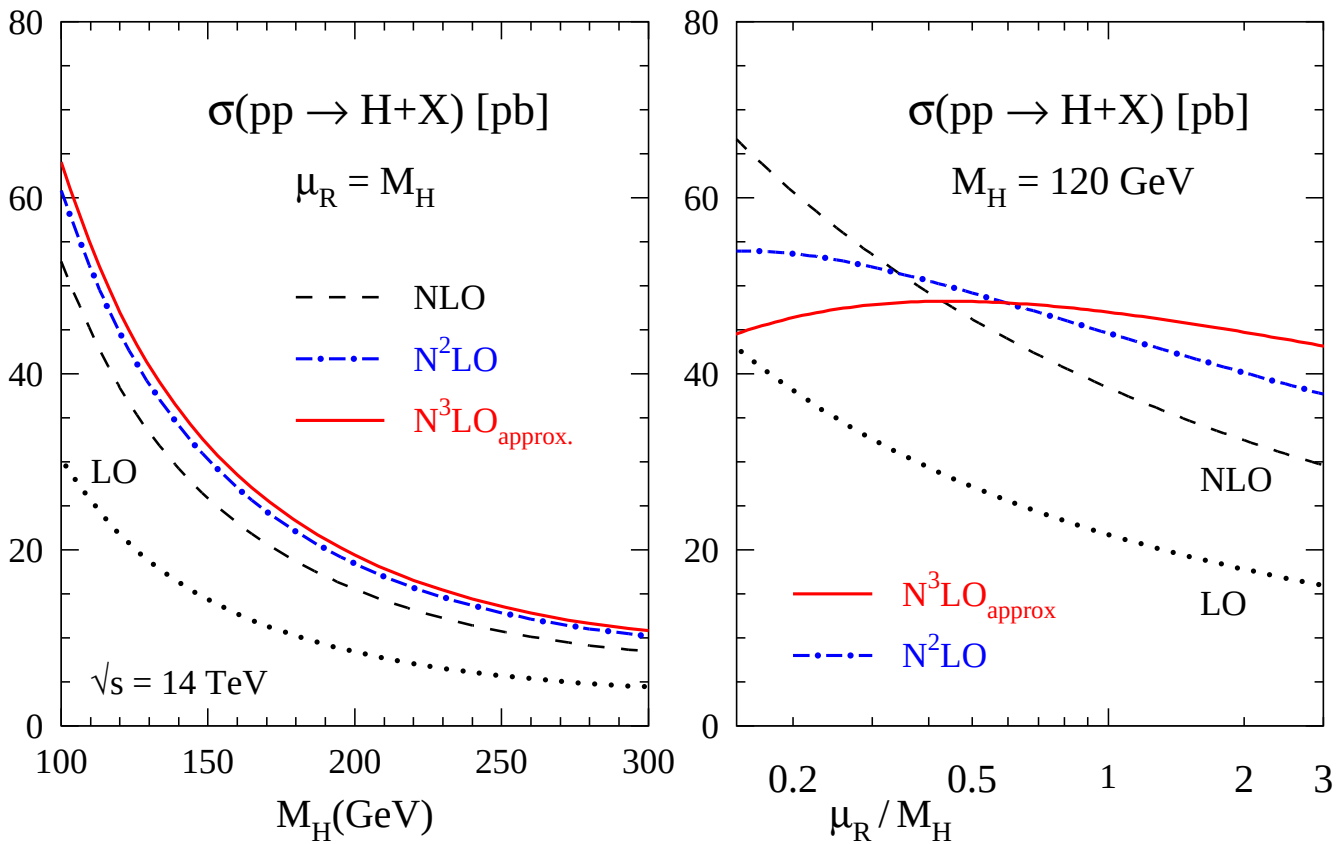
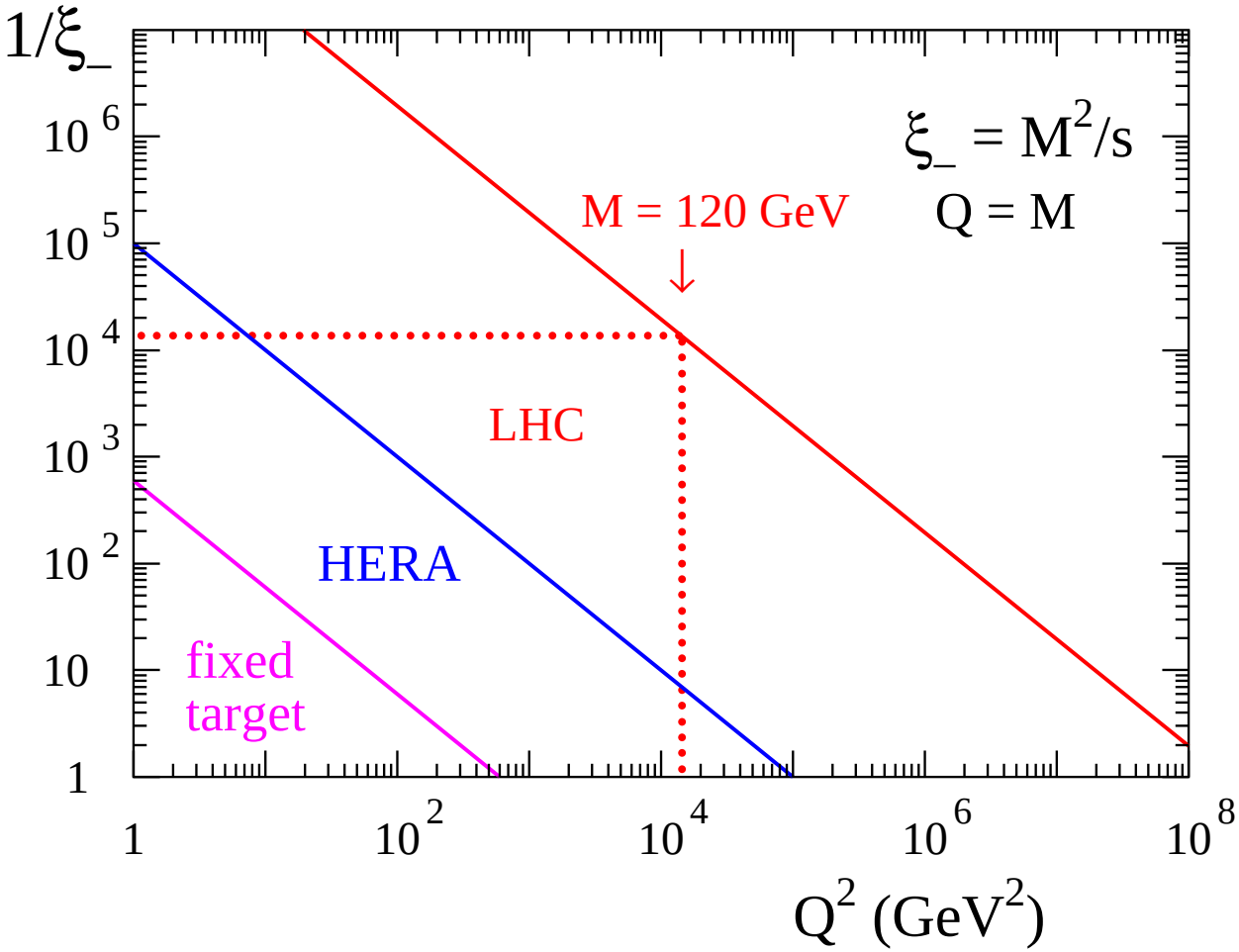


Moch, A.V. (05)

Higher-order uncertainties: $\sim 15\%$ at NNLO, $\sim 5\%$ at approx. N^{3}LO

Parton evolution from HERA to LHC

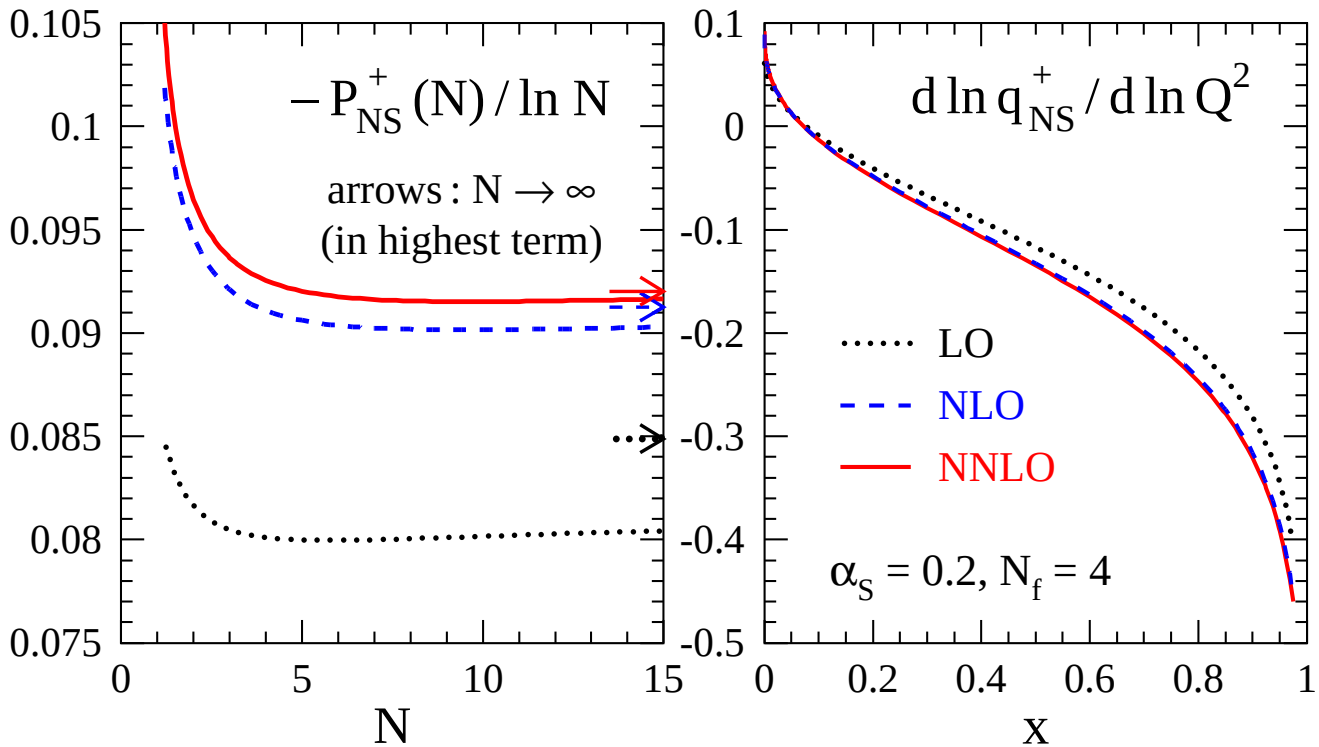
Kinematics: partons with momentum fractions $\xi_- < \xi < 1$ contribute



