Compatibility of phenomenological dipole cross sections with the Balitsky-Kovchegov equation

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based on: hep-ph/0701219

Outline

Situation:
Successful description of various data within the dipole model
Question:
Are the implied dipole cross-sections comparable with nonlinear BK evolution

1. **The dipole cross section**
   - Parameterizations and geometric scaling (violation)
   - Phenomenology
2. **The BK equation**
   - The solution
   - Definition of the saturation scale
3. **The anomalous dimension γ**
   - In momentum space
   - In coordinate space

1. The dipole cross section

- HERA data on structure function F_2 at low x ($x \lesssim 0.01$) quite well described by [Golec-Biernat, Wüsthoff]

$$N_{\mathrm{GBW}}(r, x) = 1 - \exp\left[-\frac{1}{4} r^2 Q_s^2(x)\right]$$

  – r denotes the transverse size of the dipole
  – x dependence of the saturation scale:

$$Q_s(x) = 1\,\mathrm{GeV}\left(\frac{x_0}{x}\right)^{\lambda/2}, \text{ where } x_0 \simeq 3 \times 10^{-4} \text{ and } \lambda \simeq 0.3$$

  Consistent with NLO BFKL evolution and LO BK with running coupling
  e.g. [Müller & Triantafyllopoulos, 2002]

- Basic feature of GBW model: geometric scaling $N(rQ_s) \Rightarrow F_s(Q^2/Q_s^2(x))$

- **But** more precise data require at large Q^2 scaling violating modifications
  e.g. by taking DGLAP evolution into account [Bartels et al 2002], [Gotsman et al 2002]

Geometric scaling violation

- Theoretical implications from evolution equations
  - Saturation regime $Q^2 < Q_s^2(x)$: geometric scaling expected
  - Above Q_s: a growing region $Q_s^2(x) < Q^2 < Q_{gs}^2$ where scaling holds approx.

- Scaling violation can be introduced by modifying the GBW model ($\gamma = 1$):

$$N_{\text{pheno}}(r, x) = 1 - \exp\left[-\frac{1}{4}(r^2 Q_s^2(x))^{\gamma(r,x)}\right]$$

  - **Small $\boldsymbol{r}$**: BFKL limit is recovered and γ is related to the anom. dimension:

$$N(r, x) \sim x\, g(x, \mu(r)^2) \quad \Rightarrow$$

$$\frac{d\, x\, g(x, \mu(r)^2)}{d\, \log x_0/x} \sim \gamma(r, x)\, x\, g(x, \mu(r)^2)$$

- From linear BFKL evol. with satur. bound. condition: $\gamma(r = 1/Q_s) = 0.628 \equiv \gamma_s$
  - Note, not from complete non-linear BK evolution

- **Expectations on $\boldsymbol{\gamma(r,x)}$**
  - Fixed x and $r_t \to 0$: $\gamma \to 1$ to reproduce the limit $N \sim r^2$
  - At Q_s: γ is a constant $\gamma(r_t = 1/Q_s, x) = \gamma_s \Rightarrow$ geometric scaling for N
  - $\gamma_s \simeq 0.628$: the BFKL saddle point with sat. bound. cond.
    e.g. [Iancu et al 2002, Mueller et al 2002, Triantafyllopoulos 2002]

- $\Rightarrow$ A good description of hadron production in $d + Au$ collisions at **RHIC** with [Dumitru et al 2006]

$$\gamma(r,x) = \gamma_s + (1-\gamma_s)\frac{\log(1/(r^2 Q_s^2(x)))}{\lambda y + d\sqrt{y} + \log(1/(r^2 Q_s^2(x)))}, \quad y = \log x_0/x$$

- Ansatz $N(r,x) = 1 - \exp[-1/4(r^2 Q_s^2(x))^{\gamma(r,x)}]$ with similar forms for γ used in various models (also in DIS)

- **Question we want to address**:
  Are these expectations compatible with the numerical solution of the BK equation?

