Are generalized and transverse momentum dependent parton distributions related ?

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- Definition of GPDs and TMDs
- Impact parameter representation of GPDs
- Model-independent considerations
- Model results
- Summary

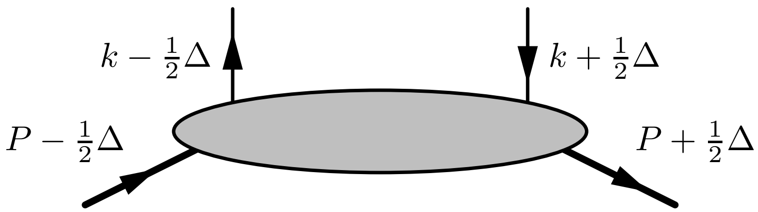
Based on: S. Meißner, A. Metz, K. Goeke, hep-ph/0703176

Definition of GPDs and TMDs

1. GPDs

- Appear in QCD-description of hard exclusive reactions

- Kinematics



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GPD-correlator

$$\begin{aligned} F^q &= \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda' | \, \bar{\psi}\Big(-\frac{z}{2}\Big) \, \gamma^+ \, \mathcal{W}_{GPD} \, \psi\Big(\frac{z}{2}\Big) \, |p;\lambda\rangle \Big|_{z^+ = z_T = 0} \\ &= \frac{1}{2P^+} \, \bar{u}(p',\lambda') \left(\gamma^+ \, H^q(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} \, E^q(x,\xi,t) \right) u(p,\lambda) \end{aligned}$$

$$x = \frac{k^+}{P^+} \qquad \xi = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

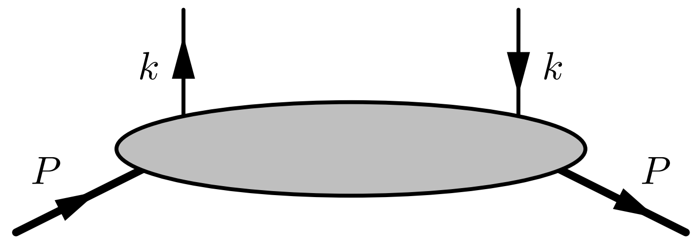
- Leading twist for

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

2. TMDs

- Appear in QCD-description of hard semi-inclusive reactions

- Kinematics



- TMD-correlator

$$\Phi^q = \frac{1}{2}\int \frac{dz^-}{2\pi}\,\frac{d^2\vec{z}_T}{(2\pi)^2}\,e^{ik\cdot z}\,\langle P;S|\,\bar{\psi}\Big(-\frac{z}{2}\Big)\,\gamma^+\,\mathcal{W}_{TMD}\,\psi\Big(\frac{z}{2}\Big)\,|P;S\rangle\Big|_{z^+=0}$$

$$= f_1^q(x,\vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij}k_T^i S_T^j}{M}\,f_{1T}^{\perp q}(x,\vec{k}_T^{\,2})$$

- Leading twist for

$$\bar{\psi}\,\gamma^+\,\psi \qquad \bar{\psi}\,\gamma^+\gamma_5\,\psi \qquad \bar{\psi}\,i\sigma^{j+}\gamma_5\,\psi$$

3. Leading twist GPDs and TMDs

	Quarks				Gluons			
Forward	f_1	g_1	h_1		g	Δg		
k_T-dependent	f_1^q	$f_{1T}^{\perp q}$	g_{1L}^q	g_{1T}^q	f_1^g	$f_{1T}^{\perp g}$	g_{1L}^g	g_{1T}^g
	h_{1T}^q	$h_{1L}^{\perp q}$	$h_{1T}^{\perp q}$	$h_1^{\perp q}$	h_{1T}^g	$h_{1L}^{\perp g}$	$h_{1T}^{\perp g}$	$h_1^{\perp g}$
Generalized	H^q	E^q	$\tilde{H}^q$	$\tilde{E}^q$	H^g	E^g	$\tilde{H}^g$	$\tilde{E}^g$
	H_T^q	E_T^q	$\tilde{H}_T^q$	$\tilde{E}_T^q$	H_T^g	E_T^g	$\tilde{H}_T^g$	$\tilde{E}_T^g$

- Trivial relations:

$$\int d^2\vec{k}_T\, f_1^q(x,\vec{k}_T^{\,2}) = f_1^q(x) = H^q(x,0,0) \quad \text{etc.}$$

- Are there non-trivial relations ?

