

# Hadron structure in lattice QCD

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XV International Workshop on Deep-Inelastic Scattering and Related Subjects

- Numerical simulations
- Effective field theory analysis: physics from lattice results
- New techniques

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# Lattice light-cone structure

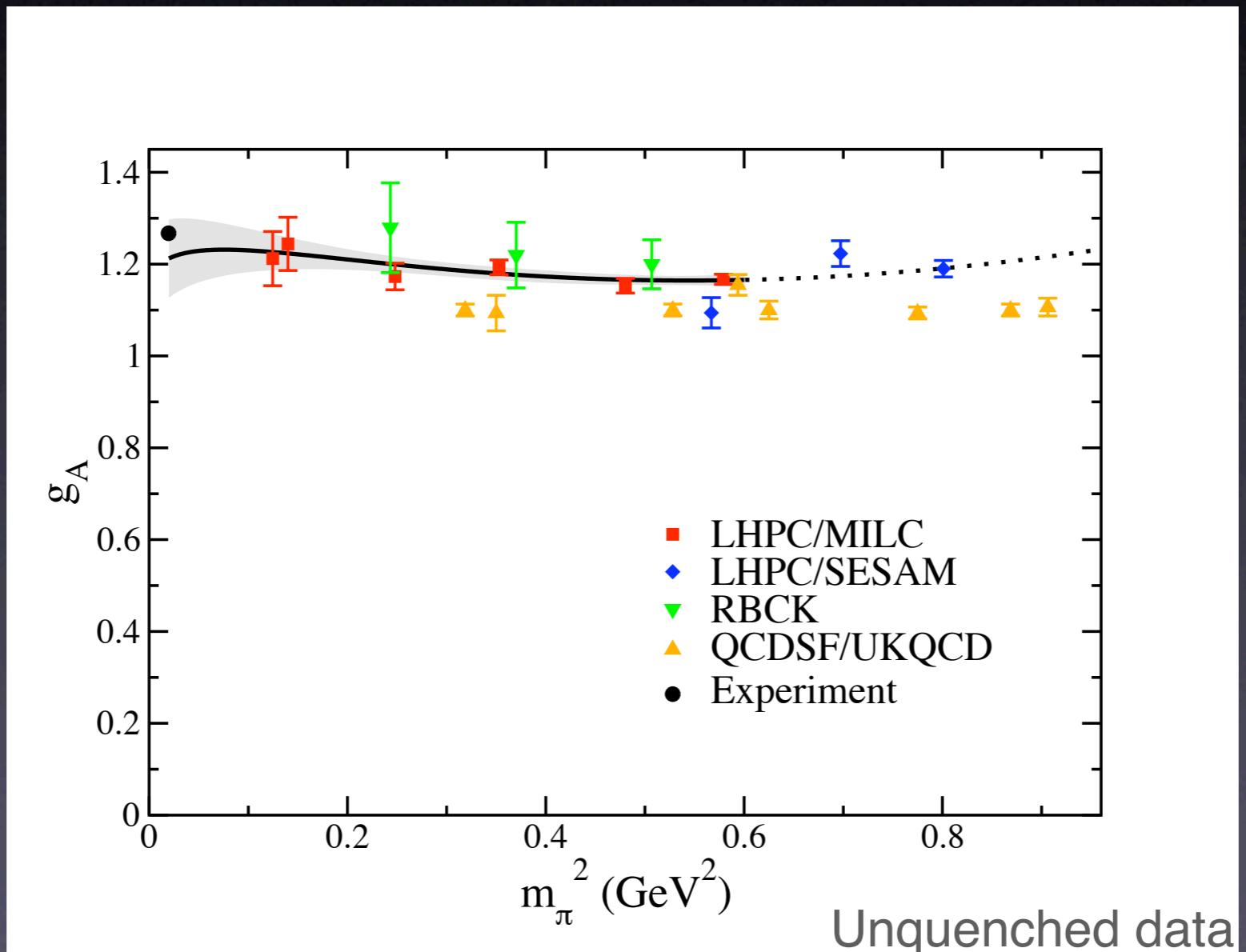
- Two difficulties:
  - Euclidean space
  - Reduced symmetry:  $O(4) \rightarrow H(4)$
- Calculate twist-2 operators matrix elts  
 $OPE \Rightarrow$  Mellin moments of PDFs
$$\langle x^n \rangle_q = \int_{-1}^1 dx x^n q(x)$$
- Limited to  $n \leq 3$  by complicated operator mixing and renormalisation

# Lattice results

- QCDSF, LHP and RBC collaborations
- Lowest three moments of  $q(x), \Delta q(x), \delta q(x)$
- Also calculate moments of GPDs [see P. Hägler SPIN-5]
- Only *isovector* moments: flavour singlet → disconnected contractions
- Large quark masses: progress in  $\chi$ PT calculations for extrapolation ( $m, L, a$ )

# Example: axial charge

- $g_A = \langle x^0 \rangle_{\Delta u - \Delta d}$  simplest twist-two op.



