

# **One gluon, two gluon: multi-gluon production at high energies**

Michael Lublinsky

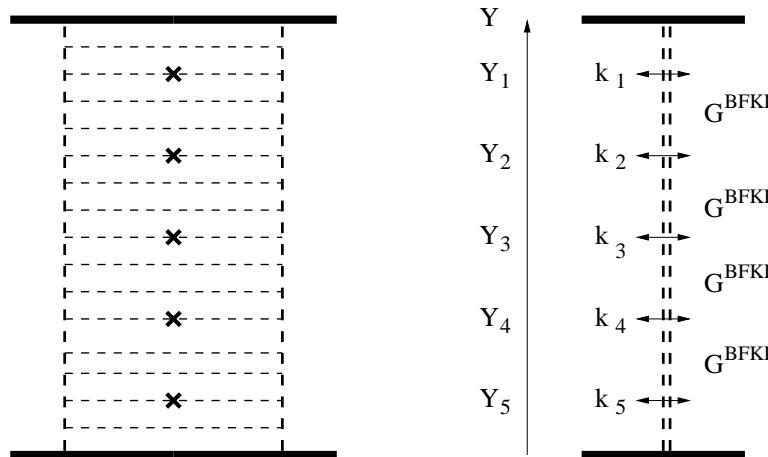
Stony Brook

based on: Alex Kovner, M.L., Heribert Weigert, Phys.Rev.D74:114023,2006

Alex Kovner and M.L., JHEP 0611:083,2006

## Multi gluon production in dilute limit

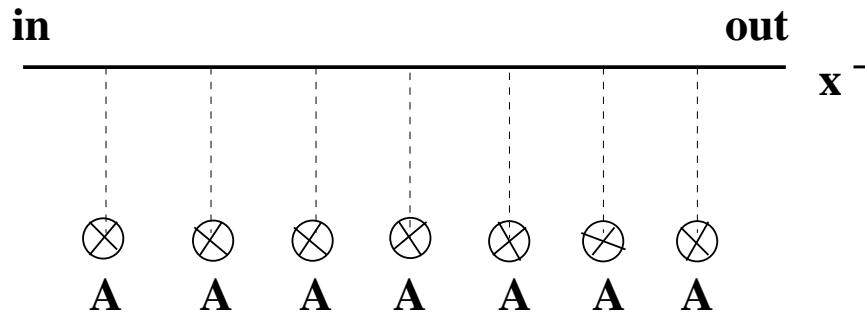
For the BFKL approximation:



Very schematically:

$$\frac{d\sigma}{dY_1 dk_1^2 \dots dY_n dk_n^2} \sim \Phi^T \ G_{Y_n - Y_0}^{\text{BFKL}} \ L(k_n) \ \dots \ G_{Y_1 - Y_2}^{\text{BFKL}} \ L(k_1) \ G_{Y - Y_1}^{\text{BFKL}} \ \Phi^P$$

What if we have multiple rescatterings? many letters: GLR-BK-JIMWLK and BKP



Multiple scattering via eikonal approximation  
Wilson line  $S$  as scattering matrix of a fast gluon

$$S(x) = \mathcal{P} \exp \left\{ i \int dx^- T^a \mathbf{A}_t^a(x, x^-) \right\} .$$

Useful trick (similar to Schwinger-Keldish formalism):  
introduce one target ( $S$ ) for the amplitude and another target ( $\bar{S}$ ) for the conjugate:

$$\frac{d\sigma}{dY_1 dk_1^2 \dots dY_n dk_n^2} \sim \int DSD\bar{S} W^T[S] \delta(S - \bar{S}) U_{Y_n - Y_0} \mathcal{O}_g^{k_n} \dots U_{Y_1 - Y_2} \mathcal{O}_g^{k_1} U_{Y - Y_1} \Sigma^P[S, \bar{S}]$$

