

# The Combined PDF+ $P_T$ fit

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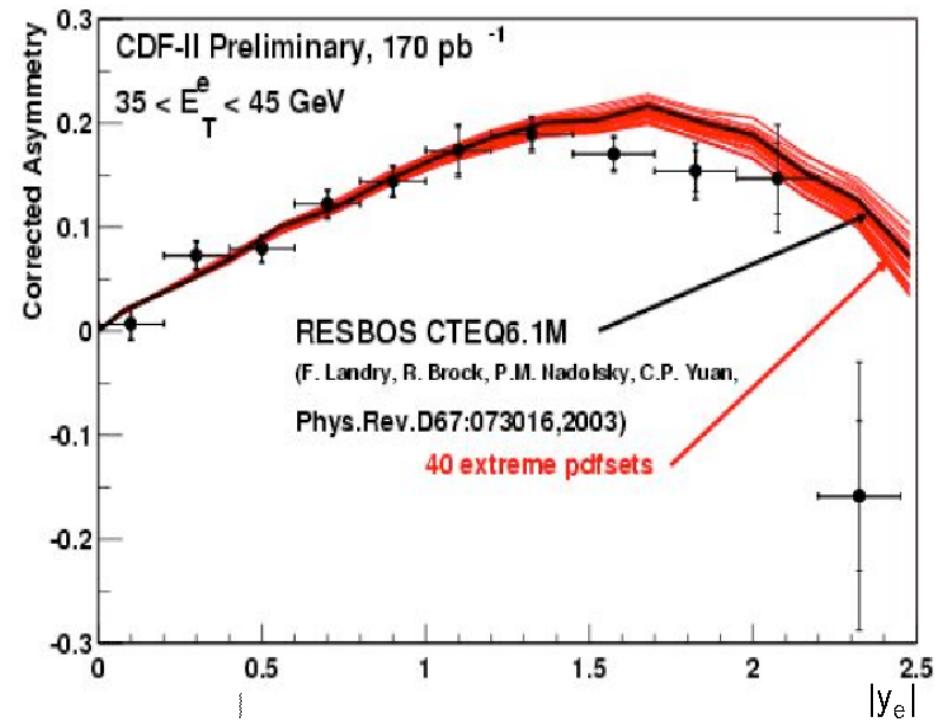
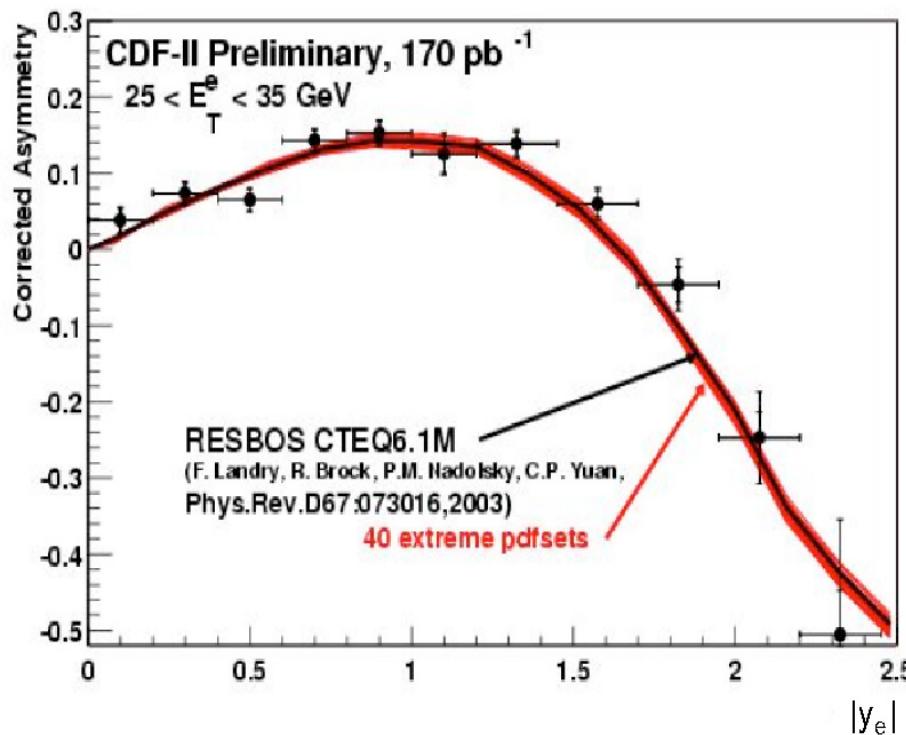
April 18, 2007 @ DIS 2007

- Conventional PDF Global Analysis
- QCD  $P_T$  Resummation Global Analysis
- Potential of Combined Analysis

# Conventional PDF Global Analysis

Tung: SF-2

- Data: DIS, Drell-Yan, CDF/D0 inclusive jet, and CDF charged lepton asymmetry (from  $W$ -decay)



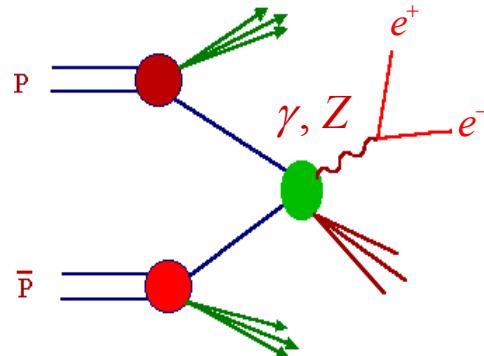
Use lepton rapidity distributions to determine PDFs.

# New Task of Global Analysis

Include Transverse Momentum  $p_T$  distributions

- New Data: include not only rapidity ( $y$ ) but also

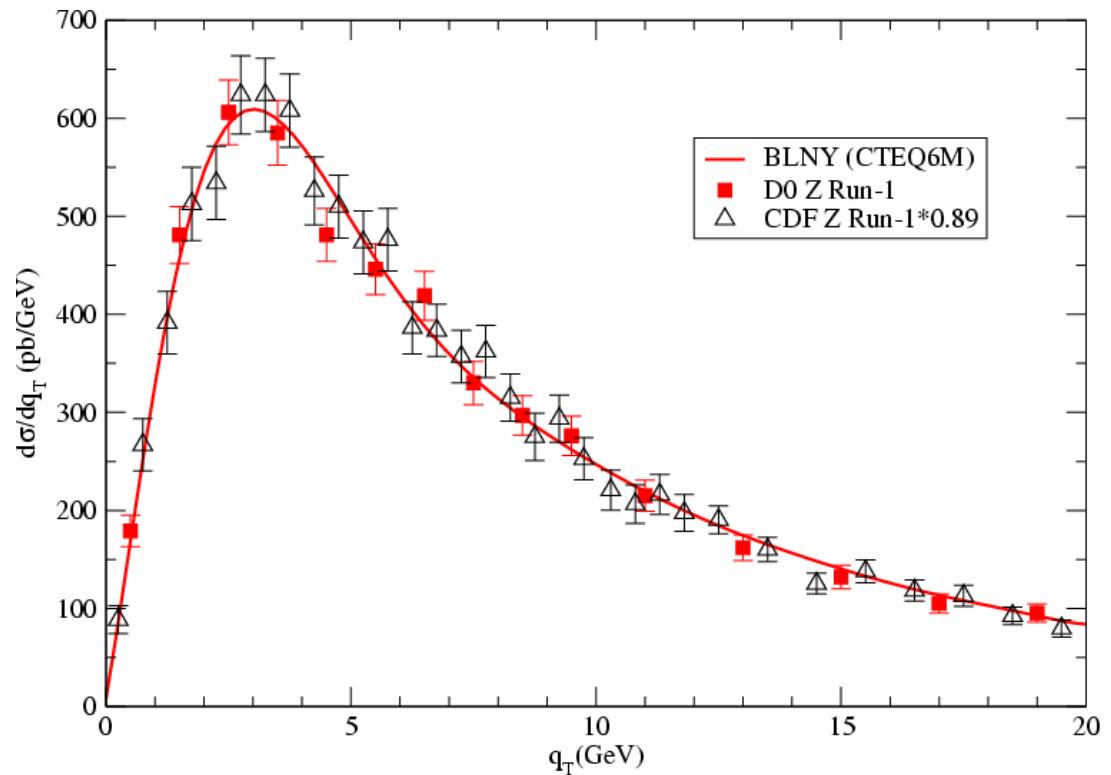
$p_T$  of Drell-Yan pairs and  $Z$  bosons



