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# **Status of $e^+e^- \rightarrow 3j$ at NNLO**

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# $e^+e^- \rightarrow 3 \text{ jets and event shapes}$

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## Classical QCD observable

- Testing ground of QCD in  $e^+e^-$  annihilation: perturbation theory, logarithmic resummation
- Precise determination of  $\alpha_s$
- Current error on  $\alpha_s$  from jet observables dominated by theoretical uncertainty:  
S. Bethke, 2006

$$\alpha_s(M_Z) = 0.121 \pm 0.001(\text{experiment}) \pm 0.005(\text{theory})$$

- theoretical uncertainty largely from missing higher orders
- NNLO corrections to the 3-Jet rate are needed !

# $e^+e^- \rightarrow 3 \text{ jets and event shapes}$

## Event shape variables

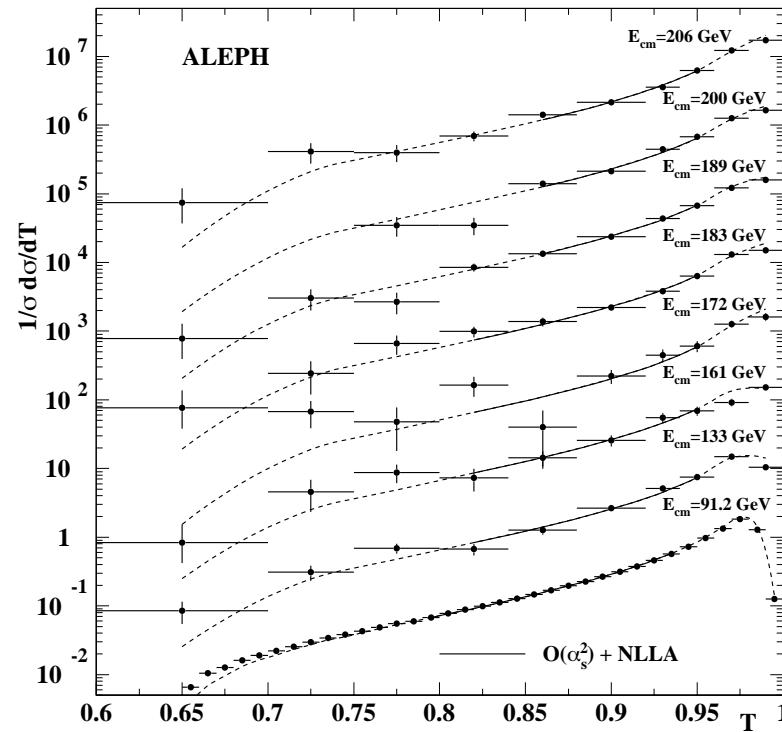
assign a number  $x$  to a set of final state momenta:  $\{p\}_i \rightarrow x$

e.g. Thrust in  $e^+e^-$

$$T = \max_{\vec{n}} \frac{\sum_{i=1}^n |\vec{p}_i \cdot \vec{n}|}{\sum_{i=1}^n |\vec{p}_i|}$$

limiting values:

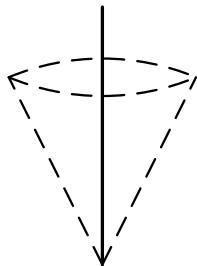
- back-to-back (two-jet) limit:  $T = 1$
- spherical limit:  $T = 1/2$



# Jets in perturbative QCD

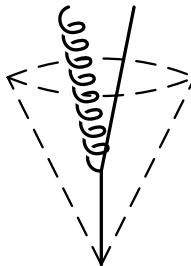
Partons are combined into jets using the same jet algorithm (recombination procedure) in theory as in experiment

LO



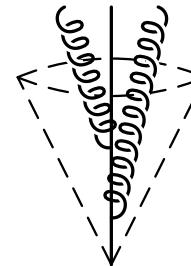
each  
parton  
forms 1 jet  
on its own

