

# An Update of the **MRST (MSTW)** Parton Distributions

Robert Thorne

April 17, 2007



University College London

Royal Society Research Fellow

## Major update in people involved.

Dick Roberts completely retired from project.

Graeme Watt started as responsive RA on parton distributions from April 1st 2006. Now making major contribution to project – responsible for many of these new results.

## Major changes in theory.

Implementation of updated heavy flavour VFNS, particularly at NNLO.

Already used a general VFNS since 1998 but change in details.

Inclusion of NNLO corrections to Drell-Yan data.

Some important changes as NLO  $\rightarrow$  NNLO.

Most important change compared to previous NNLO – new VFNS.  $\rightarrow$  significant change in partons.

Implementation of fastNLO – fast perturbative QCD calculations Kluge, Rabbertz, Wobisch. Allows easy inclusion of new data.

## New data included.

NuTeV and Chorus data on  $F_2^{\nu,\bar{\nu}}(x, Q^2)$  and  $F_3^{\nu,\bar{\nu}}(x, Q^2)$  replacing CCFR.

NuTeV and CCFR dimuon data included directly. Leads to a direct constraint on  $s(x, Q^2) + \bar{s}(x, Q^2)$  and on  $s(x, Q^2) - \bar{s}(x, Q^2)$ . Affects other partons.

CDFII lepton asymmetry data in two different  $E_T$  bins –  $25\text{GeV} < E_T < 35\text{GeV}$  and  $35\text{GeV} < E_T < 45\text{GeV}$ .

HERA inclusive jet data (in DIS).

New CDFII high- $E_T$  jet data.

Direct high- $x$  data on  $F_L(x, Q^2)$ .

Update to include all recent charm structure function data.

Look at dependence of fit on  $m_c$  – defined as pole mass.

Obtain **NNLO** partons with uncertainties due to experimental errors for the first time.

Reported last year.

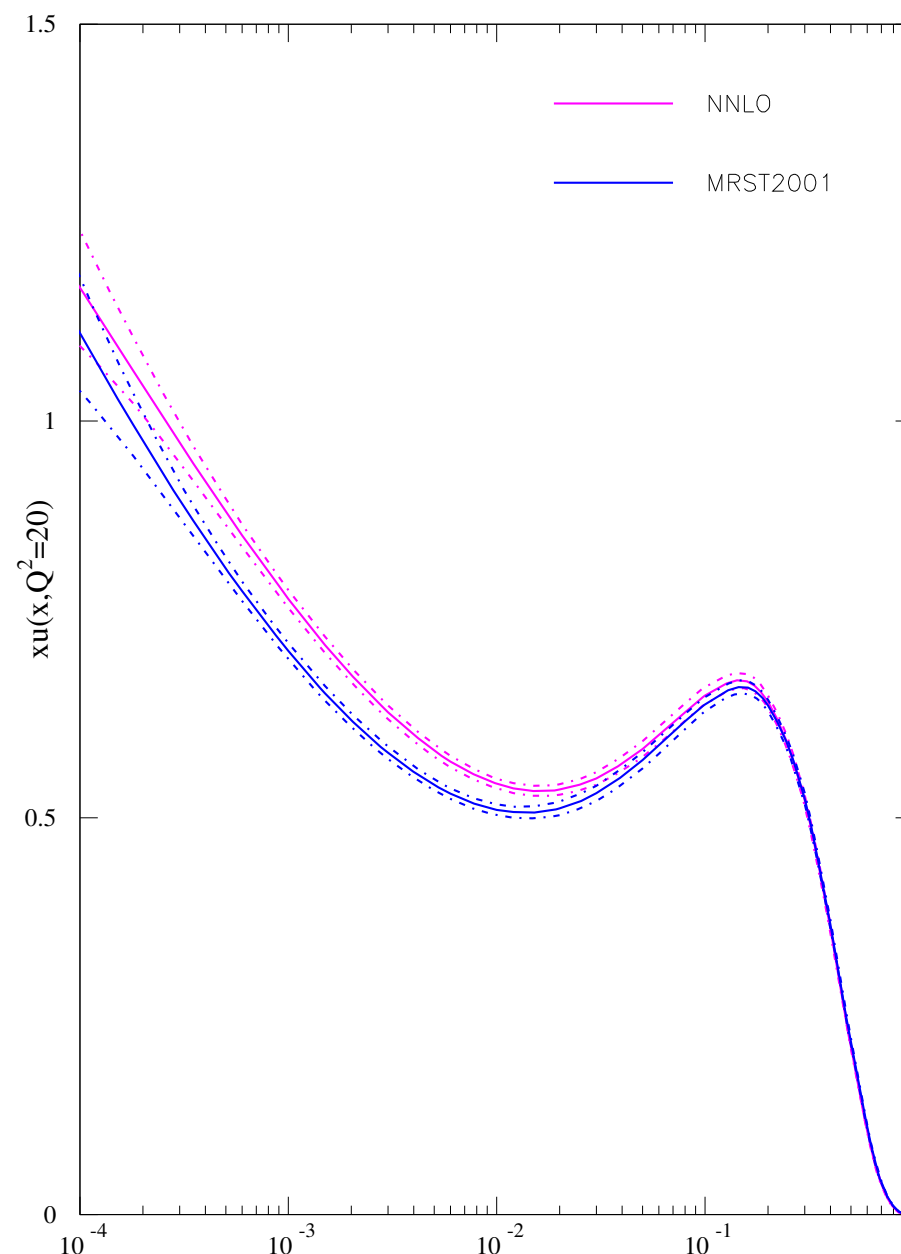
Same procedure as before – 15 eigenvector sets of partons and  $\Delta\chi^2 = 50$  for 90% confidence limit.

First time we have full **NNLO** with no major approximations. (Heavy flavours a major issue.)

In general size of uncertainties similar (perhaps a little smaller) to at **NLO**.

Change from **NLO** to **NNLO** greater than uncertainty in each.

**NNLO** fit consistently better than **NLO**.



Comparison to other (Alekhin) NNLO gluon.

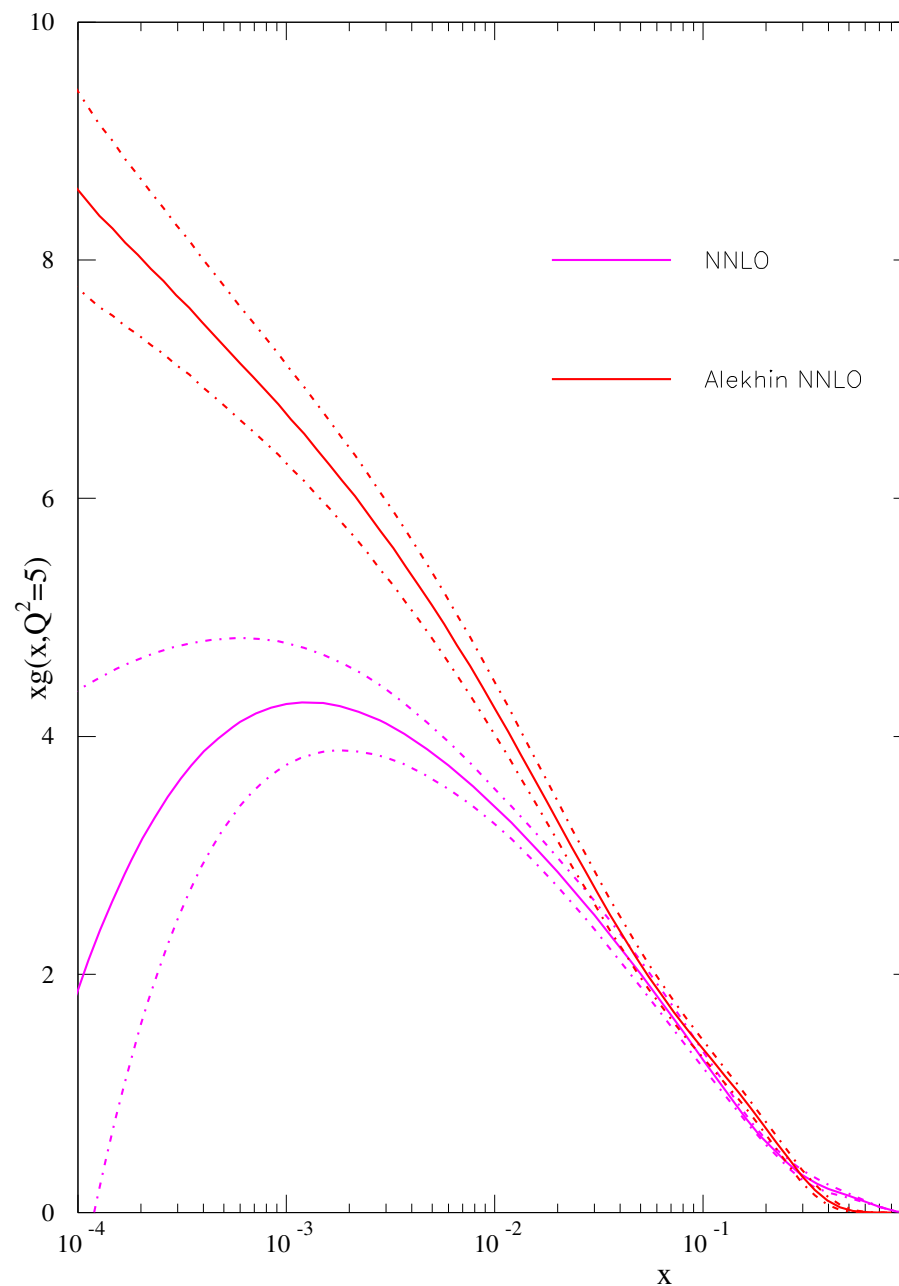
Hugely different at small  $x$ .

Differences much bigger than uncertainties.

Differences in heavy flavour treatments  
- disagreement on what constitutes definition of NNLO.

Differences in data fit and also in  $\alpha_S(M_Z^2)$ .

Note difference in uncertainty at low  $x$   
not just in shape.

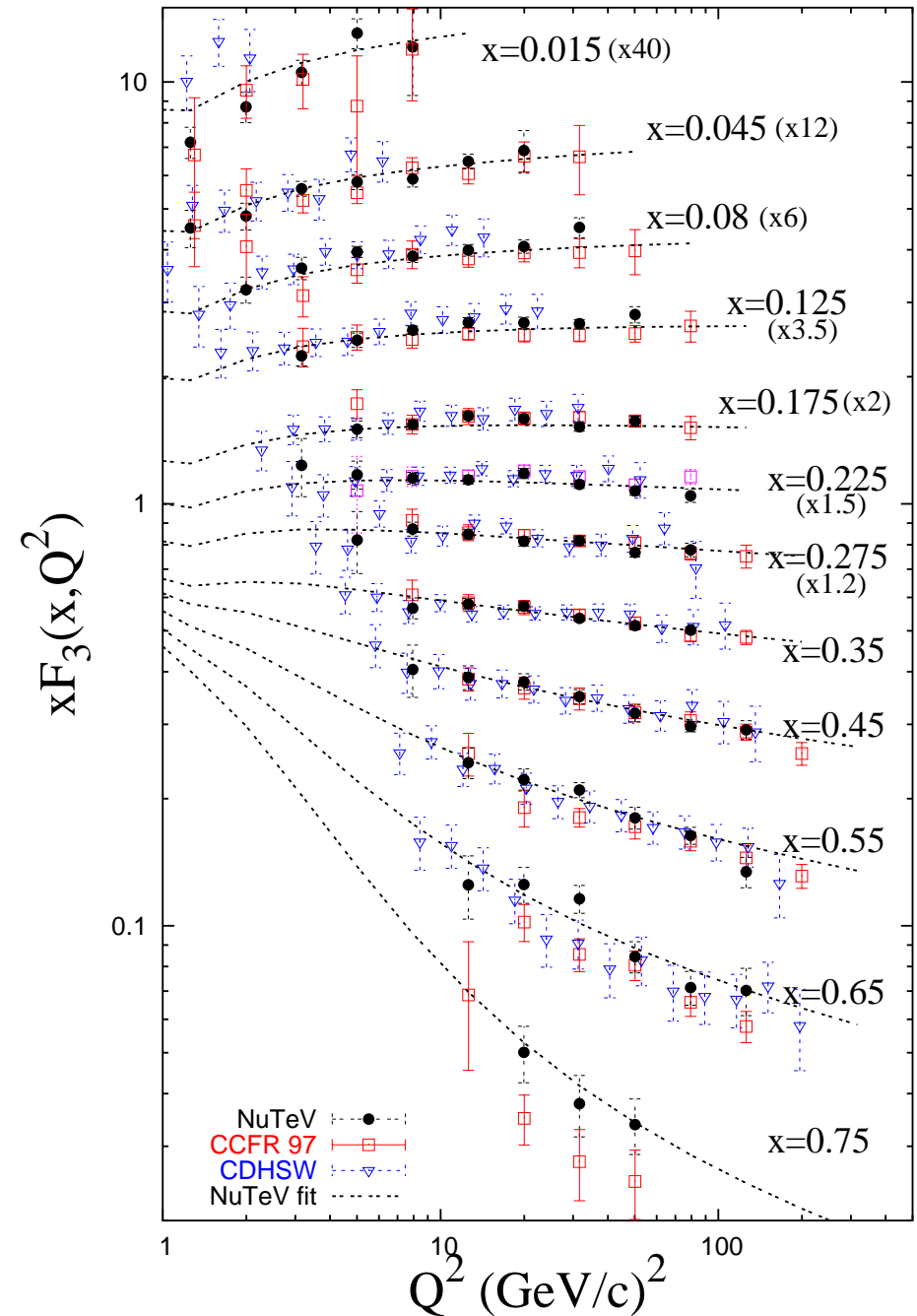


## New Data

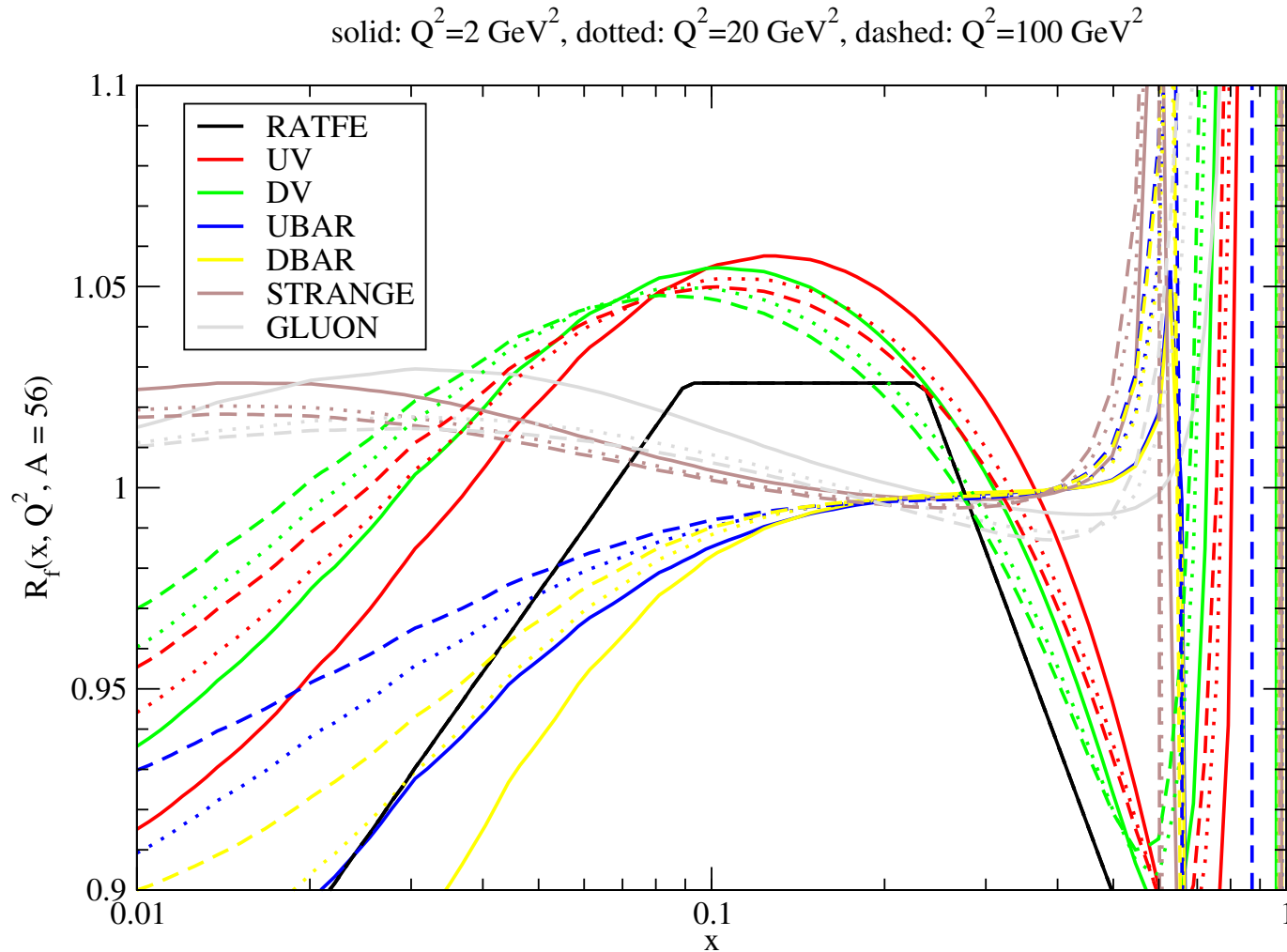
New **NuTeV** data not completely compatible with the older **CCFR** data.