$$U(Y_1 - Y_2) = \text{Exp}[-H_3(Y_1 - Y_2)]$$

The gluon production (and scattering) amplitude

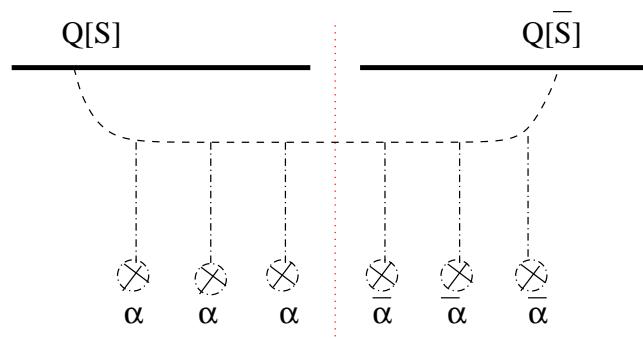
$$Q_i^a(z) = g \int_x \frac{(x-z)_i}{(x-z)^2} \left[ J_L^a(x) - S^{ab}(z) J_R^b(x) \right]$$

The generators of the left/right color rotations

$$J_R^a(x) = -\text{tr} \left\{ S(x) T^a \frac{\delta}{\delta S^\dagger(x)} \right\}, \quad J_L^a(x) = -\text{tr} \left\{ T^a S(x) \frac{\delta}{\delta S^\dagger(x)} \right\}$$

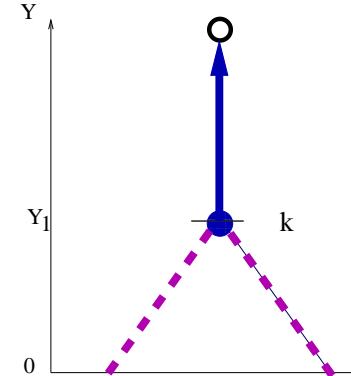
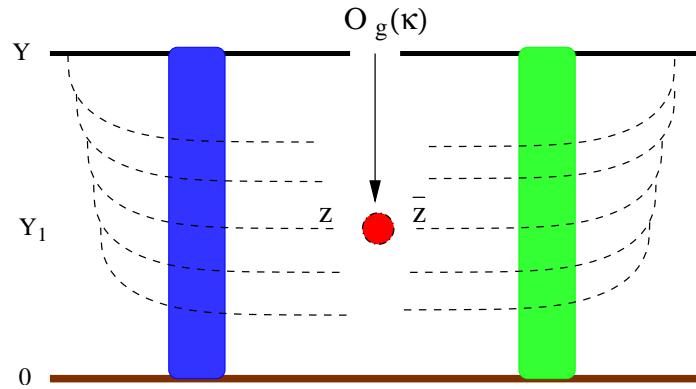
The gluon emission operator

$$\mathcal{O}_g^k[S, \bar{S}] = \int \frac{d^2 z}{2\pi} \frac{d^2 \bar{z}}{2\pi} e^{i k (z - \bar{z})} Q_i^a(z, [S]) Q_i^a(\bar{z}, [\bar{S}])$$



$$H_3[S, \bar{S}] \equiv \int_z [Q_i^a(z, [S]) + Q_i^a(z, [\bar{S}])]^2$$

## Single inclusive gluon production



Yu. Kovchegov  
and  
K. Tuchin, 2001

$$\frac{d\sigma}{dY_1 dk^2} = \int DS D\bar{S} W^T[S] \delta(\bar{S} - S) U_{Y_1} \mathcal{O}_g^k U_{Y-Y_1} \Sigma_Y^P$$

$$\begin{aligned} \frac{d\sigma}{dY_1 dk^2} &= \frac{\alpha_s}{\pi} \int_{z,\bar{z}} e^{i k(z - \bar{z})} \int_{x,y} \frac{(z - x)_i}{(z - x)^2} \frac{(\bar{z} - y)_i}{(\bar{z} - y)^2} G^{BFKL}(x, y; Y - Y_1) \times \\ &\quad \times [\langle T_{z,y} \rangle_{Y_1} + \langle T_{x,\bar{z}} \rangle_{Y_1} - \langle T_{z,\bar{z}} \rangle_{Y_1} - \langle T_{x,y} \rangle_{Y_1}] \end{aligned}$$

$\langle T \rangle$  denoting  $S$ -matrix of a gluonic dipole:

$$\langle T_{x,y} \rangle_{Y_1} \equiv \int DS W_{Y_1}^T[S] \text{tr}[S_x^\dagger S_y]$$

can be deduced from solutions of the BK-JIMWLK eqn

## The dipole limit

Introduce new degrees of freedom.