NLO



2 partons in  
1 jet, 1 parton  
experimentally  
unresolved

NNLO



3 partons in  
1 jet, 2 partons  
experimentally  
unresolved

Current state-of-the-art: NLO

Need for NNLO:

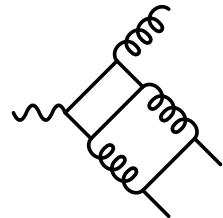
- reduce error on  $\alpha_s$
- better matching of parton level and hadron level jets

# Colour structure of NNLO 3-jet

Decomposition into leading and subleading colour terms

$$\begin{aligned}\sigma_{NNLO} = & (N^2 - 1) \left[ N^2 A_{NNLO} + B_{NNLO} + \frac{1}{N^2} C_{NNLO} + N N_F D_{NNLO} \right. \\ & \left. + \frac{N_F}{N} E_{NNLO} + N_F^2 F_{NNLO} + N_{F,\gamma} \left( \frac{4}{N} - N \right) G_{NNLO} \right]\end{aligned}$$

- last term: closed quark loop coupling to vector boson, numerically tiny



$$N_{F,\gamma} = \frac{\left( \sum_q e_q \right)^2}{\sum_q e_q^2}$$

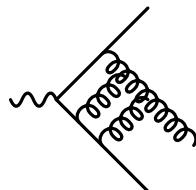
- most subleading colour:  $C_{NNLO}$ ,  $E_{NNLO}$ ,  $F_{NNLO}$ ,  $(G_{NNLO})$   
QED-type contributions: gluons  $\rightarrow$  photons
- simplest term:  $F_{NNLO}$ , only 3 parton and 4 parton contributions

# $e^+e^- \rightarrow 3 \text{ jets at NNLO}$

## Ingredients

- Two-loop matrix elements

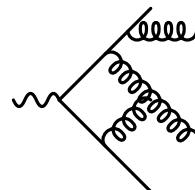
$$|\mathcal{M}|_{\text{2-loop}, 3 \text{ partons}}^2$$



explicit infrared poles from loop integrals

- One-loop matrix elements

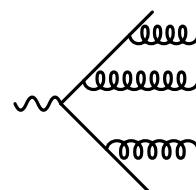
$$|\mathcal{M}|_{\text{1-loop}, 3+1 \text{ partons}}^2$$



explicit infrared poles from loop integral and  
implicit infrared poles due to single unresolved  
radiation

- Tree level matrix elements

$$|\mathcal{M}|_{\text{tree}, 3+2 \text{ partons}}^2$$



implicit infrared poles due to double unresolved  
radiation

- Infrared Poles cancel in the sum

- Divergencies extracted before the jet algorithm can be applied  
→ Subtraction formalism needed

# NLO Subtraction

Structure of NLO  $m$ -jet cross section (subtraction formalism):

Z. Kunszt, D. Soper

$$d\sigma_{NLO} = \int_{d\Phi_{m+1}} \left( d\sigma_{NLO}^R - d\sigma_{NLO}^S \right) + \left[ \int_{d\Phi_{m+1}} d\sigma_{NLO}^S + \int_{d\Phi_m} d\sigma_{NLO}^V \right]$$