LHPC, PRL 06

# Moments $\Rightarrow$ PDFs

- Well defined problem: inverse Mellin transform
- Requires all integer moments
- How useful are just 3 moments?
- Fit parametric form for PDF
- Standard PDFs have 6+ parameters

# Higher moments

[WD & CJD Lin PRD 73, 014501 (2006)]

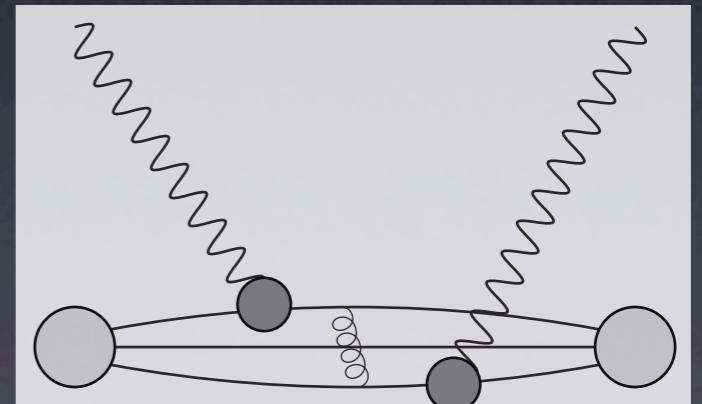
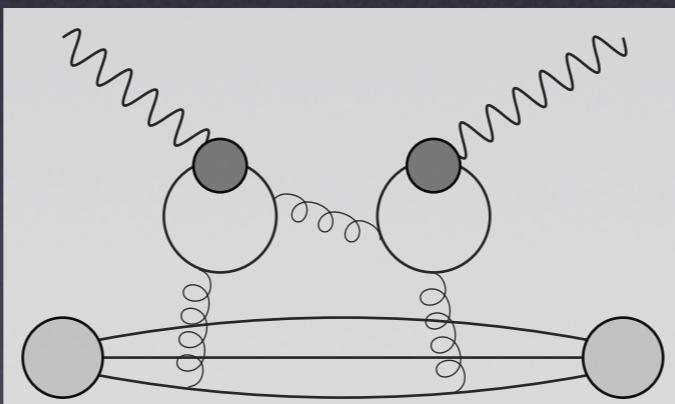
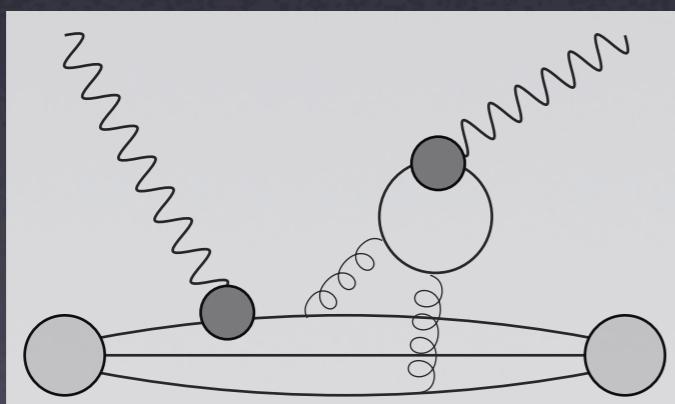
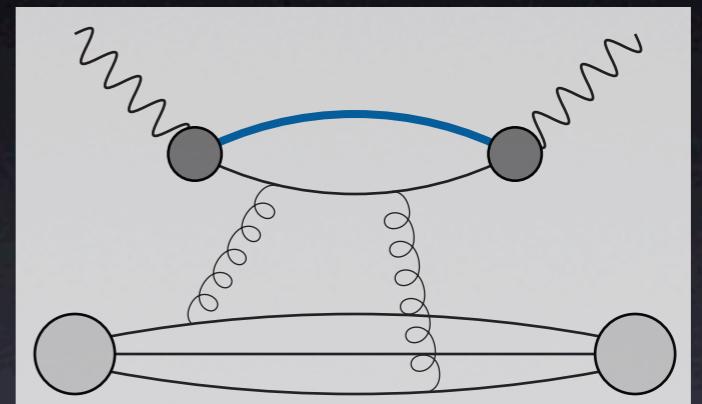
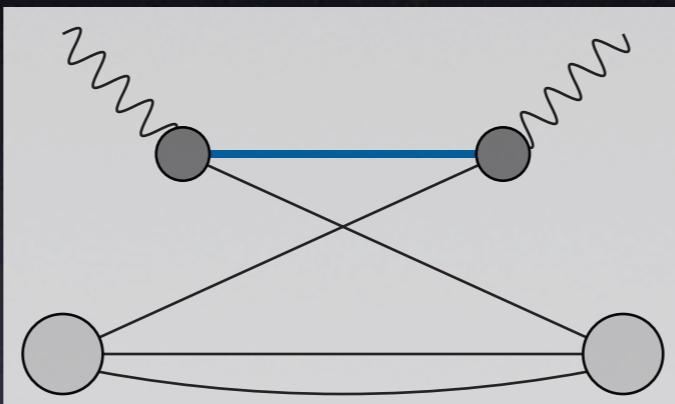
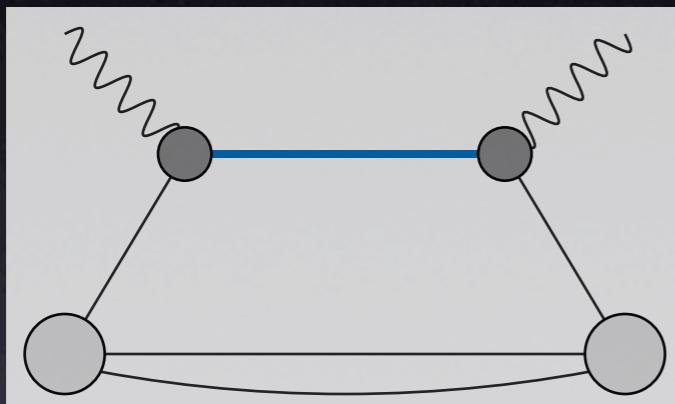
- OPE of Compton tensor in Euclidean space
  - 1. Calculate current-current commutator
  - 2. Extrapolate to continuum - restore  $O(4)$
  - 3. Match to Euclidean OPE to extract matrix elements of local operators
- Determines the same moments as in Minkowski space.