The dipole creation operator

$$s_{x,y} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y)]; \quad \bar{s}_{x,y} = \frac{1}{N} \text{tr}[\bar{S}_F(x) \bar{S}_F^\dagger(y)]$$

The quadrupole operator

$$q_{x,y,u,v} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y) S_F(u) S_F^\dagger(v)]; \quad \bar{q}_{x,y,u,v} = \frac{1}{N} \text{tr}[\bar{S}_F(x) \bar{S}_F^\dagger(y) \bar{S}_F(u) \bar{S}_F^\dagger(v)].$$

No other higher multiplet operators if the projectile at rest is made only out of dipoles!

The quadrupoles of the mixed type

$$q_{x,y,v,u}^{s\bar{s}} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y) \bar{S}_F(u) \bar{S}_F^\dagger(v)] = q_{x,y,v,u} + t_{x,y,v,u}$$

Remember that we have to set  $\bar{S} = S$  at the end of our computation  
→ perturbation theory in  $t$

Re-express the Hamiltonian  $H_3$  in new degrees of freedom

$$H_3 = H_s + H_q + H_t + V_{t \rightarrow tt}$$

$H_s$  is the dipole Hamiltonian which generates the Balitsky Kovchegov eq. for  $s$ .

$$\partial_y s(x, y) = K^{BFKL} \otimes (s - s s)$$

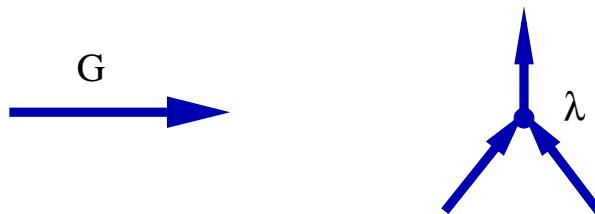
$H_q$  generates a linear evolution of  $q$  (similar to BKP) which is also coupled to  $s$ .

$$\partial_y q(x, y, u, v) = K_1 \otimes q + K_2 \otimes q s + K_3 \otimes s s$$

$H_t$  generates a linear evolution of  $t$  which is coupled to  $s$

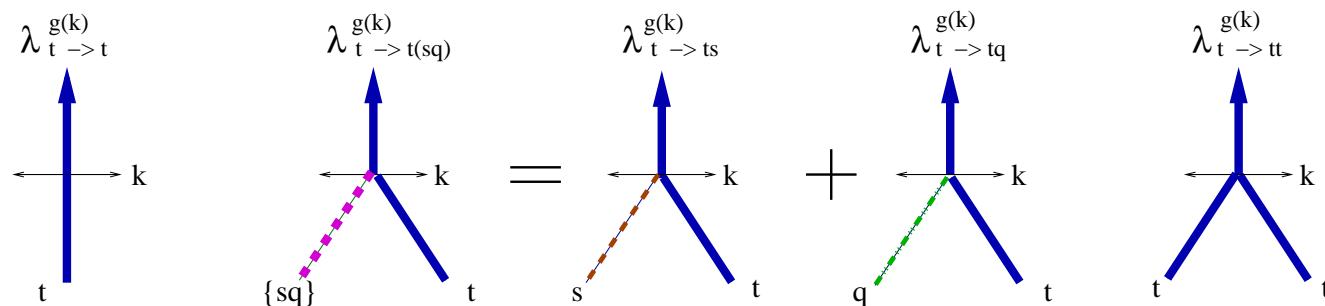
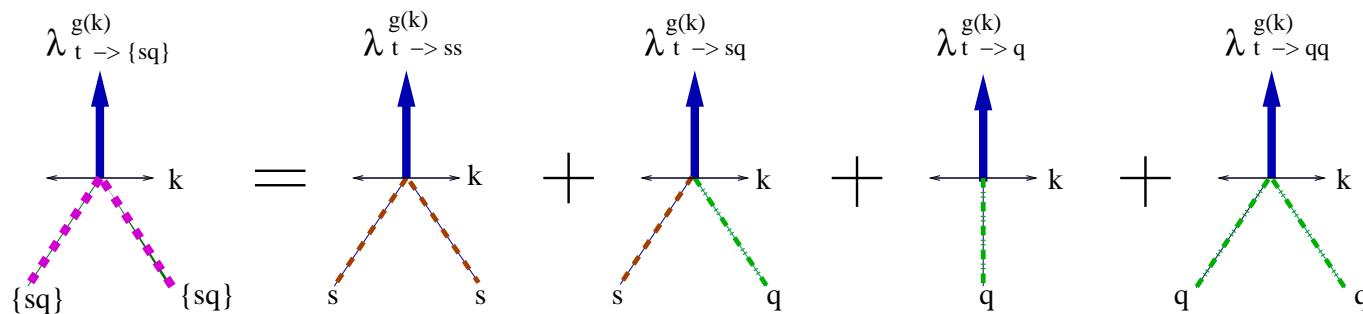
$$\partial_y t(x, y, u, v) = G^{-1}[s] \otimes t + \lambda \otimes t t$$

