DIS07 Workshop
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Endpoint singularities in unintegrated parton distributions

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I. Motivation

II. How to characterize updf's with precision?

III. Endpoint singularities in coordinate space

IV. Updf's with subtractive regularization

I. Motivation

Hard processes acquire qualitatively new features at the LHC

phase space opening up at large $\sqrt{s}$

$\Downarrow$

large number of events with multiple hard scales

QCD provides methods to treat multi-scale problems, based on

- generalized factorization formulas
- parton distributions less integrated in momentum than the ordinary parton distributions

Well-known examples:

▷ Sudakov processes

▷ small-x physics

▷ Monte-Carlo's reconstructing fully exclusive final states

sensitive to parton distributions at fixed transverse momentum

(+ virtuality/angle)

⇒ Precise characterizations of unintegrated parton distributions are highly desirable

- But this is much more difficult than for ordinary pdf's

  ▷ gauge invariance
  ▷ lightcone divergences from endpoint region

II. Can we characterize precisely updf's as matrix elements of field operators?

Collins & Zu, 2005; Collins, 2003

Boer & Mulders, 2003, 1998

Belitsky et al., 2004; Brodsky et al., 2001

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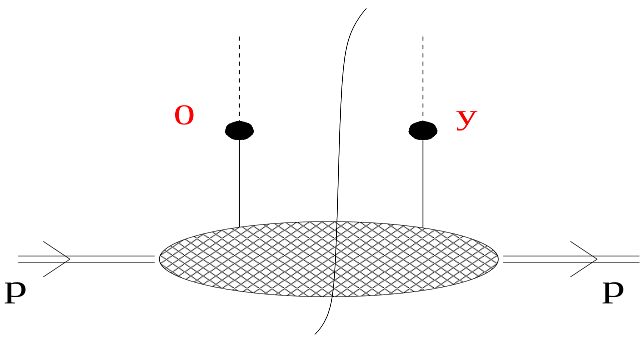


$p = (p^+, m^2/2p^+, 0_\perp)$

Ex. : $\widetilde{f}(y) = \langle\, P \mid \overline{\psi}(y)\; V_y^\dagger(n)\; \gamma^+\; V_0(n)\; \psi(0) \mid P\, \rangle$, $y = (0, y^-, y_\perp)$

$$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau\, n) \right)$$

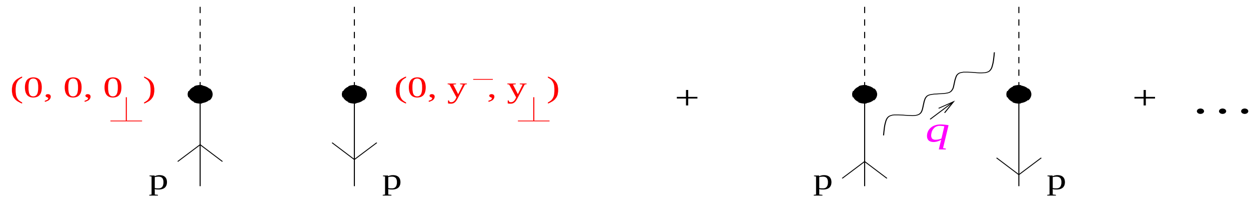
$\longrightarrow$ unintegrated quark distribution from Fourier transform in y^- and $y_\perp$

- Fine at tree level
- Difficulties arise beyond this level

$\hookrightarrow$

$n = (0, 1, 0$

◇ Suppose a gluon is absorbed or emitted by eikonal line:



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp\, P_R(x', k'_\perp)$$

$$\text{where} \qquad P_R = \frac{\alpha_s\, C_F}{\pi^2} \Bigg[\underbrace{\frac{1}{1-x}\ \frac{1}{k_\perp^2 + \rho^2}}_{\uparrow\ \textit{endpoint singularity}} + \{\text{regular} \quad \text{at} \quad x \to 1\} \Bigg] \qquad \rho\text{=IR regulator}$$

$(q^+ \longrightarrow 0,\ \forall\ k_\perp)$

◇ Physical observables:

$$\mathcal{O} = \int dx\ dk_\perp\ f_{(1)}(x, k_\perp)\ \varphi(x, k_\perp)$$
$$= \int dx\ dk_\perp\ [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

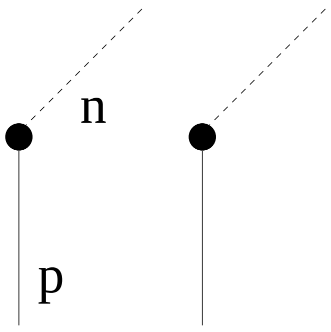
general case: endpoint divergences from incomplete KLN cancellation