Main source of discrepancy's calibration of magnetic field map of muon spectrometer → muon energy scale.

However, previous parton distribution fits were perfectly compatible with **CCFR** data using **EMC** inspired  $Q^2$  independent nuclear correction



Now implement far more sophisticated nuclear correction [De Florian, Sassot](#). Extracted using [NLO](#) partons.



Same general shape as before. Allow  $\sim 3\%$  uncertainty on corrections. Cannot match high [NuTeV](#) data.

Chorus data also consistent with CCFR (lead not iron).

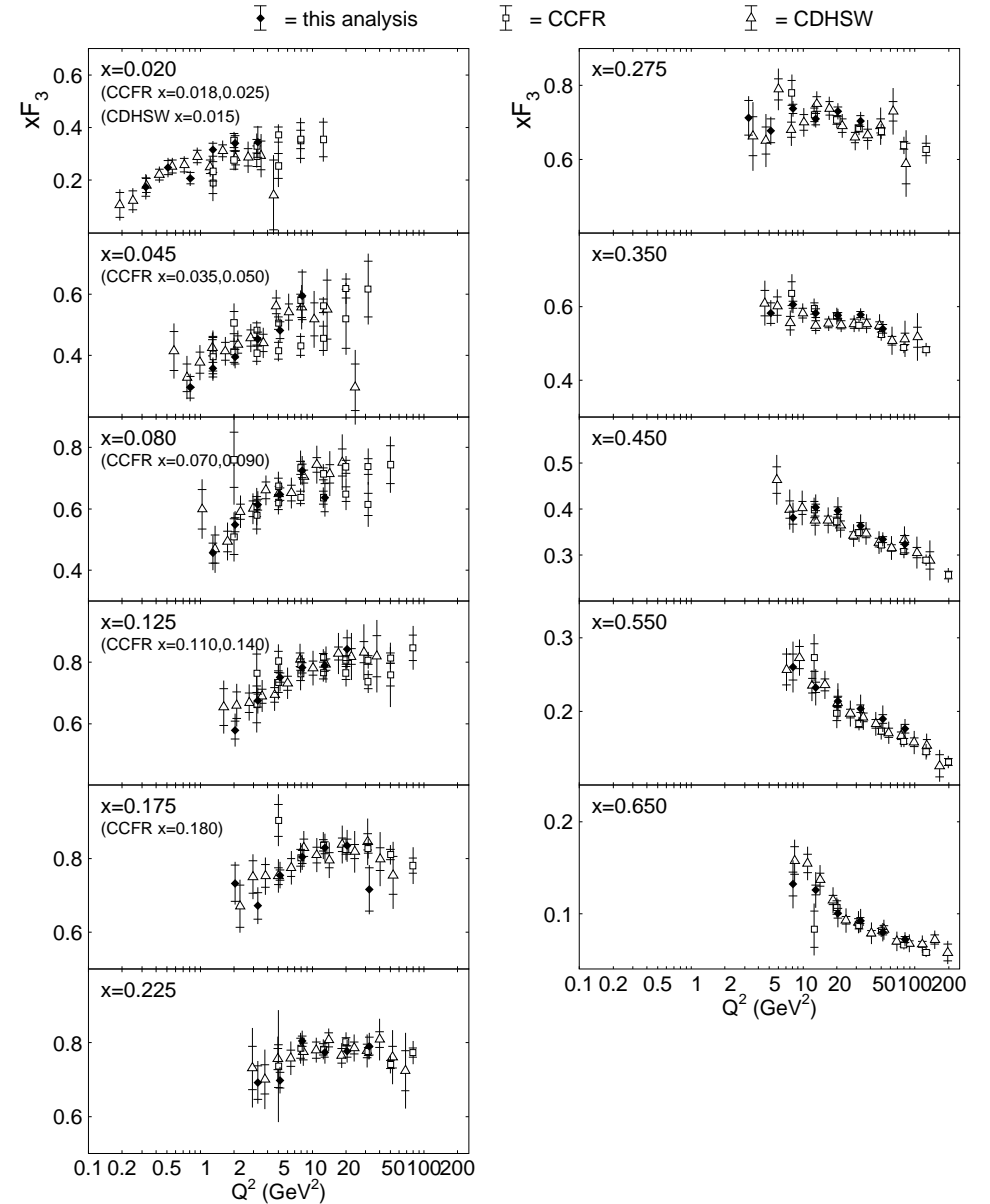
Inconsistencies at high  $x$ .

Partons in region of high  $x$  already well-determined from charged lepton structure functions.

Important information in the region  $x < 0.3$ , e.g. low  $x$  valence quarks - general consistency here.

Choose to cut neutrino structure function data for  $x \geq 0.5$ .

Also Chorus data at lower  $W^2$ .  $F_3(x, Q^2)$  expected to have larger higher twist corrections than  $F_2(x, Q^2)$ . Cut for  $W^2 \leq 20 \text{ GeV}^2$ .





## CCFR/NuTeV dimuon cross-sections and strange quarks

$$\frac{d\sigma}{dx dy}(\nu_\mu(\bar{\nu}_\mu)N \rightarrow \mu^+\mu^-X) = B_c \mathcal{N} \mathcal{A} \frac{d\sigma}{dx dy}(\nu_\mu s(\bar{\nu}_\mu \bar{s}) \rightarrow c\mu^-(\bar{c}\mu^+)X),$$

$B_c$  = semileptonic branching fraction

$\mathcal{N}$  = nuclear correction

$\mathcal{A}$  = acceptance correction.

$\nu_\mu$  and  $\bar{\nu}_\mu$  cross-sections probe  $s$  and  $\bar{s}$  (small mixing with  $d$  and  $\bar{d}$ ).

Have previously indirectly used CCFR data to parameterise strange according to

$$s(x, Q_0^2) = \bar{s}(x, Q_0^2) = \frac{\kappa}{2}[\bar{u}(x, Q_0^2) + \bar{d}(x, Q_0^2)] \quad \kappa \approx 0.5$$

Now fit strange directly rather than assuming same shape as average of  $\bar{u} + \bar{d}$  at input and some **fixed** fraction.

Also allow possibility of  $s(x, Q_0^2) \neq \bar{s}(x, Q_0^2)$ .

Make definitions at input

$$s^+(x, Q_0^2) \equiv s(x, Q_0^2) + \bar{s}(x, Q_0^2) = A_+(1-x)^{\eta_+} S(x, Q_0^2)$$

$$s^-(x, Q_0^2) \equiv s(x, Q_0^2) - \bar{s}(x, Q_0^2) = A_-(1-x)^{\eta_-} x^{-1+\delta_-} (1-x/x_0)$$

where  $S(x, Q_0^2)$  is the total sea distribution and  $x_0$  is determined by zero strangeness of proton, i.e.

$$\int_0^1 dx s^-(x, Q_0^2) = 0.$$

Extra freedom in both  $s^+$  and  $s^-$  confirmed by fit.

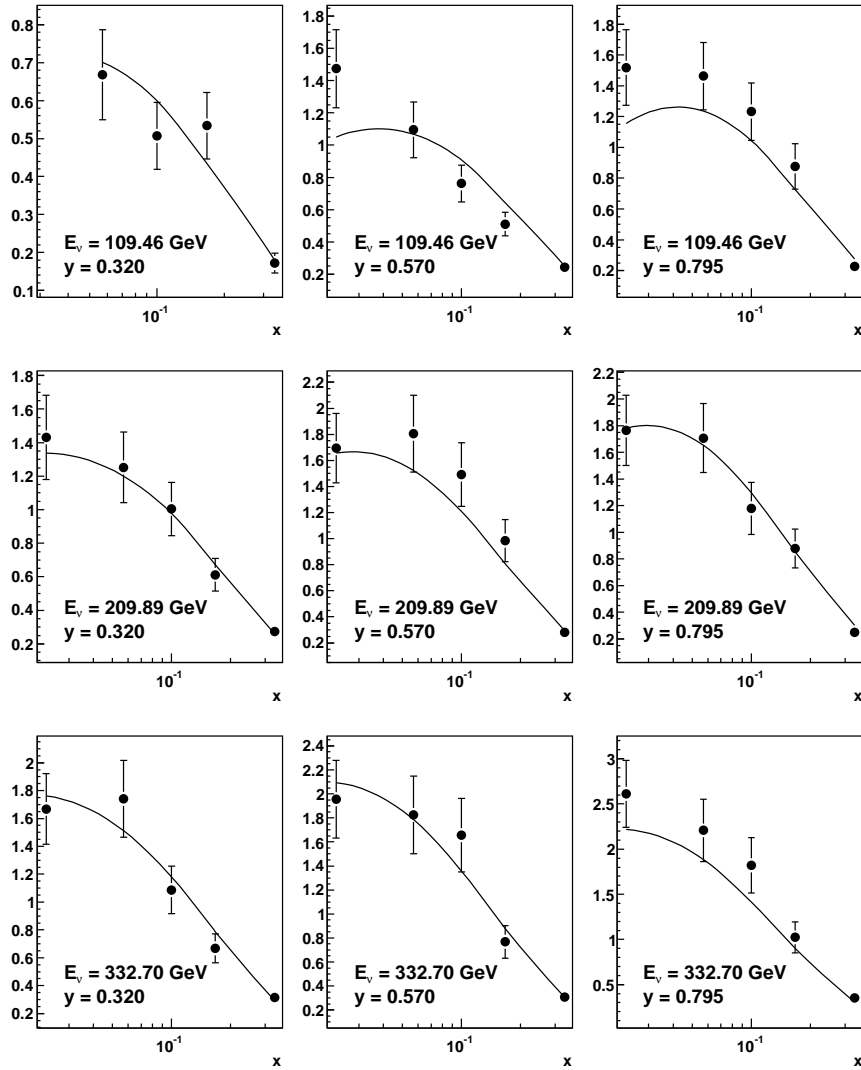
	$\chi_{CCFR}^2$ 86 pts	$\chi_{NuTeV}^2$ 84 pts	$\chi_{global}^2$ 2659 pts
$s = \bar{s} = (\bar{u} + \bar{d})/4$	71	63	2501
$s^+$ free, $s^- = 0$	65	52	2486
$s^+$ free, $s^-$ free	65	38	2472

No improvement with further parameters.

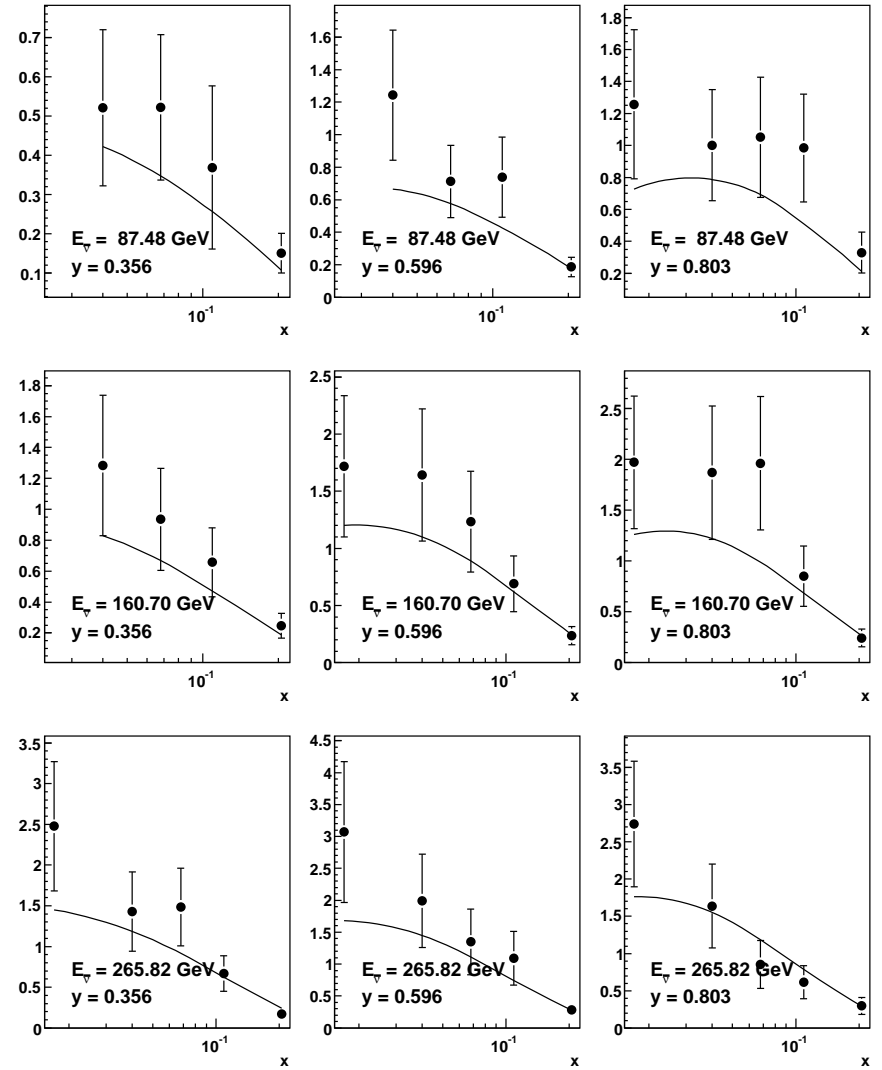
Data generally prefer  $s^+$  free. Dimuon data only affected by  $s^-$ . Decoupled from other parameters to good approximation.  $\delta_- = 0.2$  fixed, i.e. valence-like value.

Fit to data clearly very good.