# Fictitious heavy quarks

- Compton tensor for currents that couple light quarks to heavy *fictitious* quark  $\Psi$
- No disconnected contractions  $\Rightarrow$  practical
- Heavy quark mass acts like photon virtuality to suppress higher twists
- Heavy quark integrated out after OPE
  - Determine same PDF moments
  - Effects in perturbative Wilson coefficients

# Lattice correlators

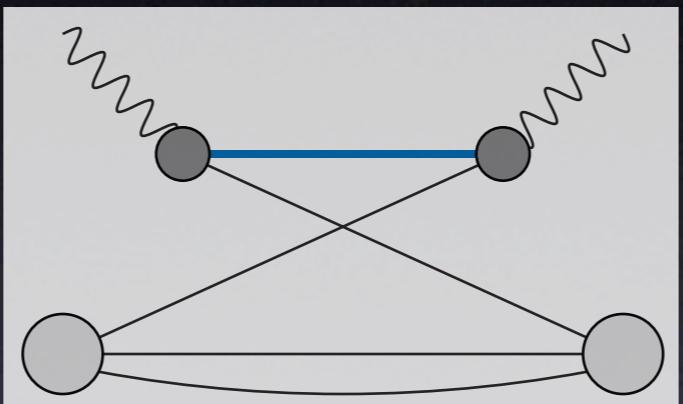
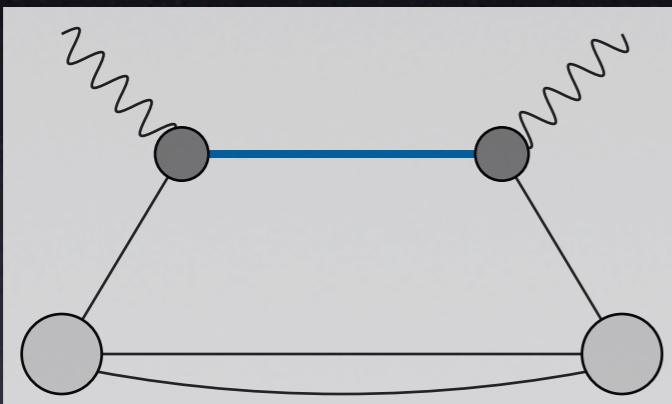
- Quark contractions in Compton correlator