2. The BK equation

- Mean-field approximation: dipole evolution described by the BK equation [Balitsky 1995, Kovchegov 1999]:

$$\frac{\partial N(r,y)}{\partial y} = \frac{\bar{\alpha}_s}{2\pi}\int \frac{d^2\boldsymbol{z}\; r^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}$$

$$[N(|\boldsymbol{x}-\boldsymbol{z}|,y) + N(|\boldsymbol{z}-\boldsymbol{y}|,y) - N(r,y) - N(|\boldsymbol{x}-\boldsymbol{z}|,y)N(|\boldsymbol{z}-\boldsymbol{y}|,y)]$$

$$\bar{\alpha}_s = \alpha_s\frac{N_c}{\pi}\,, \qquad r = |\boldsymbol{x}-\boldsymbol{y}|\,, \qquad y = \log x_0/x$$

  – Evolution depends effectively on combination $Y = y\,\bar{\alpha}_s$

- **Solution** taken from a program [Enberg et al 2005] in terms of the Fourier transf.

$$\mathcal{N}(k,y) \equiv \int \frac{d^2\boldsymbol{r}}{2\pi\, r^2}\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} N(r,y) = \int_0^\infty dr\, r\, \mathrm{J}_0(kr)\, N(r,y)$$

  – In terms of $\mathcal{N}$ the BK equation reads

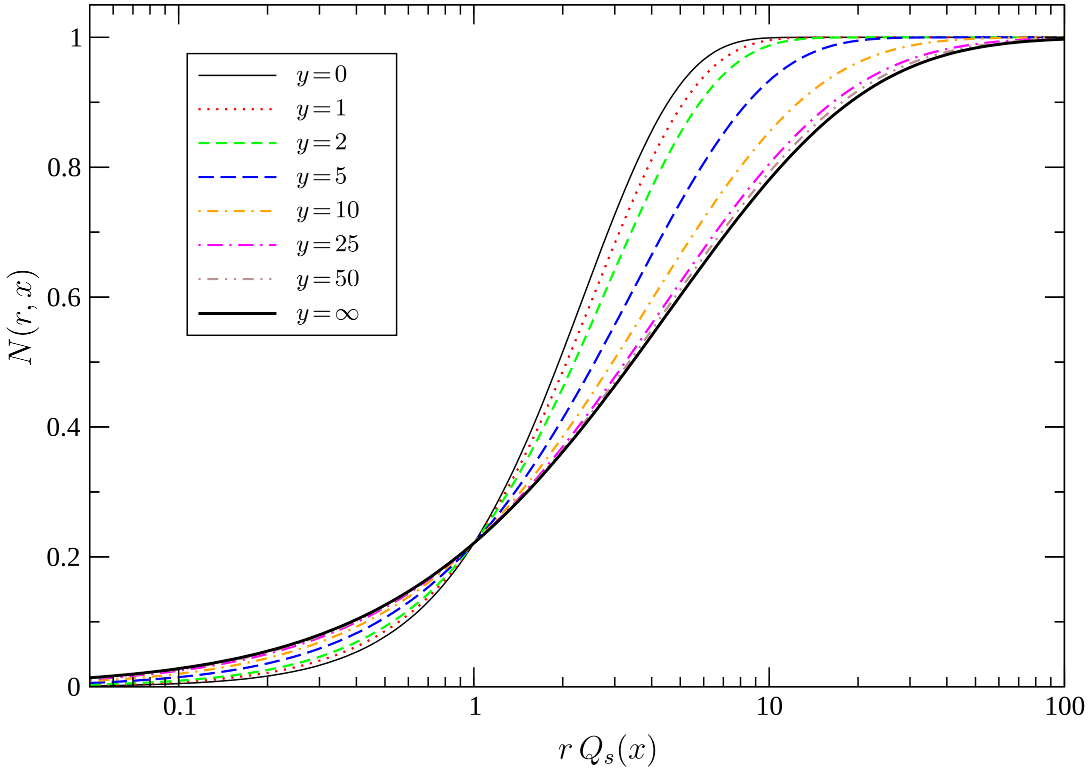
$$\partial_Y \mathcal{N} = \underbrace{\chi(-\partial_L)}_{\text{BFKL kernel}}\,\mathcal{N} - \mathcal{N}^2\,, \qquad L = \log(k^2/k_0^2)$$