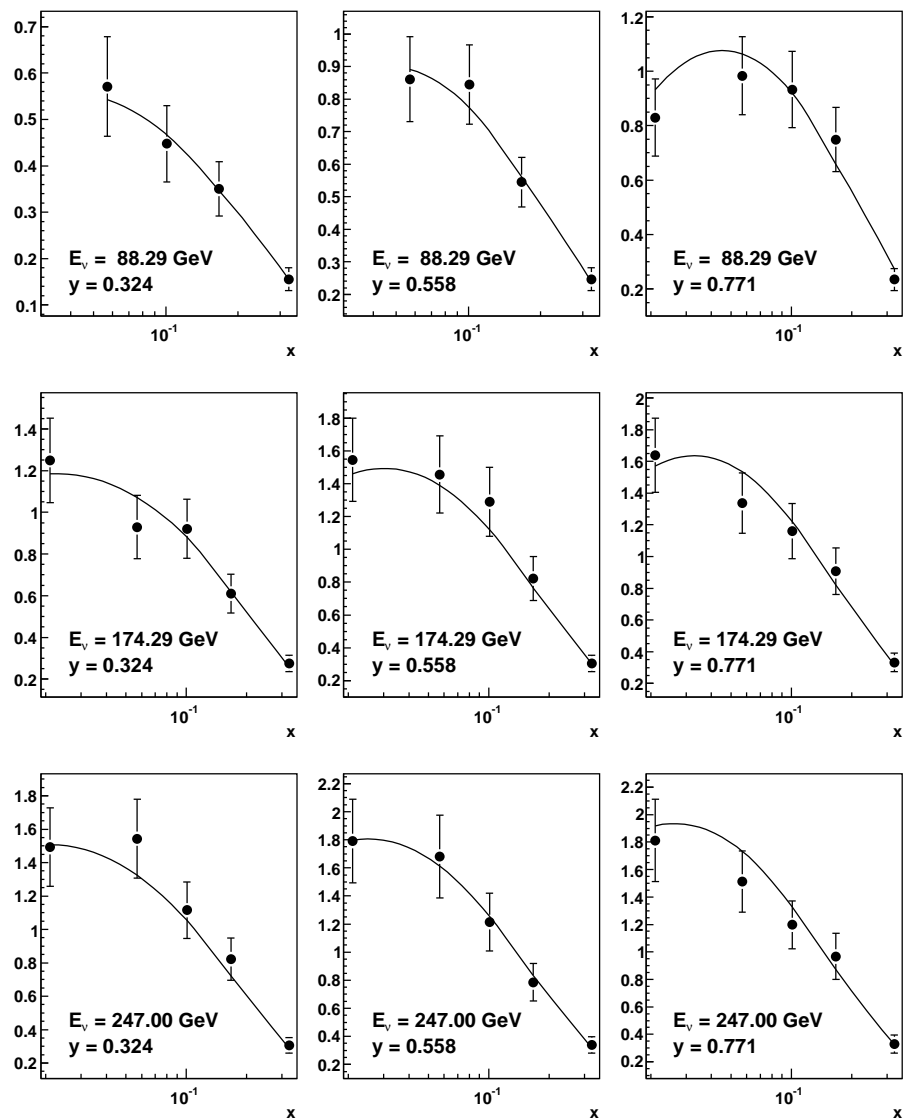
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 33/44 \text{ pts.}$$



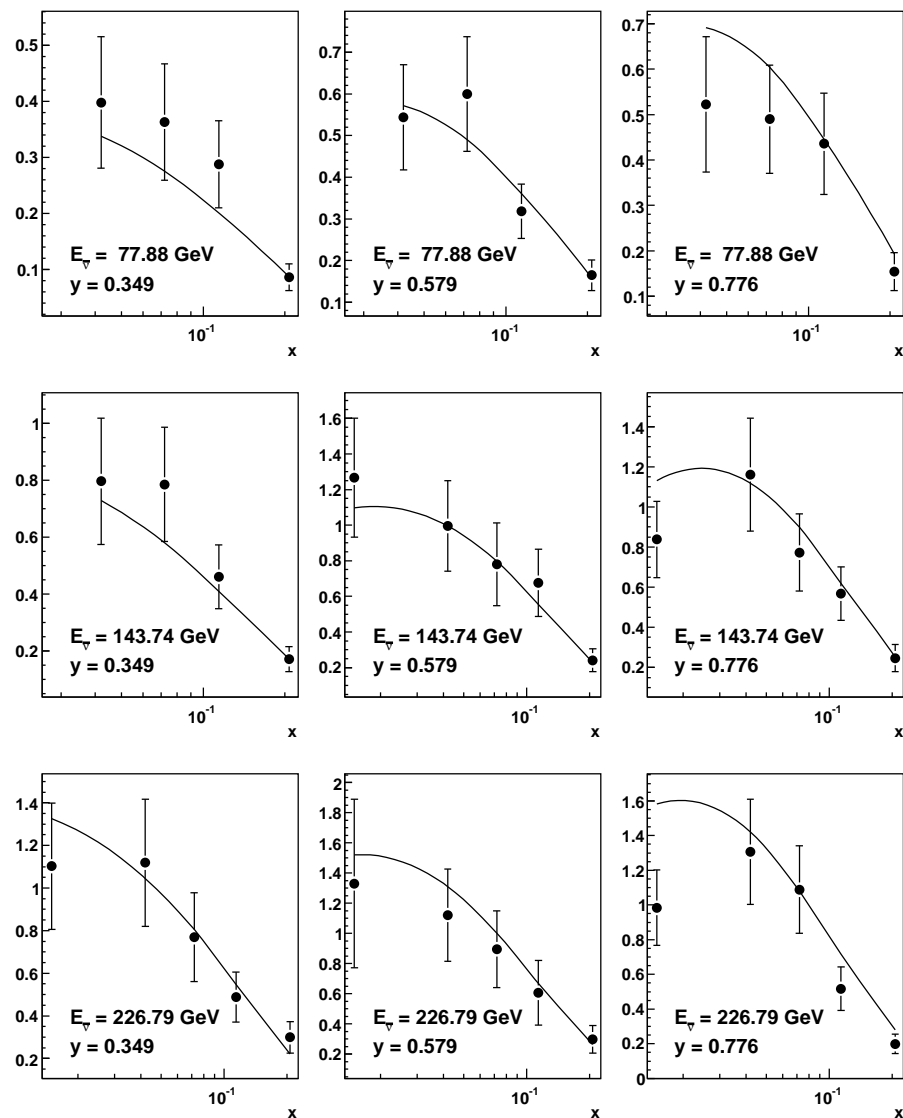
$$\text{CCFR } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy} (\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 32/42 \text{ pts.}$$



$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy}(\nu_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 11/21 \text{ DOF}$$

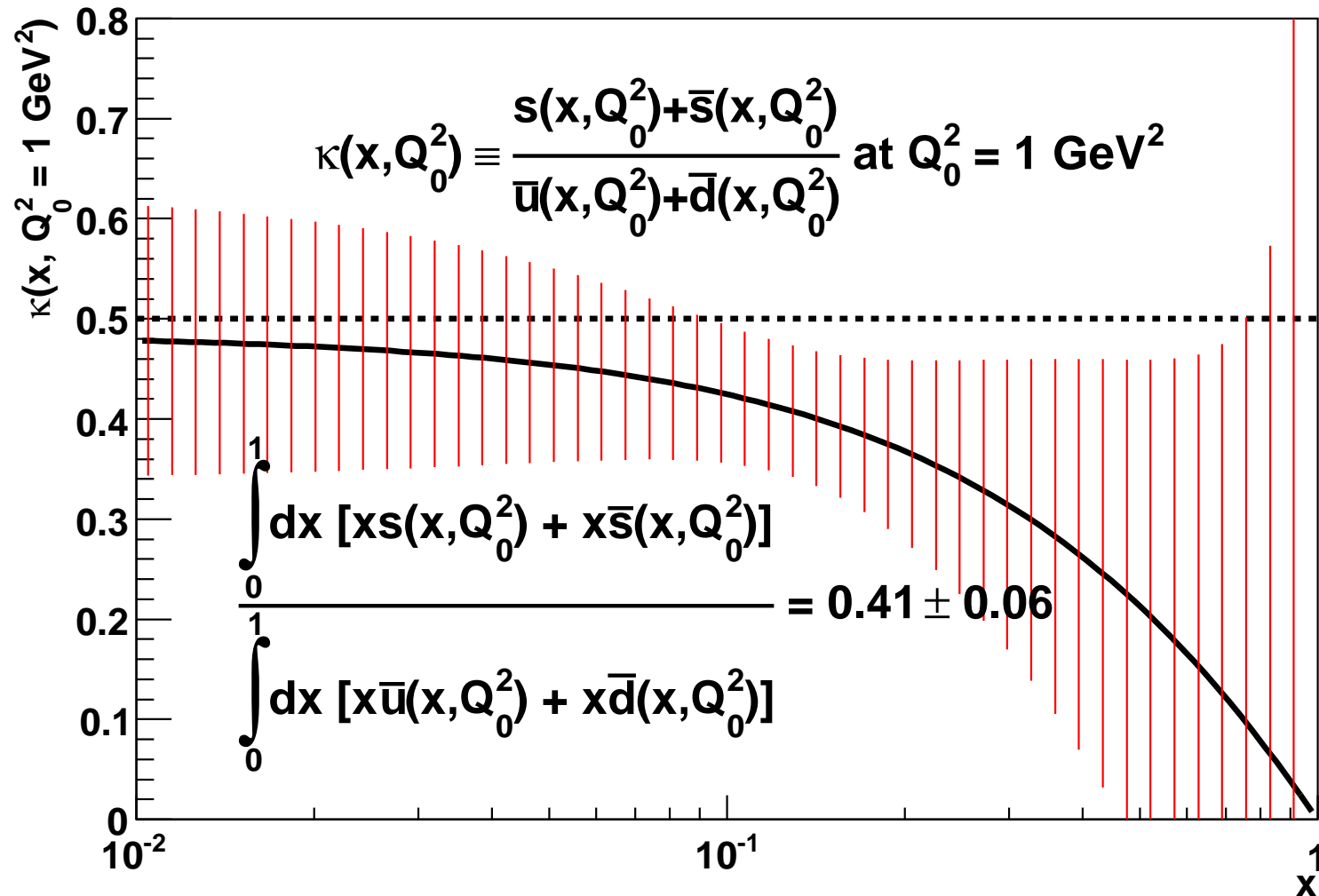


$$\text{NuTeV } \frac{100\pi}{G_F^2 M_N E_\nu} \frac{d\sigma}{dx dy}(\bar{\nu}_\mu N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^{-2}, \chi^2 = 27/19 \text{ DOF}$$



Find reduced ratio of strange to non-strange sea compared to previous default  $\kappa = 0.5$ .

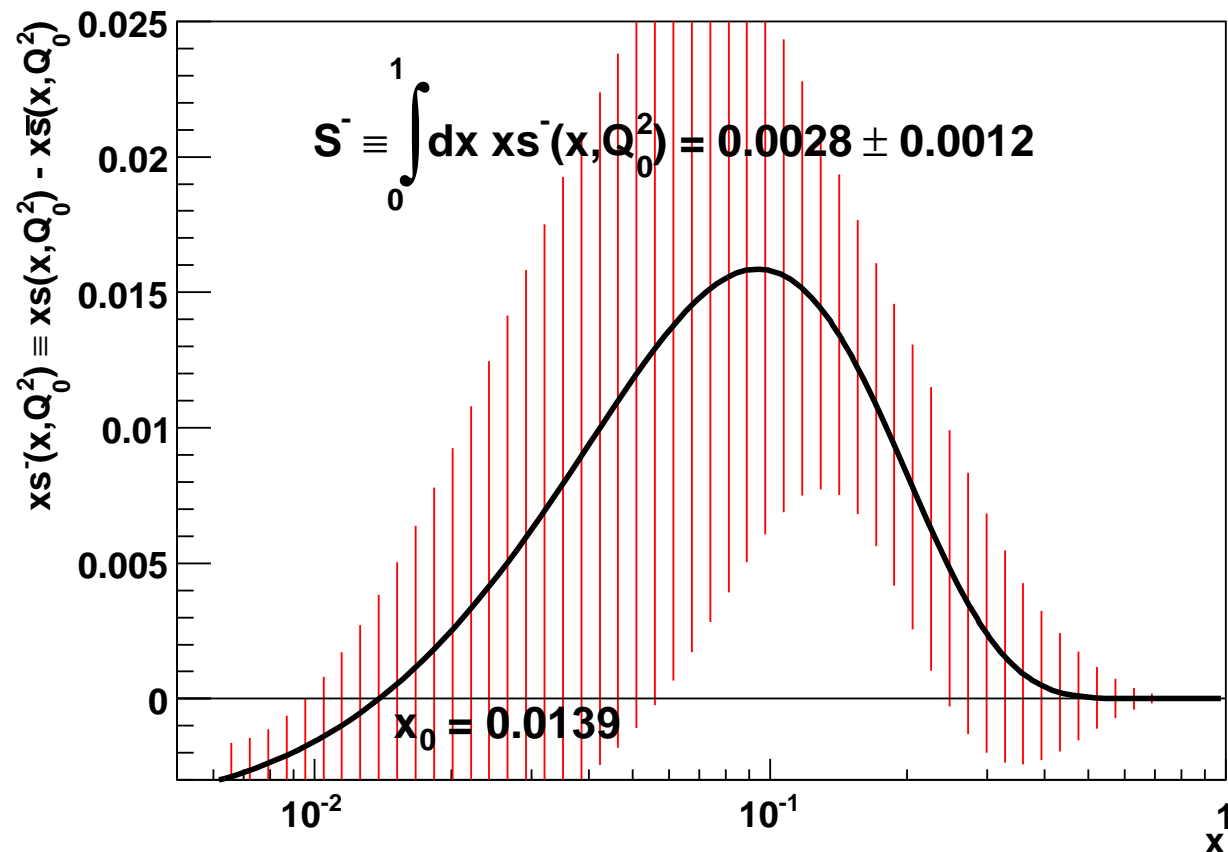
Suppression at high  $x$ , i.e. low  $W^2$ . Effect of  $m_s$ ?



Strange sea asymmetry  $xs(x, Q_0^2) - x\bar{s}(x, Q_0^2)$  constrained by dimuon data for  $0.01 \geq x \geq 0.2$ .

Positive, with central value  $0.0028 \pm 0.0012$  ( $1\sigma$ ). Nonzero value significantly greater than  $1\sigma$  significance. At  $Q^2 = 10\text{GeV}^2$  asymmetry of  $0.0021 \pm 0.0009$ .

Need  $S^- = 0.0068$  to bring NuTeV  $\sin^2 \theta_W$  in line with world average.



Fitting to strange from **NUTEV** dimuon data affects uncertainties on partons other than strange.

Previously for us (and everyone else) strange a fixed proportion of total sea in global fit.

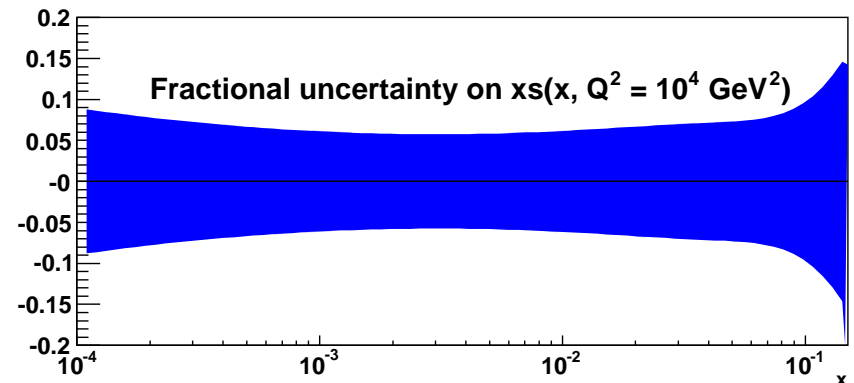
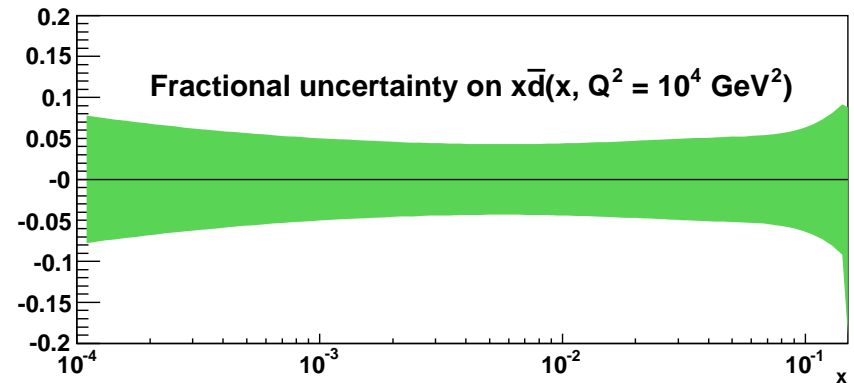
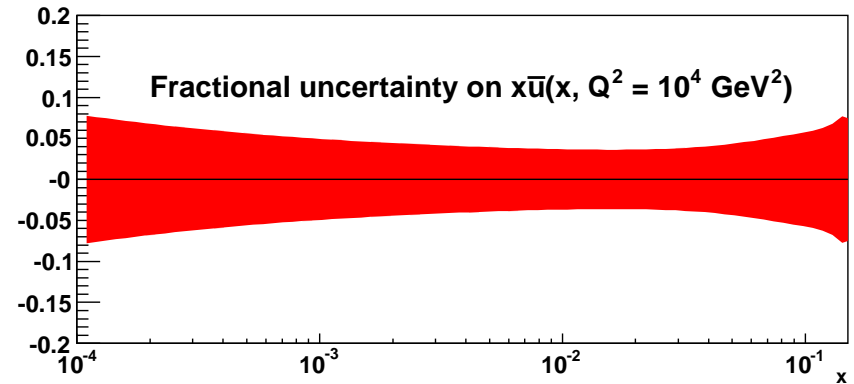
Genuine *larger* uncertainty on  $s(x)$ — feeds into that on  $\bar{u}$  and  $\bar{d}$  quarks.