W/Z, H, top, new physics: $\xi_- \gtrsim 10^{-4}$, can cut at $Q^2 \approx 10$ GeV^2

Parton evolution at large N/ large x in $\overline{\text{MS}}$

Recall $A^N = \int_0^1 dx\, x^{N-1} A(x)$. **Non-singlet$^+$:** $u+\bar{u} - (d+\bar{d})$ **etc**



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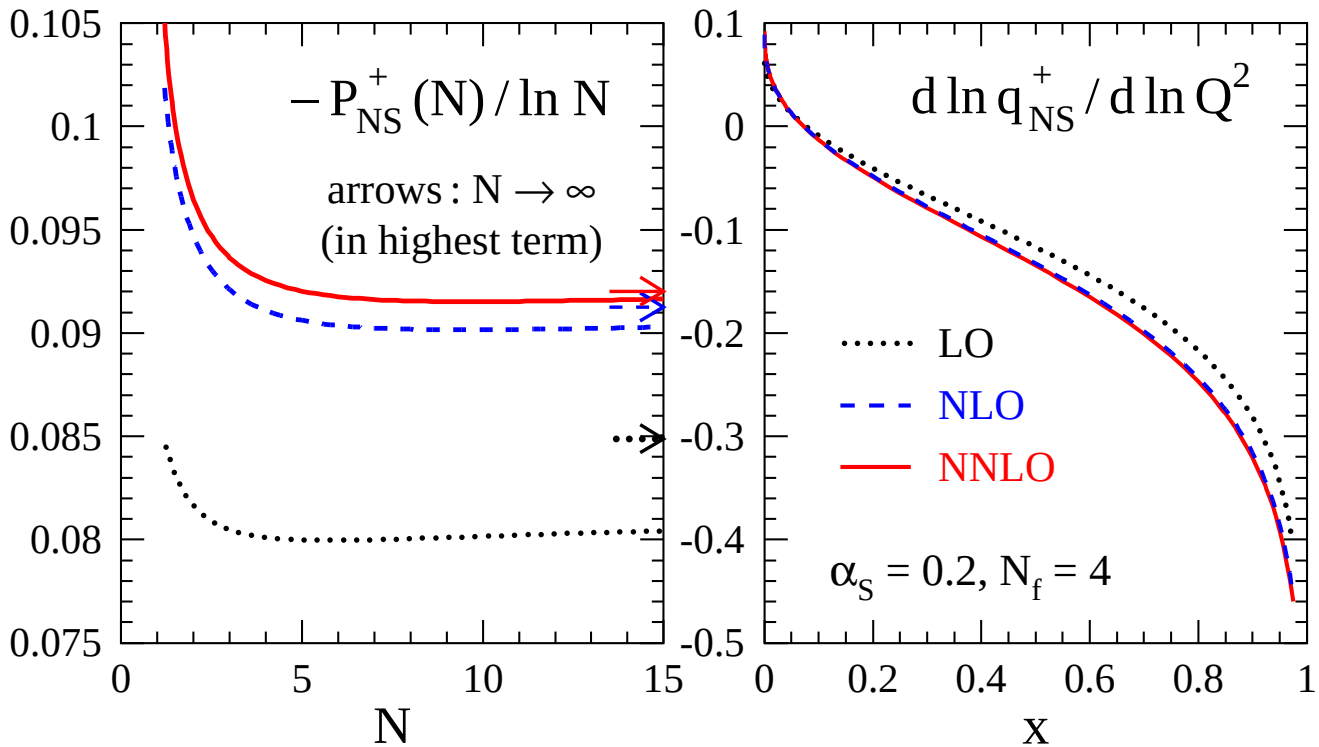


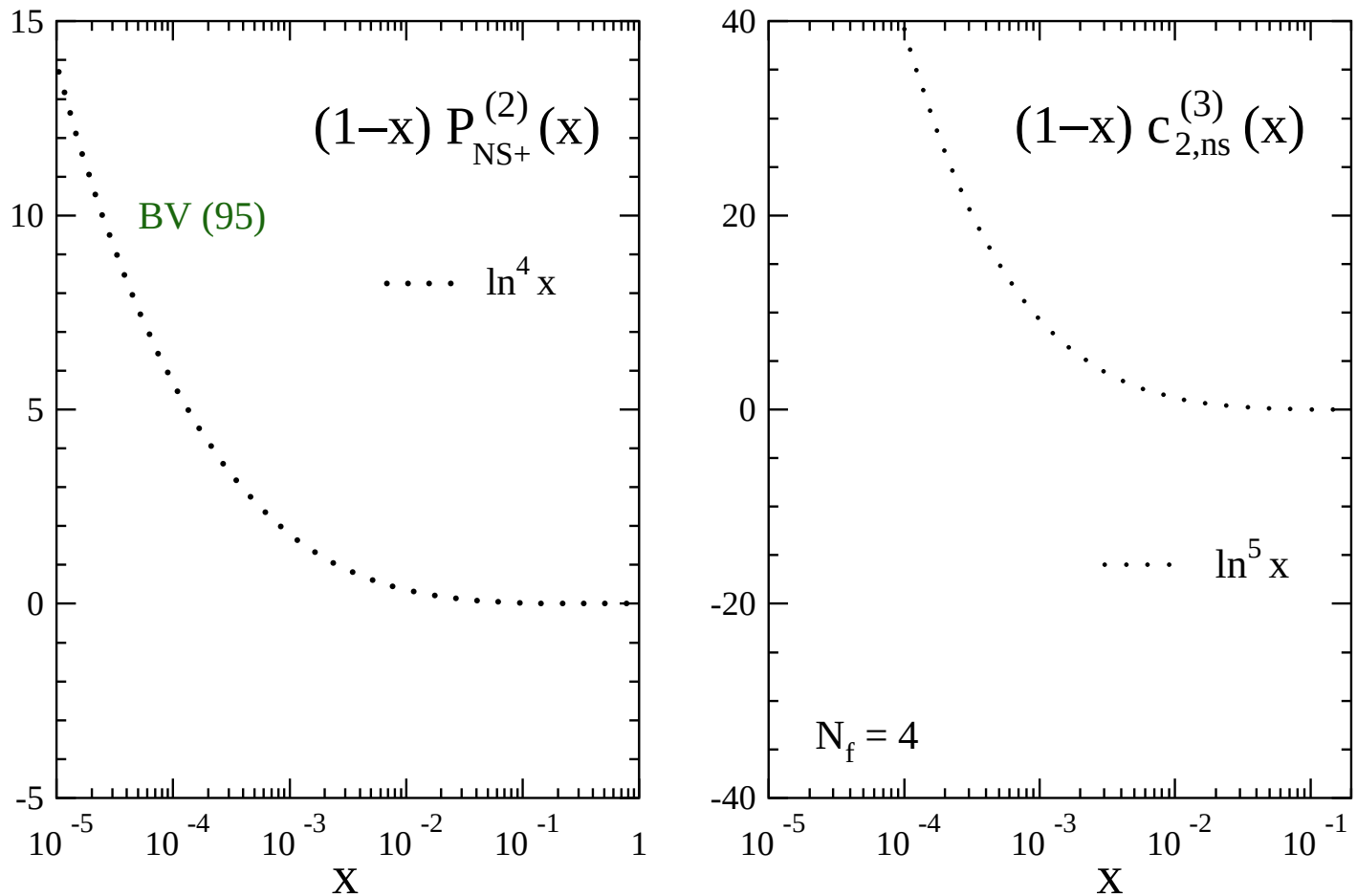
N^{3}LO: P^+_{ns} **now known for** $N=2$, $n_f = 3$ **Baikov, Chetyrkin (06)**

$$P^+_{\text{ns}} = -0.283\,\alpha_s\,[\,1 + 0.869\,\alpha_s + 0.798\,\alpha_s^2 + 0.926\,\alpha_s^3 + \ldots\,]$$

$N>2$: **similar / smaller (as** $\ln N$ **coeff's).** $n_f > 3$: **HO corr's smaller**

Non-singlet three-loop quantities at small x

Order α_s^n : small-x terms $\ln^k x$ with k up to $2n-2$ ($2n-1$) in P (c)