Solution of the BK equation and the definition of Q_s

- **First step:** calculating $N(r, x)$ via a Fourier transform of $\mathcal{N}(k, x)$

$$N(r,x) = r^2 \int \frac{d^2 k}{2\pi}\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\, \mathcal{N}(k,x) = r^2 \int_0^\infty dk\, k\, J_0(kr)\, \mathcal{N}(k,x)$$

  - **Second Step:** Fixing the saturation scale: Ansatz $N(r) = 1 - \exp[-1/4(r^2 Q_s^2)^\gamma]$ requires $N(r = 1/Q_s) \approx 0.22$
  - As usual: $\log Q_s^2 \propto y = \log x_0/x$
  - $y \to \infty$: geometric scaling $N(r, x) \to N_\infty(r Q_s(x)))$



- For finite rapidities $y = \log x_0/x$ a significant scaling violation $\Rightarrow \gamma$ **is not constant!**

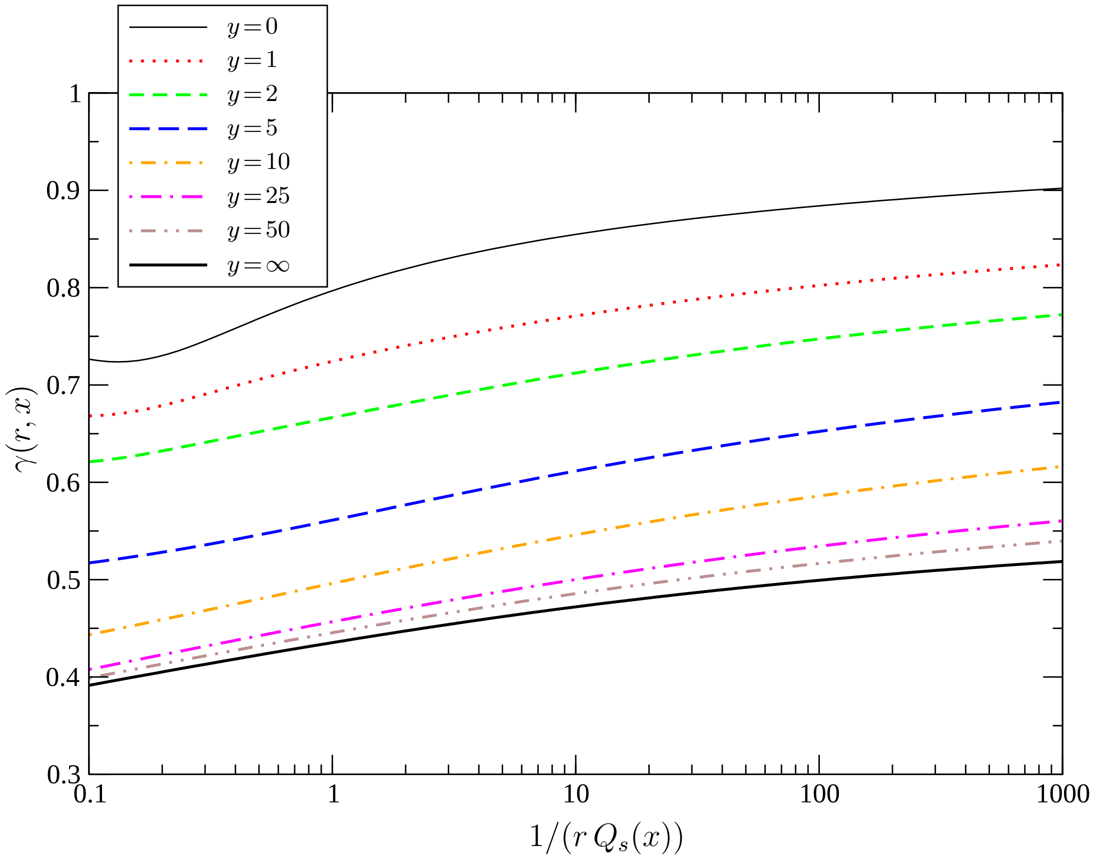
3. The anomalous dimension $\gamma(r, y)$