- Heavy quark: no disconnected diagrams for isovector

# Lattice correlators

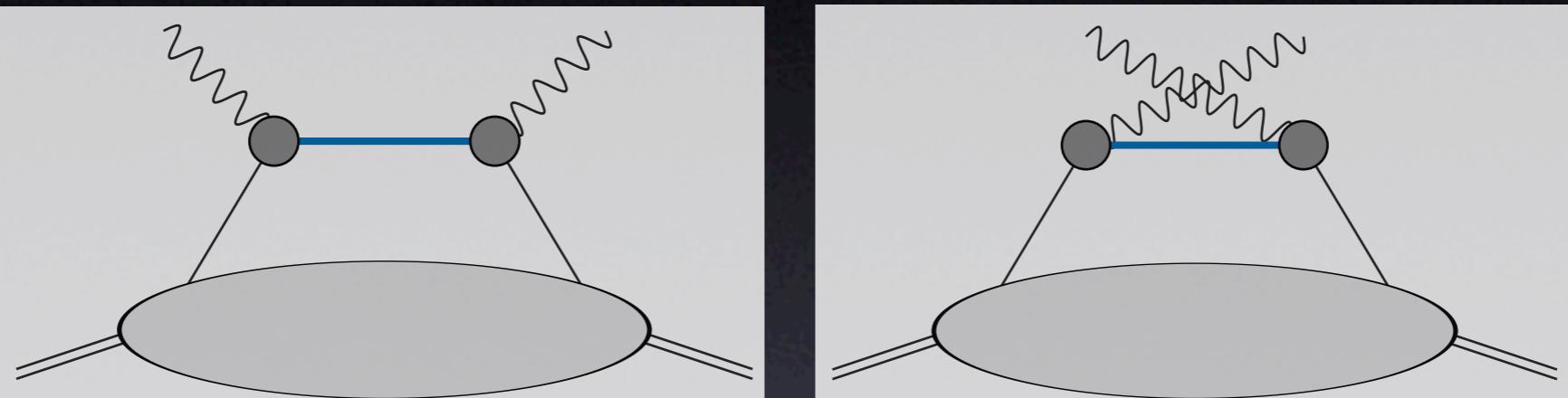
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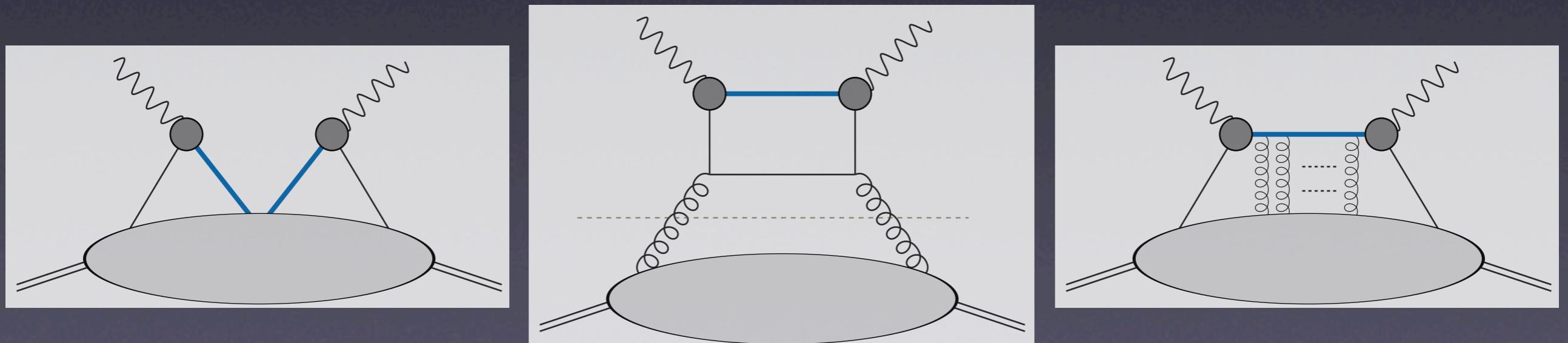
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# Compton tensor

- Leading twist contributions



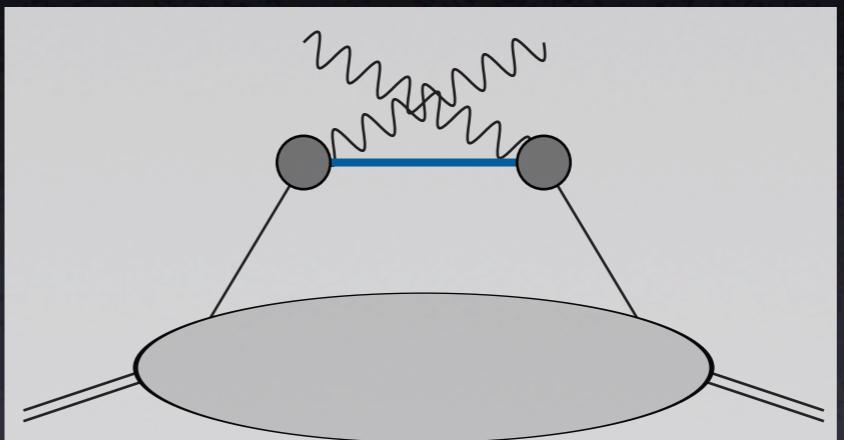
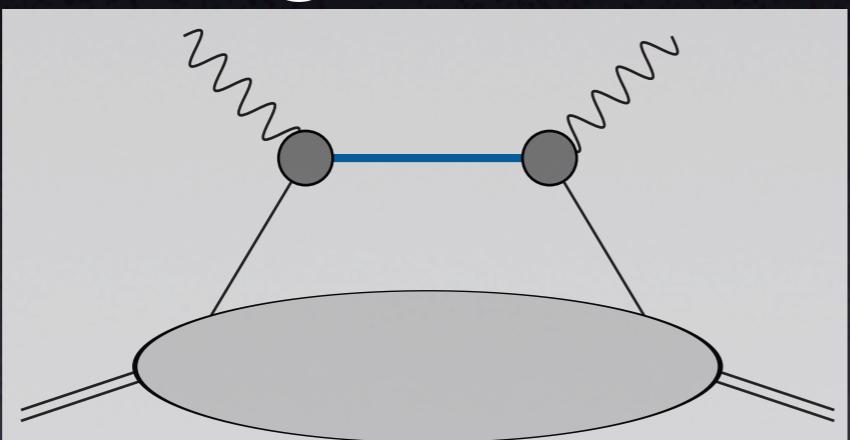
- Higher twist/gluonic contributions