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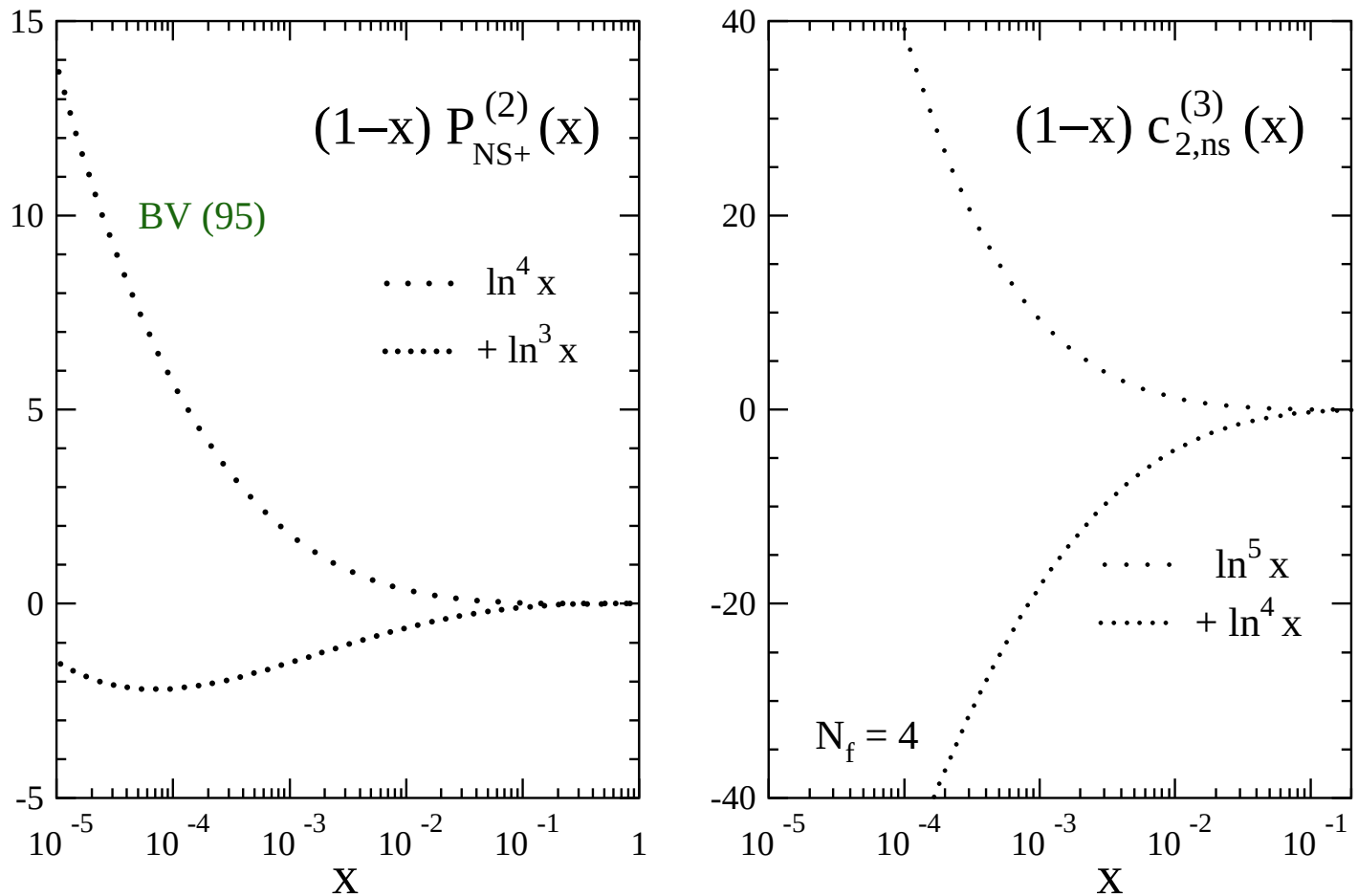

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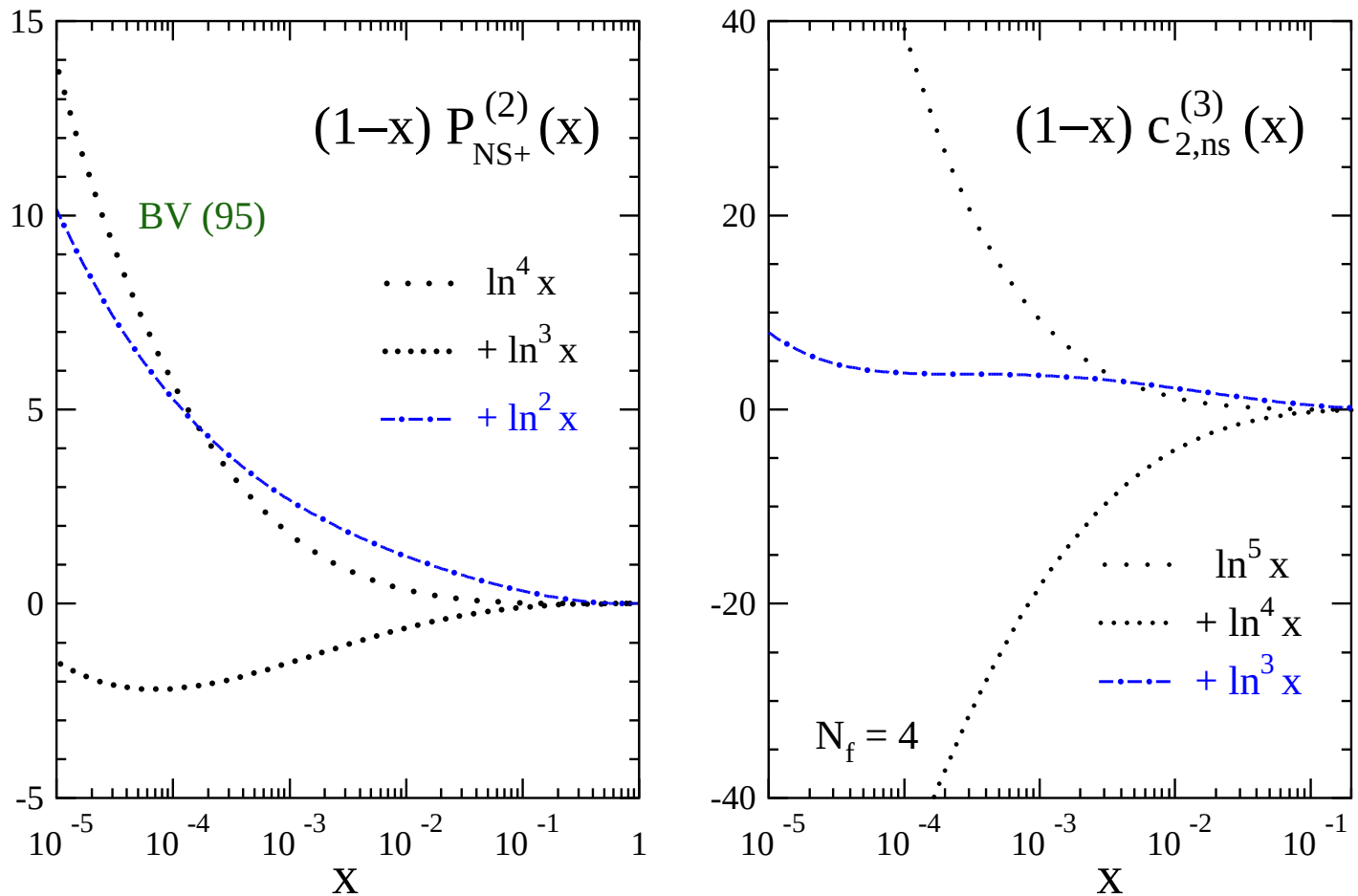