Low  $x$  data on  $F_2(x, Q^2)$  constrains sum  $4/9(u + \bar{u}) + 1/9(d + \bar{d} + s + \bar{s})$ .

Changes in fraction of  $s + \bar{s}$  affects size of  $\bar{u}$  and  $\bar{d}$  at input.

The size of the uncertainty on the small  $x$  anti-quarks roughly doubles –  $\sim 1.5\% \rightarrow \sim 3\%$ . (Remember uncertainties quoted as 90% confidence limits.)

## MSTW 2007 NLO PDFs (preliminary)



## $W$ -asymmetry

The  $W$ -asymmetry at the Tevatron is defined by

$$A_W(y) = \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy} \approx \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)},$$

where  $x_{1,2} = x_0 \exp(\pm y)$ ,  $x_0 = \frac{M_W}{\sqrt{s}}$ .

In practice it is the final state leptons that are detected, so it is really the lepton asymmetry

$$A(y_l) = \frac{\sigma(l^+) - \sigma(l^-)}{\sigma(l^+) + \sigma(l^-)}$$

which is measured. Defining angle of lepton in  $W$  rest frame

$$\cos^2 \theta^* = 1 - 4E_T^2/m_W^2 \quad \rightarrow \quad y_{lep} = y_W \pm 1/2 \log((1 + \cos \theta^*)/(1 - \cos \theta^*))$$

In practice at highish  $y_{lep}$

$$\sigma(l^+) - \sigma(l^-) \propto u(x_1)d(x_2)(1 - \cos \theta^*)^2 + \bar{u}(x_1)\bar{d}(x_2)(1 + \cos \theta^*)^2 - u(x_2)d(x_1)(1 + \cos \theta^*)^2$$

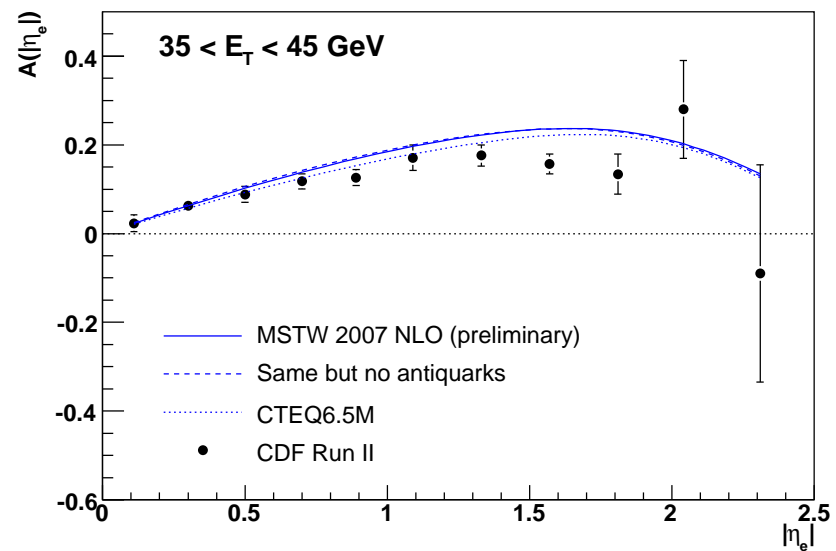
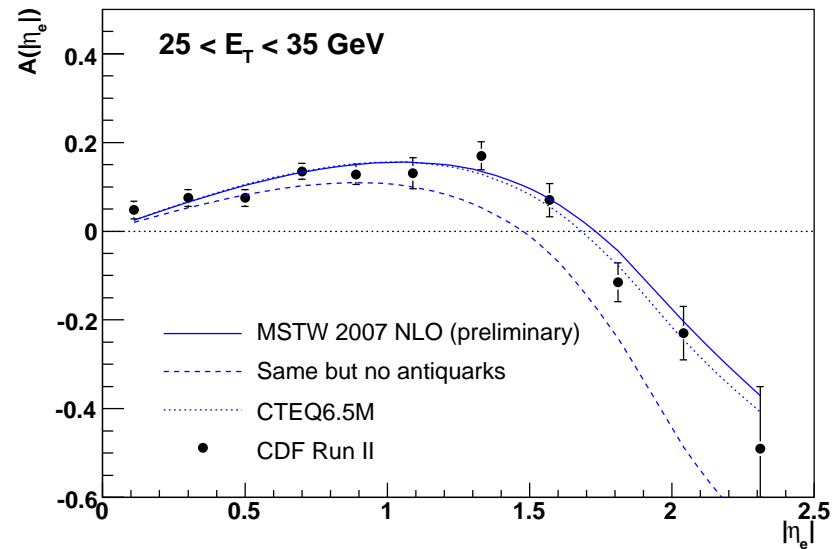
so fairly sensitive to anti-quarks at lower  $E_T$ .



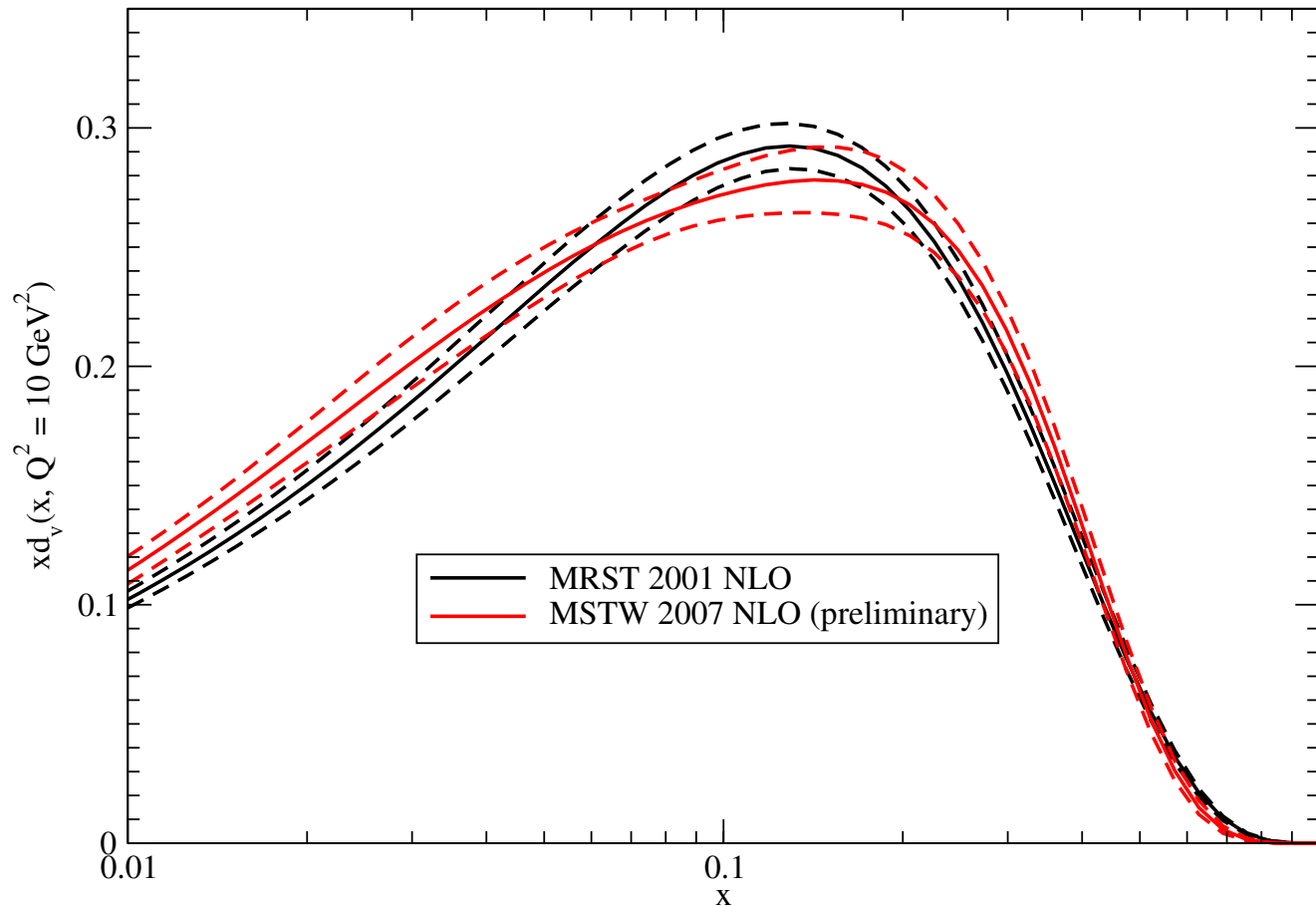
Comparison of fits with various partons.  
Some tension with neutrino structure function data.

CTEQ seems to be better shape for some reason.

New CDF data does influence  $d(x, Q^2)$  in MSTW fit.



Overall  $d_V(x, Q^2)$  now chooses a different type of shape.

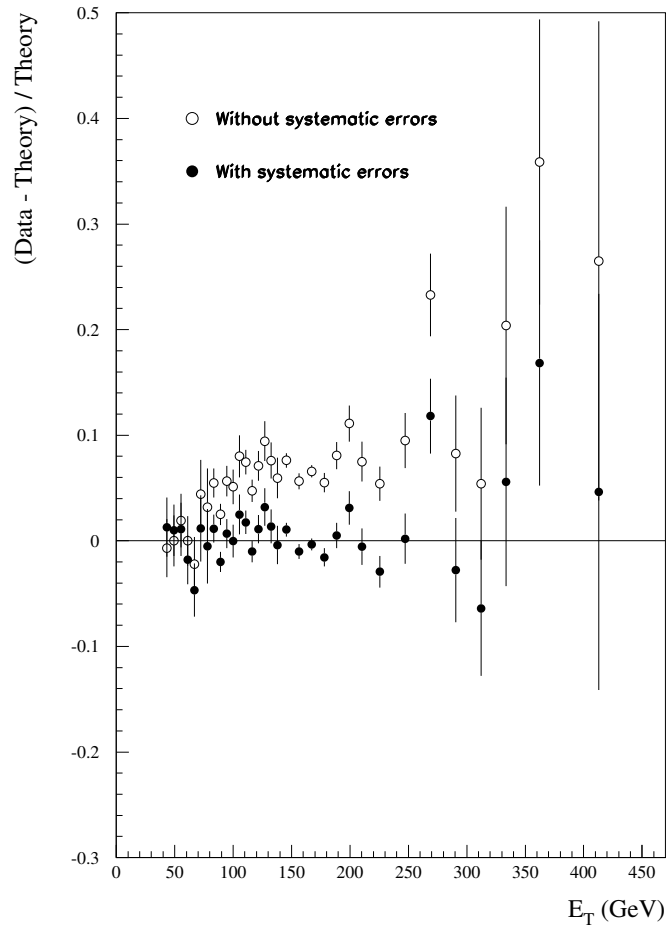


Uncertainty growing more quickly as  $x \rightarrow 0$  than before due to better parameterization in determining uncertainty eigenvectors.

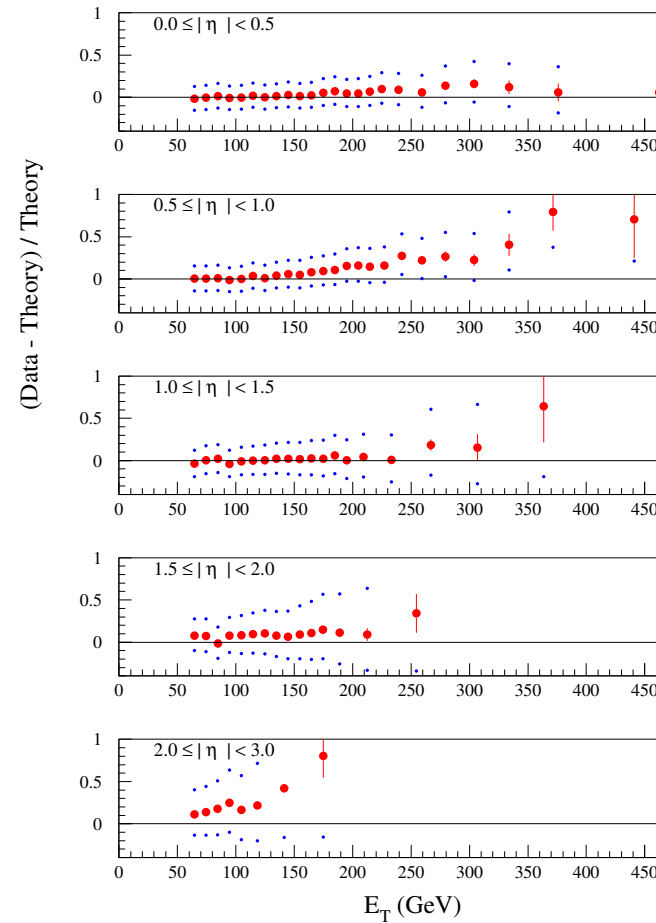
Now use **fast NLO** to implement **NLO** hard cross-section corrections to both **Tevatron** and **HERA** jets. Replaces previous “K-factors” and “pseudo-gluon data”.

No major effect on speed of fitting program. Slight influence on shape of gluon even using just **Tevatron** Run I data. (Hadronization corrections now included).

CDF Run I inclusive jet data,  $\chi^2 = 50/33$  pts.