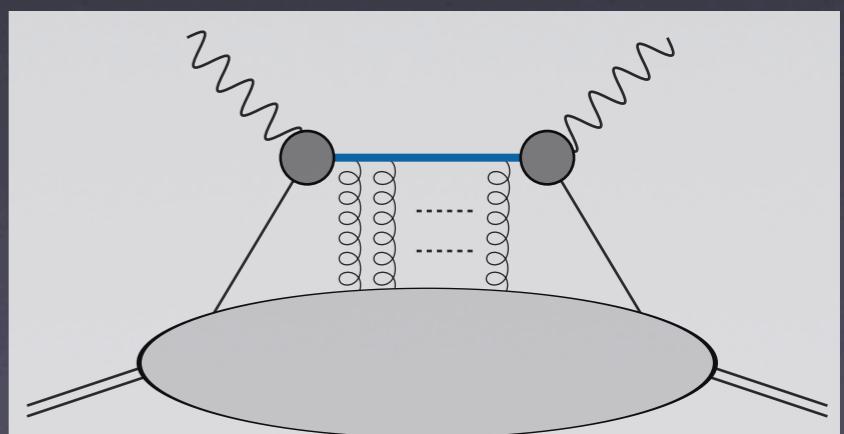
- Significantly reduced by heavy quark

# Compton tensor

- Leading twist contributions



- Higher twist/gluonic contributions



- Significantly reduced by heavy quark

# Heavy quark DIS

- Compton tensor with heavy quark

$$T^{\mu\nu}(p, q) = \sum_S \int d^4x e^{iq \cdot x} \langle p, S | T \left[ J_{\Psi, \psi}^\mu(x) J_{\Psi, \psi}^\nu(0) \right] | p, S \rangle$$

- Heavy-light vector current  $J_{\Psi, \psi}^\mu = \bar{\psi} \Gamma^\mu \Psi + \bar{\Psi} \Gamma^\mu \psi$
- OPE: heavy propagator

$$\frac{-i(iD+q)+m_\Psi}{(iD+q)^2+m_\Psi^2} = -\frac{-i(iD+q)+m_\Psi}{Q^2+D^2-m_\Psi^2} \sum_{n=0}^{\infty} \left( \frac{-2i q \cdot D}{Q^2+D^2-m_\Psi^2} \right)^n$$

- Denominator  $\Rightarrow \tilde{Q}^2 = Q^2 - M_\Psi^2 + \alpha M_\Psi + \beta_n$

Heavy-light meson mass

Binding energy  $\sim \Lambda$

Higher twists  $\sim \Lambda^2$

# Heavy quark DIS

- Usual twist-two operators

$$\mathcal{O}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} (iD)^{\mu_2} \dots (iD)^{\mu_n\}} \psi - \text{traces}$$

- Twist-3 scalar operators contribute:  $e(x)$

$$\hat{\mathcal{O}}_{\psi}^{\mu_1 \dots \mu_n} = \bar{\psi} (iD^{\mu_1}) \dots (iD^{\mu_n}) \psi - \text{traces}$$

- Matrix elements

$$\sum_S \langle p, S | \mathcal{O}_{\psi}^{\mu_1 \dots \mu_n} | p, S \rangle = A_{\psi}^n(\mu^2) [p^{\mu_1} \dots p^{\mu_n} - \text{traces}]$$

$$\sum_S \langle p, S | \hat{\mathcal{O}}_{\psi}^{\mu_1 \dots \mu_n} | p, S \rangle = i M \hat{A}_{\psi}^n(\mu^2) [p^{\mu_1} \dots p^{\mu_n} - \text{traces}]$$

