On Diffraction and JIMWLK evolution

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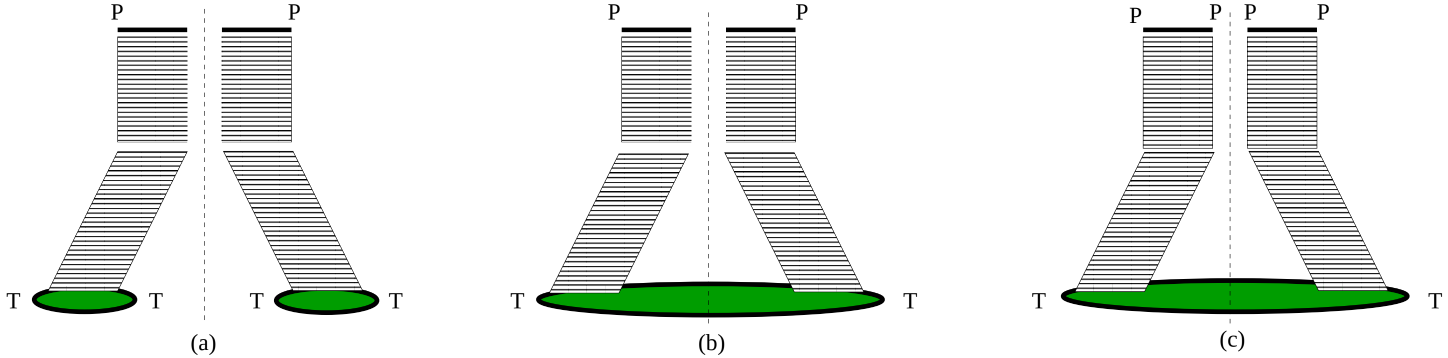
based on: Alex Kovner, M.L., Heribert Weigert, Phys.Rev.D74:114023,2006

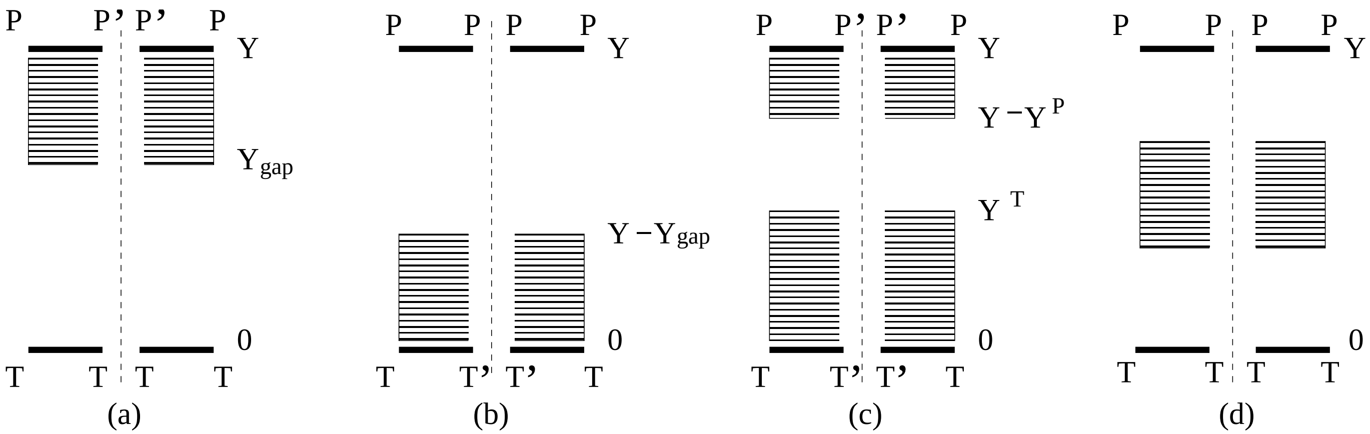
Alex Kovner and M.L., JHEP 0611:083,2006

- Develop general formalism to address semi-inclusive processes at high energies and including multiple rescatterings. Inclusive multiple gluon production is in my next talk
- High energy diffraction beyond the dipole (large N_c and target factorization) approximation



(a) Projectile diffraction with target scattered elastically Yu. Kovchegov and G. Levin; 2001; (b) Projectile diffraction with target diffracting in a small rapidity interval; (c) Projectile scatters elastically.