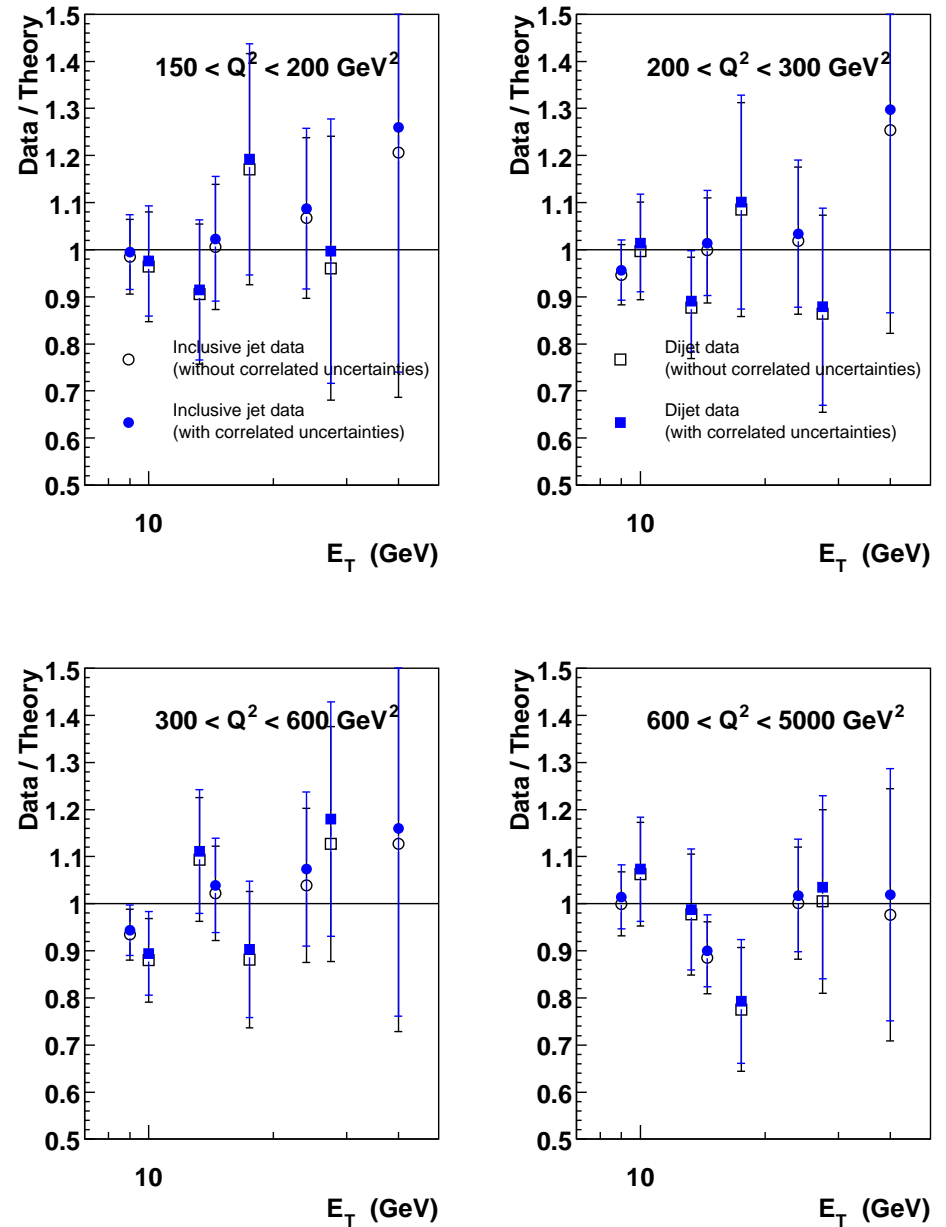
$D\bar{D}$  Run I inclusive jet data,  $\chi^2 = 58/90$  pts.



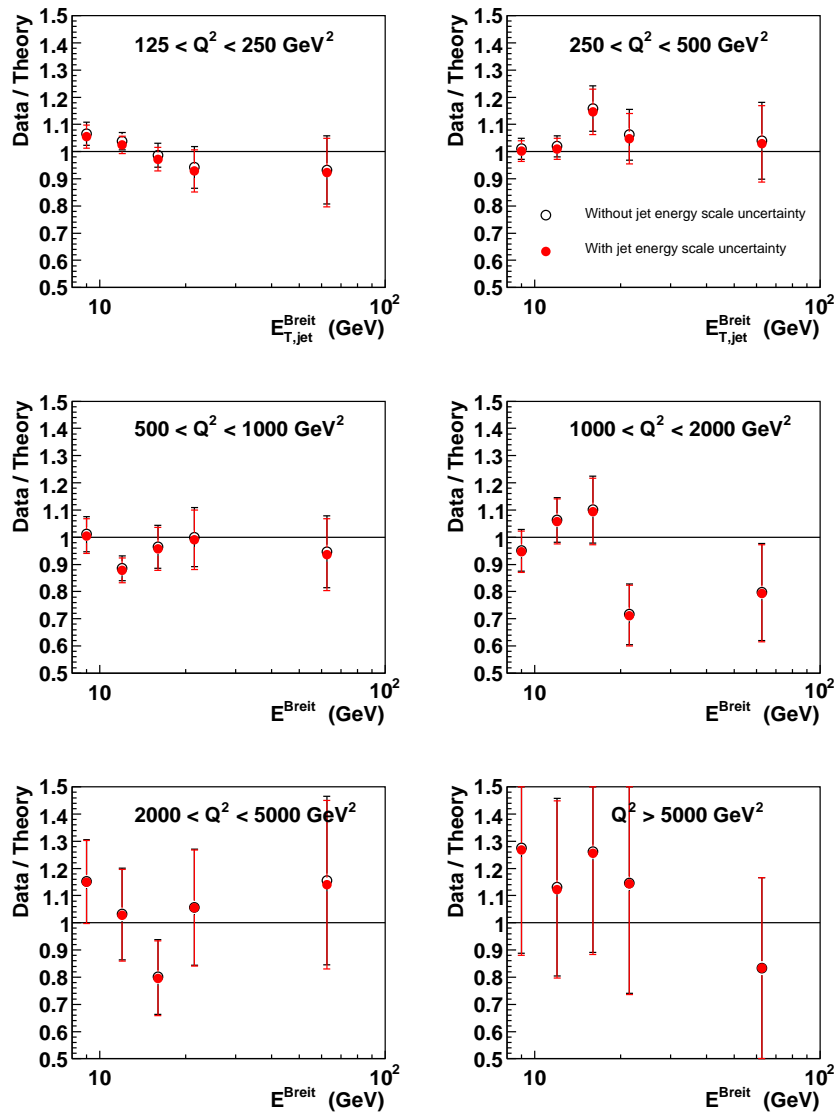
Also now include **HERA** inclusive and dijet **DIS** data using **fastNLO**.

Fit generally excellent. Correlated systematic uncertainties have little effect in this case.

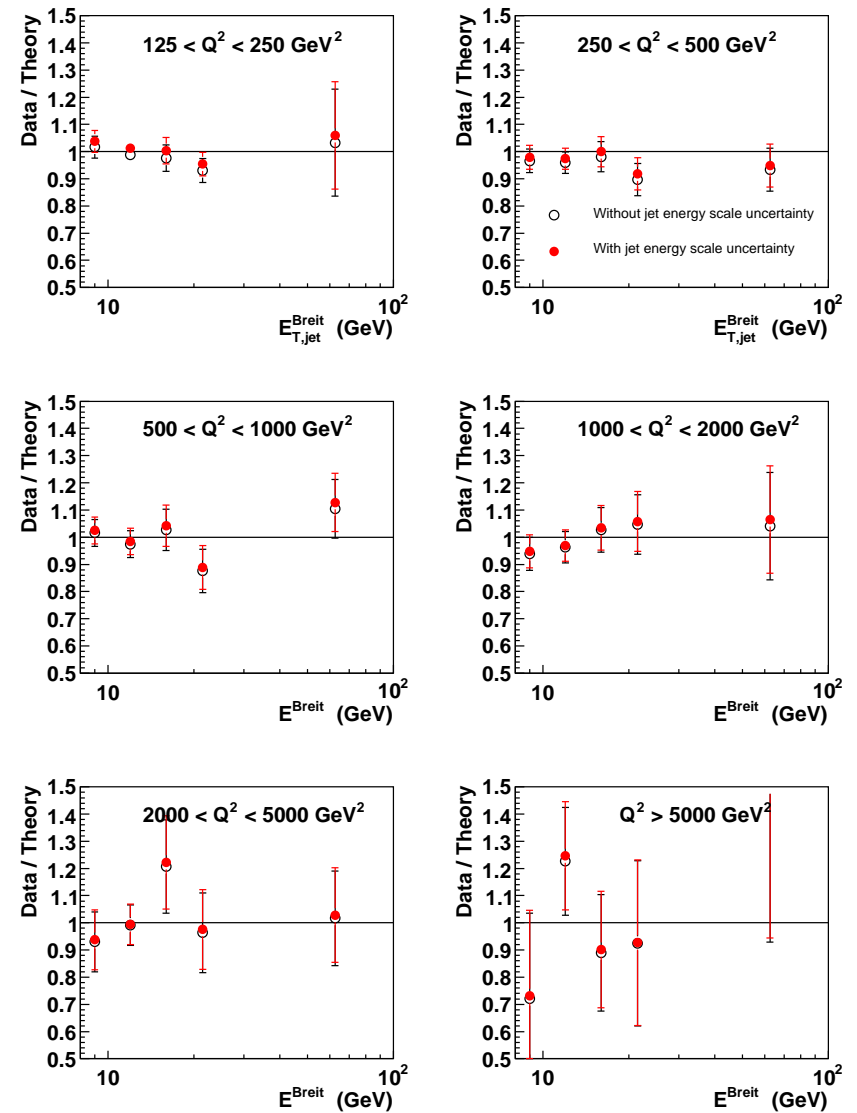
**H1 95-97 incl. jet and dijet data,  $\chi^2 = 14/32$  pts.**



## ZEUS 96-97 inclusive jet data, $\chi^2 = 30/30$ pts.

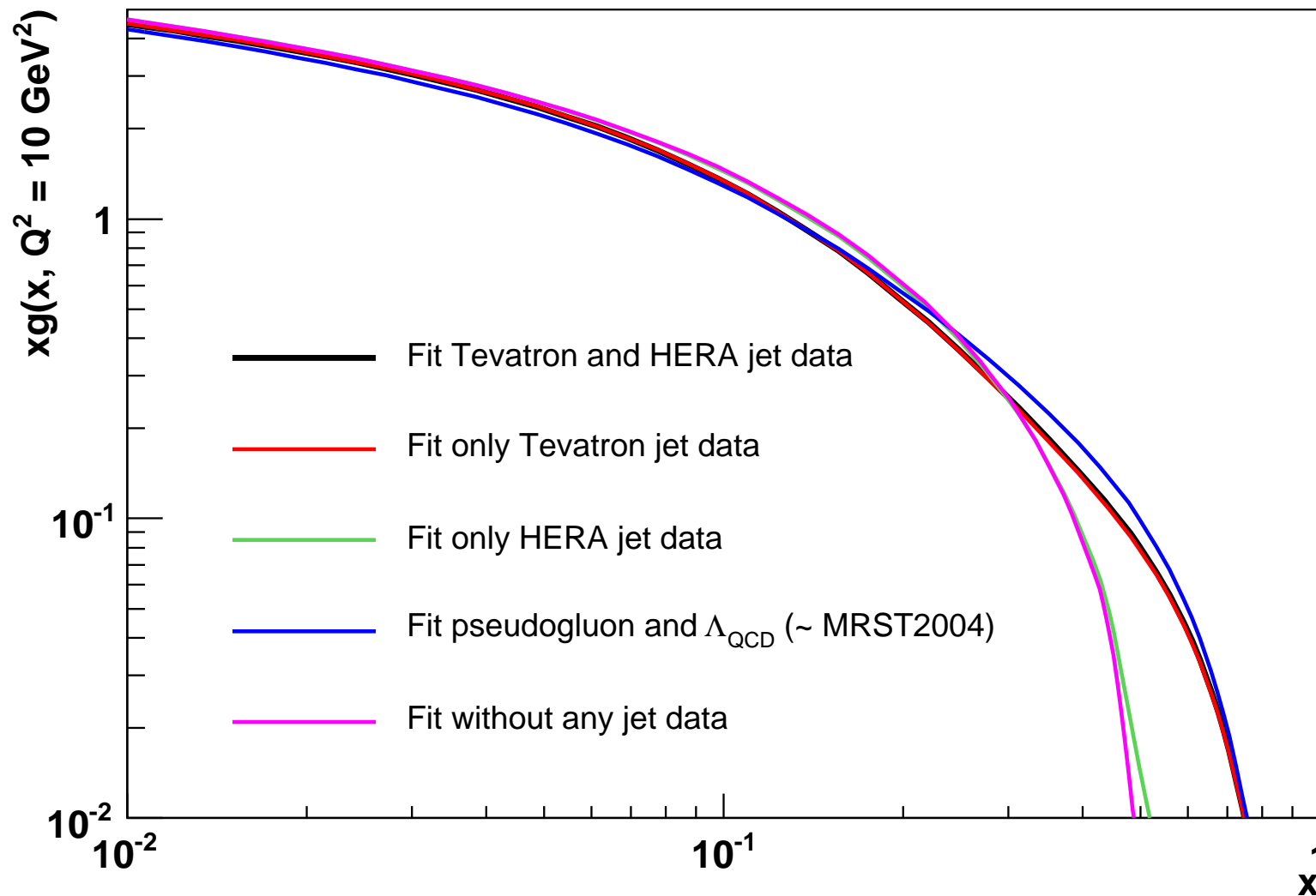


## ZEUS 98-00 inclusive jet data, $\chi^2 = 16/30$ pts.



Perhaps more constraint from photoproduction data, but requires (rather uncertain) photon distributions.

Tevatron jet data are essential for constraining high  $x$  gluon – HERA jet data not sensitive to these  $x$  values and have much less pull.

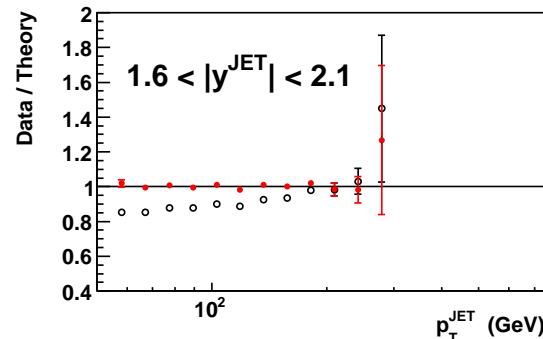
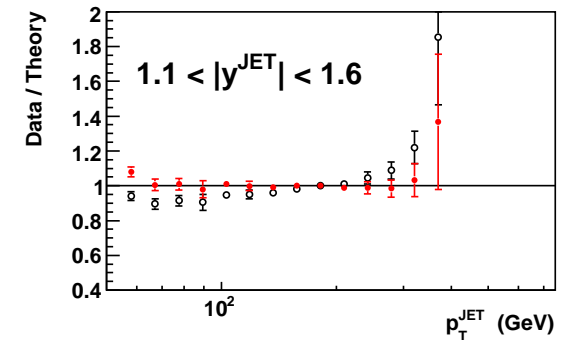
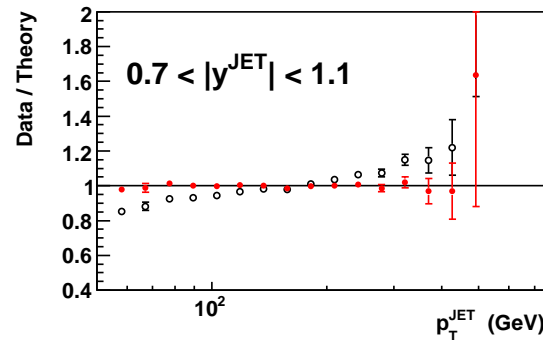
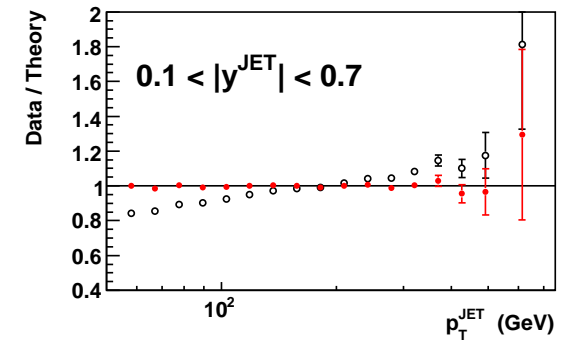
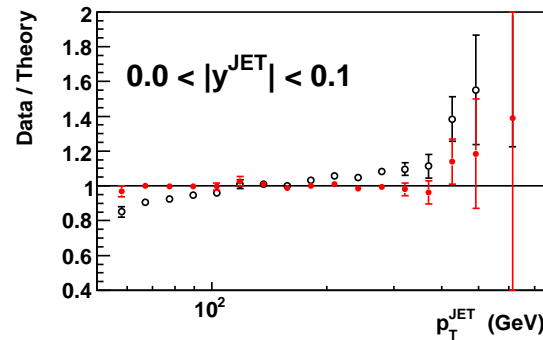


Now also include CDF Run II inclusive jet data in different rapidity bins using  $k_T$  jet algorithm (mid-point cone algorithm data seems very similar, but numbers not yet available).

Very good fit –  $\chi^2 = 56/76$ .

Full use of correlated systematic errors required for any sensible result.