# Heavy quark DIS

- Leads to

**Wilson coefficients**      **Gegenbauer polynomials**

$$\begin{aligned}
 T_{\Psi,\psi}^{\{\mu\nu\}} = & i \sum_{n=2, \text{even}}^{\infty} A_{\psi}^n(\mu) \zeta^n \left\{ \delta^{\mu\nu} \left[ \mathcal{C}_n \frac{\tilde{Q}^2}{q^2} \frac{n C_n^{(1)}(\eta) - 2\eta C_{n-1}^{(2)}(\eta)}{n(n-1)} + \mathcal{C}'_n C_n^{(1)}(\eta) \right] + \frac{p^\mu p^\nu \tilde{Q}^2}{(p \cdot q)^2} \mathcal{C}_n \left[ \frac{8\eta^2 C_{n-2}^{(3)}(\eta)}{n(n-1)} \right] \right. \\
 & + 4 \frac{p^{\{\mu} q^{\nu\}}}{p \cdot q} \left[ \mathcal{C}_n \frac{\tilde{Q}^2}{q^2} \frac{(n-1)\eta C_{n-1}^{(2)}(\eta) - 4\eta^2 C_{n-2}^{(3)}(\eta)}{n(n-1)} - \mathcal{C}''_n \frac{\eta}{n} C_{n-1}^{(2)}(\eta) \right] \\
 & \left. + \frac{q^\mu q^\nu}{q^2} \left[ \mathcal{C}_n \frac{\tilde{Q}^2}{q^2} \frac{n(n-2)C_n^{(1)}(\eta) - 2\eta(2n-3)C_{n-1}^{(2)}(\eta) + 8\eta^2 C_{n-2}^{(3)}(\eta)}{n(n-1)} - 2\mathcal{C}''_n \left( C_n^{(1)}(\eta) - 2\frac{\eta}{n} C_{n-1}^{(2)}(\eta) \right) \right] \right\} \\
 & - 2i \frac{M(m_\Psi - m)}{\tilde{Q}^2} \delta^{\mu\nu} \sum_{n=0, \text{even}}^{\infty} \hat{\mathcal{C}}_n \hat{A}_{\psi}^n(\mu) \zeta^n C_n^{(1)}(\eta)
 \end{aligned}$$

**PDF moments**

- where  $\zeta = \frac{\sqrt{p^2 q^2}}{\tilde{Q}^2}$     $\eta = \frac{p \cdot q}{\sqrt{p^2 q^2}}$
  - Gegenbauer polynomials from target mass corrections: powers of  $(p^2/q^2)^j$
- Not small

# Heavy quark DIS

- Simple in target rest frame:

$$T_{\Psi,\psi}^{\{34\}}(p, q) = \sum_{n=2, \text{even}}^{\infty} A_{\psi}^n(\mu) f(n)$$