- HE diffraction with multiple gaps. Evol. eqs. with respect to total rapidity and gap(s).



The results are complex. Let us focus on formalism instead ...

High energy evolution of hadronic wavefunction

Hadron wave function in the gluon Fock space

$$|\Psi\rangle \;=\; \Psi[a_i^{\dagger a}(x)]|0\rangle \qquad\qquad |\Psi\rangle \;=\; |v\rangle$$

The evolved wave function

$$|\Psi_{in}\rangle \;=\; \Omega_Y(\rho,\, a)\,|v\rangle\,; \qquad\qquad |v\rangle \;=\; |v\rangle\,\otimes\,|0_a\rangle$$

Gluon cloud operator in the dilute limit

$$C_Y \;\equiv\; \Omega_Y(\rho\;\rightarrow\;0) \;=\; \mathrm{Exp}\left[i\int d^2z\, b_i^a(z)\int_\Lambda^{e^Y\Lambda}\frac{dk^+}{\pi^{1/2}|k^+|^{1/2}}\left[a_i^a(k^+,z)\;+\;a_i^{\dagger a}(k^+,z)\right)\right]\,.$$

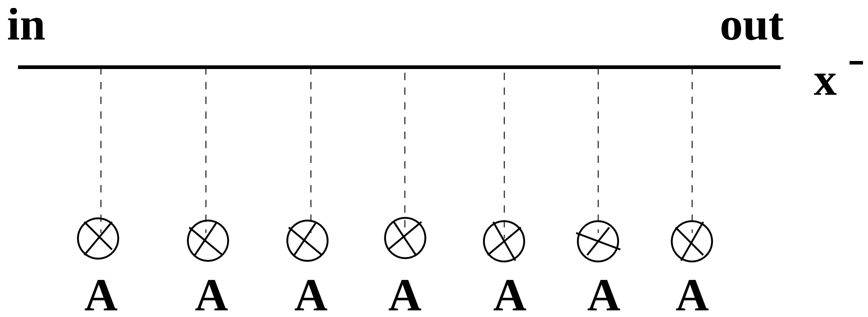
The classical WW field

$$b_i^a(z) \;=\; \frac{g}{2\pi}\int d^2x\frac{(z-x)_i}{(z-x)^2}\;\rho^a(x)$$

High energy scattering

The system emerges from the collision region with the wave function

$$|\Psi_{out}\rangle \;=\; \hat{S}\,|\Psi_{in}\rangle \;=\; \hat{S}\,\Omega_Y\,|v\rangle$$



Eikonal scattering for fast gluons
$\alpha_t \;=\; A^+$

$$S(x) \;=\; \mathcal{P}\,\exp\left\{ i \int dx^- \, T^a \, \alpha_t^a(x, x^-) \right\} \,.$$

Evolution of the diagonal element of the S-matrix operator $\Sigma^P \equiv \langle \Psi_{out} | \Psi_{in} \rangle$