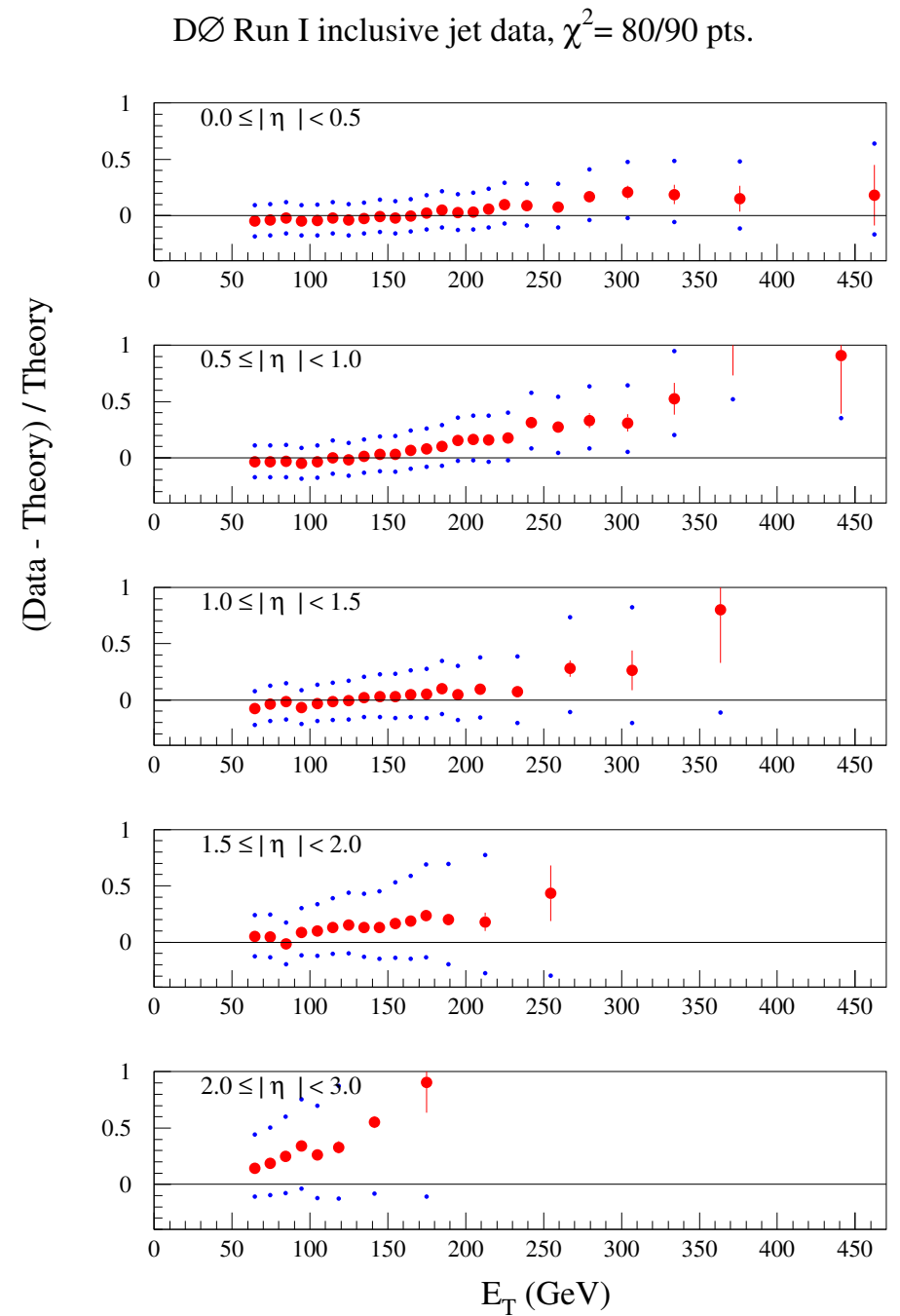
## CDF Run II inclusive jet data, $\chi^2 = 56/76$ pts.



$k_T$  algorithm with  $D = 0.7$   
MSTW 2007 NLO PDF fit (preliminary)

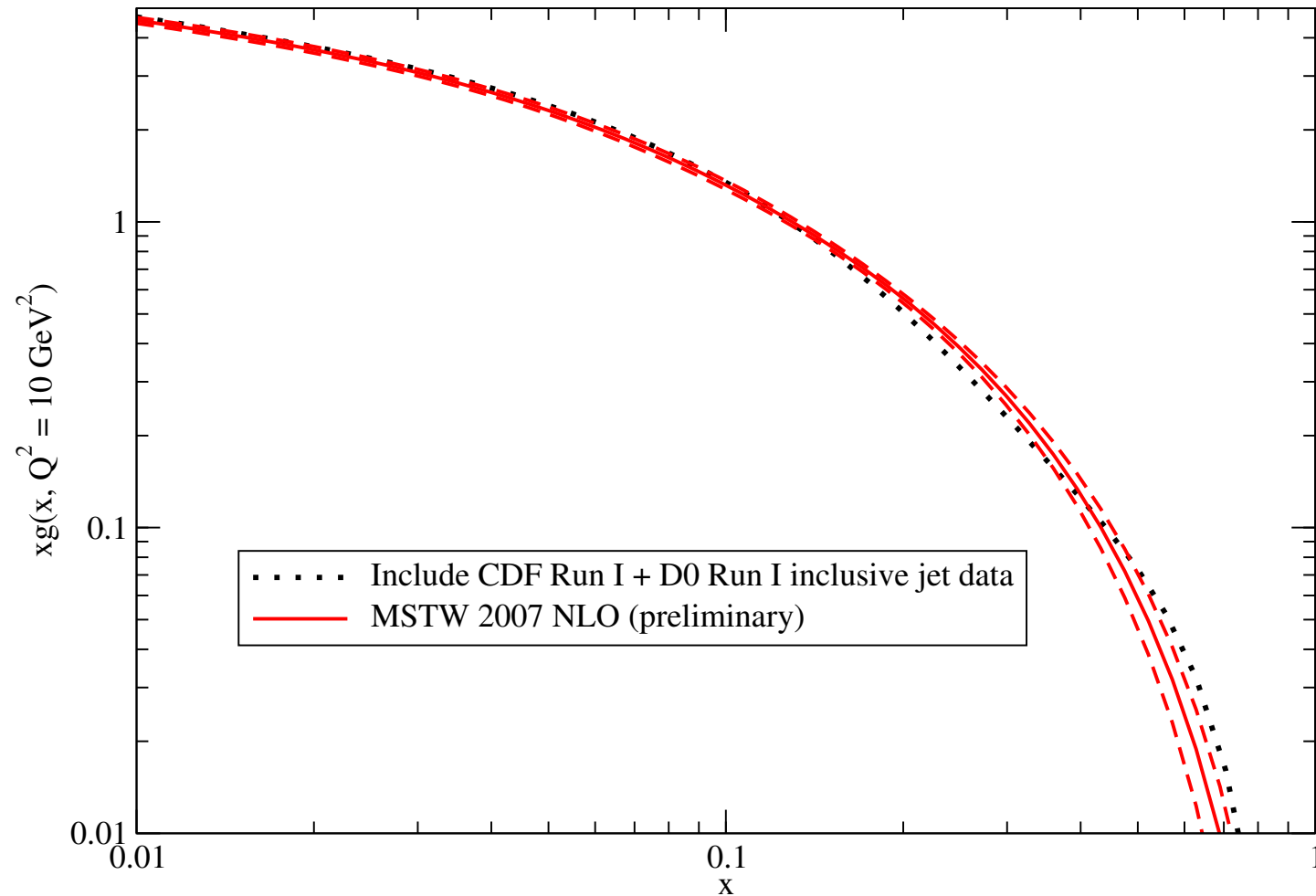
- Without systematic uncertainties
- With systematic uncertainties

Slight deterioration in fit to **D0** run I data in different rapidity bins.

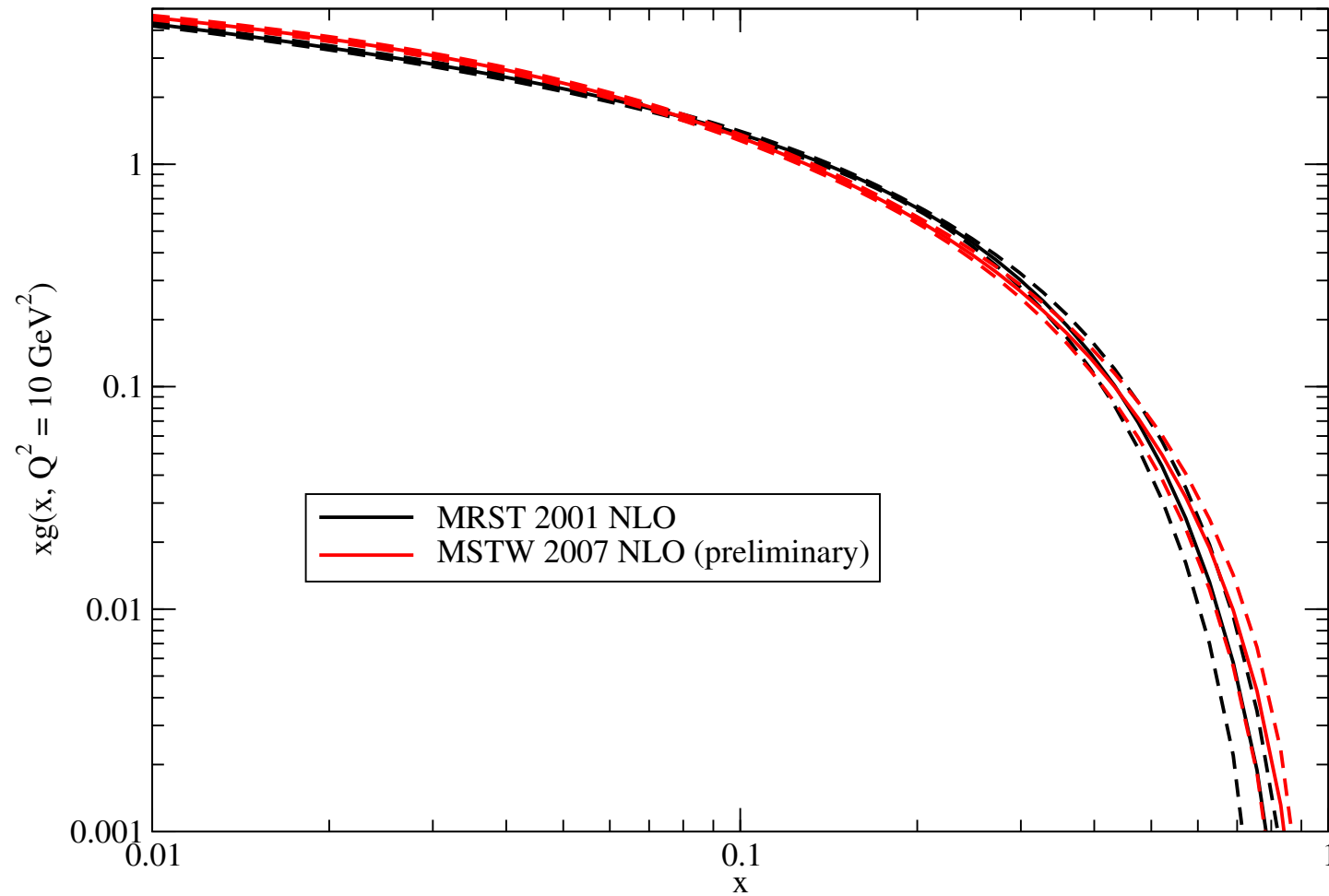




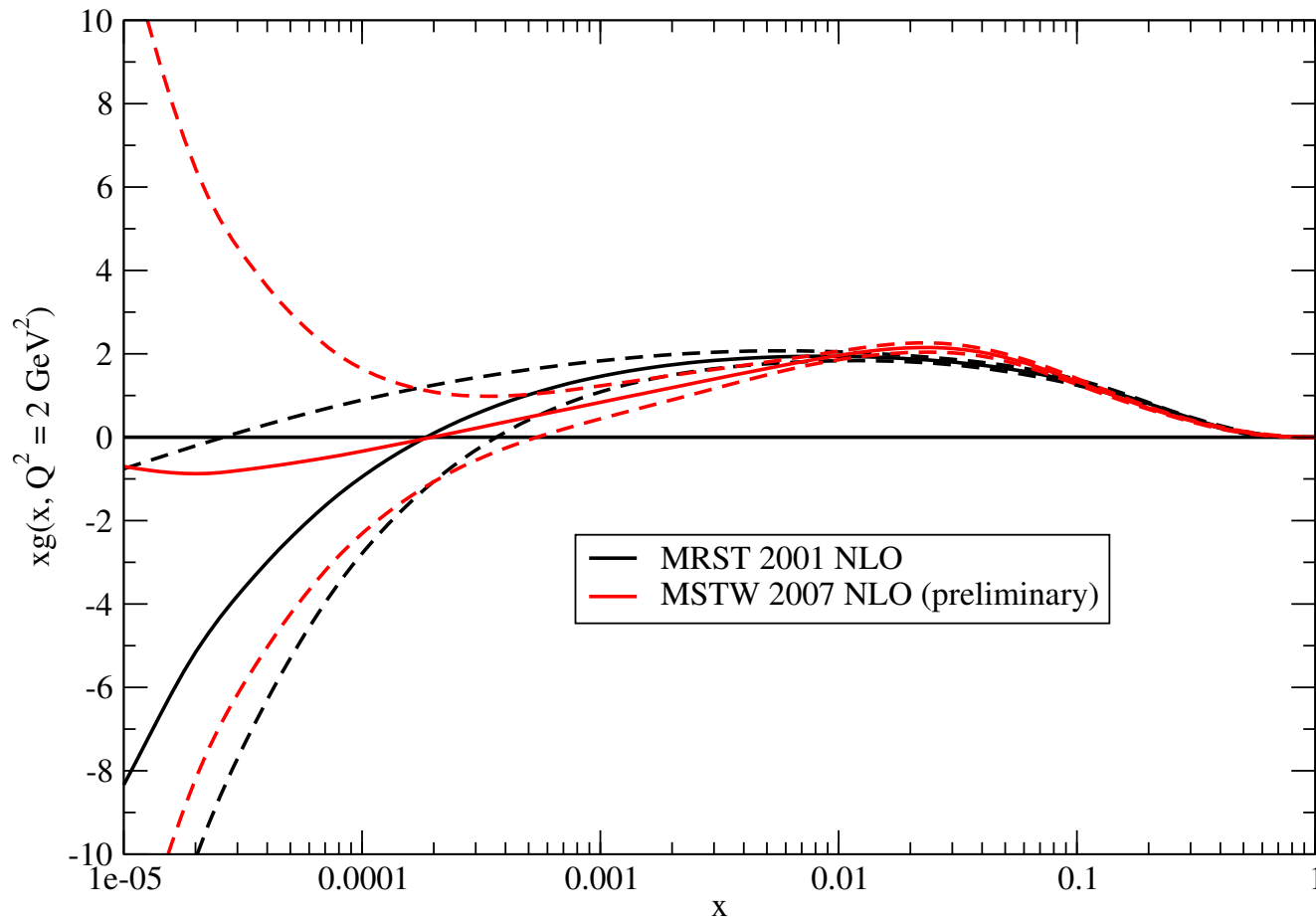
CDF run II data prefers a smaller very high  $x$  gluon distribution compared to run I data. Just outside uncertainties at our  $1\sigma$  level.



Uncertainties, shown at  $1\sigma$  level, similar at high  $x$  to those for MRST2001.



Overall input gluon of same general shape to previously. Still dips negative at low  $x$ , not quite so much. (Again  $1\sigma$  uncertainties).