QCD  $P_T$  Resummation  
Global Analysis

hep-ph/0212159

Brock, Landry, Nadolsky, CPY

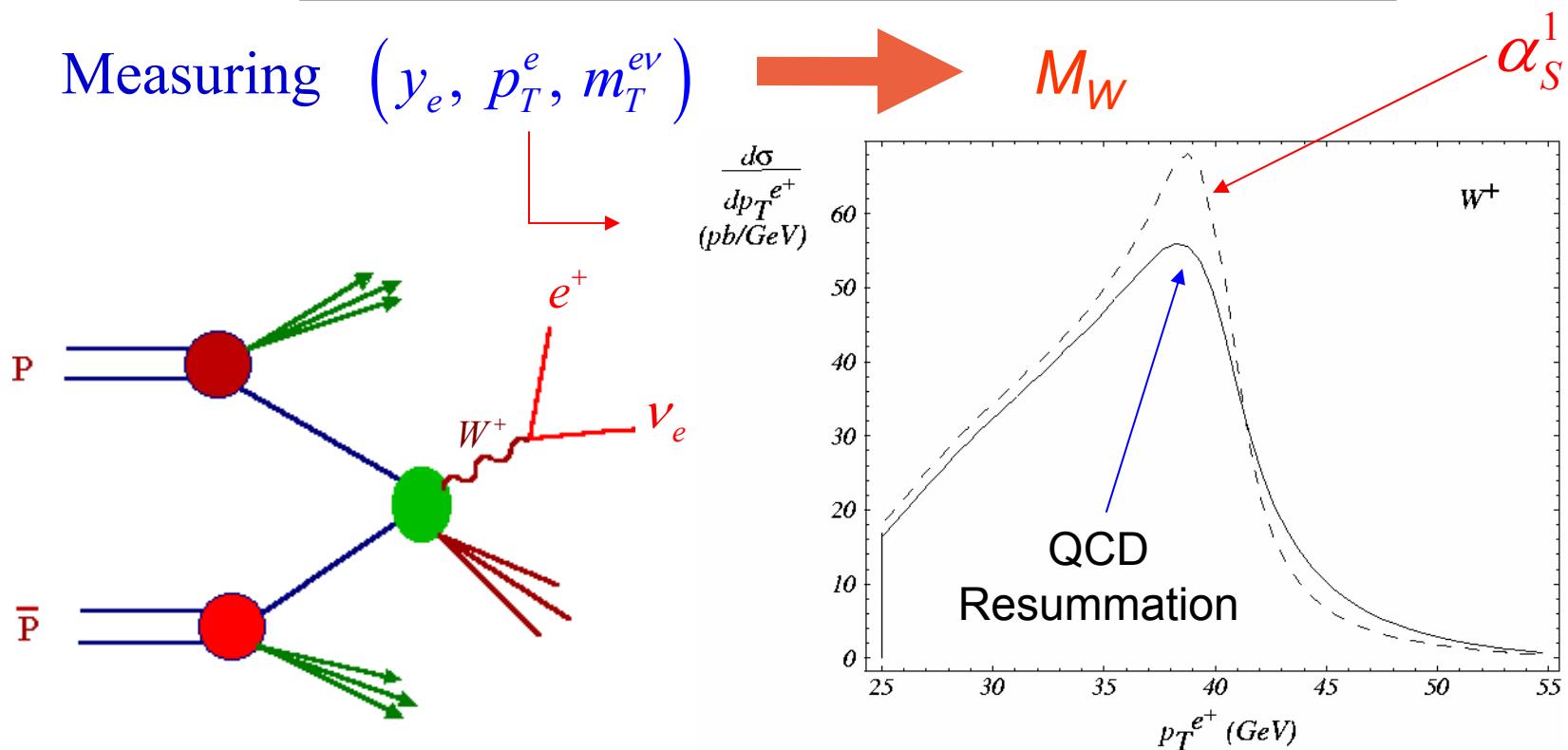


# New Task of Global Analysis:

Include Transverse Momentum  $p_T$  distributions

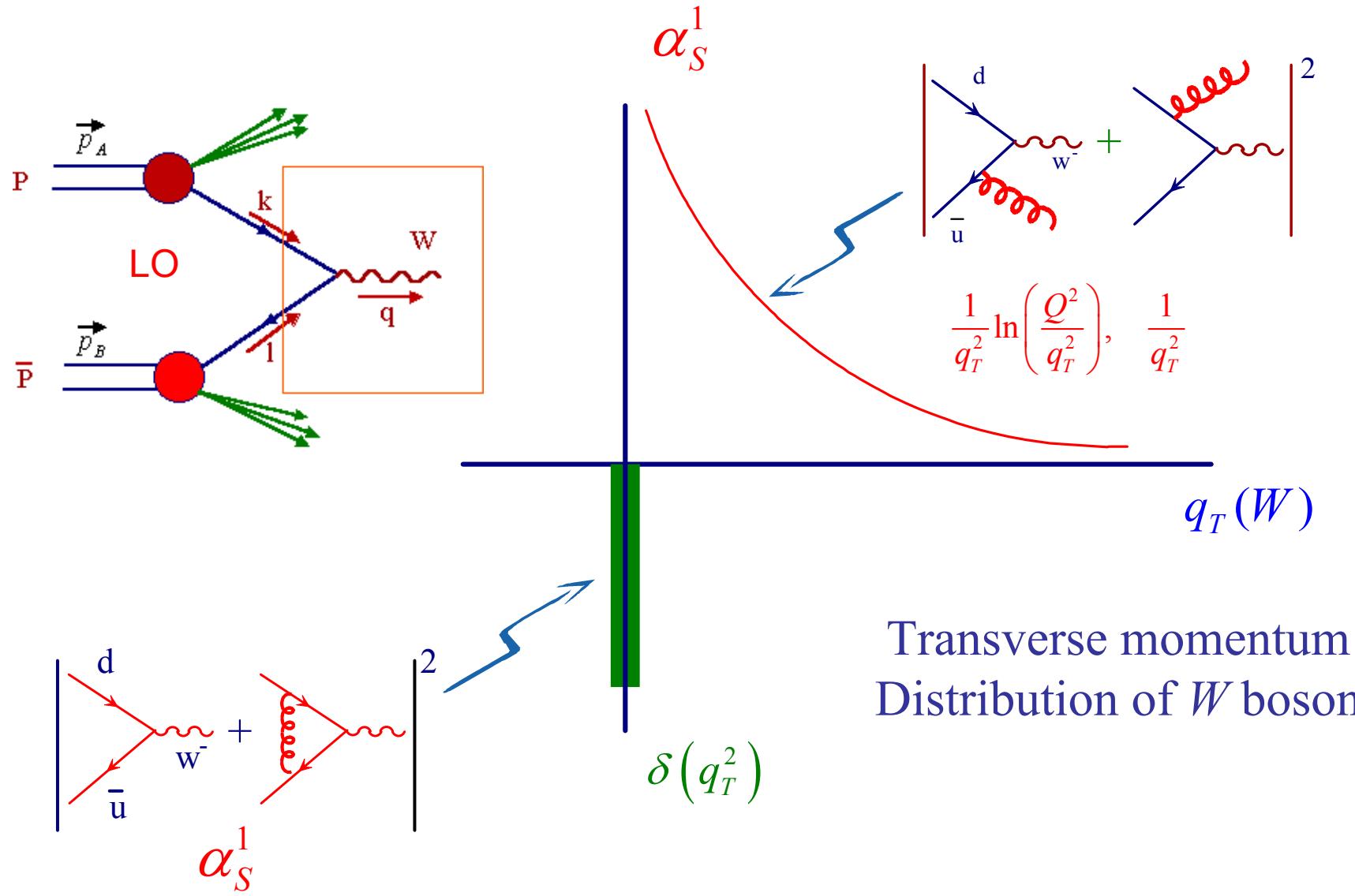


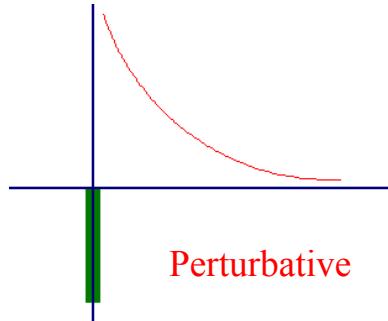
- Better determine PDFs
- Better determine, for example,  
the mass of  $W$  boson  $M_W$