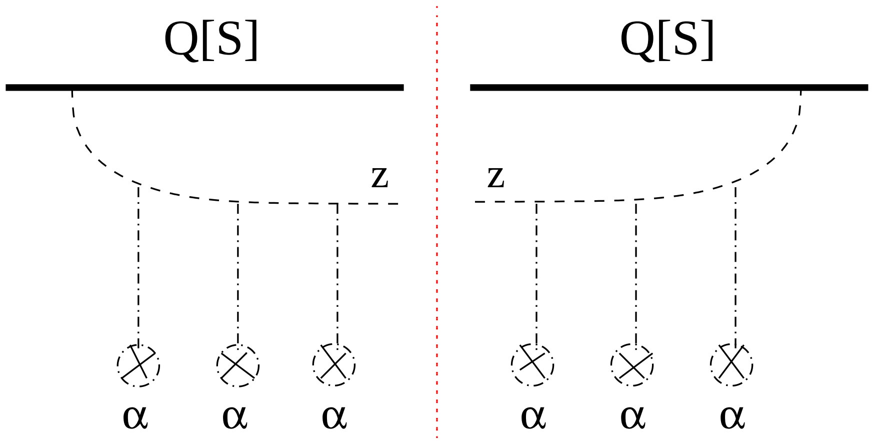
$$\partial_Y \Sigma^P = - H^{JIMWLK} \Sigma^P \, ; \qquad\qquad H^{JIMWLK} = \int_z Q_i^a(z) \, Q_i^a(z)$$

The gluon production amplitude

$$Q_i^a(z) = g \int_x \frac{(x-z)_i}{(x-z)^2} \left[J_L^a(z) - S^{ab}(x) \, J_R^b(x) \right]$$

The generators of the left/right color rotations

$$J_R^a(x) = - \mathrm{tr} \left\{ S(x) \, T^a \frac{\delta}{\delta S^\dagger(x)} \right\} , \qquad\qquad J_L^a(x) = - \mathrm{tr} \left\{ T^a \, S(x) \frac{\delta}{\delta S^\dagger(x)} \right\}$$



Semi-inclusive reactions

The system emerges from the collision at $t = 0$ and keeps evolving to the asymptotic time $t \to +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle \hat{\mathcal{O}} \rangle \; = \; \langle \, v | \, \Omega_Y^\dagger \; (1 \; - \; \hat{S}^\dagger) \, \Omega_Y \; \hat{\mathcal{O}} \;\; \Omega_Y^\dagger \; (1 \; - \; \hat{S}) \, \Omega_Y \, | v \rangle$$

Useful trick (similar to Schwinger-Keldish formalism):
introduce one target (S) for the amplitude and another target ($\bar{S}$) for the conjugate:

$$\mathcal{O}_Y[S, \bar{S}] \; = \; \langle \, P_v | \, \Omega_Y^\dagger \, (1 \; - \; \hat{S}^\dagger) \, \Omega_Y \;\; \hat{\mathcal{O}} \;\; \Omega_Y^\dagger \; (1 \; - \; \hat{\bar{S}}) \, \Omega_Y \, | P_v \, \rangle$$