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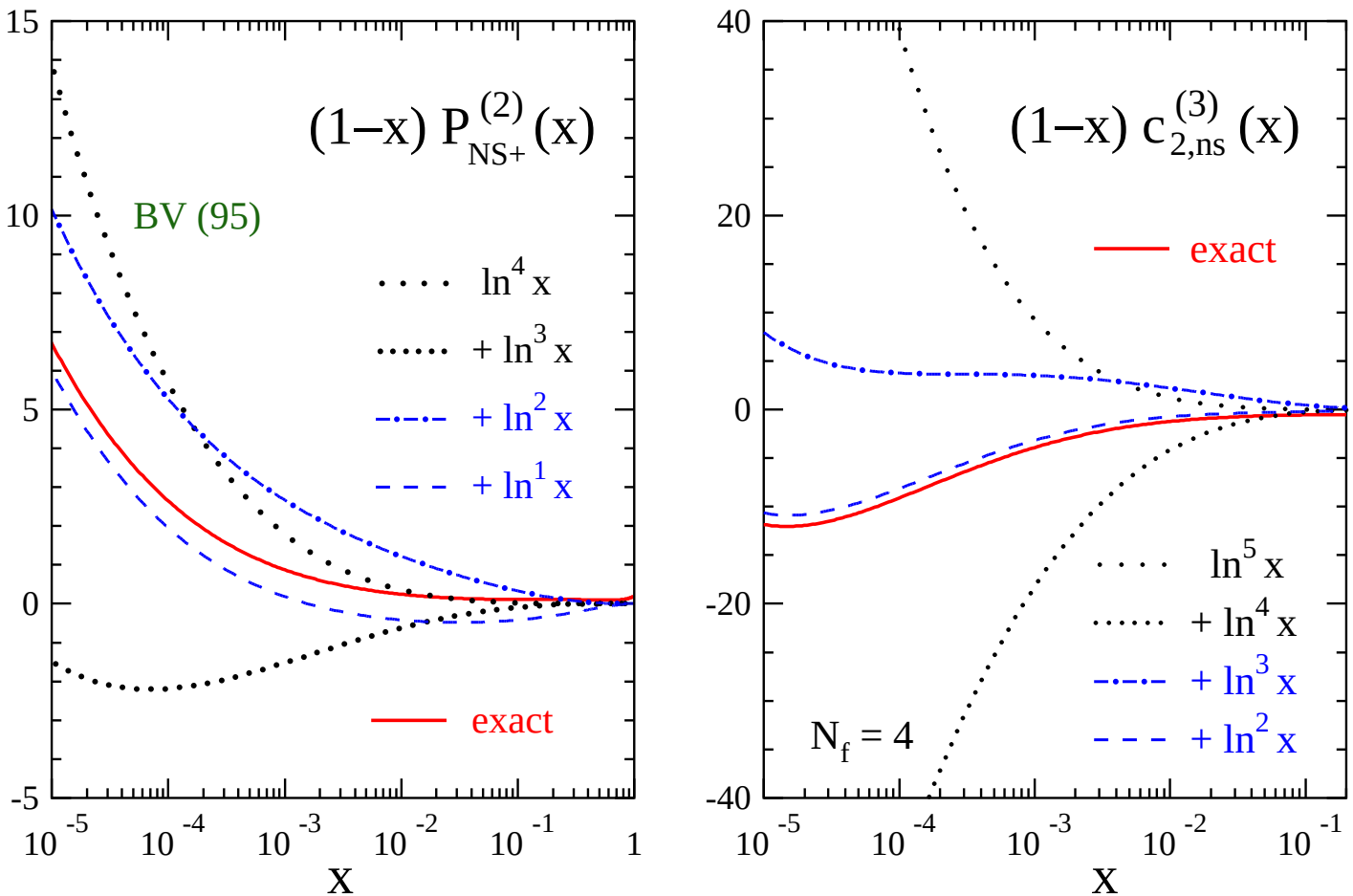
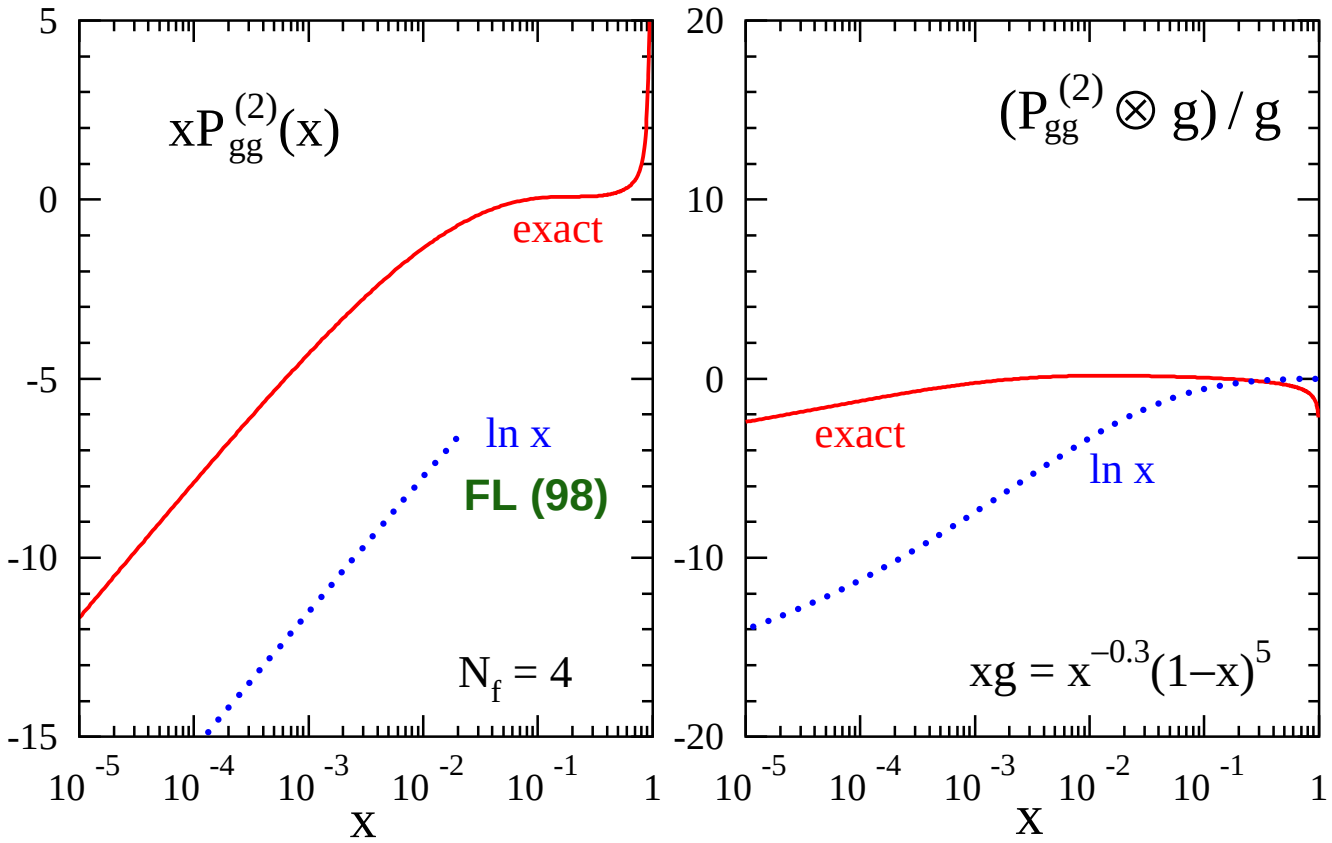


x-values for colliders: not even shape guaranteed by leading logs

Singlet splitting and evolution at small x

Splitting functions $\longrightarrow$ observables: convolutions, $\int_x^1 \frac{dy}{y}\, P(y)\, f\left(\frac{x}{y}\right)$



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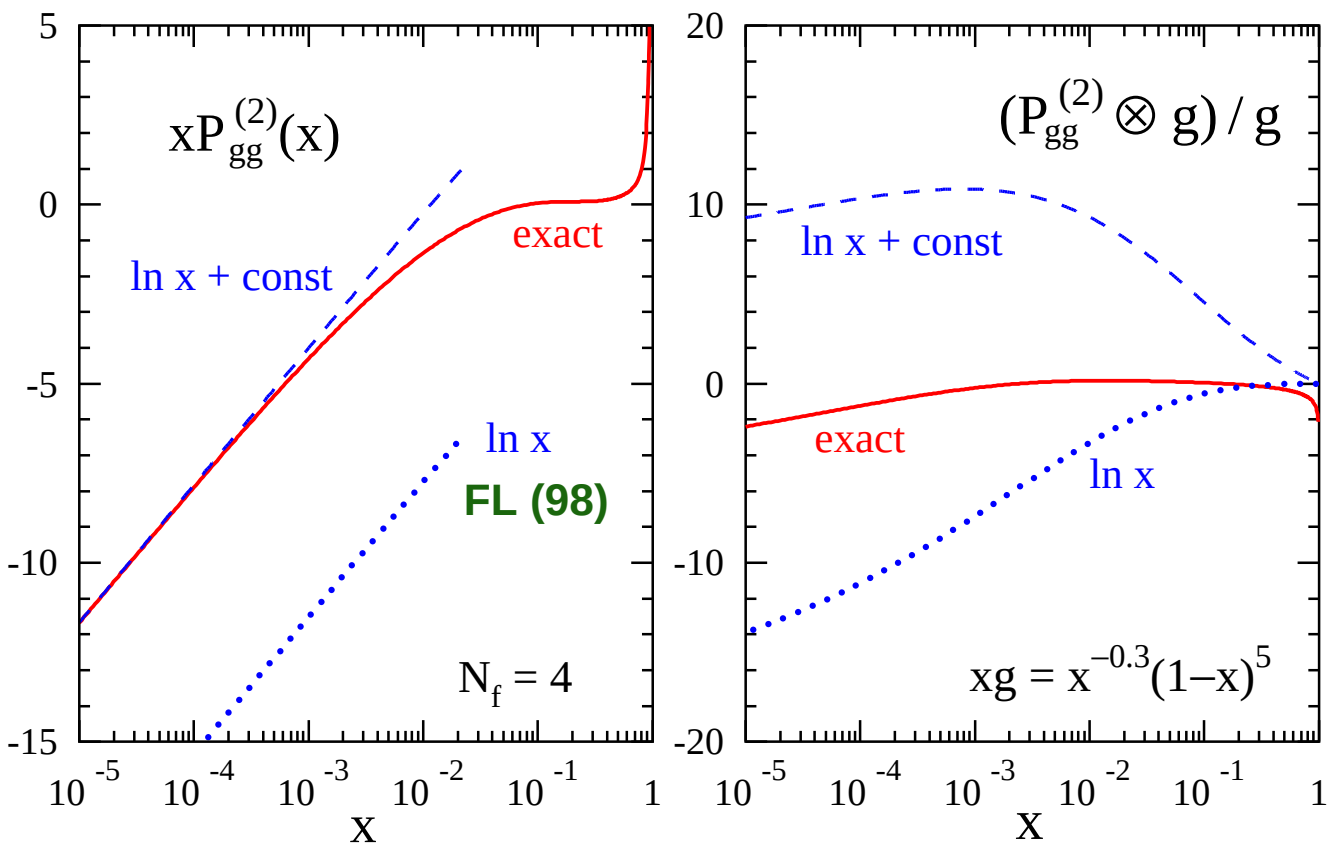
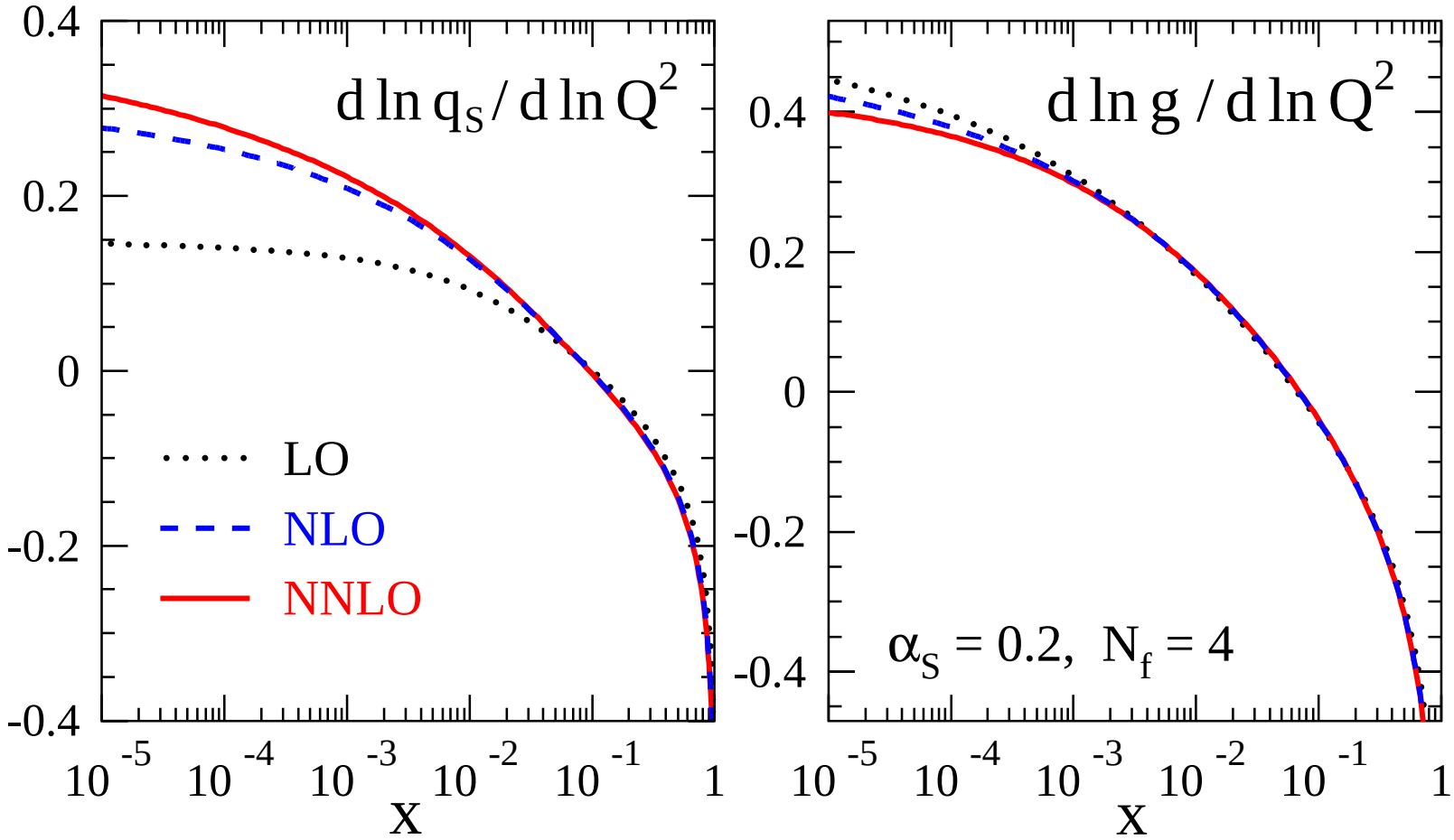