Impact parameter representation of GPDs

- Fourier transform of GPD-correlator $(\xi = 0)$ (Burkardt, 2000)

$$F^q(x, \vec{\Delta}_T, S) = \frac{1}{2} \int \frac{dz^-}{2\pi}\, e^{ik\cdot z}\, \langle p'; S|\, \bar{\psi}\Big(-\frac{z}{2}\Big)\, \gamma^+\, \mathcal{W}_{GPD}\, \psi\Big(\frac{z}{2}\Big)\, |p; S\rangle \Big|_{z^+ = z_T = 0}$$

$$\begin{aligned} \mathcal{F}^q(x, \vec{b}_T; S) &= \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2}\, e^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, F^q(x, \vec{\Delta}_T; S) \\ &= \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \Big(\mathcal{E}^q(x, \vec{b}_T^{\,2})\Big)' \end{aligned}$$

$$\text{with} \quad \mathcal{H}^q(x, \vec{b}_T^{\,2}) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2}\, e^{-i\vec{\Delta}_T\cdot\vec{b}_T}\, H(x, 0, -\vec{\Delta}_T^{\,2})$$

- Distortion of GPD-correlator in impact parameter space

$$d^{q,i} = \int dx \int d^2\vec{b}_T\, b_T^i\, \mathcal{F}^q(x, \vec{b}_T; S) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \int dx\, E^q(x, 0, 0) = -\frac{\epsilon_T^{ij} S_T^j}{2M} \kappa^q$$

$\longrightarrow$ Flavor dipole moment of about $0.2\,\mathrm{fm}$ for light flavors in nucleon

- Relation between distortion and Sivers effect

  $\longrightarrow$ Large distortion should have observable effect (Burkardt, 2002)

$$
\begin{aligned}
\langle k_T^{q,i}(x)\rangle_{UT} &= -\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, f_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) \\
&= \int d^2\vec{b}_T\, \mathcal{I}^{q,i}(x,\vec{b}_T)\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\mathcal{E}^q(x,\vec{b}_T^{\,2})\right)'
\end{aligned}
$$

(Burkardt, Hwang, 2003)

$\longrightarrow$ First non-trivial quantitative relation between GPD and TMD

$\longrightarrow$ Relation provides an intuitive picture of the Sivers effect

$\longrightarrow$ Relation is model-dependent

Model-independent considerations

$\longrightarrow$ Additonal relations by comparing the GPD-correlator with the TMD-correlator (Diehl, Hägler, 2005)

$$\Phi^q(x, \vec{k}_T; S) \quad = \quad f_1^q(x, \vec{k}_T^{\,2}) - \frac{\epsilon_T^{ij} k_T^i S_T^j}{M} \, f_{1T}^{\perp q}(x, \vec{k}_T^{\,2})$$

$$\mathcal{F}^q(x, \vec{b}_T; S) \quad = \quad \mathcal{H}^q(x, \vec{b}_T^{\,2}) + \frac{\epsilon_T^{ij} b_T^i S_T^j}{M} \left(\mathcal{E}^q(x, \vec{b}_T^{\,2}) \right)'$$

$\longrightarrow$ Comparison allows one to find (non-trivial) analogy:

$$f_{1T}^{\perp q} \leftrightarrow - \left(\mathcal{E}^q \right)'$$

$\longrightarrow$ Comparison can be extended to other quark and gluon distributions

$\longrightarrow$ No relation for GPDs $\tilde{E}, \; \tilde{E}_T$ (drop out for $\xi = 0$) and TMDs $g_{1T}, \; h_{1L}^{\perp}$

- Relations of first type

$$f_1^{q/g} \leftrightarrow \mathcal{H}^{q/g} \qquad g_{1L}^{q/g} \leftrightarrow \tilde{\mathcal{H}}^{q/g}$$

$$\left(h_{1T}^q + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp q}\right) \leftrightarrow \left(\mathcal{H}_T^q - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta\tilde{\mathcal{H}}_T^q\right)$$

- Relations of second type

$$f_{1T}^{\perp q/g} \leftrightarrow -\left(\mathcal{E}^{q/g}\right)' \qquad h_1^{\perp q} \leftrightarrow -\left(\mathcal{E}_T^q + 2\tilde{\mathcal{H}}_T^q\right)'$$

$$\left(h_{1T}^g + \frac{\vec{k}_T^{\,2}}{2M^2}\, h_{1T}^{\perp g}\right) \leftrightarrow -2\left(\mathcal{H}_T^g - \frac{\vec{b}_T^{\,2}}{M^2}\, \Delta\tilde{\mathcal{H}}_T^g\right)'$$

- Relations of third type

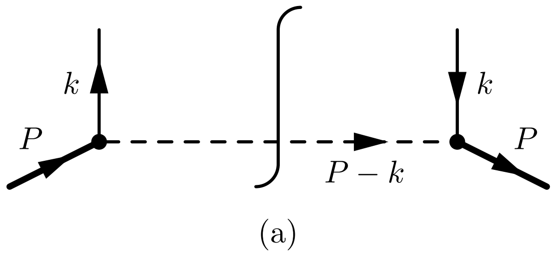
$$h_{1T}^{\perp q} \leftrightarrow 2\left(\tilde{\mathcal{H}}_T^q\right)'' \qquad h_1^{\perp g} \leftrightarrow 2\left(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g\right)''$$