- Procedures to calculate $Q_s(x)$ and $N(r,x) \stackrel{!}{=} 1 - \exp[-\frac{1}{4}(r^2 Q_s^2)^\gamma]$ are now given

  $\Rightarrow \qquad \gamma(r,x) = \log\left[\log\left[1/(1-N(r,x))^4\right]\right] / \log[r^2\, Q_s^2(x)]$

- **Remarkable differences from the discussed expectations**

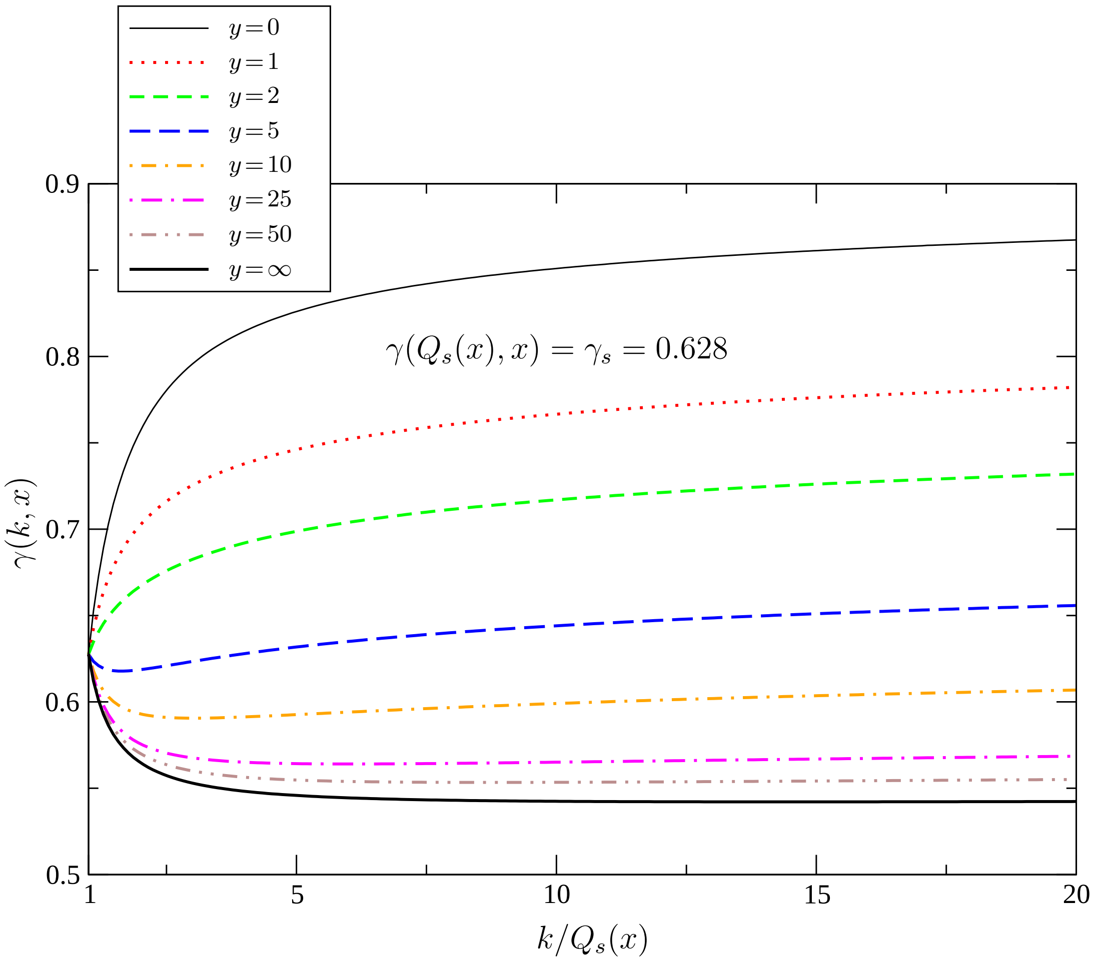
  - **Finite** y: $\gamma(r\!=\!1/Q_s, x) \neq \mathrm{const}$
    $\Rightarrow$ scaling violat. in sat. region
  - **Asympt.** $y = \log x_0/x \to \infty$:
    $\gamma(r,x) = \gamma_\infty(rQ_s) + \mathcal{O}(1/y)$
  - $\gamma_\infty \approx 0.44$ at Q_s is ≤ 0.628
  - $r \to 0$ for finite y: $\gamma \to \gamma_0 = 1$
  - Asymptotic y **and** $1/(rQ_s)$:
    $\gamma_\infty(rQ_s) \to 0.628 = \gamma_s$