Traditionally, put cut-off on the endpoint region:

▷ cut-off in Monte-Carlo generators using u-pdf's

CASCADE	www.quark.lu.se/~hannes/cascade
SMALLX	Marchesini & Webber, 90's
LDCMC	www.thep.lu.se/~leif/ariadne

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Ji, Ma & Yuan, 2005, 2006

earlier work by Collins; Korchemsky

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

Drawbacks:

- good for leading accuracy, but makes it difficult to go beyond
- $\int dk_\perp \; f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq$ ordinary pdf

III. Endpoint singularities in coordinate space

▷ Expand matrix element to one loop

▷ Feynman gauge

(a) (b)

$$\widetilde{f}_{(a)+(b)} = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \Bigg[e^{ip\cdot yv}\; 2^{d/2-1} \left(\frac{\rho^2}{\mu^2}\right)^{d/4-1}$$

$$\times \frac{1}{(-y^2\mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip\cdot y}\;\Gamma(2-\frac{d}{2})\;(\frac{\mu^2}{\rho^2})^{2-d/2}\Bigg]$$

$K =$ modified Bessel function; $\Gamma =$ Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v\lambda^2$$

- $v \to 1$: endpoint singularity
  (present even with finite masses m, λ)

H, hep-ph/0702196

∞

- can relate result to ordinary pdf by expanding in y^2:

$$\widetilde{f}_{(a)+(b)} \simeq \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} \, p^+ \int_0^1 dv \, \frac{v}{1-v} \, \Big\{ \big[e^{ip\cdot yv} - e^{ip\cdot y}\big] \, \Gamma(2-\frac{d}{2}) \, (\frac{\mu^2}{\rho^2})^{2-d/2}$$
$$+ \; e^{ip\cdot yv} \, 4^{d/2-2} \, \Gamma(\frac{d}{2}-2) \, (-y^2\mu^2)^{2-d/2}$$
$$+ \; \sum_{k=1}^{\infty} \frac{\Gamma(2-d/2)\,\Gamma(d/2-1)}{k! \, 4^k \, \Gamma(k+d/2-1)} \, e^{ip\cdot yv} \, (\frac{\rho^2}{\mu^2})^{d/2+k-2} (-y^2\mu^2)^k$$
$$+ \; \sum_{k=1}^{\infty} \frac{4^{d/2-2-k} \, \Gamma(d/2-2)\,\Gamma(3-d/2)}{k! \, \Gamma(k+3-d/2)} \, e^{ip\cdot yv} \, (\frac{\rho^2}{\mu^2})^k (-y^2\mu^2)^{2-d/2+k} \Big\}$$

$\longrightarrow$ separate long-distance contributions in $\ln(\mu^2/\rho^2)$
and short-distance contributions in $\ln(y^2\mu^2)$

◇ first line: $v \to 1$ singularity cancels for ordinary parton distributions
◇ singularity is associated with coefficient function at leading power
◇ higher order terms are $\mathcal{O}(y^2)^k$, $k \geq 1$

For $n^2 \neq 0$:

$$\widetilde{f}_{(a)+(b)} = \frac{i\ e^{-i\pi d/4}}{4^{d/2-3/2}\pi^{d/2-1}}\ \alpha_s\ C_F\ (\mu^2)^{2-d/2}\ \int_0^1 dv \int_0^\infty d\tau\ e^{i\tau p\cdot n(1-v)} \int_0^\infty d\sigma\ \sigma^{1-d/2} e^{-i\sigma}$$

$$\times \left[\left(2\ p\cdot n\ v + \frac{\tau n^2}{2\sigma} \right) p^+ + (1-v)\ m^2\ n^+ \right] \left\{ e^{ip\cdot y}\ e^{-in^2\tau^2/(4\sigma)} - e^{ip\cdot yv}\ e^{-i(y-n\tau)^2/(4\sigma)} \right\}$$

- $v \to 1$ behavior (from τ integral over the eikonal) is regularized by $n^2 \neq 0$
- parton distribution depends on cut-off parameter $\eta = (p\cdot n)^2/n^2$ also after integration over $k_\perp$

IV. Updf's with subtractive regularization

Subtractive method: more systematic. Widely used in NLO calculations.
Formulation suitable for eikonal-line matrix elements: Collins & H, 2001.