General: small-x limits of splitting fct's insufficient in convolutions

First estimate of corr's beyond NNLO: future moments at order α_s^4

Evolution of singlet parton distributions

Example: quark and gluon scale derivatives at scale $Q^2 \approx 30$ GeV^2



Expansion very stable for main LHC momentum fractions $x \gtrsim 10^{-4}$

Available evolution codes including NNLO

x-space: discretization in x, μ_{f} of coupled integro-differential eqs.

HOPPET (G. Salam, from 2001), http://hepforge.cedar.ac.uk/hoppet/

QCDNUM (M. Botje, new: v. 17), http://www.nikhef.nl/~h24/qcdnum/

N-space: ordinary diff. eqs., time-ordered exp., $N \to x$ numerical

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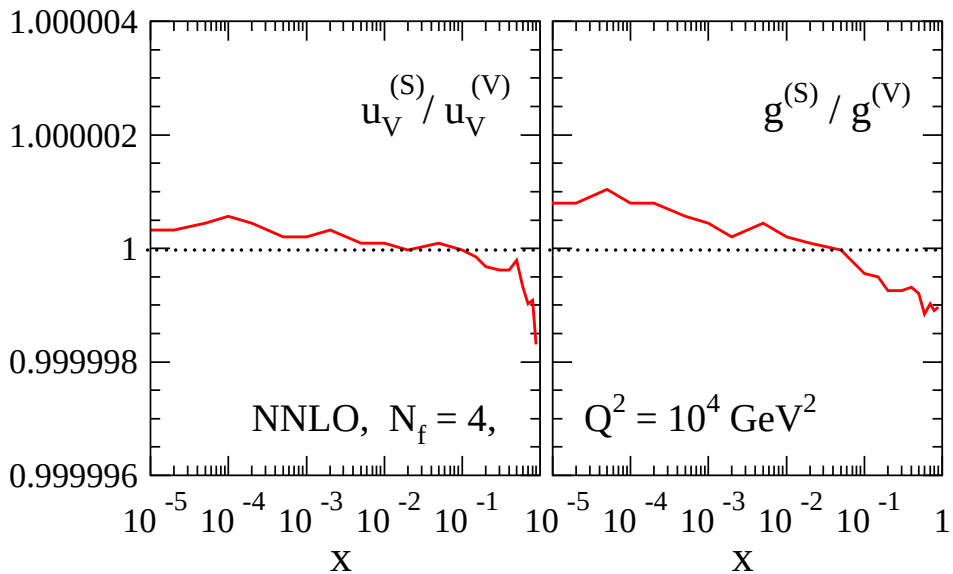
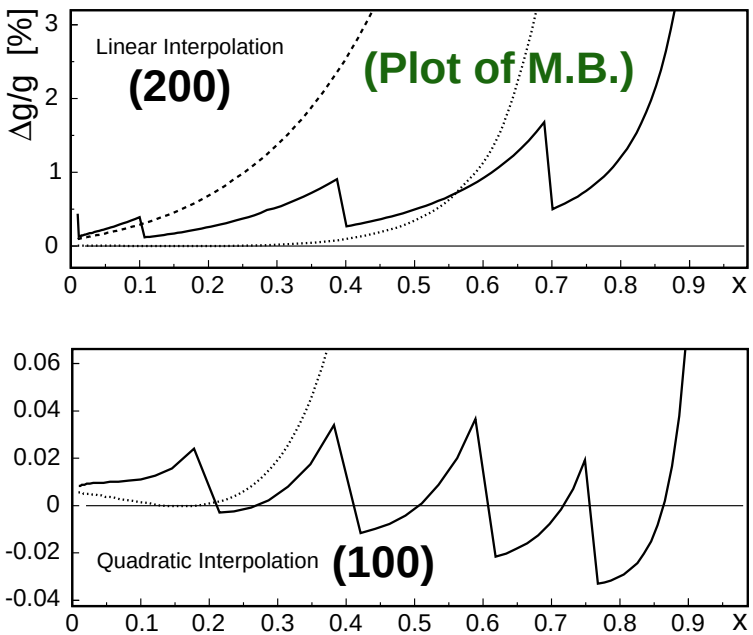
Sample comparisons





Benchmark tables for the parton evolution

Evolution of Les Houches (2001) reference input at $\mu^2_{\mathrm{f},0} = 2$ GeV^2

$$\begin{aligned} x u_v(x, \mu^2_{\mathrm{f},0}) &= 5.1072\ x^{0.8}\ (1-x)^3 \quad , \quad \ldots \\ x g\,(x, \mu^2_{\mathrm{f},0}) &= 1.7000\, x^{-0.1}\,(1-x)^5 \end{aligned}$$

with

$$\alpha_{\mathrm{s}}(\mu^2_{\mathrm{r}} = 2\ \mathrm{GeV}^2) = 0.35$$

at LO, NLO, NNLO, for $\mu_{\mathrm{r}} = \{0.5,\ 1,\ 2\}\,\mu_{\mathrm{f}}$, with fixed/variable N_f

Use of two completely different codes. G. Salam, A.V. (2002, 05)

Five-digit agreement over wide range in x, μ^2_{f} $\Rightarrow$ reference tables

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Example: (iterated) NNLO, $\mu_{\mathrm{r}} = 2\mu_{\mathrm{f}}$, $N_f = 4$ at $\mu^2_{\mathrm{f}} = 10^4$ GeV^2

$$x = 10^{-5}\ , \quad xu_v = 2.9032 \cdot 10^{-3}\ , \quad \ldots\ , \quad xg = 2.2307 \cdot 10^2$$

$$\ldots$$

$$x = 0.9\ , \quad xu_v = 3.6527 \cdot 10^{-4}\ , \quad \ldots\ , \quad xg = 1.2489 \cdot 10^{-6}$$

Input shapes and factorization schemes

$\overline{\text{MS}}$ **evolution:** convenient, stable (see above) – but pdf's not physical

Shapes for $x \to 1$, e.g., more natural in other scheme? Positivity?

NLO partons (which scheme?) best choice with LO Monte-Carlos?