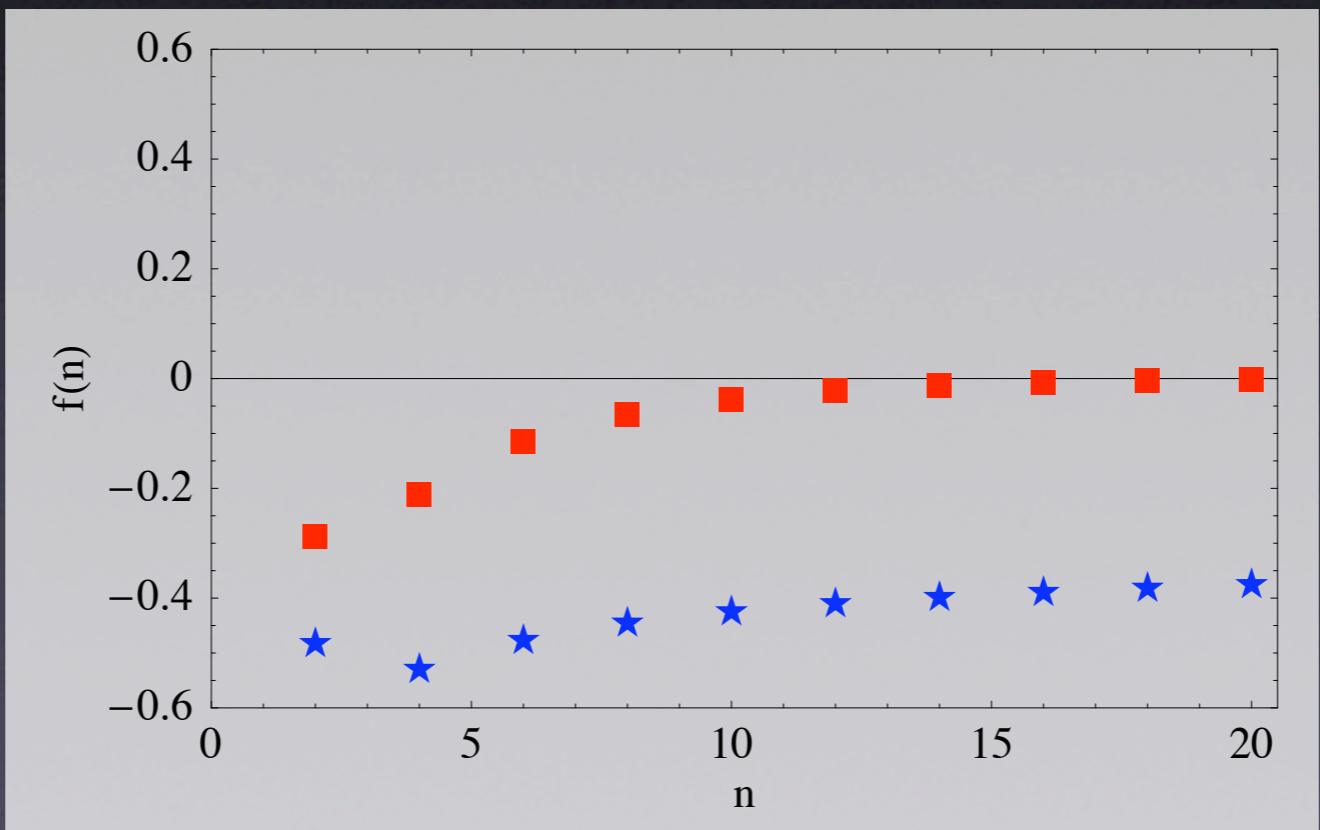
- Where form of  $f(n)$  is completely known:
  - Kinematic variables & Wilson coefficients
  - Parameters:  $\alpha, \beta_n$
  - Includes target mass effects
- Fits to calculations of LHS  $\Rightarrow A_{\psi}^n(\mu), \alpha, \beta_n$

# Lattice details

- Scale hierarchy:  $\Lambda_{\text{QCD}} \ll |Q|, m_\Psi \ll a^{-1}$ 
  - Need fine lattice spacings:  $a^{-1} > 5 \text{ GeV}$
- Heavy quark quenched: very cheap
  - Use variety of masses
  - Different  $q^\mu$  also easy
  - Lattice renormalisation and matching is simple

# Moment extraction

- Aim: extract 6-8 moments (neglect higher)
- Example of n dependence:



$M_\Psi = 2.1 \text{ GeV}, Q^2 = -3.9 \text{ GeV}, q_0 = 2.0 \text{ GeV}$

$\alpha = 0.4, 1.2 \text{ GeV}, \beta_n = 0, M_N = 1.2 \text{ GeV}$

# Other observables

- Odd moments
- Moments of helicity and transversity distributions
- Moments of GPDs
- Moments of meson distribution amplitudes

# $\pi$ distribution amplitude

- Distribution amplitudes (LCWFs) important for hard processes (QCDF)
- Moments of meson DAs: e.g.  $\phi_\pi(\xi)$

$$\langle \xi^n \rangle_\pi = \int_0^1 \xi^n \phi_\pi(\xi) d\xi$$

- Local ME method limited to one moment!
- OPE method determines higher moments
- Not related to a physical process

# $\pi$ distribution amplitude

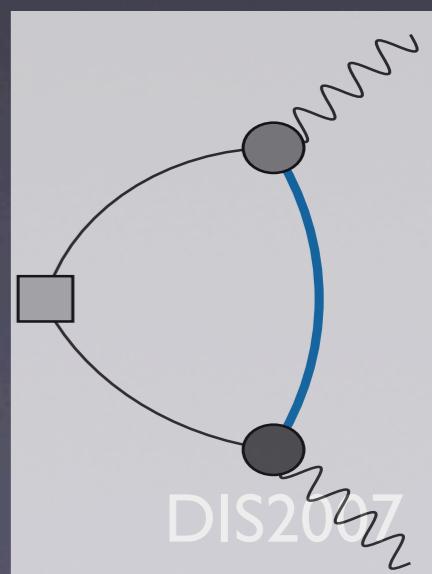
- Lattice calculations of the tensor

$$S_{\Psi,\psi}^{\mu\nu}(p, q) = \int d^4x e^{i q \cdot x} \langle \pi^+(p) | T[V_{\Psi,\psi}^\mu(x) A_{\Psi,\psi}^\nu(0)] | 0 \rangle$$

- After OPE determine same matrix elements as for distribution amplitude:

$$\langle \pi^+(p) | \bar{\psi} \gamma^{\{\mu_0} \gamma_5 (i D)^{\mu_1} \dots (i D)^{\mu_n\}} | 0 \rangle = f_\pi \langle \xi^n \rangle_\pi (p^{\mu_1} \dots p^{\mu_n} - \text{tr})$$