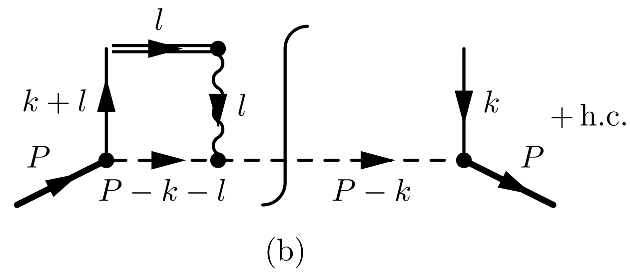
- Relation of fourth type

$$h_{1T}^{\perp g} \leftrightarrow -4\left(\tilde{\mathcal{H}}_T^g\right)'''$$

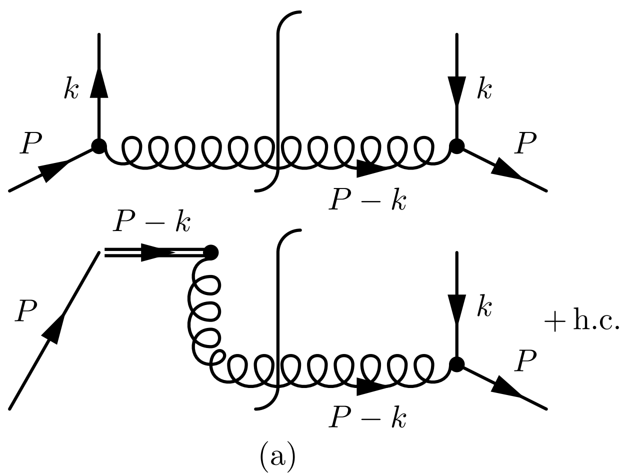
Model results

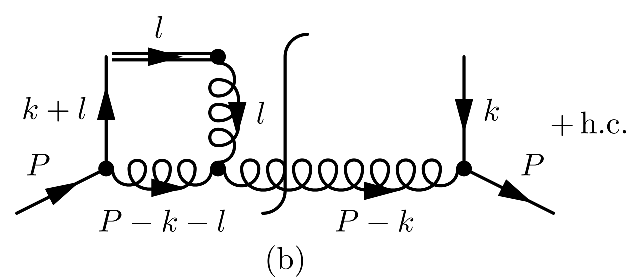
- Scalar diquark spectator model of the nucleon





- Quark target model in QCD





- Moments of GPDs and TMDs

$$X^{(n)}(x) = \frac{1}{2M^2}\int d^2\vec{\Delta}_T \left(\frac{\vec{\Delta}_T^2}{2M^2}\right)^{n-1} X\left(x, 0, -\frac{\vec{\Delta}_T^2}{(1-x)^2}\right)$$

$$Y^{(n)}(x) = \int d^2\vec{k}_T \left(\frac{\vec{k}_T^2}{2M^2}\right)^{n} Y(x, \vec{k}_T^2)$$

- Relations of second type

$$f_{1T}^{\perp q\,(n)}(x) = h_2(n)\,\frac{1}{1-x}\,E^{q\,(n)}(x) \qquad (0 \leq n \leq 1)$$

$\longrightarrow$ $h_2(n)$ is model-dependent

$\longrightarrow$ Formula holds for all relations of second type

$\longrightarrow$ Particular cases

$$f_{1T}^{\perp q\,(0)}(x) = \frac{\pi e_q e_s}{48(1-x)}\,E^q(x,0,0) \qquad \text{(Lu, Schmidt, 2006)}$$

$$f_{1T}^{\perp q\,(1)}(x) = \frac{e_q e_s}{4(2\pi)^2\,(1-x)}\,E^{q\,(1)}(x) \qquad \text{(Burkardt, Hwang, 2003)}$$

- Relations of third type

$$h_{1T}^{\perp q\,(n)}(x) = h_3(n)\,\frac{1}{(1-x)^2}\,\tilde{H}_T^{q\,(n)}(x) \qquad\qquad (0 \leq n \leq 1)$$

$\longrightarrow$ $h_3(n)$ is model-independent

$\longrightarrow$ Formula holds for all relations of third type

$\longrightarrow$ Alternative representation for $n = 1$

$$h_{1T}^{\perp q\,(1)}(x) = \int d^2\vec{k}_T\,\frac{\vec{k}_T^{\,2}}{2M^2}\,h_{1T}^{\perp q}(x,\vec{k}_T^{\,2}) = \int d^2\vec{b}_T\,\frac{\vec{b}_T^{\,2}}{2M^2}\,2\Big(\tilde{\mathcal{H}}_T^q(x,\vec{b}_T^{\,2})\Big)''$$

- Relation of fourth type

$\longrightarrow$ Trivially satisfied because

$$h_{1T}^{\perp g} = \tilde{\mathcal{H}}_T^g = 0$$

Summary

- So far no non-trivial model-independent relations between GPDs and TMDs established

- Many relations between GPDs and TMDs exist for perturbative low order spectator model calculations

- Relations of second type are likely to break down in spectator models if higher orders are considered

- Model-dependent relations may provide a good qualitative picture

  $\longrightarrow$ Example

$$f_{1T}^{\perp u/p} \sim -\kappa^{u/p} \qquad\qquad f_{1T}^{\perp d/p} \sim -\kappa^{d/p}$$