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Traditional alternative: DIS scheme, quarks physical via F_2. Singlet:

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Drawback: gluon transf. arbitrary except at $N = 2$ (momentum sum)

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Interesting old concept: DIS$_\phi$ scheme Furmanski, Petronzio (81)

Also gluon shape physical via structure fct. F_ϕ of scalar coupling to $G^{\mu\nu} G_{\mu\nu}$ ($\leftrightarrow$ Higgs in large-m_t limit) – $c_{\phi,i}^{(1,2,3)}(x)$ now computed

$\Rightarrow$ End-point constraints from positivity of F_ϕ , ...

Heavy quarks in parton evolution

Below: disregard 'intrinsic charm' – can be relevant at large x

Pumplin, Lai, Tung (2007)

$\overline{\text{MS}}$ evolution of partons and α_s with variable number of flavours:

matching of effective theories. For partons at $\mu_F = m_h$ (pole mass):

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Coefficients: **Buza et al. (95/96), [Bierenbaum, Blümlein, Klein (07)]**

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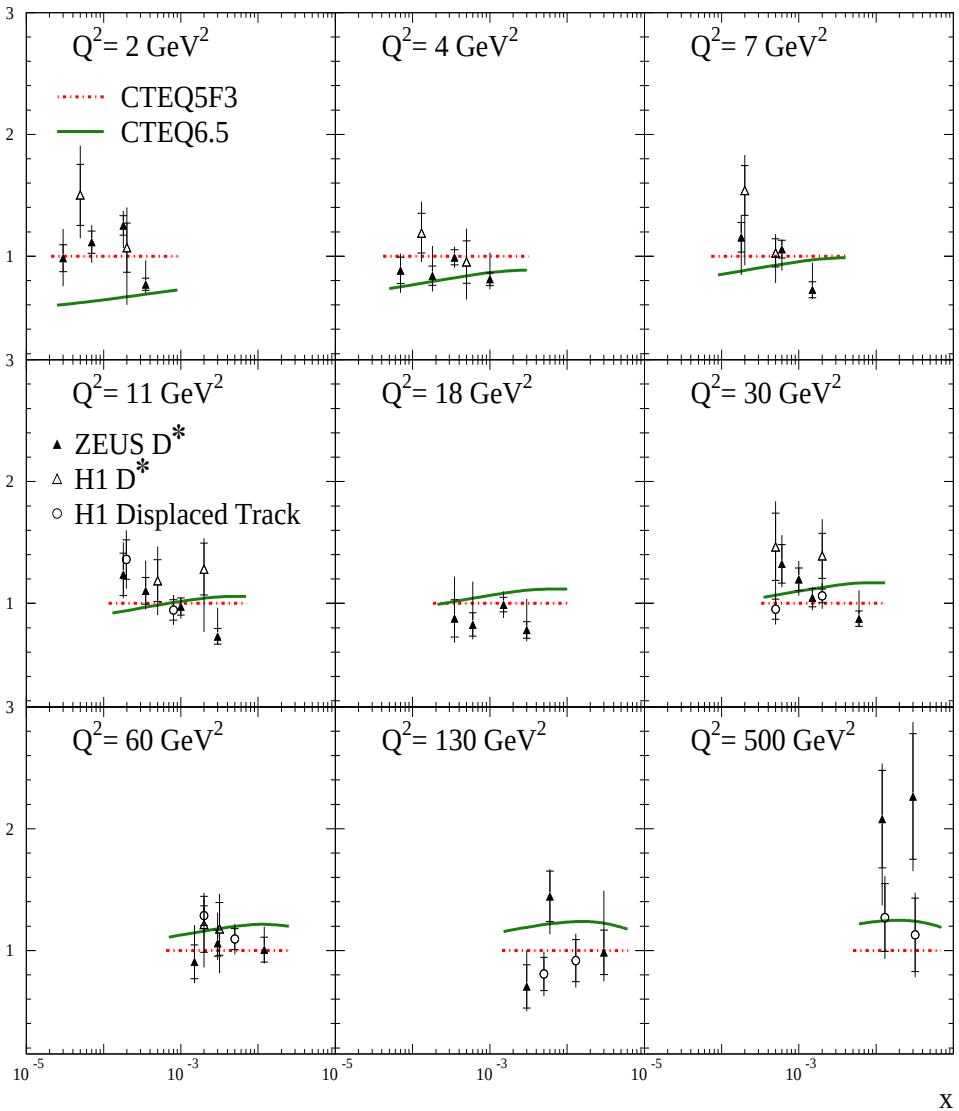
Included in the evolution codes and benchmarks discussed above

Ignored in NNLO MRST partons, now implemented for MSTW (07) sets

+ more improvements, e.g., fastNLO (Kluge, Rabberts, Wobisch) instead of K-factors

Heavy quarks in structure functions

$F_2^{c\bar{c}}$/CTEQ5F3, plot of P.D. Thompson (07)



$Q \not\gg m_c$: u, d, s, g partons

+ massive c coeff. fct's, FFNS

NLO: Laenen et al. (92); HVQDIS (95)

$Q \ggg m_c$: terms $m_c/Q \to 0$

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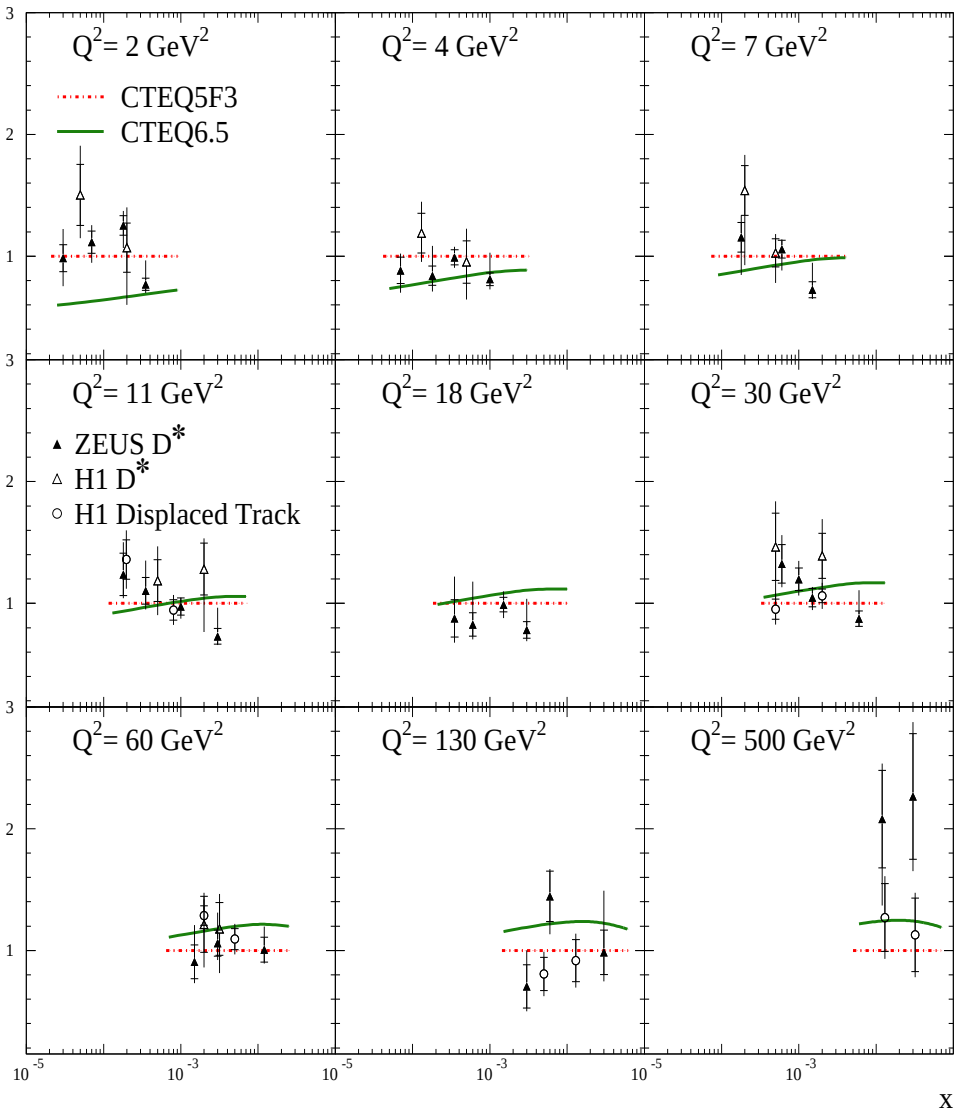

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$m = 0$ coeff. fct's, ZM-VFNS

$Q \gg m_c$: terms $m_c/Q \neq 0$,
but quasi-collinear logs large
$n_f = 4$ pdf's, 'interpolating'
coeff. functions, (GM-) VFNS

