

# Critical tests of unintegrated gluon distribution: $F_2^{c,b}$ and $F_L$

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## O U T L I N E

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2. SF with the  $k_T$ -factorization
3. Unintegrated gluon densities
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# 1. Introduction

The **SF**  $F_L(x, Q^2)$  is directly connected to **gluon** in proton. It is not equal zero in pQCD only.

However **pQCD leads to controversial results** still.

In the LO approximation  $F_L$  takes about 10-20% of the  $F_2$ .

The NLO QCD corrections are large and can be negative at small  $x$  and low  $Q^2$ .

The NNLO corrections for  $F_L$  restore positivity of  $F_L$  at low  $x$  (even with a negative gluon).

But as it was shown recently:

- The  $F_L$  exp. data at HERA seem to be inconsistent with some of the NLO theor. prediction (in particular the MRST) at small  $x$
- The BFKL effects significantly improve the description of the low  $x$  data when compared to a standard NLO  $\bar{MS}$ -scheme global fit
- NNLO global fits become better taking into account higher order terms involving powers of  $\ln(1/x)$

R.S. Thorne, hep-ph/0511351;  
C.D. White, R.S. Thorne, PR D74 (2006) 014002,  
PR D75 (2007) 034005.

⇒ We need **resummation procedure**.

It is known the BFKL effects are taken into account from the very beginning in the  $k_T$  **factorization** approach

S. Catani, M. Ciafaloni, F. Hautman,  
Nucl. Phys. B366 (1991) 135;  
J.C. Collins, R.K. Ellis, Nucl. Phys. B360 (1991) 3;  
E. Levin, M. Ryskin, Yu. Shabelski, A. Shuvaev,  
Sov. J. Nucl. Phys. 53 (1991) 657.

Applications of  $k_T$ -factorization are shown:

Bo Andersson et al. (Small  $x$  Collaboration),  
Eur. Phys. J. C25 (2002) 77,  
J. Andersen et al. (Small  $x$  Collaboration),  
Eur. Phys. J. C25 (2002) 77, C35 (2004) 67.

The  $k_T$ -factorization approach is based on BFKL

L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338;  
E.A. Kuraev, L.N. Lipatov and V.S. Fadin,  
Sov. Phys. JETP 44 (1976) 443, 45 (1977) 199;  
Ya.Ya. Balitzki and L.N. Lipatov,  
Sov. J. Nucl. Phys. 28 (1978) 822;  
L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904.

or CCFM

M. Ciafaloni, Nucl. Phys. B296 (1988) 49;  
S. Catani, F. Fiorani, G. Marchesini,  
Nucl. Phys. B336 (1990) 18;  
G. Marchesini, Nucl. Phys. B445 (1995) 49

gluon evolution equations which sum up the large logarithmic terms proportional to  $\ln(1/x)$  or  $\ln(1/(1-x))$  in the LLA.

In the framework of  $k_T$ -factorization approach the study of the longitudinal SF  $F_L$  began ten years ago

Catani and F. Hautmann, Nucl. Phys. B427 (1994) 475,  
S. Catani, hep-ph/9608310,

where the small  $x$  asymptotics of  $F_L$  has been evaluated using the BFKL results for the Mellin transform of the unintegrated gluon distribution and the longitudinal Wilson coefficient functions have been calculated analytically for the full perturbative series at asymptotically small  $x$  values.

Since we want to analyze SF data in a broader range at small  $x$  we use a more phenomenological approach in our analyses of  $F_2$  and  $F_L$  data:

B.Badelek, J.Kwiecinski and A. Stasto,  
Z. Phys. C74 (1997) 297.

A.V. Kotikov, A.V. Lipatov and N.Z.  
Eur. Phys. J. C26 (2002) 51.