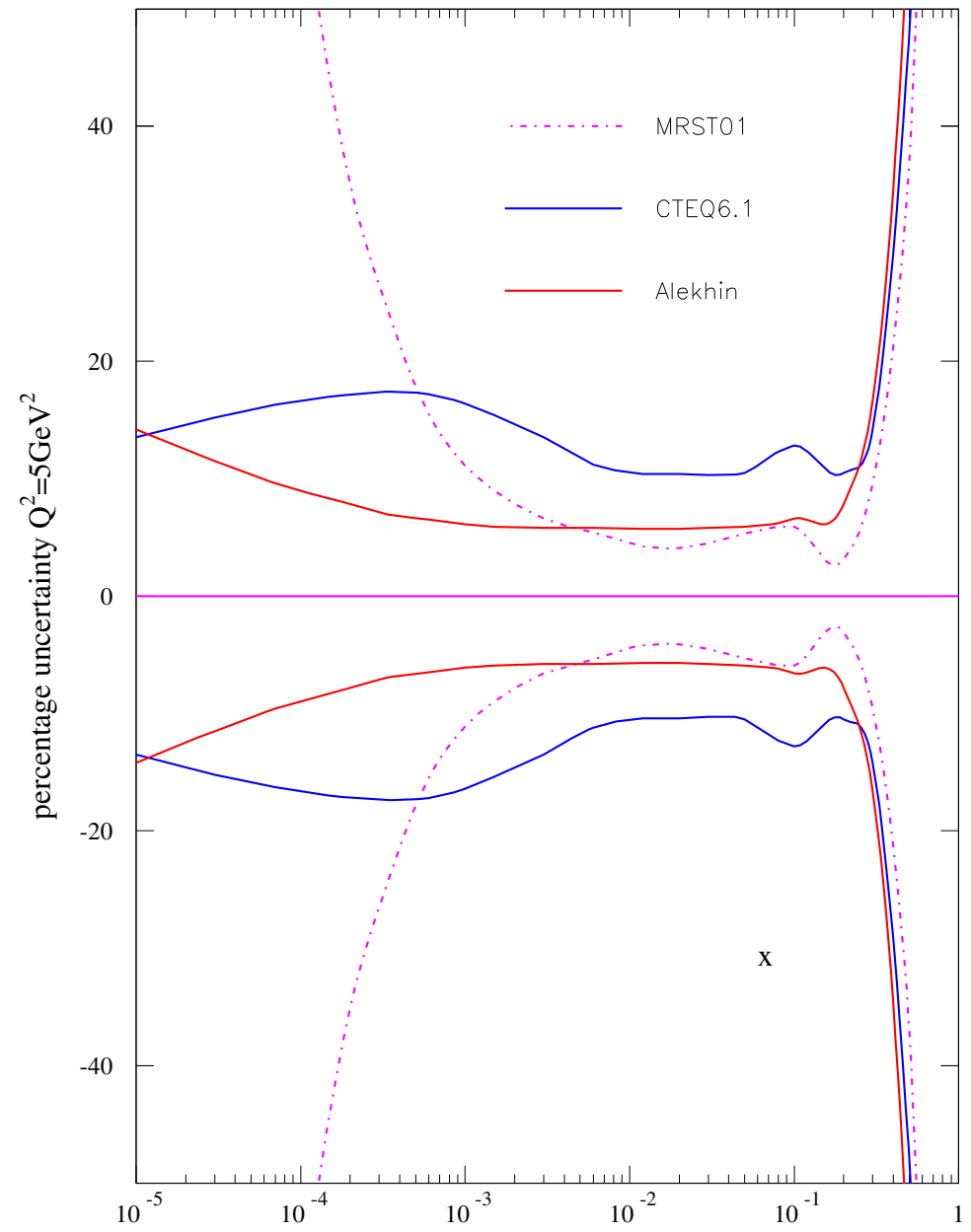


Uncertainty on gluon (without theoretical prejudice) extremely large at  $x = 10^{-5}$ .

MRST uncertainty blows up for very small  $x$ , whereas Alekhin (and ZEUS and H1) gets slowly bigger, and CTEQ saturates (or even decreases).

Related to input forms and scales.

(*Neck* in MRST gluon cured in MSTW).



MRST (MSTW) parameterise at  $Q_0^2 = 1\text{GeV}^2$  but allow negative and positive small  $x$  contributions. Very flexible. Represent true uncertainty at low  $x$ ?

Alekhin and ZEUS gluons input at higher scale – behave like  $x^{-\lambda}$  at small  $x$ . Uncertainty due to uncertainty in one parameter.

CTEQ gluons input at  $Q_0^2 = 1.69\text{GeV}^2$ . Behave like  $x^\lambda$  at small  $x$  where  $\lambda$  large and positive. Input gluon valence-like.

Requires fine tuning. Evolving backwards from steep gluon at higher scale valence-like gluon only exists for very narrow range of  $Q^2$  (if at all).

Small  $x$  input gluon tiny – very small absolute error. At higher  $Q^2$  all uncertainty due to evolution driven by higher  $x$ , well-determined gluon. Very small  $x$  gluon no more uncertain than at  $x = 0.01 - 0.001$ .

## Dependence on $m_c$

Vary  $m_c$  in steps of  $0.1\text{GeV}$ .

$m_c$ (GeV)	$\chi_{global}^2$ 2659 pts	$\chi_{F_2^c}^2$ 78 pts	$\alpha_s(M_Z^2)$
1.2	2541	179	0.1183
1.3	2485	129	0.1191
1.4	2472	100	0.1206
1.5	2479	95	0.1213
1.6	2518	101	0.1223
1.7	2576	123	0.1221

Clear correlation between  $m_c$  and  $\alpha_s(M_Z^2)$ .

For low  $m_c$  overshoot low  $Q^2$  medium  $x$  data badly.

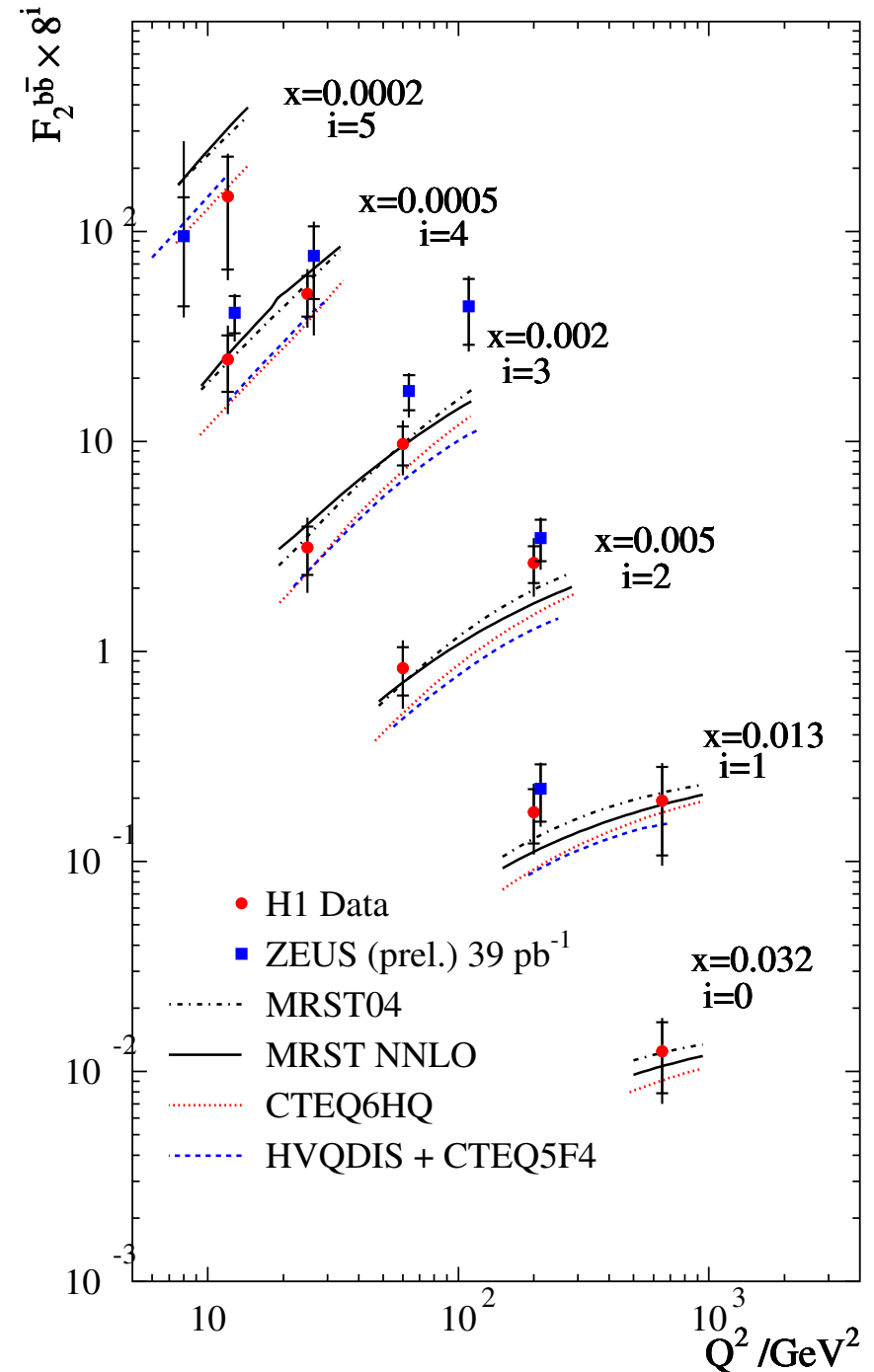
Preference for  $m_c = 1.4\text{GeV}$ . Towards lower end of pole mass determinations. Uncertainty from fit  $\sim 0.1 - 0.15\text{GeV}$ .

Also now choose  $m_b = 4.75\text{GeV}$ , i.e. reasonable pole mass value. Not determined well by fit.

Good comparison to both H1 and ZEUS data on  $F_2^b(x, Q^2)$

The difference in the NLO predictions from MRST and CTEQ is due to details of definition of VFNS near threshold.

Both VFNS curves for  $m_b = 4.3\text{GeV}$ . Should be corrected to  $m_b = 4.75\text{GeV}$ . Lowers both prediction slightly, particularly at low  $Q^2$ .



# Conclusions

**NNLO** partons in principle exist now. Fit data well but **NLO** better. Provisional update of partons, need to input full data sets. Main difference due to better **NNLO** heavy flavour prescription. This is important.

Inclusion of new data. Neutrino structure function data inconsistent at high  $x$ . Cut at  $x = 0.5$ . Important constraint at lower  $x$ . Dimuon data fitted directly. Important constraint on strange, and weak evidence for strangeness momentum asymmetry. New uncertainties on  $s + \bar{s}$  feed into other partons.

New **CDF**  $W$ -asymmetry data more constraining for  $d_V$  and to some extent  $\bar{d}$ . Slightly different shape for  $d_v(x, Q^2)$ .

**HERA** jets, and **Tevatron** high- $E_T$  jets, now fit using **fastNLO**. Works well and fit good. New run II **CDF** jet data included in fit. Small, but significant change. Smaller high- $x$  gluon.

Will have full updated **NLO** and **NNLO** partons for **LHC** complete with uncertainties – experimental and theoretical.

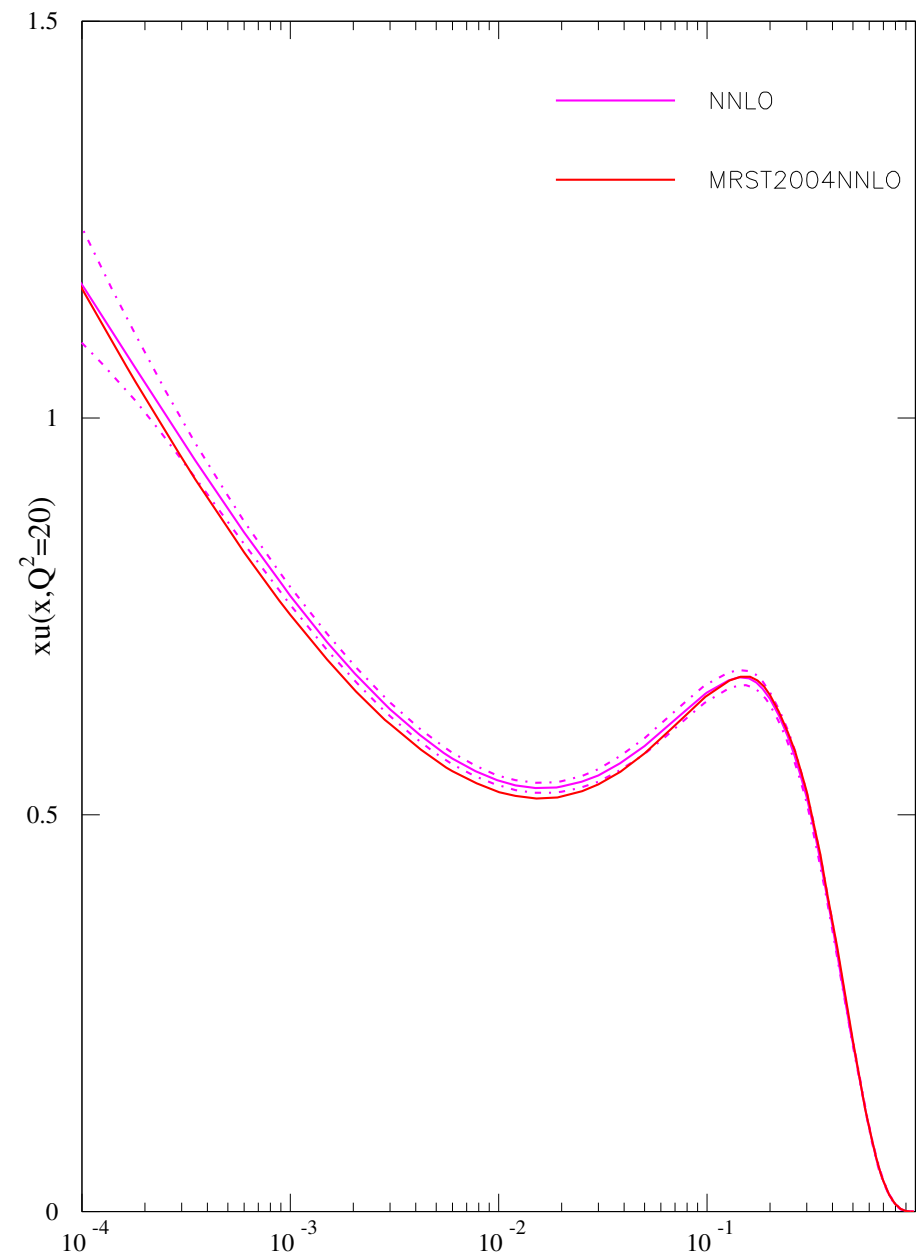


Not much change in light quarks due to these theoretical updates.

Minor change – bit bigger than **MRST2004** at small  $x$ .

Slightly lower  $s(x, Q^2) \rightarrow$  more  $u(x, Q^2)$ .

Also slightly higher  $\alpha_S(M_Z^2)$ . Negative **NNLO** correction bigger  $\rightarrow$  more  $u(x, Q^2)$ .