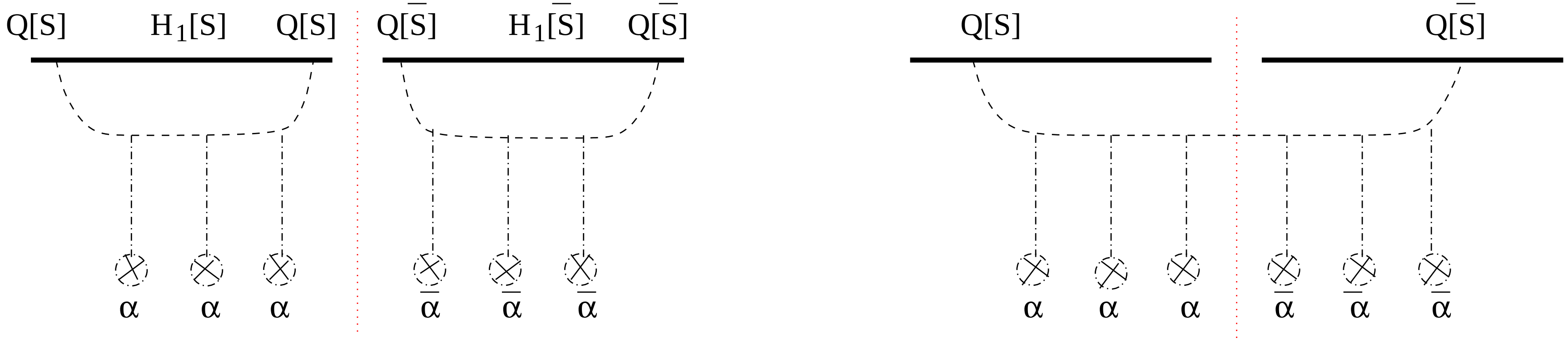
High energy evolution of the observable

$$\frac{d\mathcal{O}_Y[S, \bar{S}]}{dY} \; = \; \lim_{\Delta y \to 0} \frac{\mathcal{O}_{Y+\Delta y}[S, \bar{S}] \; - \; \mathcal{O}_Y[S, \bar{S}]}{\Delta y} \; = \; - \, H_3[S, \bar{S}] \; \mathcal{O}_Y[S, \; \bar{S}]$$

The Hamiltonian H_3 M. Hentschinski, H. Weigert and A. Schafer (2005)

$$H_3[S,\bar{S}] \;\equiv\; H_1[S] \;+\; H_1[\bar{S}] \;+\; 2\int_z Q_i^a(z,[S])\,Q_i^a(z,[\bar{S}])$$

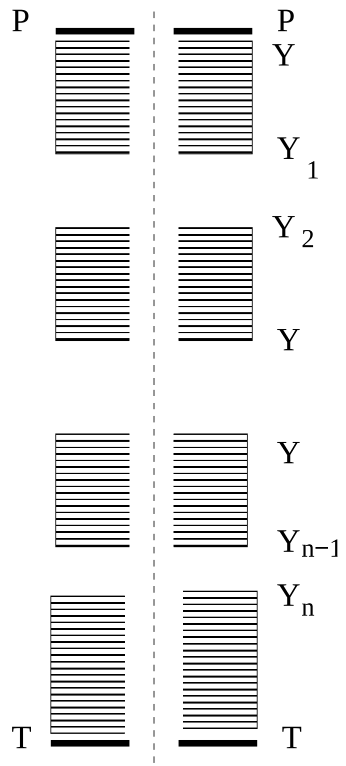
$$H_1[S] \;\equiv\; H^{JIMWLK}[S] \;=\; \int_z Q_i^a(z,[S])\,Q_i^a(z,[S])\,, \qquad\qquad H_2[S,\bar{S}] \;\equiv\; H_1[S] + H_1[\bar{S}]$$



$$U^3_{Y_1-Y_2} \;=\; Exp\,[-\,H_3\,(Y_1 \;-\; Y_2)] \qquad\qquad U^2_{Y_1-Y_2} \;=\; Exp\,[-\,H_2\,(Y_1 \;-\; Y_2)]$$

Formal solution for inclusive diffraction with multiple gaps and multiple rescatterings

$$\sigma^{diff} \sim \int DS\, D\bar{S}\, W^t[S]\, \delta(S-\bar{S})\, U^3_{Y_0-Y_n}\, U^2_{Y_n-Y_{n-1}} \cdots U^2_{Y_1-Y_2}\, U^3_{Y-Y_1}\, \Sigma^p[S,\bar{S}]$$



Things become less formal and more useful when passing to the dipole limit

Introduce dipole degrees of freedom.

$$s_{x,y} \;=\; \frac{1}{N}\, tr[S_F(x)\, S_F^{\dagger}(y)]\,;$$

However we need to remember that the factorization

$$\langle s(x,y)\, s(u,v)\rangle_T \;=\langle s(x,y)\rangle_T\, \langle s(u,v)\rangle_T$$

is not always valid. Very important in order to include target diffractive states

For processes involving transverse momentum transfer, quadrupole operator is needed

$$q_{x,y,u,v} \;=\; \frac{1}{N}\, tr[S_F(x)\, S_F^{\dagger}(y)\, S_F(u)\, S_F^{\dagger}(v)]$$

No other higher multiplet operators if the projectile at rest is made only out of dipoles!