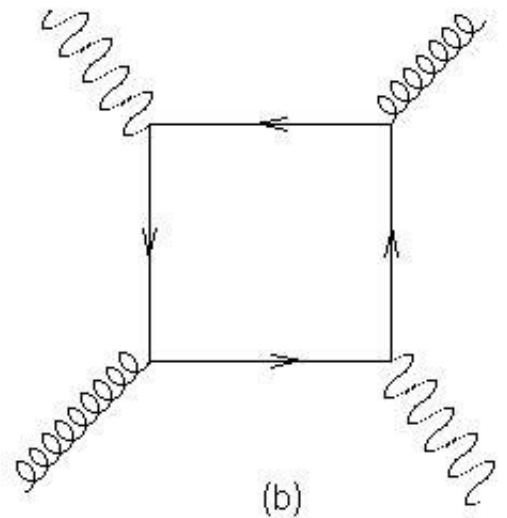
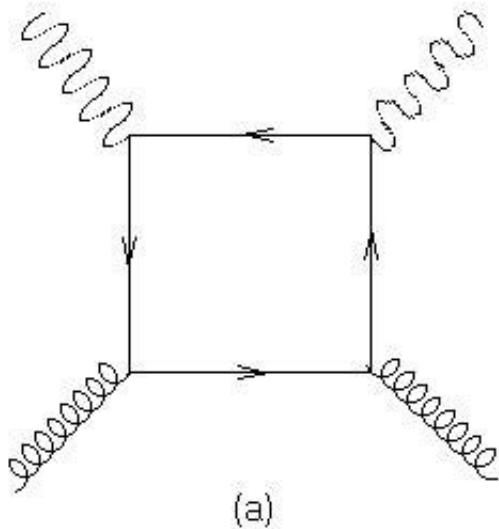
Using the  $k_T$ -factorization approach for the description of different SF at small  $x$  we hope to obtain additional information (or restrictions), in particular, about one of the main ingredients of  $k_T$ -factorization approach - **the unintegrated gluon distribution (UGD)**.

## 2. SF in the $k_T$ -factorization approach

In the  $k_T$ -factorization the SF  $F_{2,L}(x, Q^2)$  are driven at small  $x$  primarily by gluons and are related in the following way to the unintegrated distribution  $\Phi_g(x, k_T^2)$ :

$$F_{2,L}(x, Q^2) = \int_x^1 \frac{dz}{z} \int^{Q^2} dk_T^2 \sum_{i=u,d,s,c} e_i^2 \cdot \hat{C}_{2,L}^g(x/z, Q^2, m_i^2, k_T^2) \Phi_g(z, k_T^2),$$

The functions  $\hat{C}_{2,L}^g(x, Q^2, m_i^2, k_T^2)$  can be regarded as SF of the off-shell gluons with virtuality  $k_T^2$  (hereafter we call them *hard structure functions*). They are described by the sum of the quark box (and crossed box) diagram contribution to the photon-gluon interaction.



It is instructive to note that these diagrams are similar to those of the photon-photon scattering process. The corresponding QED contributions have been calculated many years ago:

V.N. Baier, V.S. Fadin, V.A. Khose,  
Sov. J. JETP 23 (1966) 104;  
V.G. Zima, Sov. J. Nucl. Phys. 16 (1973) 580;  
V.M. Budnev, I.F. Ginsburg, G.V. Meledin, V.G. Serbo,  
Phys. Report 15 (1975) 181.

To apply Eq. for SF at low  $Q^2$  we change the low  $Q^2$  asymptotics of the QCD coupling constant within hard structure functions. We apply the so called “freezing” procedure which can be done in the **hard** or the **soft** way.

In the **hard** case the strong coupling constant itself is modified: it is taken to be constant at all  $Q^2$  values less then some  $Q_0^2$ , i.e.  $\alpha_s(Q^2) = \alpha_s(Q_0^2)$ , if  $Q^2 \leq Q_0^2$ .

In the **soft** case the subject of the modification is the argument of the strong coupling constant: it is shifted  $Q^2 \rightarrow Q^2 + M^2$ . Then  $\alpha_s = \alpha_s(Q^2 + M^2)$ .

For massless quarks  $M = m_\rho$ , for massive quarks with mass  $m_Q$ ,  $M = 2m_Q$ .

We have used the **soft** version of freezing procedure.

To calculate the SF  $F_2^{c,b}$  and  $F_L(x, Q^2)$  we used:

- The hard SF  $\hat{C}_{2,L}^g(x, Q^2, m^2, k_T^2)$  from

A.V. Kotikov, A.V. Lipatov and N.Z.

Eur. Phys. J. C26 (2002) 51, C27 (2003) 219.

To note that at  $Q^2 \rightarrow 0$  there is full agreement of our results with the formulae for the photoproduction of heavy quarks from the paper

S. Catani, M. Ciafaloni and F. Hautmann

*Proc. of the Workshop on Physics at HERA, Hamburg, 1991, v.2, p.690.*

- Two unintegrated gluon distributions  $\mathcal{A}(x, \mathbf{k}_T^2, \mu^2)$  obtained in our previous paper

H.Jung, A.V. Kotikov, A.V. Lipatov and N.Z.

ICHEP'07, Moscow, hep-ph/0611093.