# Predicting $P_T$ distributions at fixed orders

LO and Next to Leading order QCD corrections

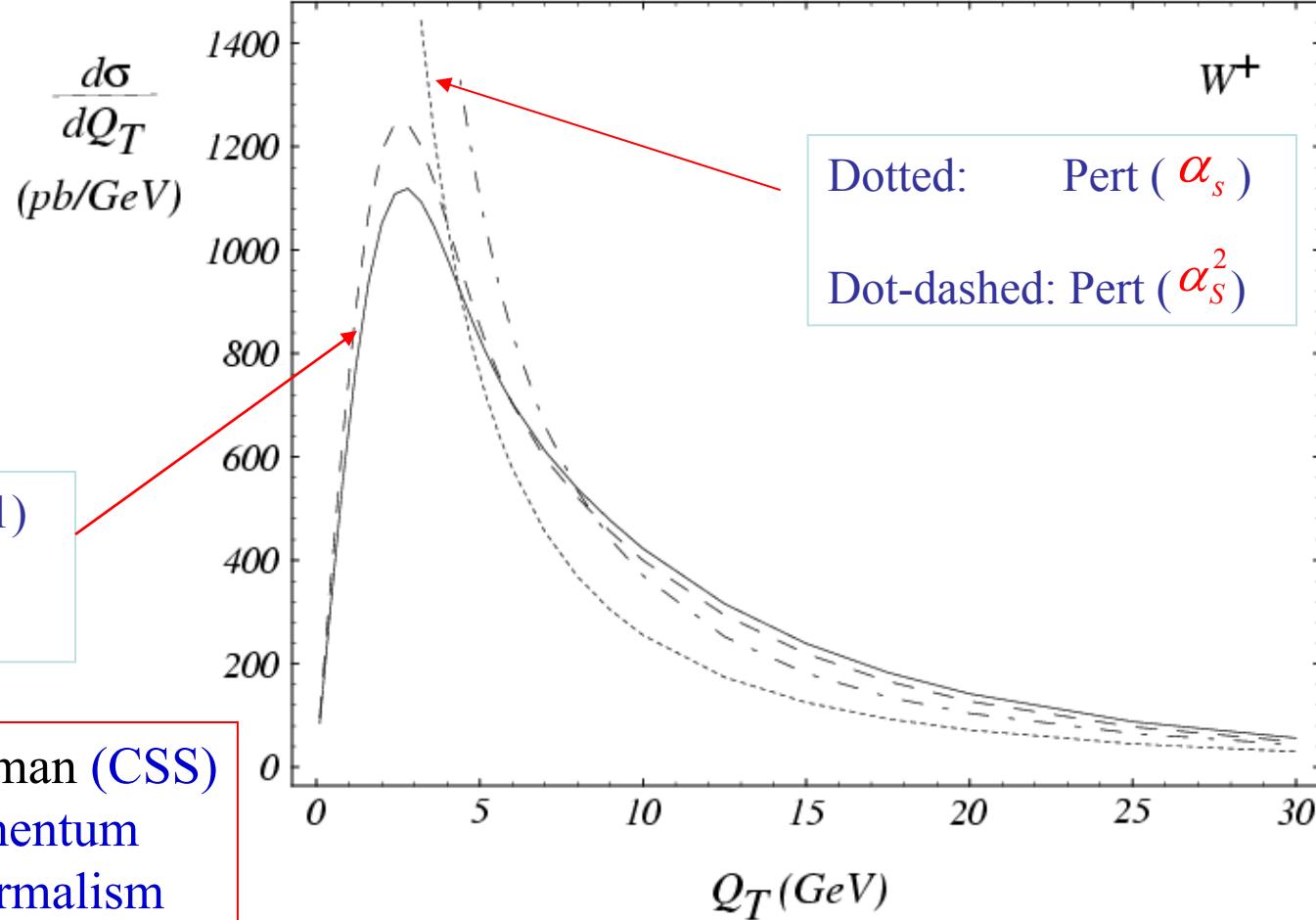




To describe data

All order QCD  
Resummation is needed.

Perturbative



Colline-Soper-Sterman (CSS)  
Transverse momentum  
Resummation formalism

# What's QCD Resummation?

- Perturbative expansion

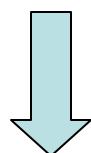
$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_s \left\{ 1 + \alpha_s + \alpha_s^2 + \dots \right\}$$

- The singular pieces, as  $\frac{1}{q_T^2}$  (1 or log's)

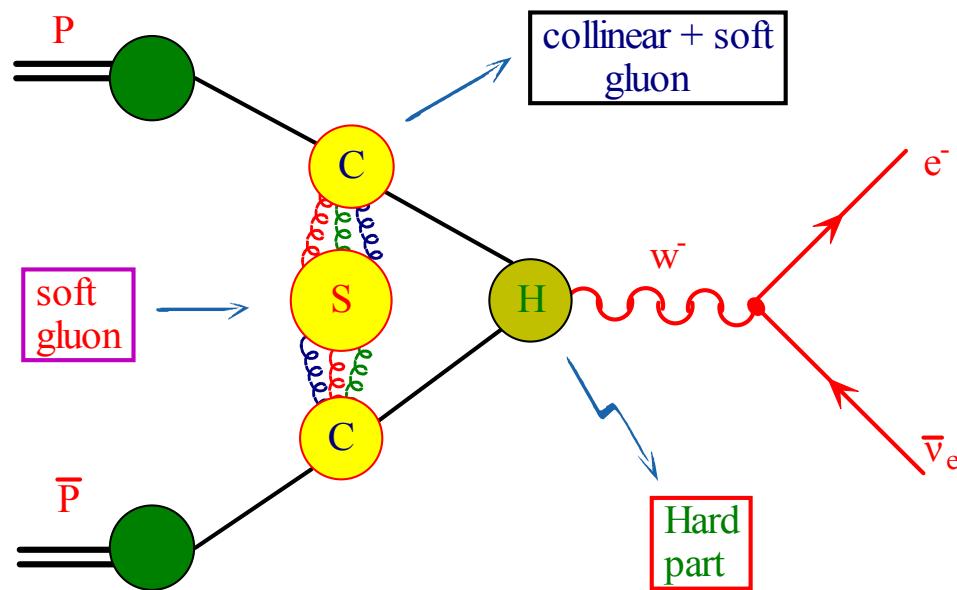
$$\begin{aligned} \frac{d\hat{\sigma}}{dq_T^2} &\sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^{(n)} \ln^{(m)} \left( \frac{Q^2}{q_T^2} \right) \\ &\sim \frac{1}{q_T^2} \left\{ \alpha_s \left( \underline{L+1} \right) \right. \\ &\quad + \alpha_s^2 \left( \underline{L^3+L^2} + \underline{L+1} \right) \\ &\quad + \alpha_s^3 \left( \underline{L^5+L^4} + \underline{L^3+L^2} + \underline{L+1} \right) \\ &\quad \left. + \dots \right\} \end{aligned} \quad L \equiv \ln \left( \frac{Q^2}{q_T^2} \right)$$

Resummation is to reorganize the results in terms of the large Log's.

# CSS Resummation Formalism



Resum all order effects in large logs



NPB 250 ('85)

Colline-Soper-Sterman (CSS)  
Transverse momentum  
Resummation formalism

# CSS Resummation Formalism

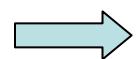
$$\frac{d\sigma}{dq_T^2 dy dQ^2} = \frac{\pi}{S} \sigma_0 \delta(Q^2 - M_W^2).$$

$$\begin{aligned}
 & \left\{ \frac{1}{(2\pi)^2} \int d^2 b \ e^{i\bar{q}_T \cdot \vec{b}} \tilde{W}(b, Q, x_A, x_B) \cdot [\text{Non-perturbative function}] \right. \\
 & \quad \left. + Y(q_T, y, Q) \right\} \\
 & \xrightarrow{\quad} \sum_j \int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj} \left( \frac{x_A}{\xi_A}, b, \mu \right) \cdot f_{j/A}(\xi_A, \mu) \\
 \tilde{W} = e^{-S(b)} \cdot C \otimes f(x_A) \cdot C \otimes f(x_B) \\
 & \quad \downarrow \\
 & \quad \xrightarrow{\quad} \sum_k \int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk} \left( \frac{x_A}{\xi_B}, b, \mu \right) \cdot f_{k/B}(\xi_B, \mu) \\
 & \quad \boxed{\text{Sudakov form factor } S(b) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A(\bar{\mu}) + B(\bar{\mu}) \right]}
 \end{aligned}$$

[Non-perturbative functions] are functions of  $(b, Q, x_A, x_B)$  which include QCD effects beyond Leading Twist.

[non-perturbative function] is a function of  $(b, Q, x_A, x_B)$ , implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan,  $W^\pm, Z^0$  data.



- Test QCD in problems involving multiple scales.
- Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative function], dependent of  $Q, b, x_A, x_B$ , is necessary to describe  $q_T$  – distribution of Drell-Yan,  $W^\pm, Z^0$  events.

Fits:

- Brock-Landry-Nadolsky-Yuan (BLNY)

[hep-ph/0212159](#)

$$\exp \left[ -g_1 b^2 - g_2 b^2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 b^2 \ln (100x_A x_B) \right]$$

$$\begin{aligned} Q_0 &= 1.6 \text{ GeV} \\ g_1 &= 0.21 \text{ GeV}^2 \\ g_2 &= 0.68 \text{ GeV}^2 \\ g_3 &= -0.60 \\ (b_{\max}) &= 0.5 \text{ GeV}^{-1} \end{aligned}$$