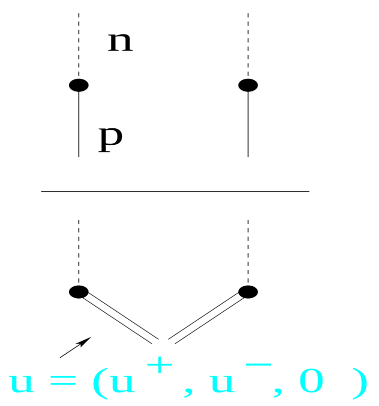
- gauge link still evaluated at n lightlike
- multiplied by "generalized-renormalization" factors designed to cancel the endpoint singularity

$\hookrightarrow$ vev's of lightlike and non-lightlike lines

Collins, 2003

cfr. IR counterterms in Sudakov form factor

E.g.:

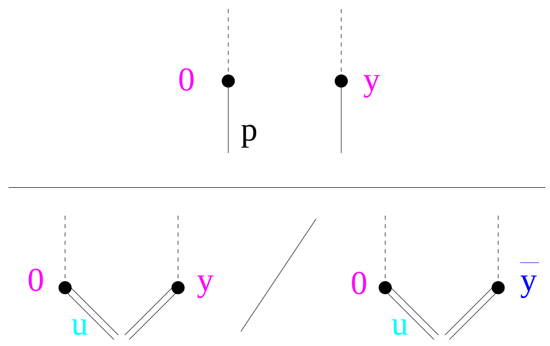


$\Longrightarrow$ cancellation of L.C. divergence (real emission)

- virtual correction terms depend on angle but not on $y_\perp$ $\quad\hookrightarrow$

Matrix element for quark distribution in subtractive regularization:

$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \;/\; \langle 0|V_{\bar{y}}(u)V_{\bar{y}}{}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, hep-ph/0702196

- denominator cancels the endpoint divergence

  (explicit form at one loop: see below)

- counterterms come from gauge-invariant operator matrix elements

Note: dependence on auxiliary eikonal u cancels in distribution integrated over $k_\perp$

Back to momentum space.

One loop expansion:

$$
\begin{aligned}
f^{(\mathrm{subtr})}_{(1)}(x,k_\perp) &= P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp) \quad (\leftarrow\text{from numerator}) \\
&- W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta) \quad (\leftarrow\text{from vev's})
\end{aligned}
$$

$\zeta{=}(p^{+2}/2)u^-/u^+$

$$
\begin{aligned}
P_R(x,k_\perp) &= \frac{\alpha_s C_F}{2\pi^2}\left\{\frac{(1-x)[(k_\perp^2+m^2(1-x)^2-2xm^2]}{[k_\perp^2+m^2(1-x)^2]^2} + \frac{2x/(1-x)}{[k_\perp^2+m^2(1-x)^2]}\right\} \\
W_R(x,k_\perp) &= \frac{\alpha_s C_F}{2\pi^2}\left\{-\frac{8\zeta(1-x)}{[k_\perp^2+4\zeta(1-x)^2]^2} + \frac{2/(1-x)}{[k_\perp^2+4\zeta(1-x)^2]}\right\}
\end{aligned}
$$

Note endpoint singularity $(1-x)^{-1}\times\alpha_s C_F/(\pi^2 k_\perp^2)$ in P_R
and corresponding subtraction term in W_R

Endpoint behavior is regularized in physical observables:

$$\begin{aligned} \mathcal{O} &= \int dx\ dk_\perp\ f^{(\mathrm{subtr})}_{(1)}(x,k_\perp)\ \varphi(x,k_\perp) \\ &= \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp)-\varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp)-\varphi(x,0_\perp)]\} \end{aligned}$$

- extension for $k_\perp \neq 0$ of the plus-distribution regularization

V. Conclusions

- U-pdf's relevant to QCD methods for

  — final states with multiple hard scales

  — infrared-sensitive processes

  Ex.: MC's with shower evolution at subleading level

Lack of complete KLN cancellation

$\Rightarrow$ need to address new problems compared to ordinary pdf's

Q: How to define a $k_\perp$ distribution gauge-invariantly?

(over the whole phase space)

- Full factorization results yet to come.

- Theory efforts on better understanding of endpoint divergences & more precise operator definitions of u-pdf's.

  ▷ more transparent representation of divergences in coordinate space

  ▷ subtractive method as an alternative to cut-off method $(x \to 1)$

  ▷ more systematic, helpful connection with ordinary pdf's and lightcone limit