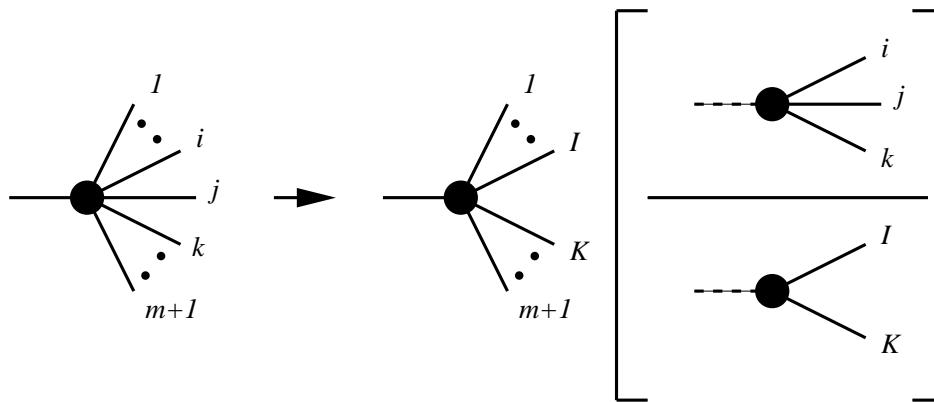
- $d\sigma_{NLO}^S$ : local counter term for  $d\sigma_{NLO}^R$
- $d\sigma_{NLO}^R - d\sigma_{NLO}^S$ : free of divergences, can be integrated numerically

General methods at NLO

- Dipole subtraction  
S. Catani, M. Seymour; NNLO: S. Weinzierl
- $\mathcal{E}$ -prescription  
S. Frixione, Z. Kunszt, A. Signer; NNLO: S. Frixione, M. Grazzini
- Antenna subtraction  
D. Kosower; J. Campbell, M. Cullen, N. Glover; NNLO: this talk

# NLO Antenna Subtraction

Building block of  $d\sigma_{NLO}^S$ : NLO-Antenna function  $X_{ijk}^0$  and phase space  $d\Phi_{X_{ijk}}$



$$d\sigma_{NLO}^S = \mathcal{N} \sum_{m+1} \sum_j d\Phi_m(p_1, \dots, \tilde{p}_I, \tilde{p}_K, \dots, p_{m+1}; q) \cdot d\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K)$$

$$\times X_{ijk}^0 |\mathcal{M}_m(p_1, \dots, \tilde{p}_I, \tilde{p}_K, \dots, p_{m+1})|^2 J_m^{(m)}(p_1, \dots, \tilde{p}_I, \tilde{p}_K, \dots, p_{m+1})$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2 J_m^{(m)} d\Phi_m \int d\Phi_{X_{ijk}} X_{ijk}^0$$

can be combined with  $d\sigma_{NLO}^V$

# NNLO Subtraction

Structure of NNLO  $m$ -jet cross section:

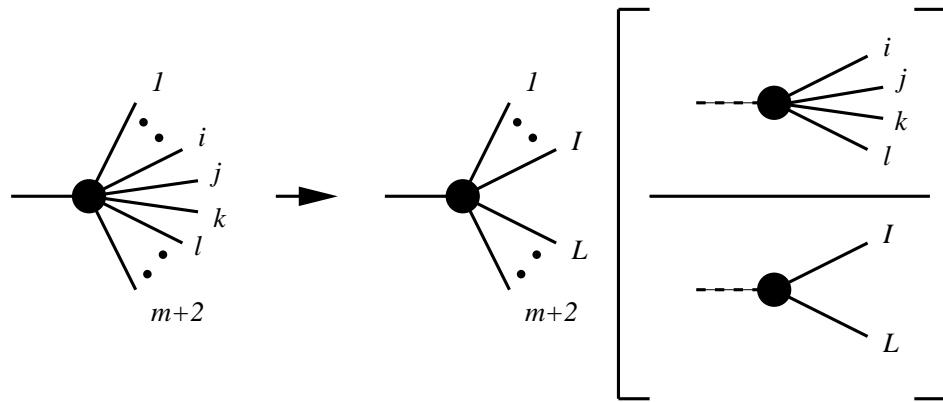
$$\begin{aligned} d\sigma_{NNLO} &= \int_{d\Phi_{m+2}} \left( d\sigma_{NNLO}^R - d\sigma_{NNLO}^S \right) \\ &\quad + \int_{d\Phi_{m+1}} \left( d\sigma_{NNLO}^{V,1} - d\sigma_{NNLO}^{VS,1} \right) \\ &\quad + \int_{d\Phi_m} d\sigma_{NNLO}^{V,2} + \int_{d\Phi_{m+2}} d\sigma_{NNLO}^S + \int_{d\Phi_{m+1}} d\sigma_{NNLO}^{VS,1}, \end{aligned}$$