## Unintegrated gluon distributions

The unintegrated gluon distribution is determined by a convolution of the non-perturbative starting distribution  $\mathcal{A}_0(x)$  and CCFM evolution denoted by  $\bar{\mathcal{A}}(x, \mathbf{k}_T^2, \mu^2)$ :

$$x\mathcal{A}(x, \mathbf{k}_T^2, \mu^2) = \int dz \mathcal{A}_0(z) \frac{x}{z} \bar{\mathcal{A}}\left(\frac{x}{z}, \mathbf{k}_T^2, \mu^2\right),$$

where

$$x\mathcal{A}_0(x) = Nx^{-B_g}(1-x)^{C_g}(1-D_gx). \quad (1)$$

The parameters  $N, B_g, C_g, D_g$  of  $\mathcal{A}_0$  are determined in the fits to  $F_2$  and  $F_2^c$  independently.

## The measurement of $F_2(x, Q^2)$

H1 Collab., A. Adloff et al.

Eur. Phys. J. **C21** (2001) 33

was used in the range  $x < 0.005$  and  $Q^2 > 5 \text{ GeV}^2$  to determine "first" UGD.

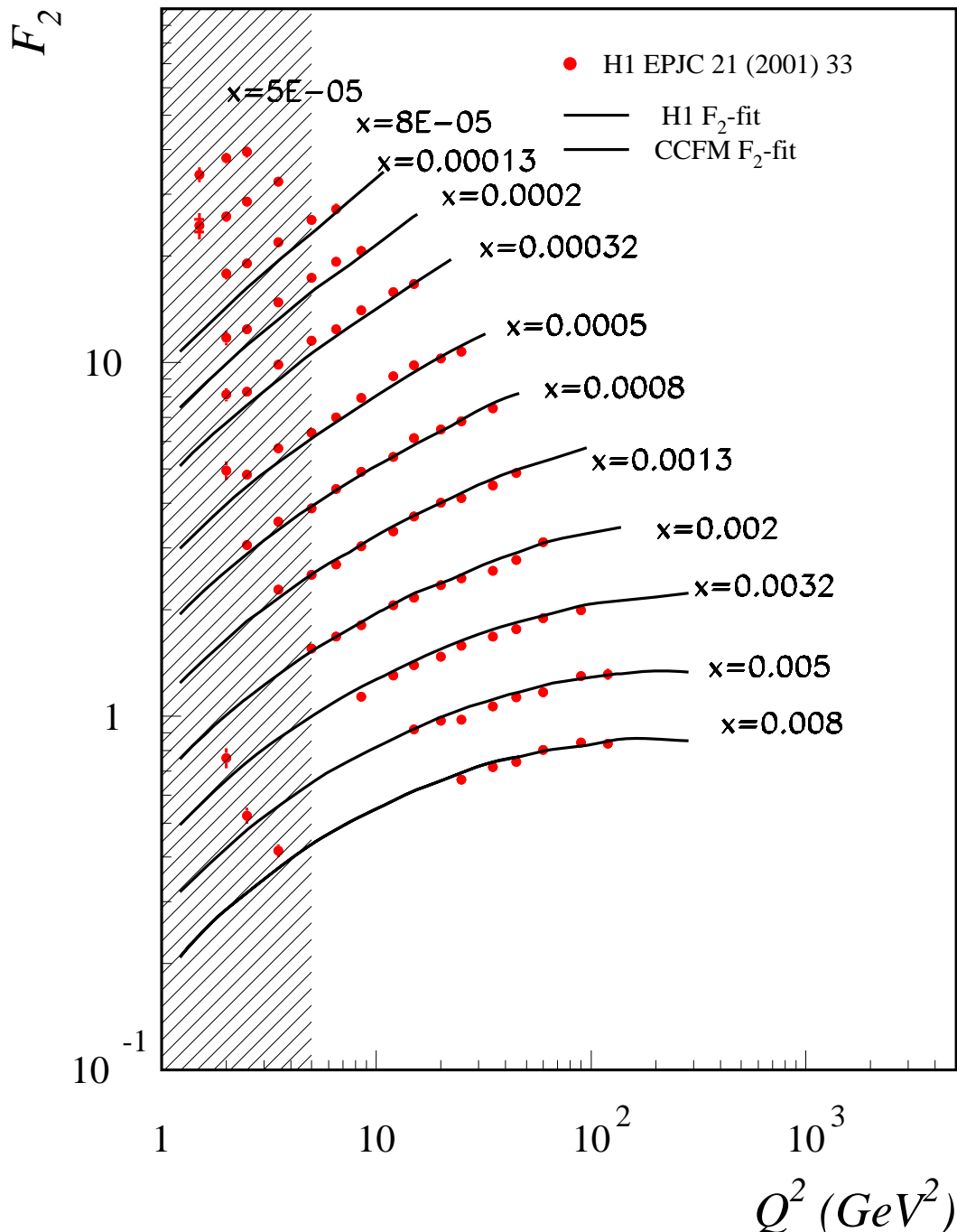
The parameters of  $\bar{A}_0$  were investigated separately. The starting distribution  $\bar{A}_0$  was parametrized at  $\bar{q}_0 = 1.2 \text{ GeV}$ .