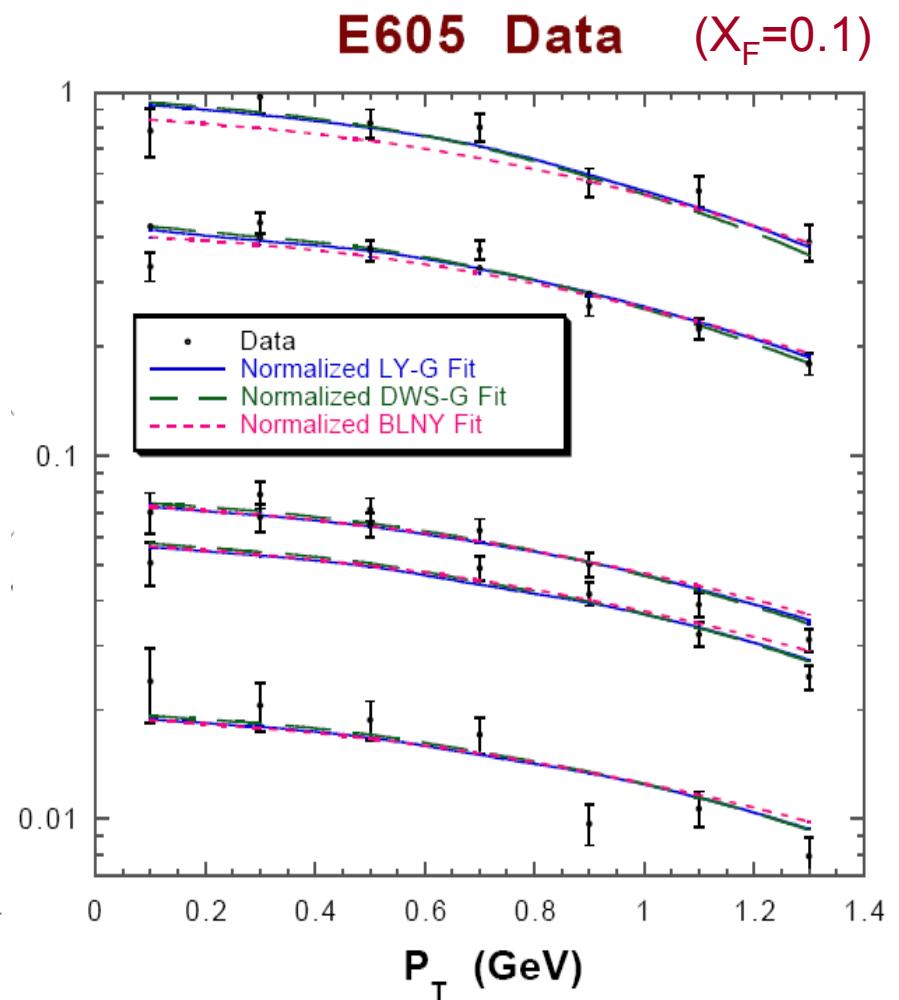
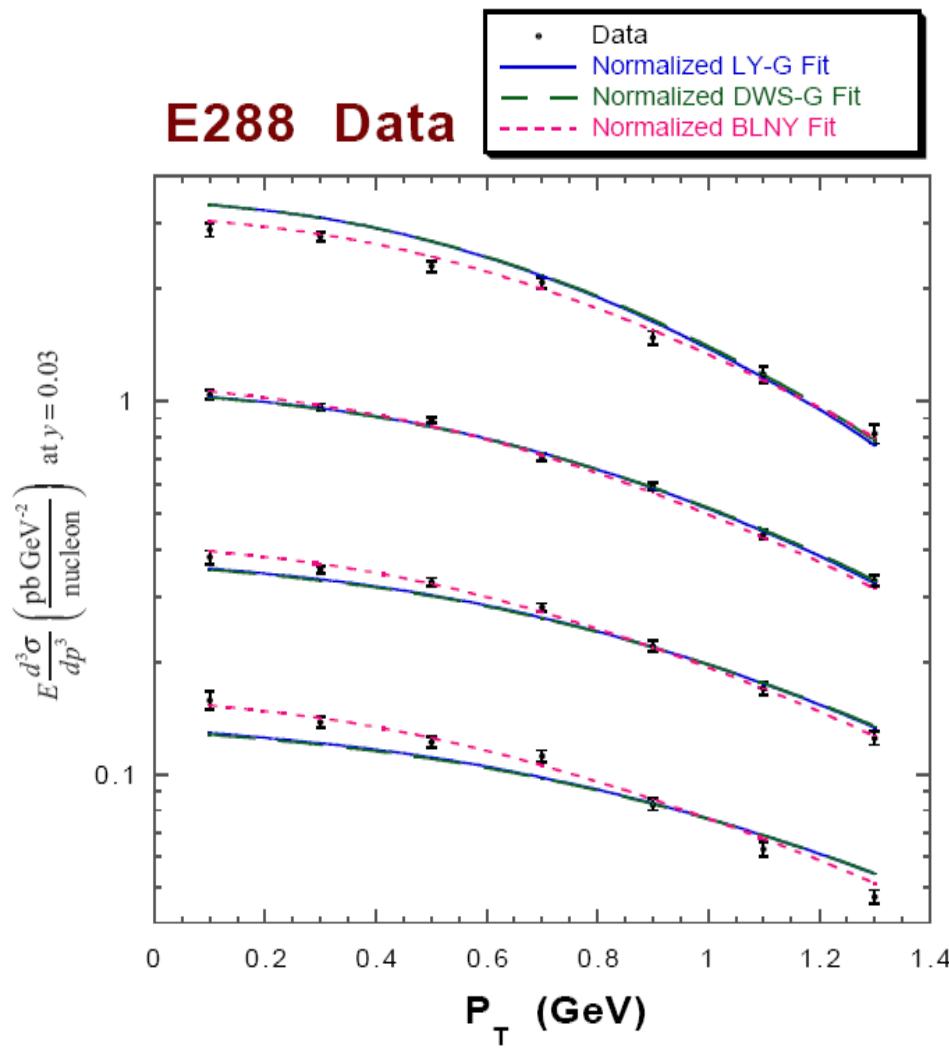
New term with  
x-dependence

# Experiments included in $P_T$ global fits

Experiment	CM Energy (GeV)	$\tau$ Range	Boson Mass (GeV)	Process
R209	62.0	0.007-0.031	5-11	$p + p \rightarrow \mu^+ \mu^- + X$
CDF Z	1800.0	0.003	91.19	$p + \bar{p} \rightarrow Z$
D $\emptyset$ Z	1800.0	0.003	91.19	$p + \bar{p} \rightarrow Z$
E288	27.4	0.033-0.110	5-9	$p + Cu \rightarrow \mu^+ \mu^- + X$
E605	38.8	0.033-0.054 0.073-0.22	7-9 10.5-18	$p + Cu \rightarrow \mu^+ \mu^- + X$

# QCD $P_T$ Resummation Global Analysis

A global analysis on Transverse Momentum  $P_T$  distributions of Drell-Yan pairs

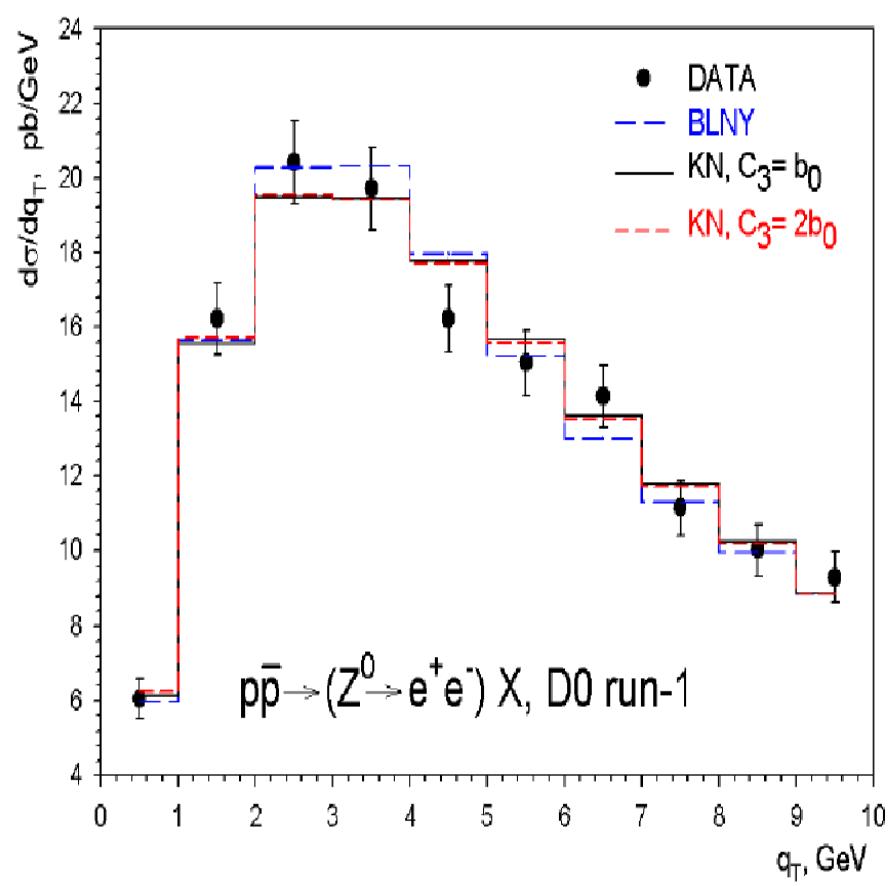
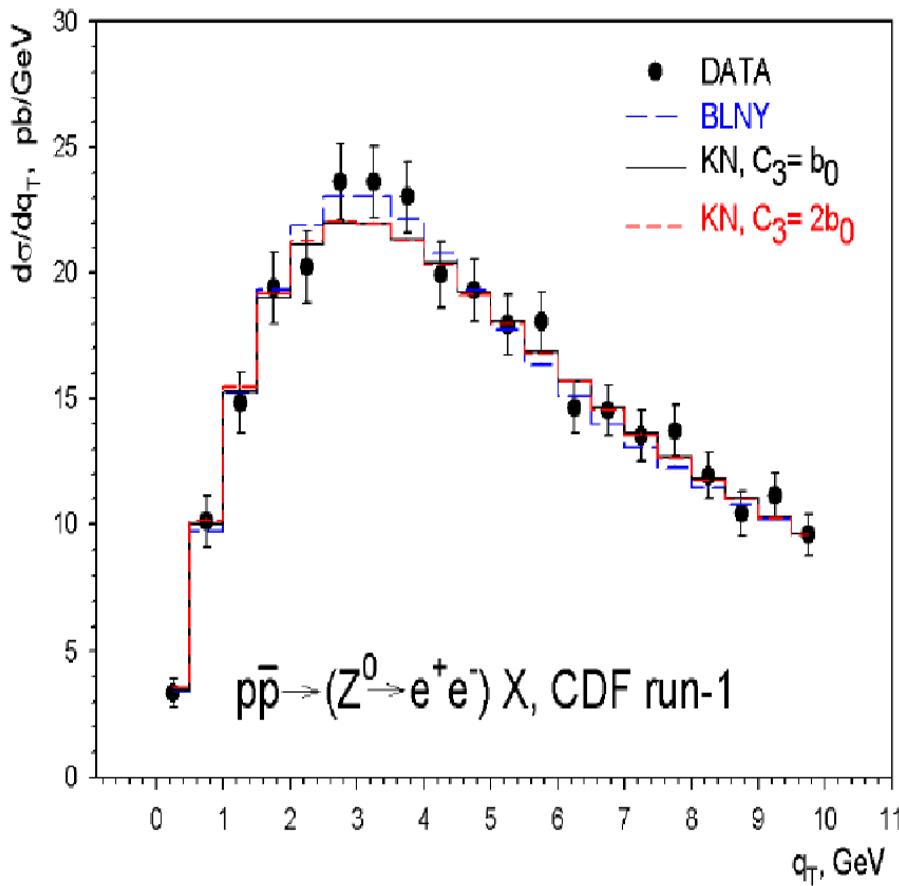