$G$  is a propagator in the external “Pomeron” field  $s$ .  $G \rightarrow G^{BFKL}$  for two-point functions.

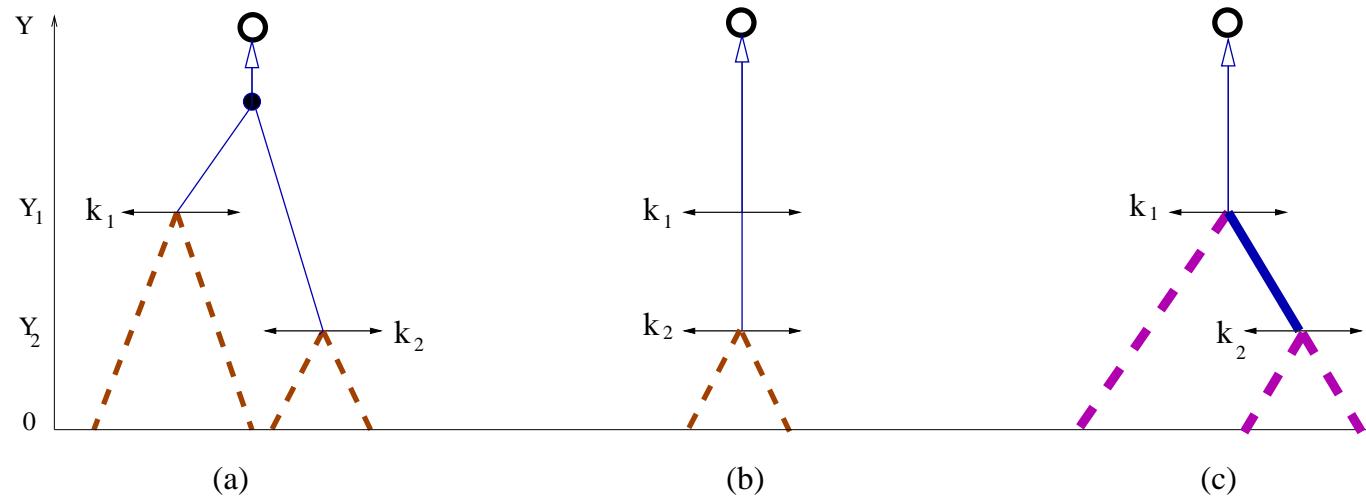


Re-express the insertion operator  $Q_g$

$$\mathcal{O}_g(k) = A_{-1}(k) + A_0(k) + A_1(k)$$



## Two gluon inclusive production



J. Jalilian-Marian and Yu. Kovchegov (2004)

Formally violates AGK cutting rules.

Formulas are computer ready. We need somebody brave to do the job.