The running coupling  $\alpha_s(\mu)$  was used in the 1-loop approximation in the region  $\mu > \bar{q}_0$ , and was kept fixed at  $\alpha(\bar{q}_0)$  at  $\mu < \bar{q}_0$ .

A acceptable fit to the measured  $F_2$  values was obtained with  $\chi^2/ndf = 118.8/61 = 1.83$  (compare to  $\chi^2/ndf \sim 1.5$  in the collinear approach at NLO).

The dependence on the choice of  $\Lambda_{QCD}$  was investigated also. A clear preference for  $\Lambda_{QCD}^{(4)} \sim 130 \text{ MeV}$  was observed.

Here the measurement is compared to the fit of the SF  $F_2(x, Q^2)$ :



The SF  $F_2(x, Q^2)$  as function of  $Q^2$  for different values of  $x$ . The shaded area shows the region which is not used in the fit.

The SF  $F_2^c$  is directly sensitive to the gluon density.

The measurements of

H1 Collab., A. Adloff et al.

Phys. Lett. B528 (2002) 199,

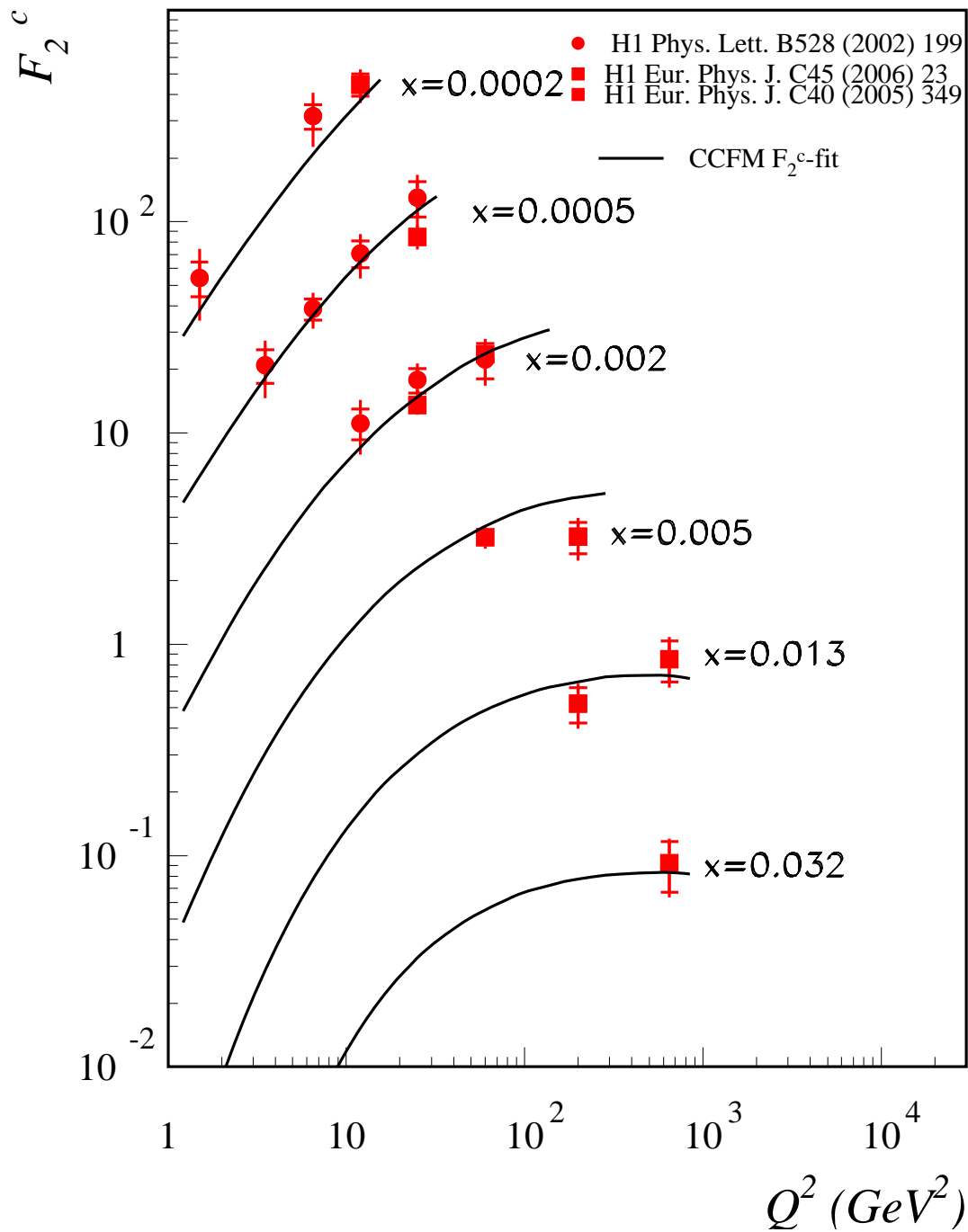
A. Aktas et al., Eur.Phys. J. C40 (2005) 349,