# Updated QCD $P_T$ Resummation Global Analysis

- Improve global analysis of Drell-Yan pair and  $Z$  boson  $P_T$  distributions
- Demonstrate the universality of the non-perturbative function in CSS  $P_T$  resummation formalism

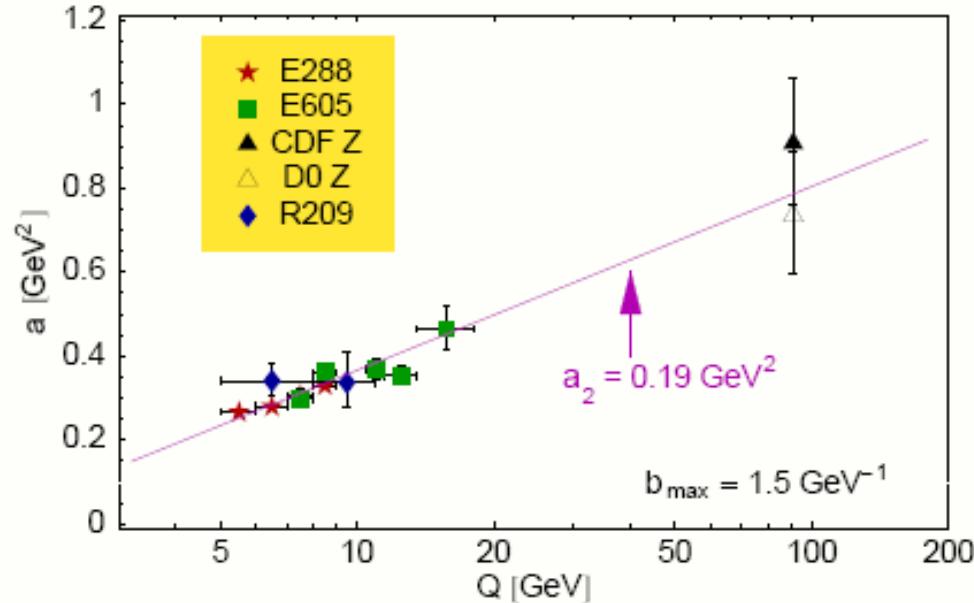
hep-ph/0506225

Konychev, Nadolsky



# Universality of the non-perturbative function in CSS resummation formalism

Nonperturbative smearing  $a(Q)$ : independent scans of 5 experiments



[non-perturbative function]

$$\mathcal{F}_{NP}(b, Q) \approx a(Q)b^{2-\beta}$$

hep-ph/0506225  
Konychev, Nadolsky

- The best fit:
  - ◆  $\beta \approx 0$  in all experiments
  - ◆  $a(Q) \approx a_1 + a_2 \ln(Q/3.2)$
- $a_2 \sim 0.18 \text{ GeV}^2$  agrees well with the IR renormalon + lattice QCD estimate,  $(a_2)_{IR} = 0.19^{+0.12}_{-0.09} \text{ GeV}^2$

hep-ph/0102237  
Tafat

# ResBos

Include      Initial state QCD soft gluon resummation  
and              Final state QED corrections

For Drell-Yan,  $W$ ,  $Z$ , Higgs, di-photon pairs, etc.

In collaboration with

hep-ph/9505203

Csaba Balazs, Pavel Nadolsky, Qing-Hong Cao, Jian-Wei Qiu  
(Michigan State, Iowa State)

**ResBos:** <http://hep.pa.msu.edu/resum/>

**ResBos-A:** including final state QED corrections

**Plotter:** on-line plotting package

(by P. Nadolsky)

hep-ph/9704258

hep-ph/0401026

# The need for combined PDF + $P_T$ Fit

Example:

To precisely predict the **rapidity distribution** of the lepton from the  $W$ -boson decay,

**ResBos** prediction is needed.

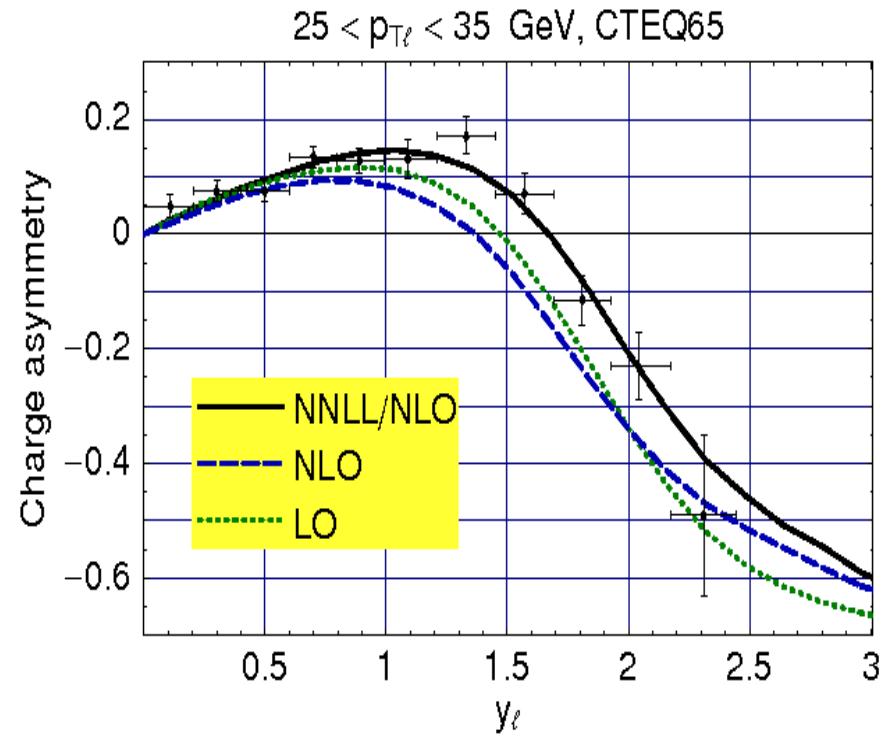
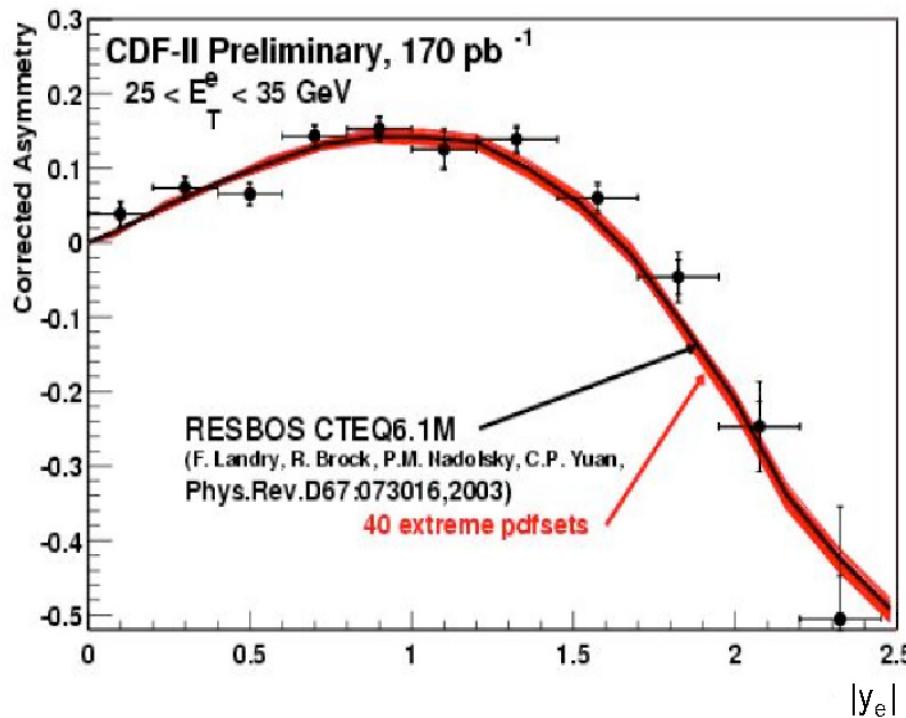
- Lepton rapidity asymmetry is sensitive to quark PDFs
- Transverse momentum distributions in all  $P_T$  range can only be reliably predicted by resummation calculation.



Combined fit can further constrain the PDFs

# To predict rapidity distribution requires ResBos calculation

Charged lepton asymmetry (from  $W$  decay)



All recent CTEQ and MSTW PDF fits include the effects of soft gluon resummation predicted by ResBos.

# Potential of Combined $P_T$ resummation and PDF global QCD analysis

- New constraints on PDFs,
- Better predictions for precision  
 $W$ ,  $Z$ , Top, and Higgs physics at the Tevatron and LHC.