- $d\sigma_{NNLO}^S$ : real radiation subtraction term for  $d\sigma_{NNLO}^R$
- $d\sigma_{NNLO}^{VS,1}$ : one-loop virtual subtraction term for  $d\sigma_{NNLO}^{V,1}$
- $d\sigma_{NNLO}^{V,2}$ : two-loop virtual corrections

Each line above is finite numerically and free of infrared  $\epsilon$ -poles → numerical programme

# Double Real Subtraction

Two colour-connected unresolved partons



Phase space factorisation

$$X_{ijkl}^0 = S_{ijkl,IL} \frac{|M_{ijkl}^0|^2}{|M_{IL}^0|^2}$$

$$d\Phi_{X_{ijkl}} = \frac{d\Phi_4}{P_2}$$

$$d\Phi_{m+2}(p_1, \dots, p_{m+2}; q) = d\Phi_m(p_1, \dots, \tilde{p}_I, \tilde{p}_L, \dots, p_{m+2}; q) \cdot d\Phi_{X_{ijkl}}(p_i, p_j, p_k, p_l; \tilde{p}_I + \tilde{p}_L)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2 J_m^{(m)} d\Phi_m \int d\Phi_{X_{ijkl}} X_{ijkl}^0 \sim |\mathcal{M}_m|^2 J_m^{(m)} d\Phi_m \int d\Phi_4 |M_{ijkl}^0|^2$$

Four-particle inclusive phase space integrals are known

T. Gehrmann, G. Heinrich, AG

# Numerical Implementation

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## Parton-level event generator

Starting point:  $e^+e^- \rightarrow 4 \text{ jets}$  at NLO (EERAD2: J. Campbell, M. Cullen, N. Glover)

- contains already 4-parton and 5-parton matrix elements
- is based on NLO antenna subtraction

additions:

- NNLO subtraction terms (5-parton channel)
- 1-loop-single unresolved integrated subtraction term (4-parton channel)
- 2-loop matrix element (3-parton channel)

checks:

- independence on phase space cut  $y_0$
- local cancellations along phase space trajectories
- cancellation of infrared poles

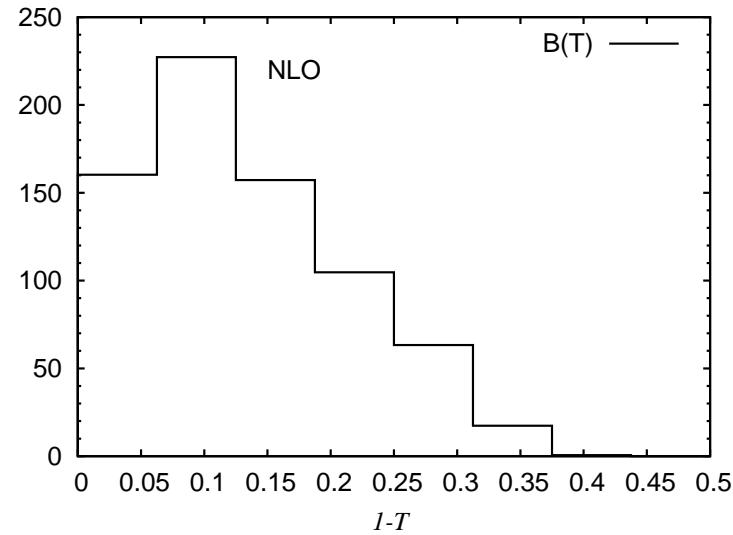
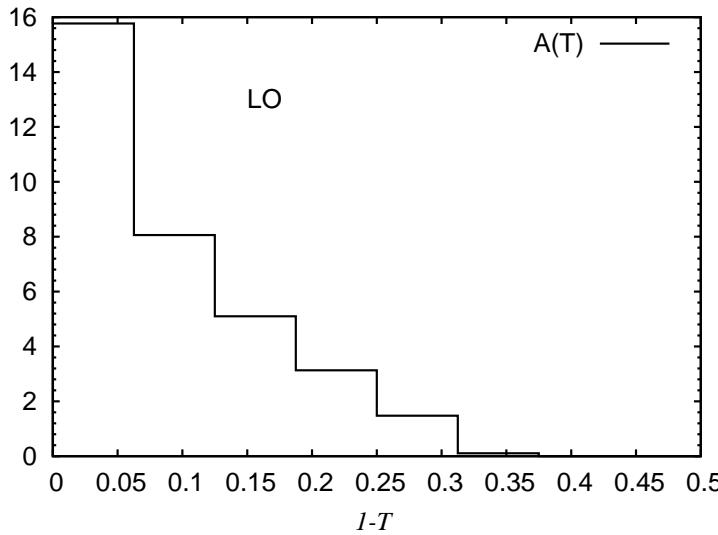
# Thrust distribution