Eur.Phys. J. C45 (2006) 23,

were used to determine the UGD in the range  $Q^2 \geq 1.5 \text{ GeV}^2$ .

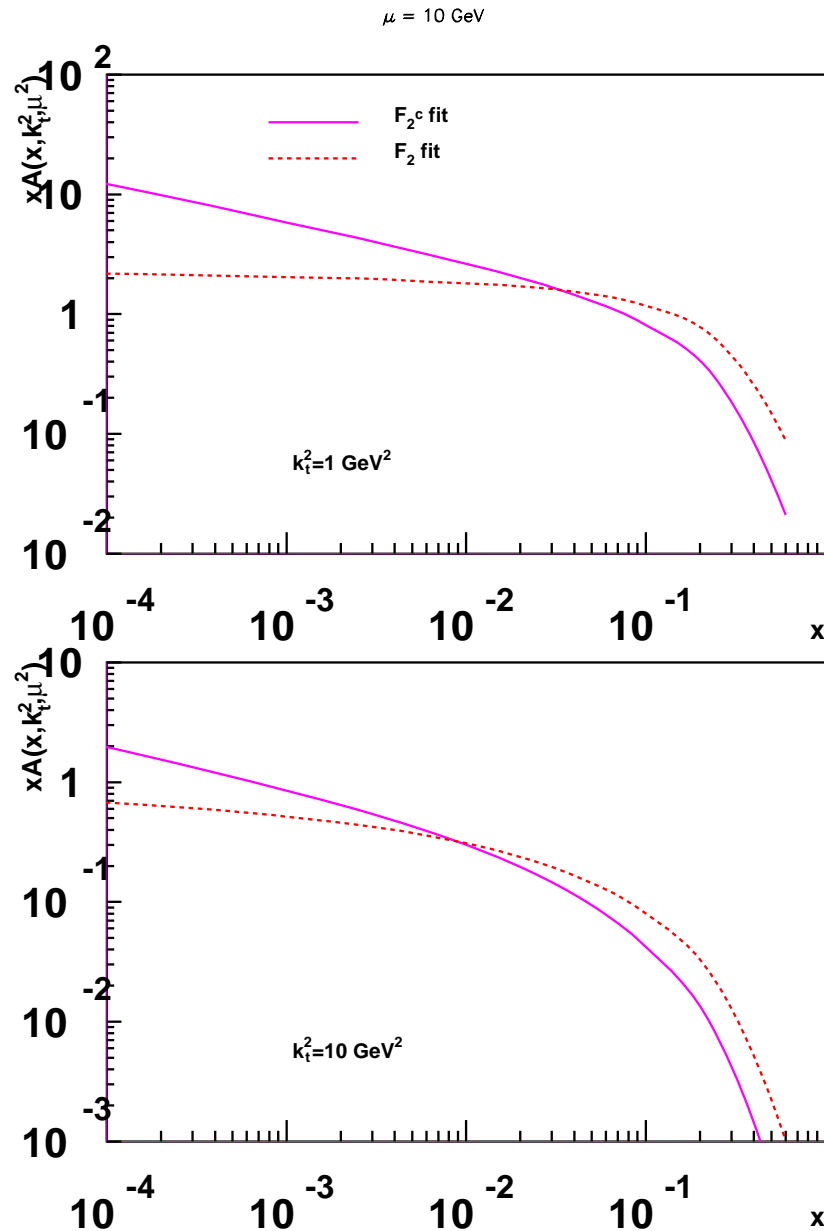
A acceptable fit to the measured  $F_2^c$  values was obtained with  $\chi^2/ndf = 18.8/20 = 0.94$  using statistical and systematic uncertainties.