$\gamma(k, x)$ in momentum space

- Essential part of former phenomel. approach: $\gamma(r) \approx \gamma(\langle r \rangle)$ where $\langle r \rangle \sim 1/k$
  $\Rightarrow \gamma$ depends effectiv. on $k \Rightarrow N(r, x; \gamma)$ is not only a Fourier transf. of $\mathcal{N}(k, x)$
- $\Rightarrow$ New freedom in fixing Q_s, e.g. $\mathcal{N}(k = Q_s(x)) = \text{const} \Rightarrow \boldsymbol{\gamma}$ **is const. at** $\boldsymbol{Q_s}$**!**

  - Obvious choice $\mathcal{N}(Q_s) \approx 0.19$
    $\Rightarrow \gamma(Q_s(x), x) = 0.628$
  - Small y: γ rises monot. with k
  - Larger y: different then DHJ:
    γ drops towards smaller values
  - $y = \log x_0/x \to \infty$:
    $\gamma(r, x) = \gamma_\infty(rQ_s) + \mathcal{O}(1/y)$
  - For $k < Q_s(x)$:
    with the given Ansatz no description of $\mathcal{N}(k)$ possible

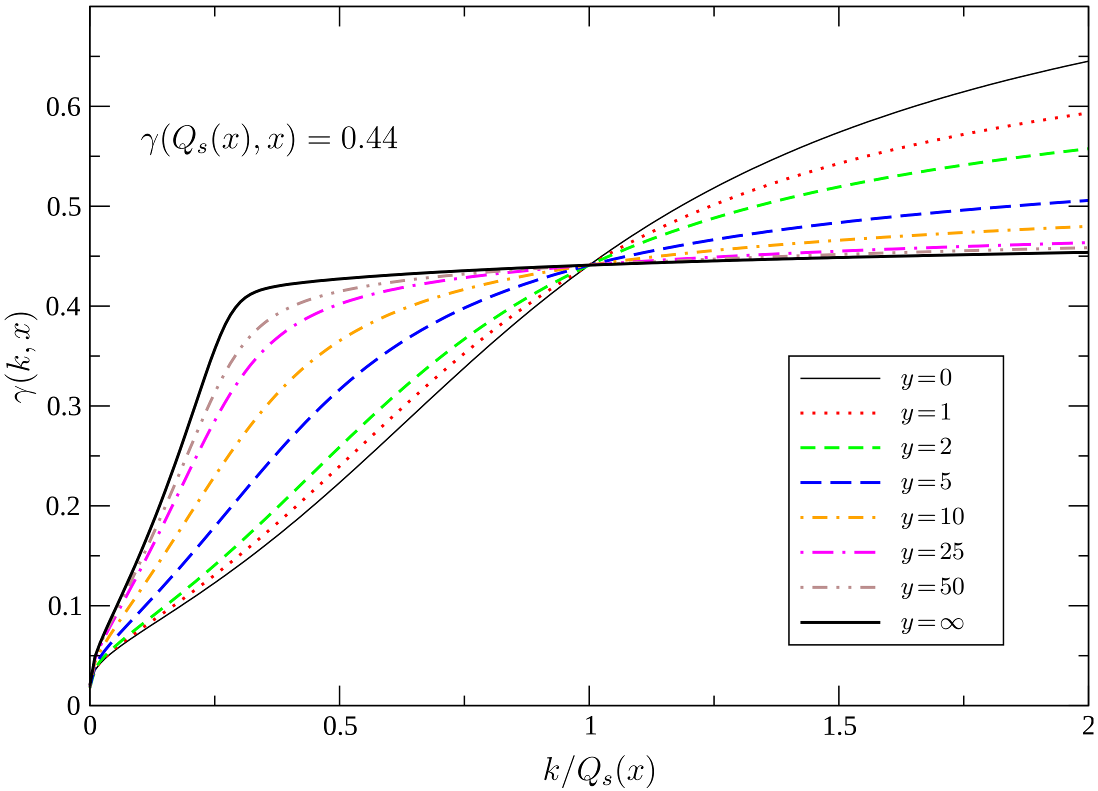


$\gamma(k, x)$ for lower γ_s

- Possible reason for these problems: γ tends towards smaller values then 0.628
  $\Rightarrow$ Fix $\gamma(x, k)$ at $k = Q_s$ to be smaller

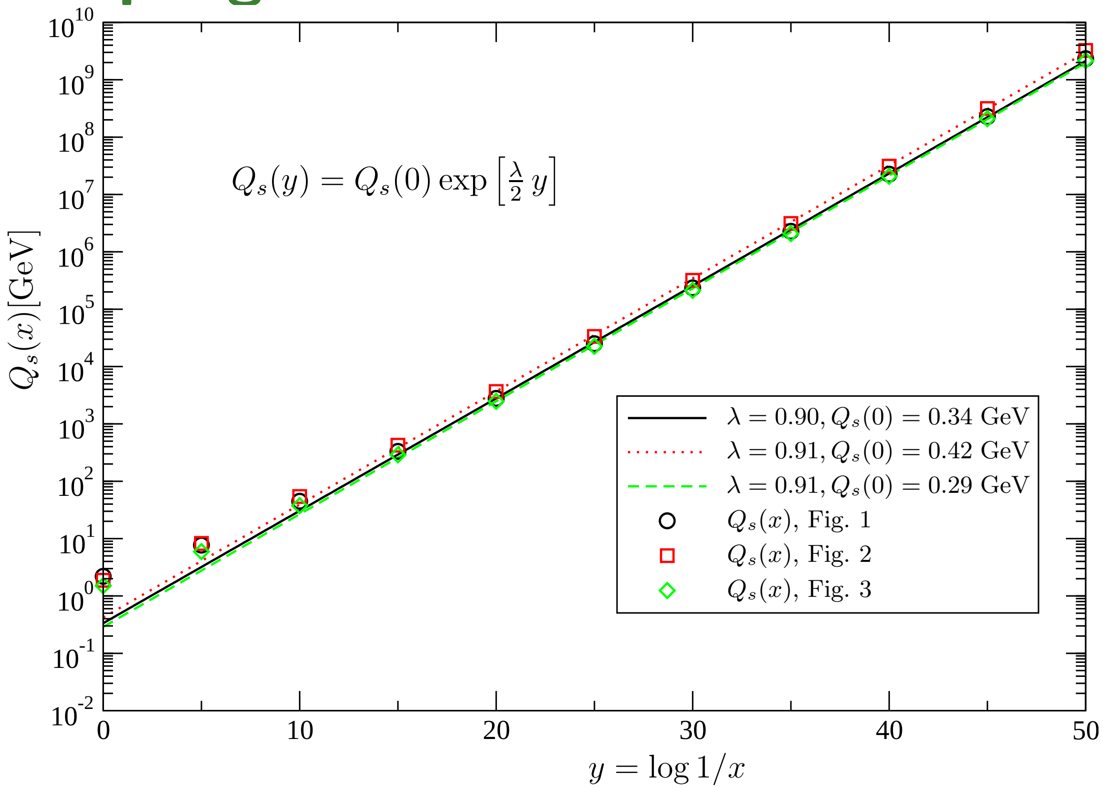
- **Implied choice** from investigating $\gamma(r, x)$:
  $\gamma(k = Q_s(x), x) = \lim_{x\to 0} \gamma(r = 1/Q_s(x), x) = 0.44 \Rightarrow \mathcal{N}(Q_s) = 0.28$