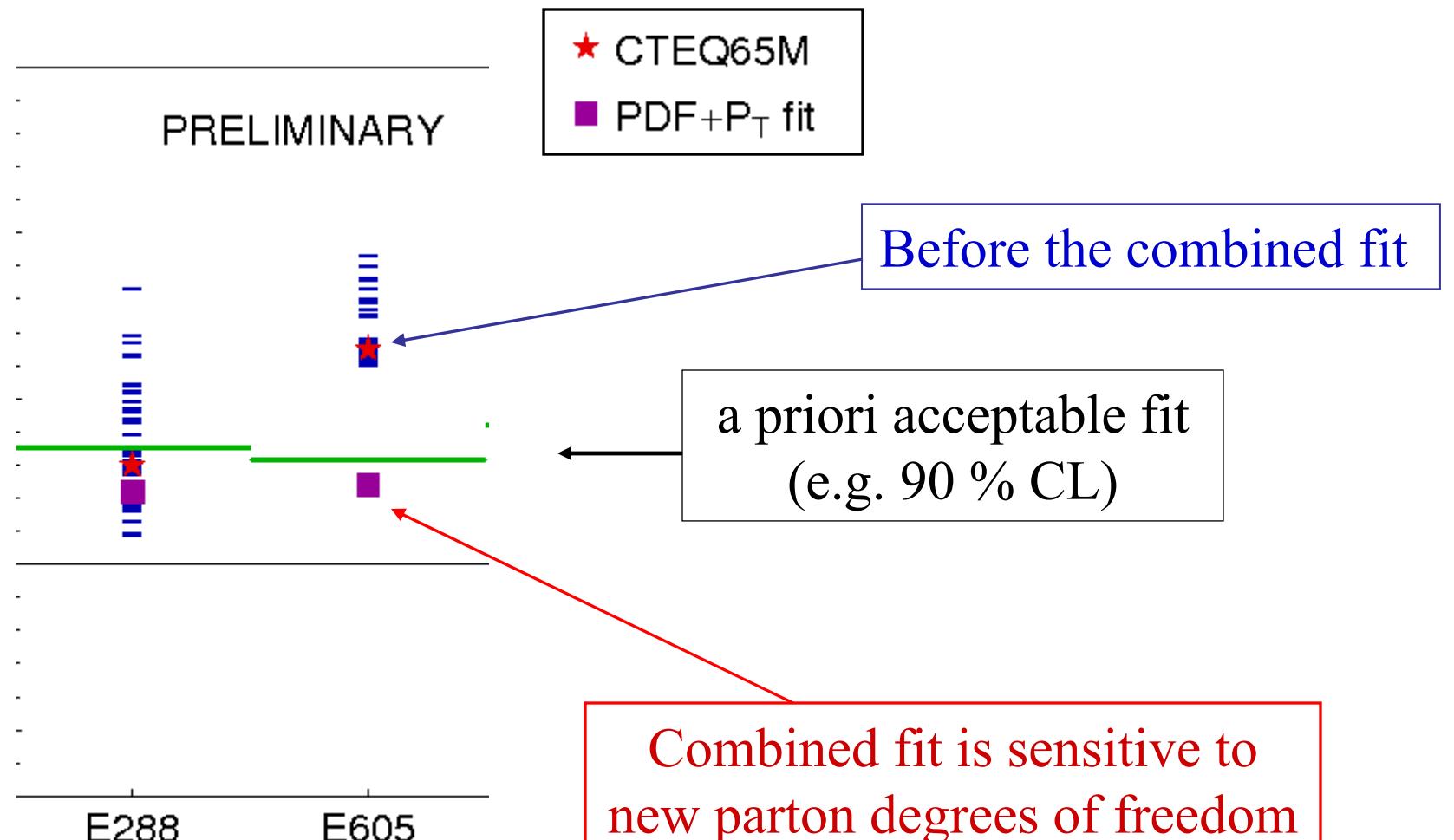
We have recently succeeded in performing the combined PDF+ $P_T$  fits

(Overcome the problem of speed and precision of calculations.)

In collaboration with

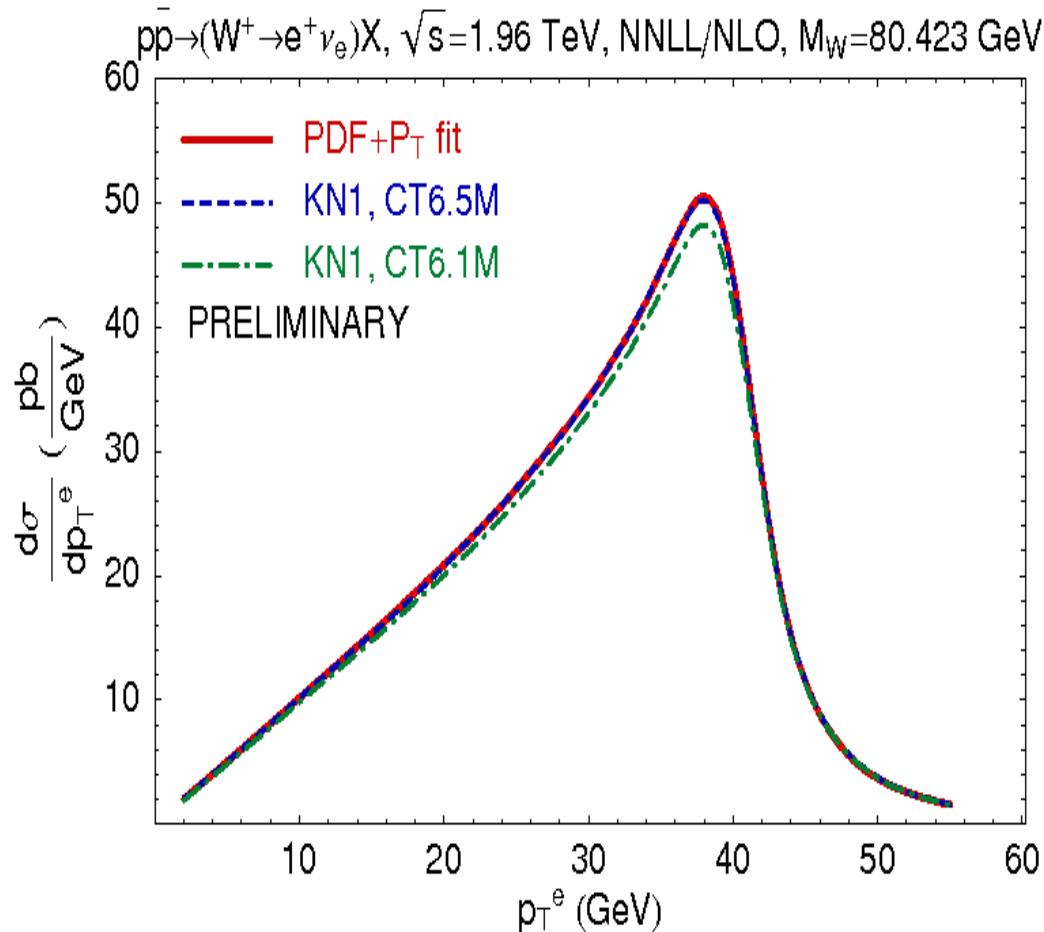
Lai, Nadolsky, Pumplin, Tung  
(Argonne, Michigan State, U Washington)