Here the measurement is compared to the prediction of the SF  $F_2^c(x, Q^2)$  as obtained from the fit:



The SF  $F_2^c(x, Q^2)$  as function of  $Q^2$  for different values of  $x$ .

⇒ We have obtained two unintegrated gluon densities:

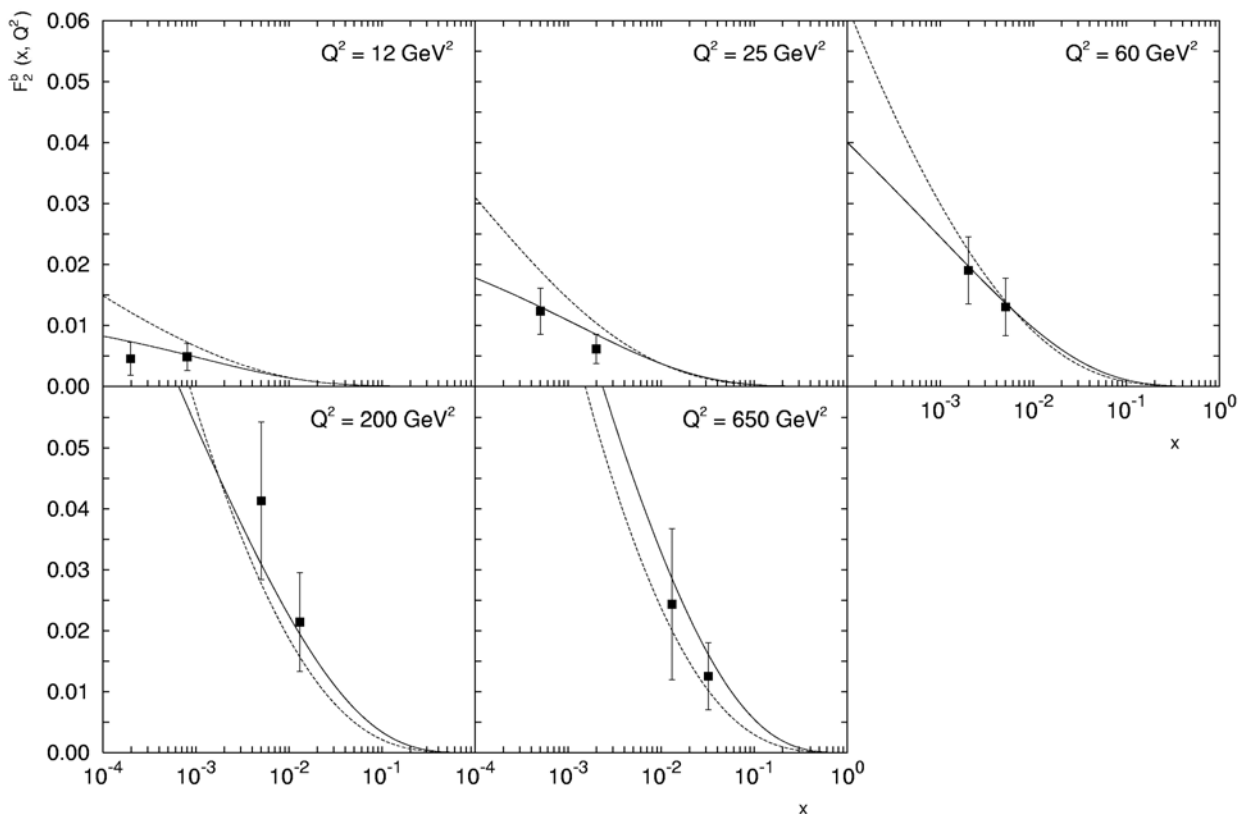


The small  $x$  behaviour of these UGD is very different. See also

H. Jung, talk in HFS working group.

## Numerical results

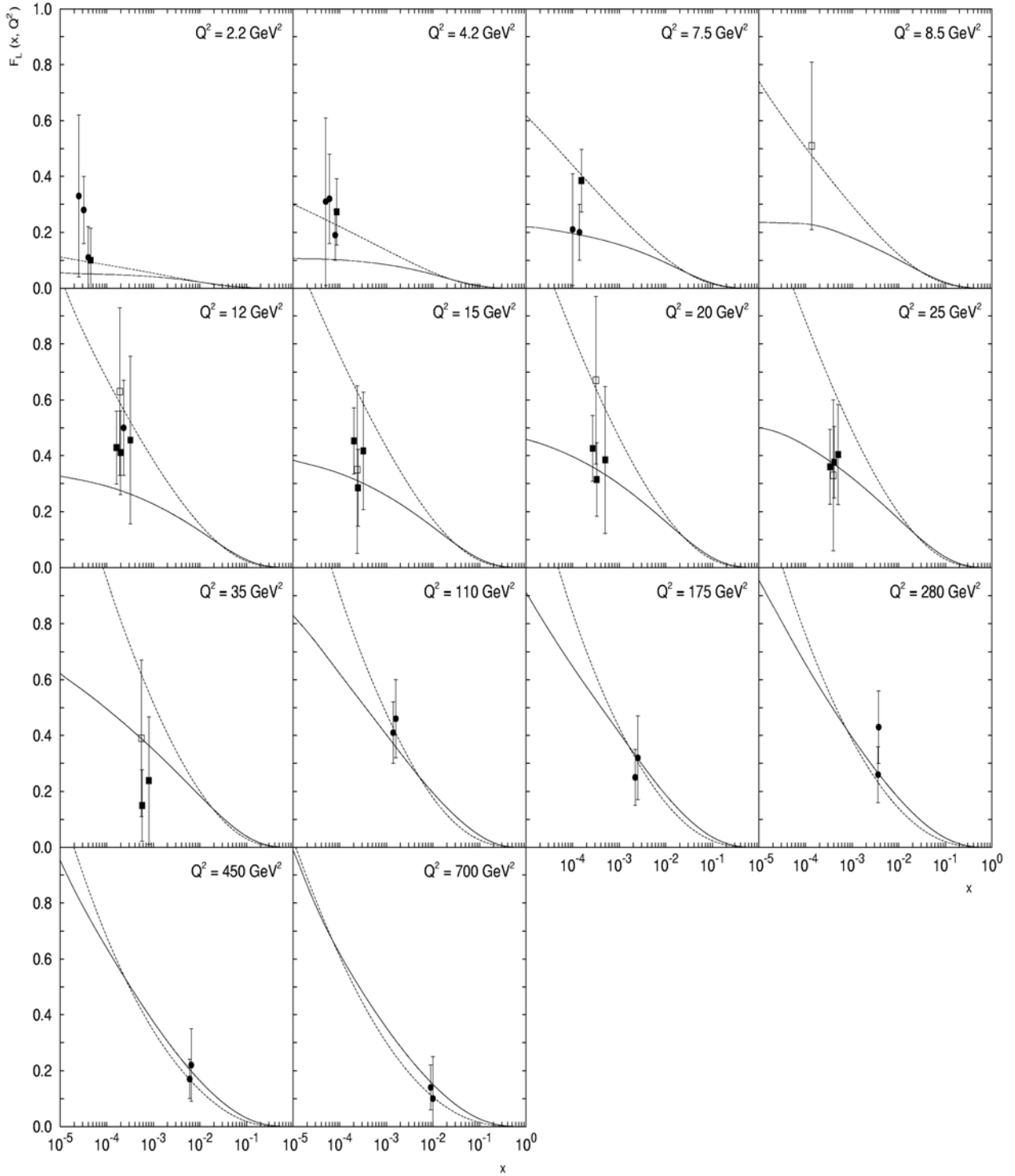
To calculate the SF  $F_2^b(x, Q^2)$  and  $F_L(x, Q^2)$  we took  $m_c = 1.4$  GeV and  $m_b = 4.75$  GeV and used the  $m^2 = 0$  limit of the above formulae to evaluate the corresponding light quark contributions to the SF.



The SF  $F_2^b$  as a function of  $x$  at fixed  $Q^2$  compared to the H1 data:

*H1 Collab., A. Aktas et al. EPJ C40 (2005) 349, C45 (2006) 23.*

The solid and dashed lines are from CCFM-evolved UGD obtained from the fits to  $F_2(x, Q^2)$  and  $F_2^c(x, Q^2)$ .