- Numerical work under consideration
- Computationally similar to pion FF



# Summary

- LC structure from lattice calculations
  - Moments of parton distributions
  - Moments of generalised parton distributions
  - Moments of distribution amplitudes
- OPE method for higher moments  
Fictitious heavy quark  $\rightarrow$  practical for isovector distributions
- $\chi$ PT calculations for extrapolations



# Supplementary Slides

# Hadron structure in $\chi$ PT

- Much progress also in moments of PDFs/GPDs/DAs in chiral perturbation theory
- Forward matrix elements (PDFs): NLO calculations, FV, lattice spacing effects
- Non-forward:
  - N: no  $\Delta s$ , infinite volume, continuum
  - $\pi$ : finite volume, lattice spacing effects
- DAs: NLO infinite volume, continuum

# Higher twists in OPE

- Derivative squared generates towers of higher twist operators

$$\sum_{n=0}^{\infty} \bar{\psi} \left( \frac{-2i q \cdot D + D^2}{Q^2 - m_\Psi^2} \right)^n \psi \rightarrow \dots q_{\mu_1} \dots q_{\mu_3} \bar{\psi} D^{\mu_1} D^4 D^{\mu_2} D^{\mu_3} D^2 \psi \dots$$

- Corrections should be  $\mathcal{O}((\Lambda_{\text{QCD}}^2/Q^2)^j)$
- Depend on  $n$  since non-Abelian
- Can be include in lattice analysis

# Compton correlator

$$\begin{aligned} G_{(4)}^{\mu\nu}(\mathbf{p}, \mathbf{q}, t, \tau; \Gamma) &= \sum_{\mathbf{x}, \mathbf{z}} \sum_{\mathbf{y}} e^{i\mathbf{p}\cdot\mathbf{x}} e^{i\mathbf{q}\cdot\mathbf{y}} \Gamma_{\beta\alpha} \langle 0 | \chi_\alpha(\mathbf{x}, t) ) \bar{J}_{\Psi, \psi}^\mu(\mathbf{y} + \mathbf{z}, \tau + \frac{\tau}{2}) \bar{J}_{\Psi, \psi}^\nu(\mathbf{z}, \frac{\tau}{2}) \bar{\chi}_\beta(\mathbf{0}, 0) | 0 \rangle \\ &= \sum_{\mathbf{x}, \mathbf{z}} \sum_{\mathbf{y}} \sum_{N, N'} \sum_{s, s'} e^{i(\mathbf{p} - \mathbf{p}_N) \cdot \mathbf{x}} e^{i(\mathbf{p}_N - \mathbf{p}_{N'}) \cdot \mathbf{z}} e^{i\mathbf{q}\cdot\mathbf{y}} e^{-(E_N + E_{N'})\frac{\tau}{2}} \Gamma_{\beta\alpha} \\ &\quad \times \langle 0 | \chi_\alpha(0) | E_N, \mathbf{p}_N, s \rangle \langle E_N, \mathbf{p}_N, s | \bar{J}_{\Psi, \psi}^\mu(\mathbf{y}, \tau) \bar{J}_{\Psi, \psi}^\nu(0) | E_{N'}, \mathbf{p}_{N'}, s' \rangle \langle E_{N'}, \mathbf{p}_{N'}, s' | \bar{\chi}_\beta(0) | 0 \rangle \\ &\xrightarrow{t \rightarrow \infty} e^{-E_0 t} \sum_{\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{y}} \sum_{s, s'} \Gamma_{\beta\alpha} \langle 0 | \chi_\alpha(0) | E_0, \mathbf{p}, s \rangle \langle E_0, \mathbf{p}, s | \bar{J}_{\Psi, \psi}^\mu(\mathbf{y}, \tau) \bar{J}_{\Psi, \psi}^\nu(0) | E_0, \mathbf{p}, s' \rangle \langle E_0, \mathbf{p}, s' | \bar{\chi}_\beta(0) | 0 \rangle \end{aligned}$$

$$G_{(2)}(\mathbf{p}, t; \Gamma) = \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}} \Gamma_{\beta\alpha} \langle 0 | \chi_\alpha(0) \bar{\chi}_\beta(\mathbf{x}, t) | 0 \rangle$$

$$T_{\Psi, \psi}^{\{\mu\nu\}}(p, q) = 4M a \sum_{\tau} e^{iq_4\tau} \left[ \lim_{t \rightarrow \infty} \frac{G_{(4)}^{\{\mu\nu\}}(\mathbf{p}, \mathbf{q}, t, \tau; \Gamma_4)}{G_{(2)}(\mathbf{p}, t; \Gamma_4)} \right]$$