  - γ rises for all x with $k/Q_s(x)$
  - γ exists also below Q_s
  - $k > Q_s$ fit similar to DHJ:
    $\gamma(k, x) = 0.44 + 0.56 \frac{\log(k^2/Q_s^2)}{\lambda y + d\sqrt{y} + \log(k^2/Q_s^2)}$, $d \approx 3, \lambda \approx 0.9$
  - $y = \log x_0/x \to \infty$:
    $\gamma(r, x) = \gamma_\infty(rQ_s) + \mathcal{O}(1/y)$



saturation scale , running coupling case and initial conditions

- Definitions of Q_s are consistent with each other and with usual expectations $\log Q_s^2 \propto y$



- The running coupling case was also investigated
  - As expected, the saturation scale is signif. smaller $\log Q_s^2(y) \propto \sqrt{y}$
  - $\gamma(rQ_s(y), y)$ and $\gamma(k/Q_s(y), y)$ are almost unchanged
- Initial conditions at $y = \log x/x_0 = 0$:
  - $\mathcal{N}(k, x = x_0)$ inspired by the MV model were used $\Rightarrow \gamma \to 1$ for $r \to 0$
  - In general: $\gamma_\infty(rQ_s)$ is independent of i.c. as long as $\gamma(x = x_0) < \gamma_s \approx 0.628$

Conclusion & Outlook

- Finite $y = \log x_0/x$: solut. of the BK eq. does not show exact geometric scaling
  - Therefore $\gamma(r,x)$ is not a function of $rQ_s(x)$ exclusively
  - In particular not even a constant at the saturation scale ($r = 1/Q_s$)
- $y \to \infty$ geometric scaling is recovered: $\gamma(r,x) \to \gamma_\infty(rQ_s(x))$
  - γ_∞ reads 0.44 at $r = 1/Q_s$
  - Only for $y \to \infty$ and $rQ_s \to 0$, $\gamma_\infty \approx \gamma_s \approx 0.628$ is recovered
- **No conflict** to theo. consid. since only for small r, γ_{pheno} and γ_{BFKL} are equal
  But used parameterizations of $\gamma(r,x)$ or $N(r,x)$ in the models are questionable
- $\gamma(r,x) \to \gamma(1/k,x)$ leads to a solution with a fixed value $\gamma(k = Q_s, x)$
  - Usual choice $\gamma(k = Q_s, x) = \gamma_s = 0.628$ yields some unwanted features
  - Keeping $\gamma(k = Q_s, x)$ fixed at a smaller value, e.g. 0.44, seems more suitable
- **In the future**, modification of e.g. the DHJ model compatible with BK equation and the data can be considered