ACOT; BMSN/CSN; (Roberts-) Thorne

Transition process-dependent

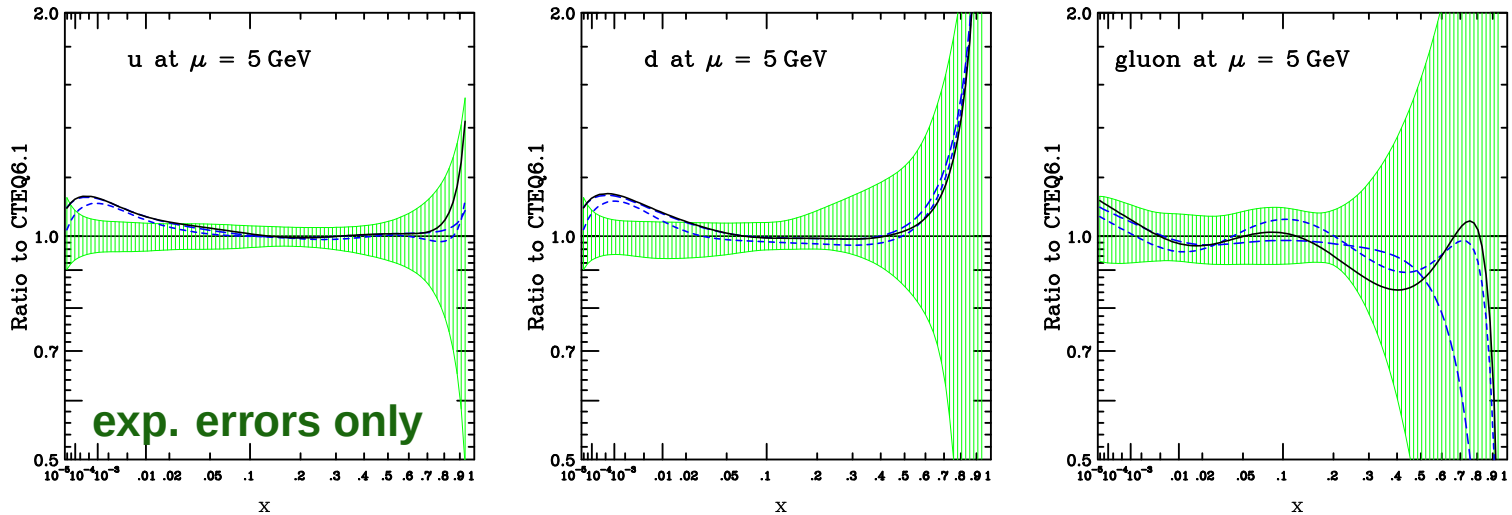
Exp.+ th.: "$Q \not\gg m_c$" for HERA $F_2^{c\bar{c}}$

Highlights of recent parton analyses (I)

CTEQ6.5, . . . , W.K. Tung et al. (06)

Charm mass suppression in DIS finally implemented in fits

Improved iterative treatment of inconsistent data under way, . . .



Less $F_2^{c\bar{c}}$ $\Rightarrow$ more u, d $\Rightarrow$ higher W/Z cross section ($\sim 8\%$) at LHC

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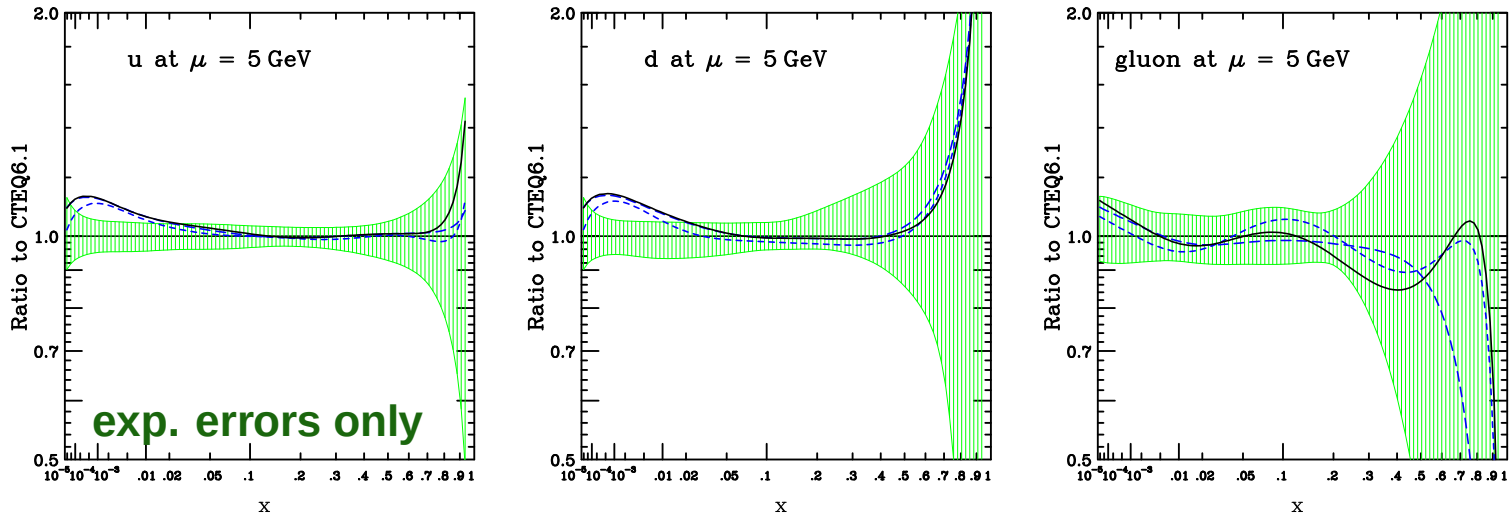

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Alekhin, Melnikov, Petriello (06)

Extension of fits (S.A., 02: DIS only) to consistent subset of DY data

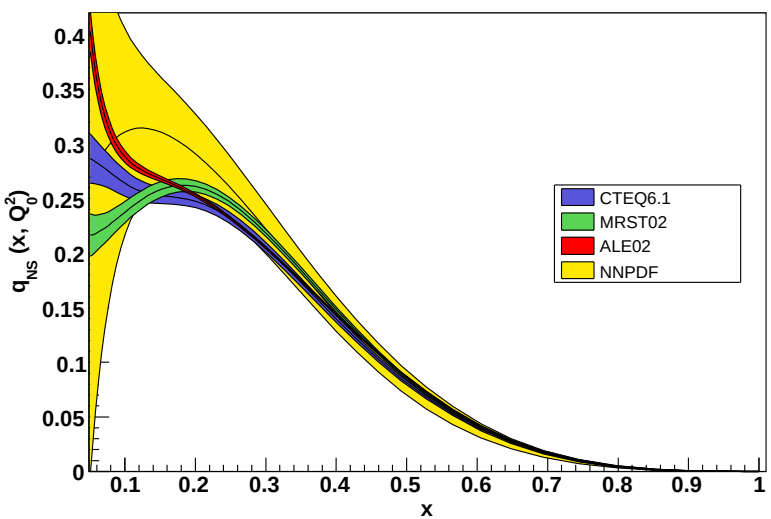
NNLO corr's to Drell-Yan cross sections crucial; preferred α_s low

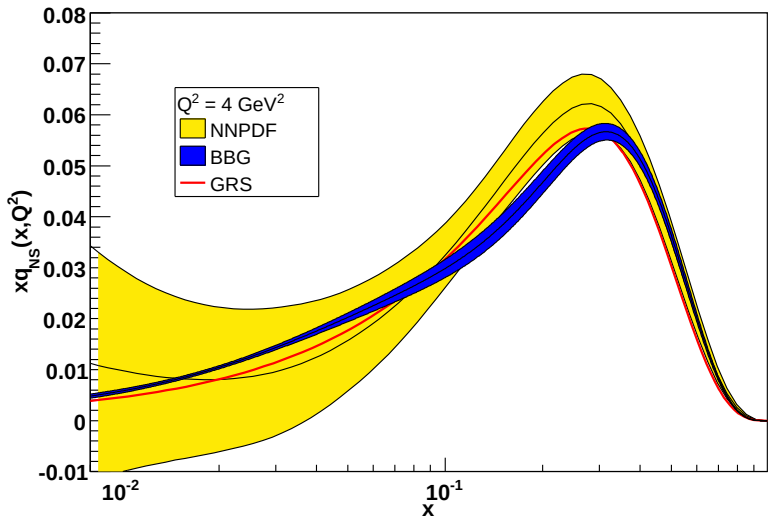
Highlights of recent parton analyses (II)

NNPDF, L. Del Debbio et al (07)

Hybrid evolution combining advantages of N- and x-space

Non-singlet (so far); neural networks to avoid parametrization bias





Width diff. at, e.g., $x \approx 0.2$ looks too big for param. bias – data sets?

Highlights of recent parton analyses (II)

NNPDF, L. Del Debbio et al (07)

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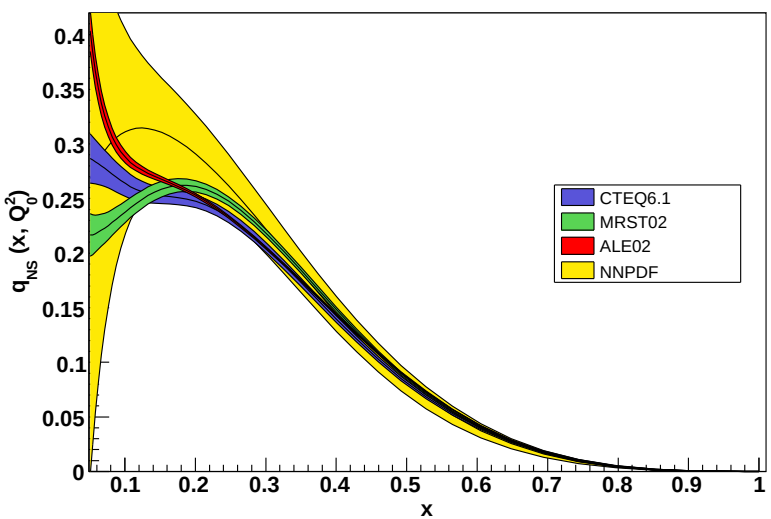
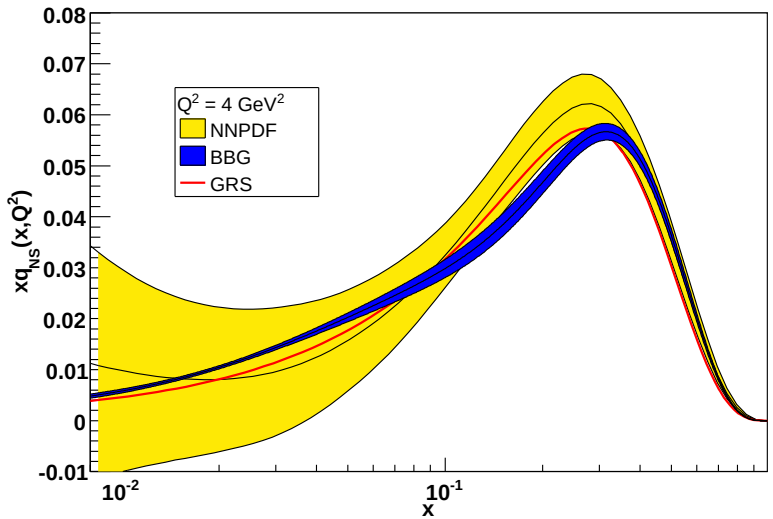