# “Goodness-of-fit”: before and after the combined fit



Fit to  $P_T$  of Drell-Yan pairs

## $P_T$ of lepton: before and after the combined fit



Cause the change in  
the shape of  $P_T$   
near the Jacobian peak

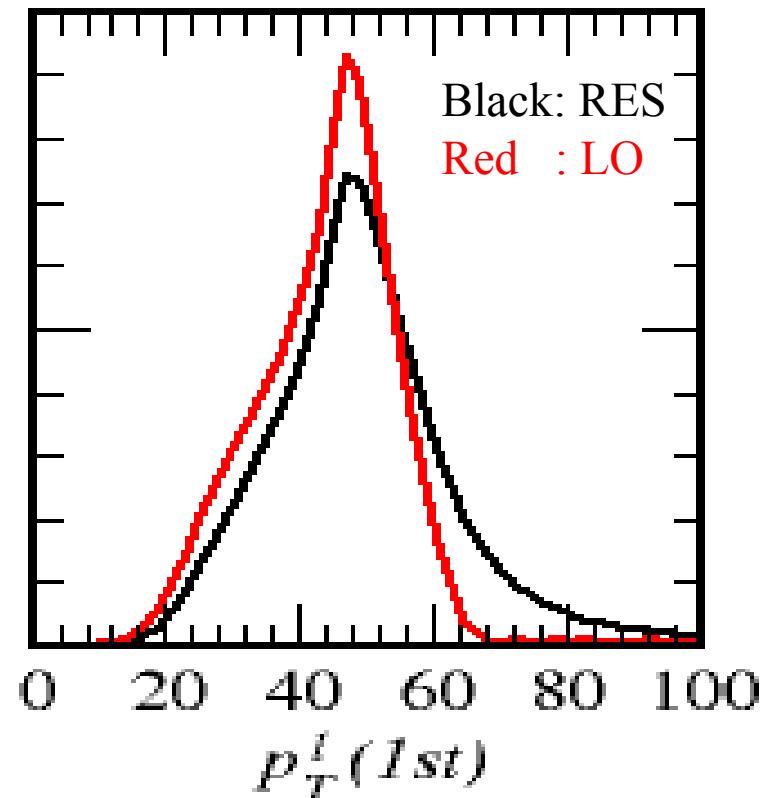
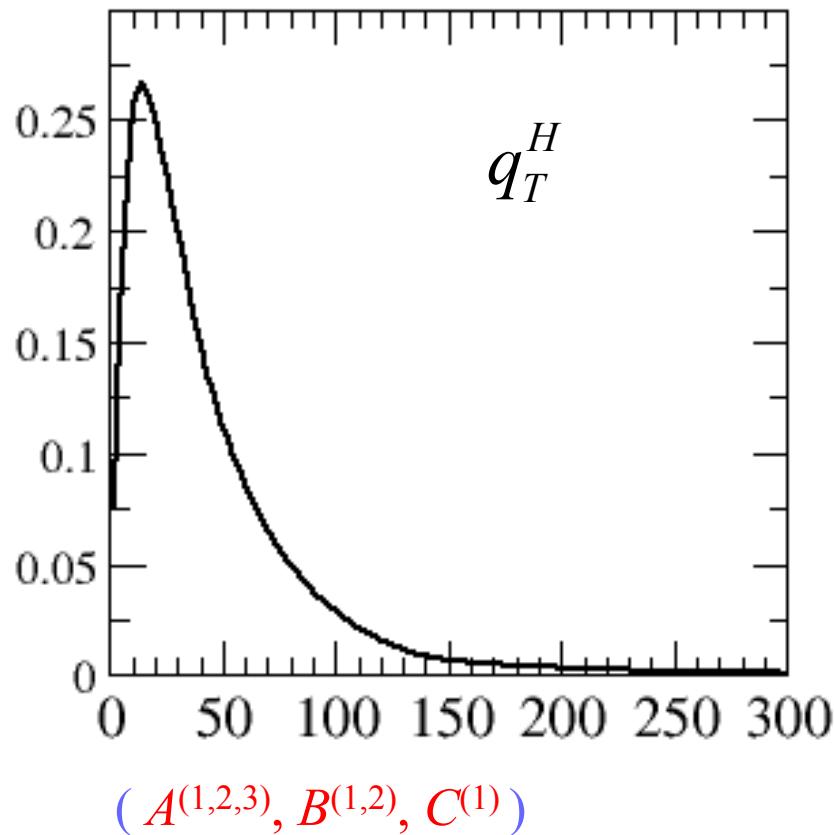
Affect  $M_W$  measurement

# ResBos: Higgs Phenomenology

$gg \rightarrow H \rightarrow ZZ \rightarrow 4l's$

hep-ph/0704.1344  
Cao, Chen

The spin correlation effects in  $H \rightarrow ZZ$  have been kept.



The acceptance of the event is modified.



Affect the measured inclusive cross section of Higgs boson.

# Conclusion

- Effect of QCD  $P_T$  resummation (implemented in ResBos code) is important for precision measurements at Tevatron and LHC:

- PDFs (e.g., from lepton rapidity distribution)
- precision electroweak measurement (e.g.,  $M_W$ )  
(Drell-Yan,  $W$ ,  $Z$ , Higgs, di-photon, etc.)



Combined PDF +  $P_T$  fit

(Stay tuned!)

- A similar ResBos code exists for Semi-Inclusive DIS processes. (Ask me about it)

# Backup Slides

# Heavy Flavor Production at HERA

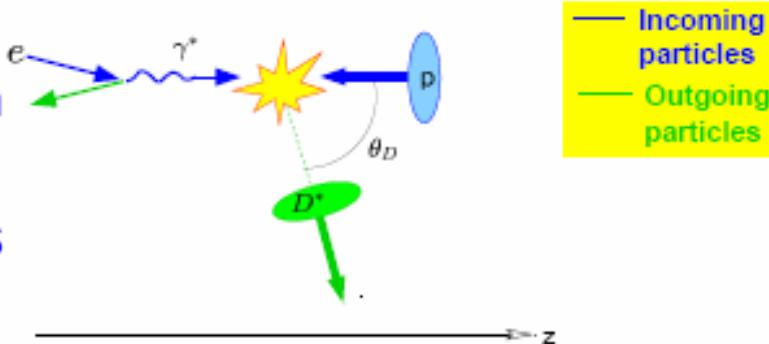
Hep-ph/0210082

Pavel Nadolsky, Kidonakis, Olness, CPY

- ✓ total cross section (or heavy flavor component of DIS structure functions  $F_2^H(x, Q^2)$ ,  $H = c, b, \dots$ )

$$\frac{d\sigma(e + p \rightarrow e + D^*(B) + X)}{dx dQ^2} \propto a_1 F_1^H(x, Q^2) + a_2 F_2^H(x, Q^2)$$

- ✓ differential distributions
  - ◆ transverse momentum distributions
  - ◆ polar angle distributions in the  $\gamma^* p$  c.m. frame

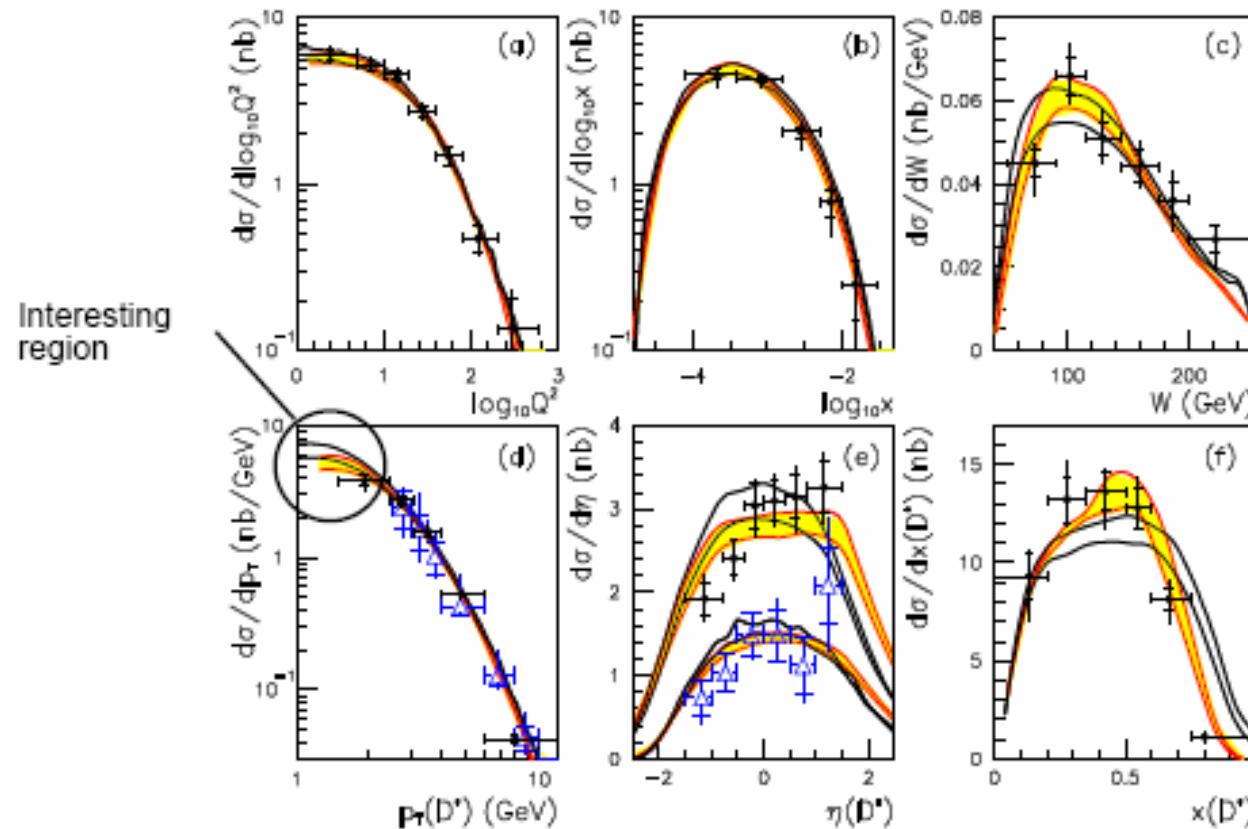


#2 is needed to derive #1

Fixed-order calculations in any scheme fail in the region  $\theta_D \rightarrow 0$   
The problem also affects  $p_T$  distributions, etc.