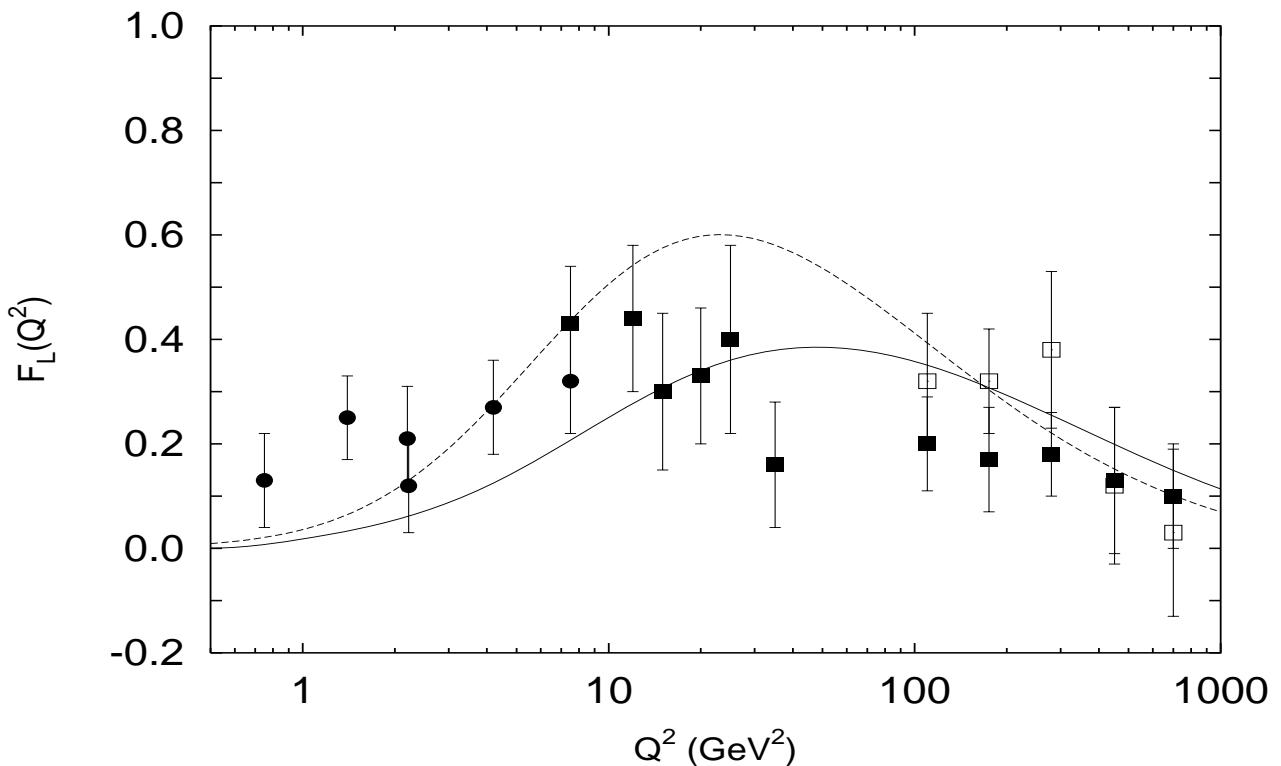


The SF  $F_L$  as a function of  $x$  at fixed  $Q^2$  compared to the H1 data. The solid and dashed lines are from CCFM-evolved UGD obtained from the fits to  $F_2(x, Q^2)$  and  $F_2^c(x, Q^2)$ .



The H1 exp. data points are from

H1 Collab., S. Aid et al.  
 Phys. Lett. B393 (1997) 452,  
 A. Adloff et al., Eur.Phys. J. C21 (2001) 33,  
 N. Gogitidze, J. Phys. G 28 (2002) 751.



The  $Q^2$  dependence of SF  $F_L$  (at  $W = 276$  GeV) compared to the H1 data.

The solid and dashed lines are from CCFM-evolved UGD obtained from the fits to  $F_2(x, Q^2)$  and  $F_2^c(x, Q^2)$ .

The H1 exp. data points are from

E.M. Lobodzinska  
 in *Proc. of the DIS 2003, Gatchina, Russia*, p. 93.

## Conclusions

- The  $k_T$ -factorization approach with the CCFM-evolved UGD obtained from the fits to the  $F_2(x, Q^2)$  and  $F_2^c(x, Q^2)$  data reproduces the H1 data for SF  $F_2^b(x, Q^2)$ ,  $F_L(x, Q^2)$  and  $F_L$  at fixed  $W$ .
- The new exp. data for  $F_L(x, Q^2)$  and at fixed  $W$  are very important for the UGD.
- We need more precise experimental data for the SF  $F_2^c(x, Q^2)$  and  $F_2^b(x, Q^2)$  also.