## NNLO expression for Thrust

$$(1 - T) \frac{1}{\sigma_{\text{had}}} \frac{d\sigma}{dT} = \left( \frac{\alpha_s}{2\pi} \right) A(T) + \left( \frac{\alpha_s}{2\pi} \right)^2 (B(T) - 2A(T)) \\ + \left( \frac{\alpha_s}{2\pi} \right)^3 (C(T) - 2B(T) + 1.64 A(T))$$

with LO contribution  $A(T)$ , NLO contribution  $B(T)$

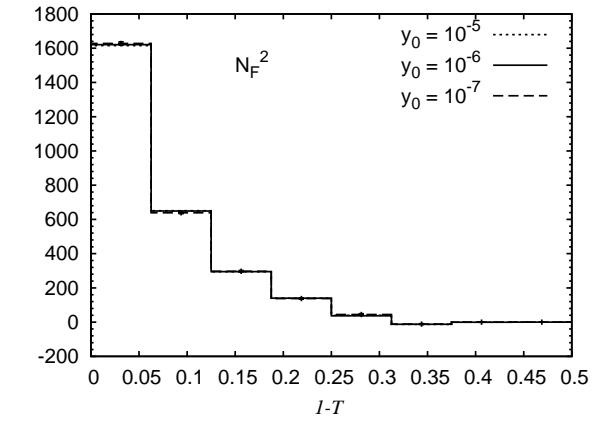
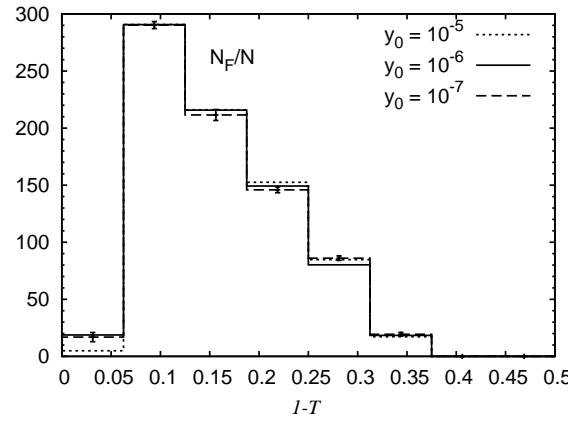
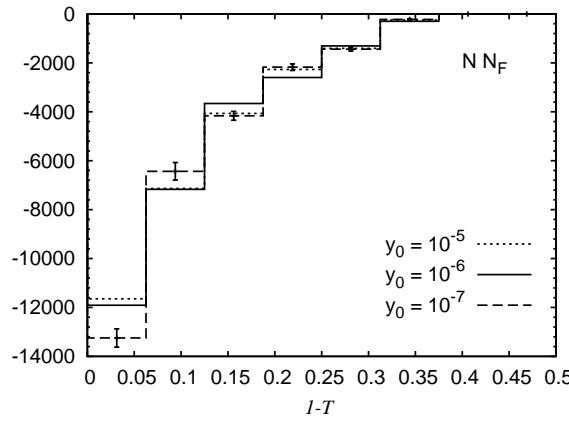
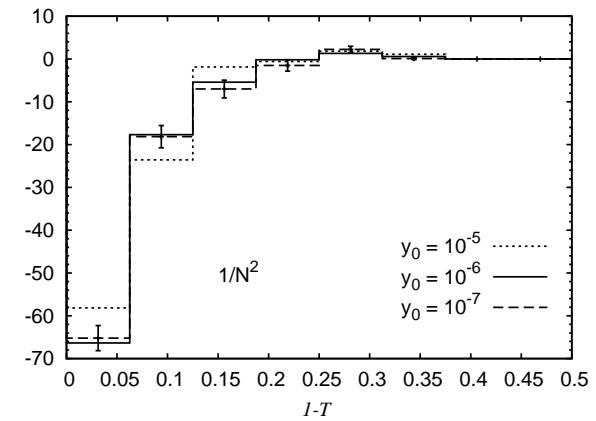
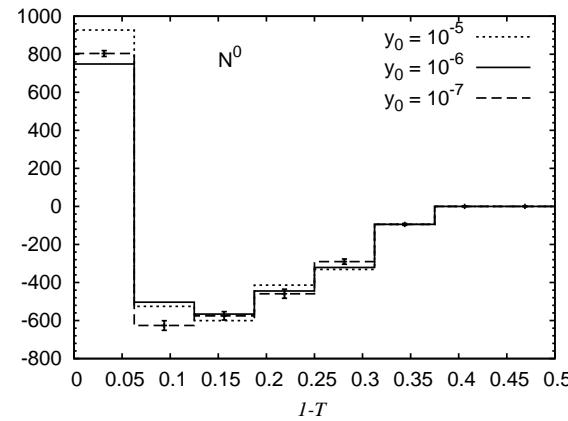
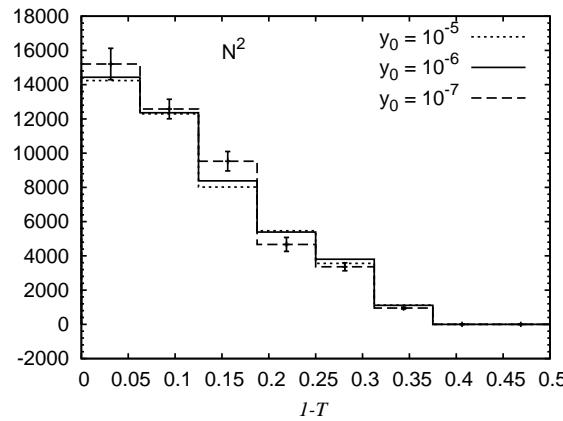
R.K. Ellis, D.A. Ross, A. Terrano



# Results

## NNLO coefficient $C(T)$ of thrust (preliminary)

T. Gehrmann, E.W.N. Glover, G. Heinrich, AG



# Conclusions

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- Antenna subtraction method at NNLO
  - building blocks of subtraction terms: 3 and 4 parton antenna functions, derived from physical matrix elements
  - subtraction terms correctly approximate the full  $|\mathcal{M}|^2$  (double real, one-loop/real) in all unresolved limits
- Applied to  $e^+e^- \rightarrow 3j$  at NNLO
  - implementation completed and checked
  - first results obtained for NNLO thrust distribution
  - ongoing: verification and production of high-precision results
- Outlook
  - implementation of all event shapes and  $\sigma_{3j}$
  - NNLO determination of  $\alpha_s$  from event shape data