## Differential distributions of charm quarks

ZEUS 1996–97

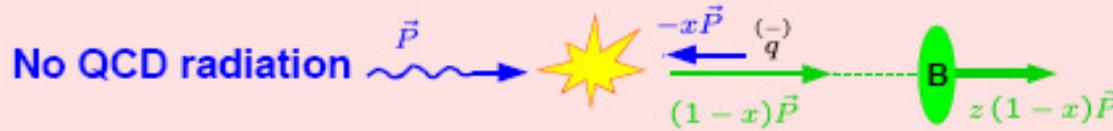
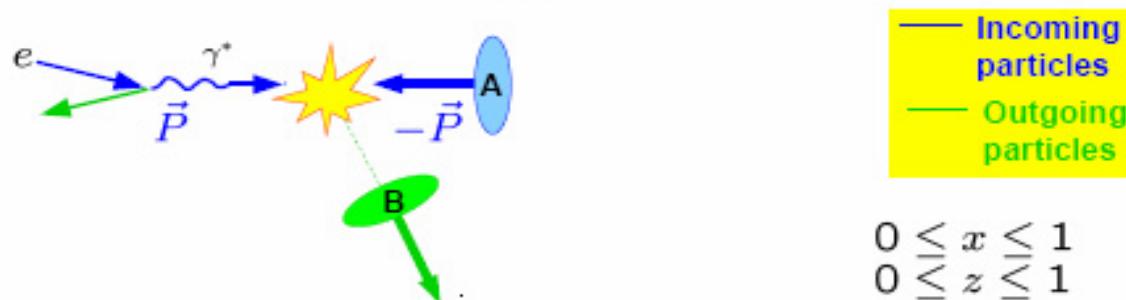


- ✓ Curves:  
2-loop  
finite-order  
calculation;  
PYTHIA
- ✓ 30 ÷ 60% of  
 $F_2^c(x, Q^2)$  is  
derived from  
the unseen  
region
- ✓ Quality of  
the data will  
improve in  
HERA Run II

I discuss a more accurate formalism for extrapolation to the small  $p_T$  region; it leads to sizeable differences in the extrapolated cross section

## The culprit: intensive radiation of soft & collinear gluons

Semi-inclusive DIS in  $\gamma^* p$  c.m. frame

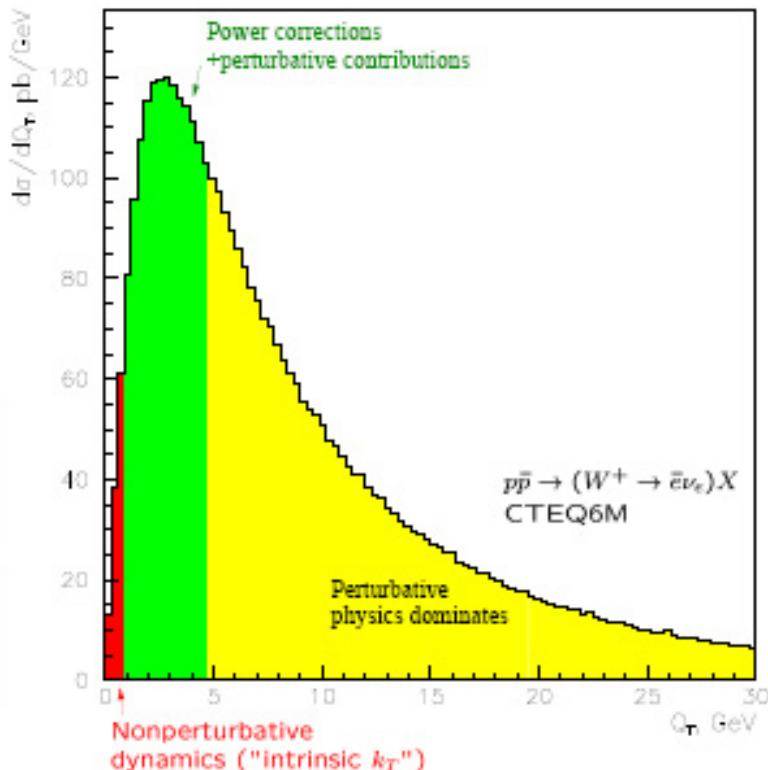
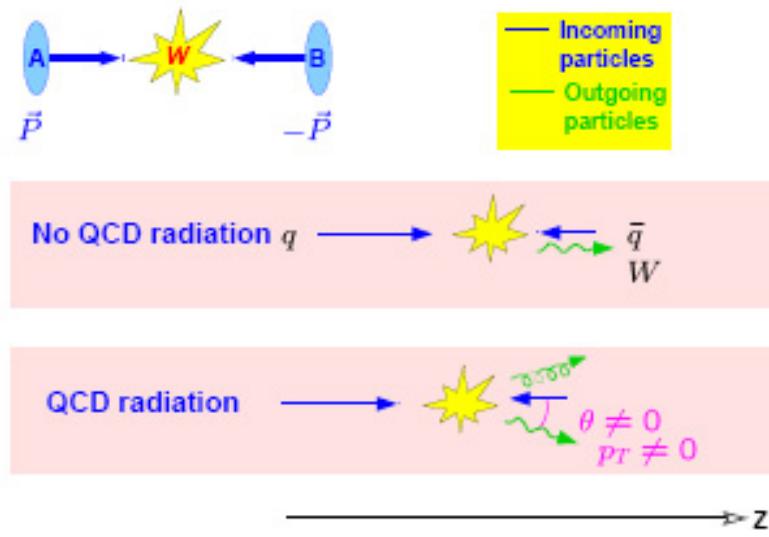


- ✓ Does not affect inclusive distributions (KLN theorem)
- ✓ Strongly affects differential cross sections

Fixed-order calculations in any scheme fail in the region  $\theta_D \rightarrow 0$   
The problem also affects  $p_T$  distributions, etc.

## $q_T$ resummation for vector boson production

### Resummation: W boson production at the Tevatron



Needed to precisely measure W-boson mass

Different  $q_T$  ranges  $\Leftrightarrow$  different dynamical mechanisms

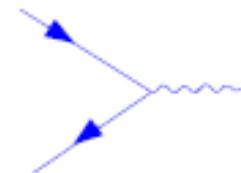
CSS formalism describes all  $q_T$  range in one elegant framework

DIS hadroproduction is related to Drell-Yan process through crossing

DIS

Drell-Yan

Due to its Lorentz invariance, CSS formalism should be straightforwardly applicable to DIS hadroproduction



Small- $q_T$  cross section is related to a form factor in the impact parameter ( $b$ ) space

$$\frac{d\sigma}{dx dz dQ^2 dq_T^2} \Big|_{q_T \rightarrow 0} \propto \int \frac{d^2 b}{(2\pi)^2} e^{-i \vec{q}_T \cdot \vec{b}} D(z, b) e^{-S(b, Q)} \mathcal{F}(x, b)$$

$S(b, Q)$ : soft (Sudakov) factor

$\mathcal{F}(x, b) = (\mathcal{C} \otimes f)(x, b)$  :  $b$ -dependent parton distribution

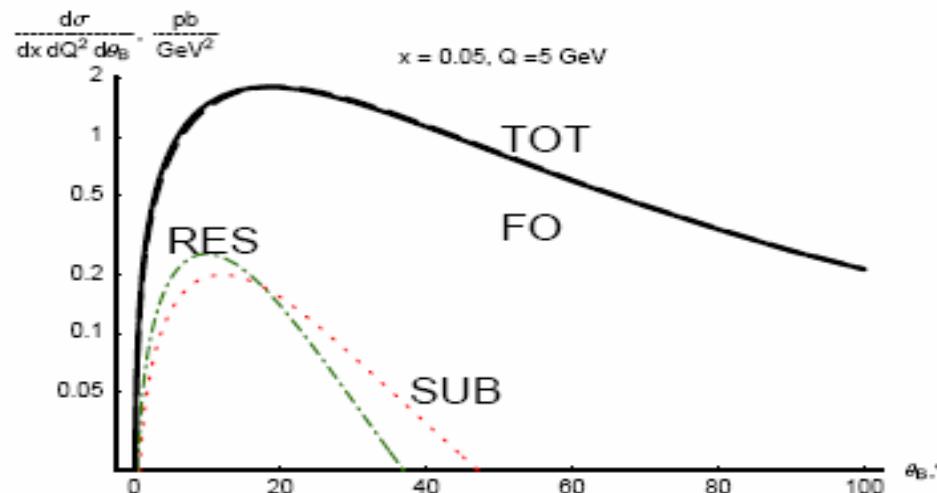
$D(z, b) = (D \otimes \mathcal{C})(x, b)$ :  $b$ -dependent fragmentation function

What is  $q_T$  in DIS?

It is a function of  $\theta_D$

$$q_T^2 = \frac{(p_T)_{D^*}^2}{z^2} = Q^2 \frac{(1-x)}{x} \left( \frac{\theta_D^2}{4} + \dots \right)$$

Distributions of bottom quarks in the  $\gamma^* p$  c.m. frame  
Threshold region  $Q = 5$  GeV



$\sqrt{s_{ep}} = 300$  GeV;  $M_b = 4.5$  GeV; CTEQ5HQ PDFs; Peterson FF's;  $S^{NP}(b, Q) = 0$ ;  $b_{max} = 1.23$  GeV<sup>-1</sup>

$\sigma_{TOT}(\text{solid}) = \sigma_{RES}(\text{dot-dashed}) + \sigma_{FO}(\text{dashed}) - \sigma_{SUB}(\text{dotted}) \approx \sigma_{FO}(\text{dashed})$

# ResBos for SIDIS at HERA

- Consistent with CTEQ6.5 (ACOT,  $\chi$ ) scheme
- Available for SIDIS **charm** and **bottom** meson productions.
- Also available for SIDIS **energy flow** and **charged particle multiplicity**.

hep-ph/0012261  
Nadolsky, Stump, CPY

ResBos for SIDIS:

<http://hep.pa.msu.edu/resum/>

# What can we learn from the $P_T$ of Z-Boson at the Tevatron/LHC

- QCD

- \* Calibrate  $P_T$  of W-boson

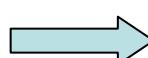
 Improve the measurement of  $M_W$

- \* Measure PDF

Small x,      Large Q  
 $(\sim 10^{-4}-10^{-3})$        $(\sim 100 \text{ GeV})$

- Electroweak

Forward-Backward Asymmetry of lepton  
on- or off- Resonance

  $\sin^2 \theta_W$ ,  $Z'$ , etc.