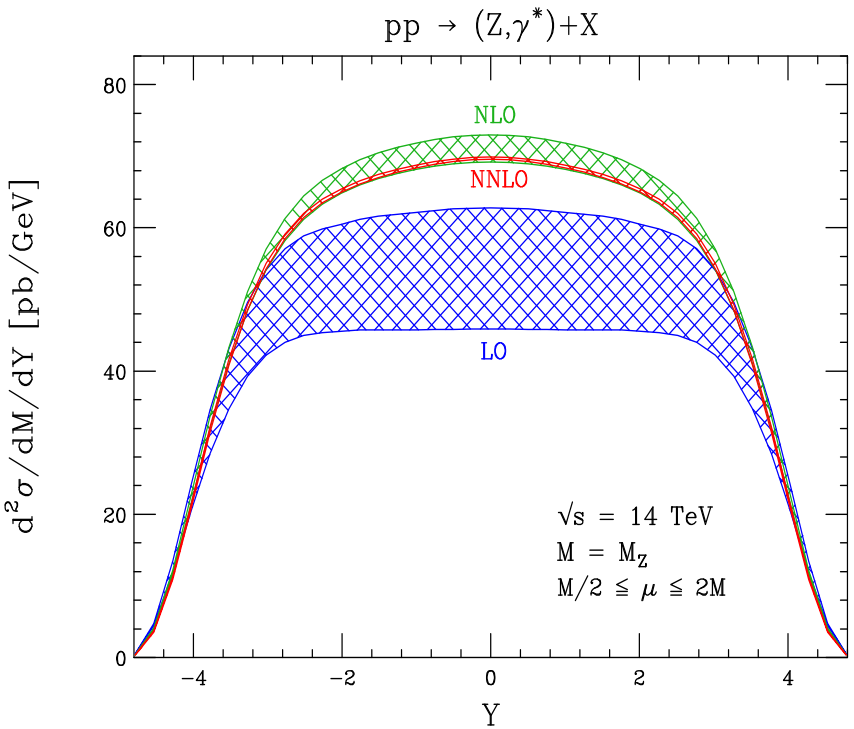
Width diff. at, e.g., $x \approx 0.2$ looks too big for param. bias – data sets?

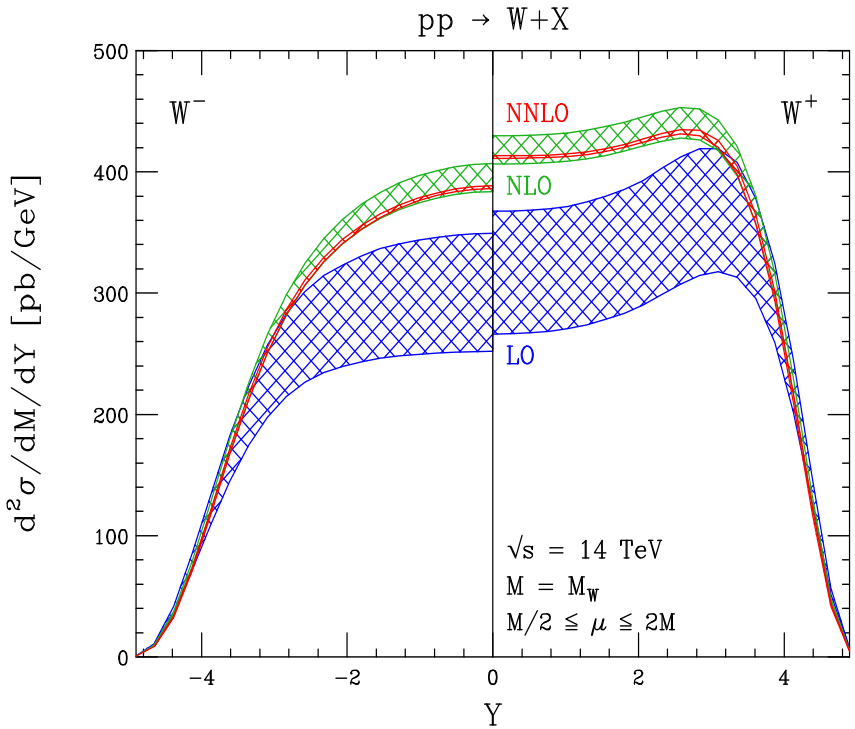
Blümlein, Böttcher, Guffanti (06)

Global non-singlet DIS analysis with determination of α_s to N^{3}LO

$\alpha_s(M_Z) = 0.1134{,}41 \pm 0.002$ at N^{2,3}LO, consistent with AMP (06)

Constraints of LHC gauge-boson production





diff. $\hat{\sigma}_{\mathrm{NNLO}}$: Anastasiou, Dixon, Melnikov, Petriello (03)

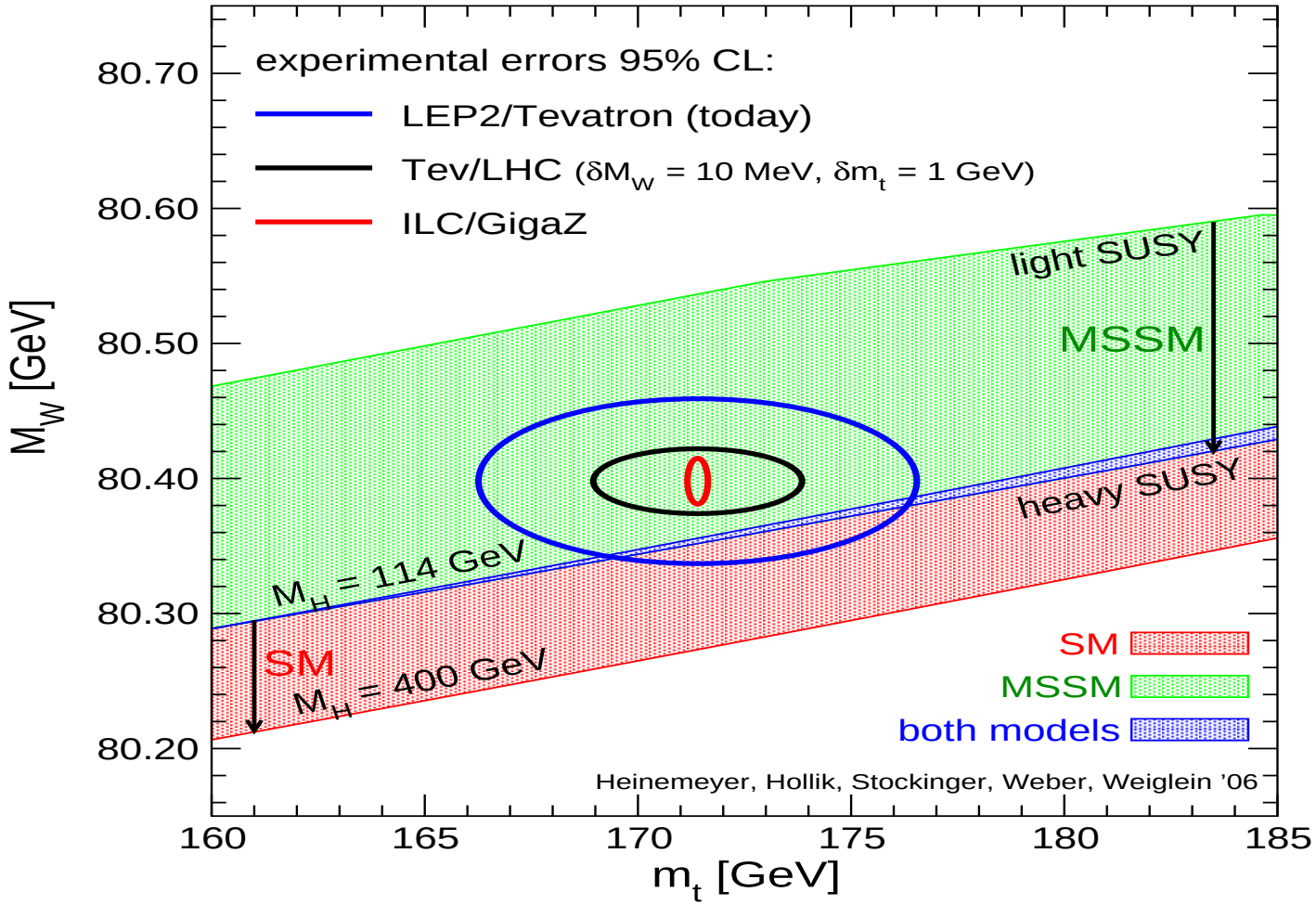
'Gold-plated' processes: NNLO perturbative accuracy better than 1%

$\Rightarrow$ use to improve upon pre-LHC determinations of parton densities

A. Cooper-Sarkar, M. Klein, . . . (this workshop)

Instead of a summary: W-mass at the LHC

M_W as function of τ_μ, M_Z, ... can discriminate between theories



Precision QCD and partons required to make the black ellipse happen