Previously used correction applied to theoretical prediction.

$$x < 0.0903 \quad R = 1.238 + 0.203 \log_{10} x$$

$$x > 0.2340 \quad R = 0.783 - 0.385 \log_{10} x$$

$$0.234 > x > 0.0903 \quad R = 1.026$$

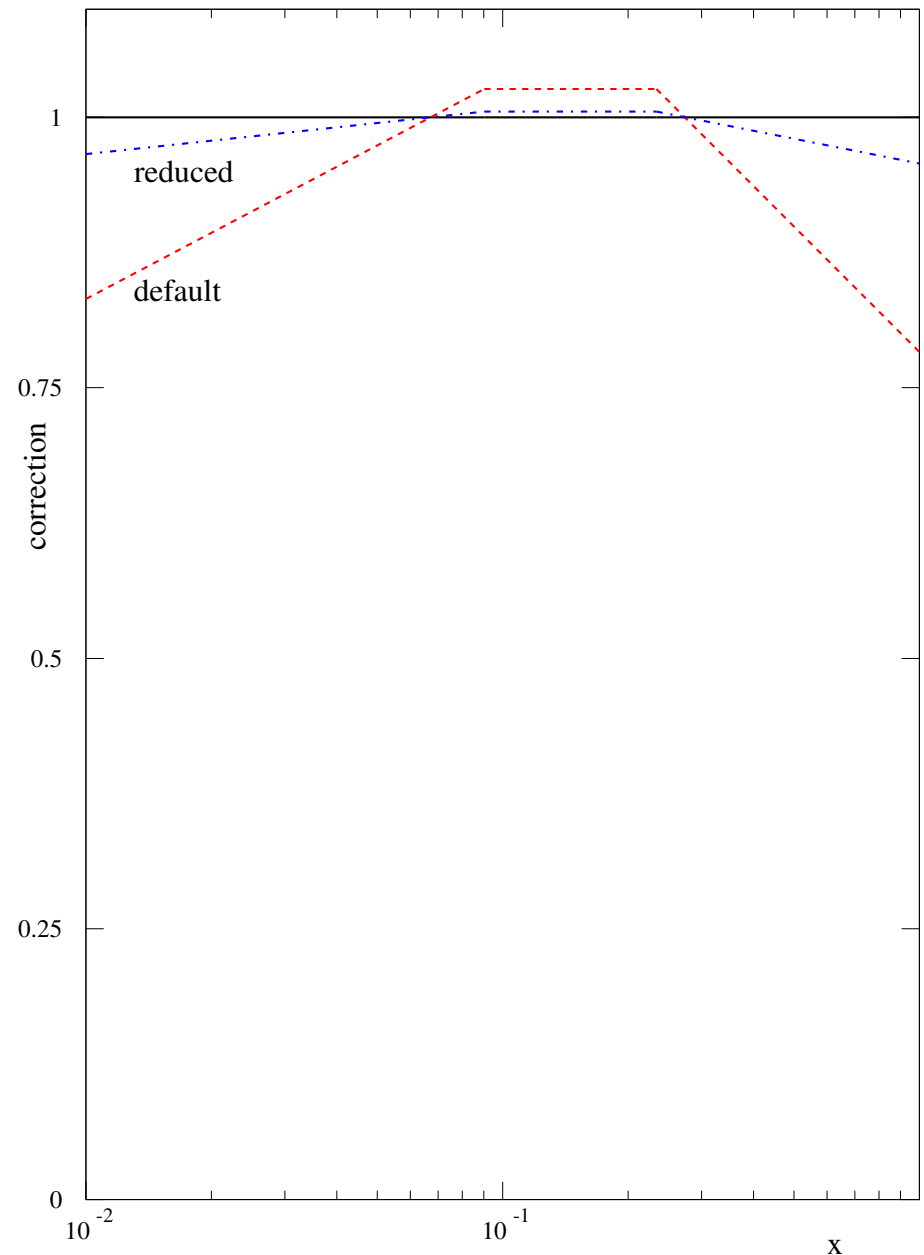
Far too large for new NuTeV data.  
High- $x$  completely determined by valence quarks for both  $F_2^{\nu, \bar{\nu}}(x, Q^2)$  and  $F_3^{\nu, \bar{\nu}}(x, Q^2)$ .

These well known from fixed target  $F_2^p(x, Q^2)$  and  $F^d(x, Q^2)$ .

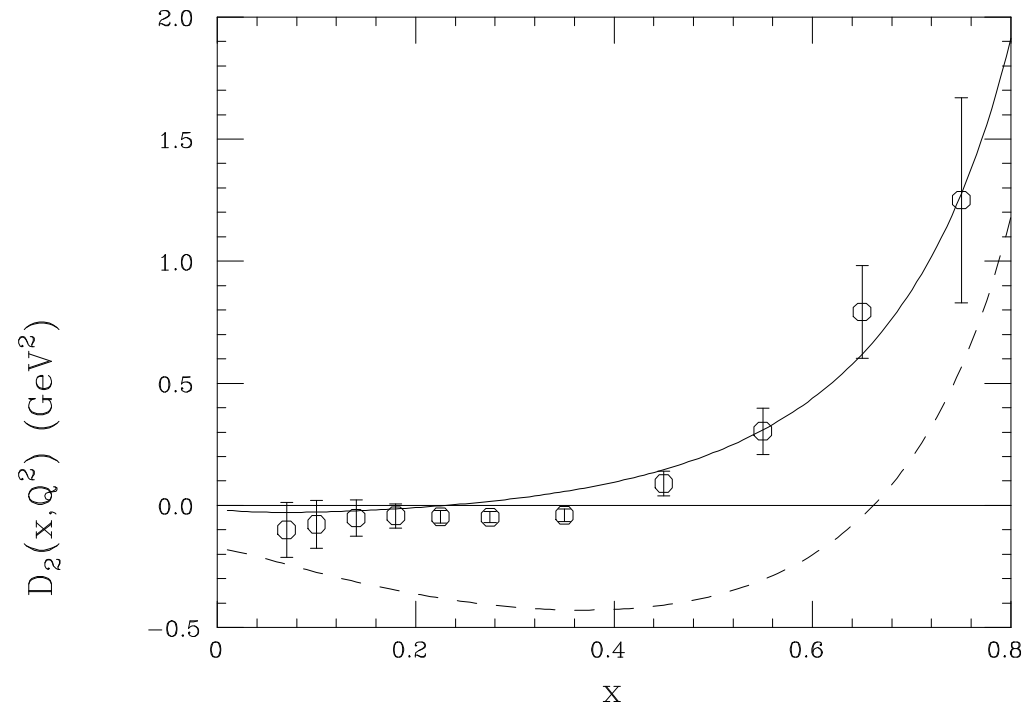
Try form  $R^{eff} = 1 + A * (R - 1)$ .

Best fit  $A = 0.2$ .

Nuclear corrections for NuTeV data

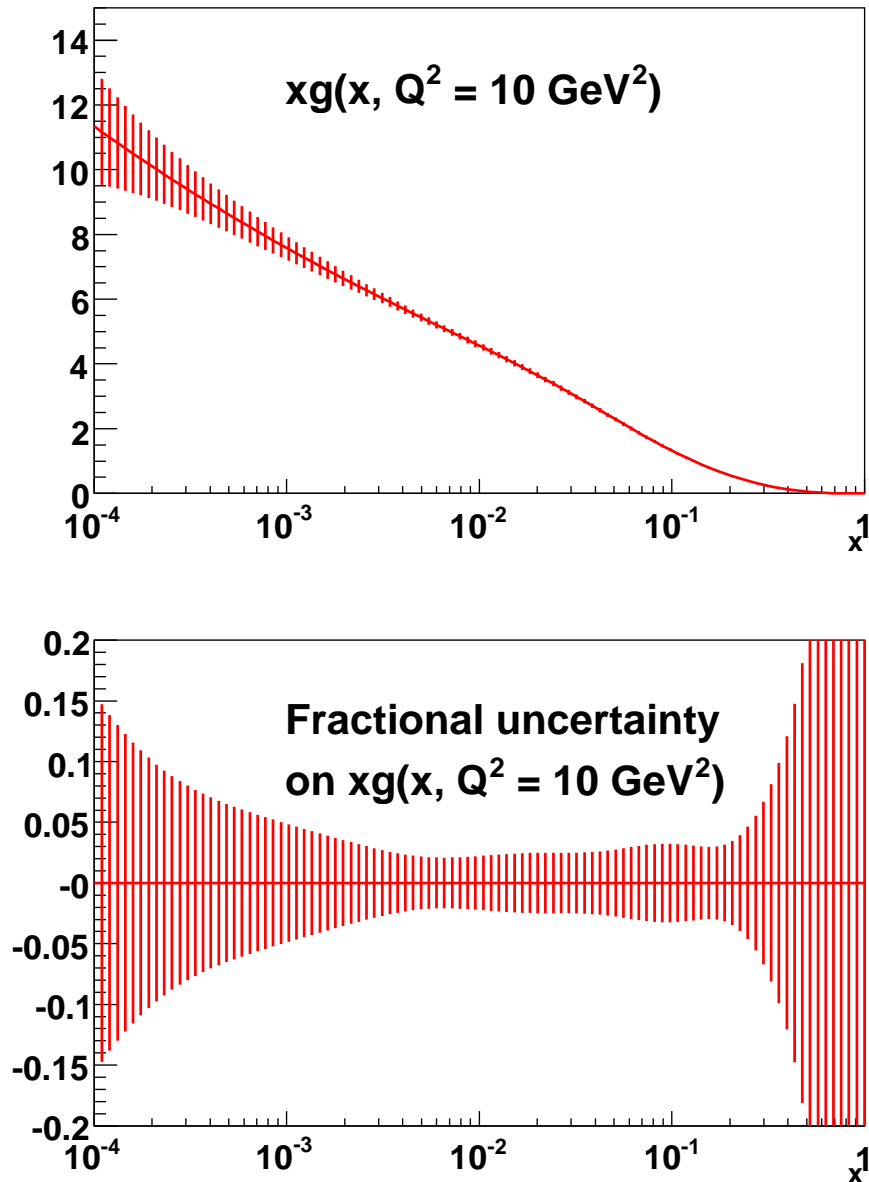


Renormalon prediction for  $1/Q^2$   
 corrections for  $F_2(x, Q^2)$  (solid  
 line) and  $xF_3(x, Q^2)$  (dashed line)  
 Dasgupta and Webber.



Fractional uncertainty (at one- $\sigma$  level) for MSTW gluon.

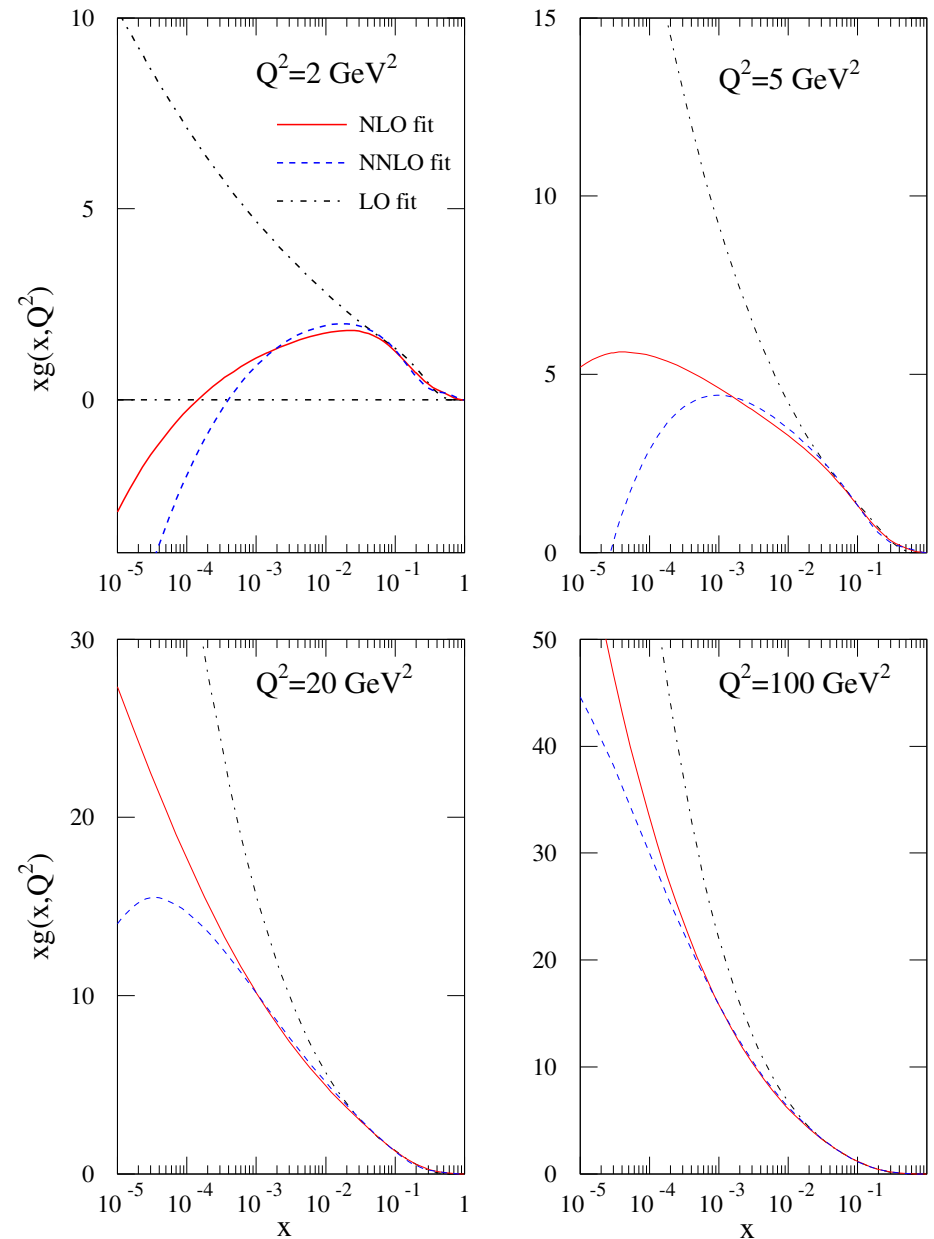
### MSTW 2007 NLO PDFs (preliminary)



The gluon extracted from the global fit at **LO**, **NLO** and **NNLO**.

Additional and positive small- $x$  contributions in  $P_{qg}$  at each order lead to smaller small- $x$  gluon at each order.

Note - this conclusion relied on correct application of flavour thresholds in a General Variable Flavour Number Scheme at **NLO** not present in earlier approximate **NNLO MRST** fits. Correct treatment of flavour particularly important at **NNLO** because discontinuities in unphysical quantities appear at this order.

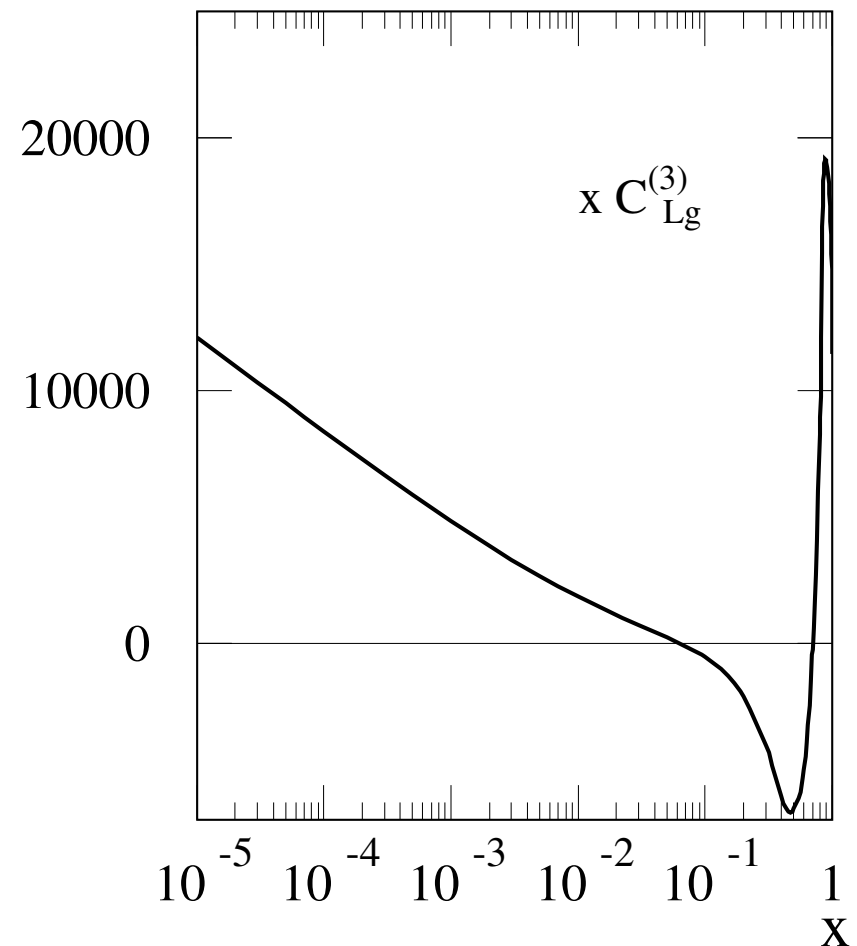


The NNLO  $\mathcal{O}(\alpha_s^3)$  longitudinal coefficient function  $C_{Lg}^3(x)$  given by

$$C_{Lg}^3(x) = n_f \left( \frac{\alpha_s}{4\pi} \right)^3 \left( \frac{409.5 \ln(1/x)}{x} - \frac{2044.7}{x} - \dots \right).$$

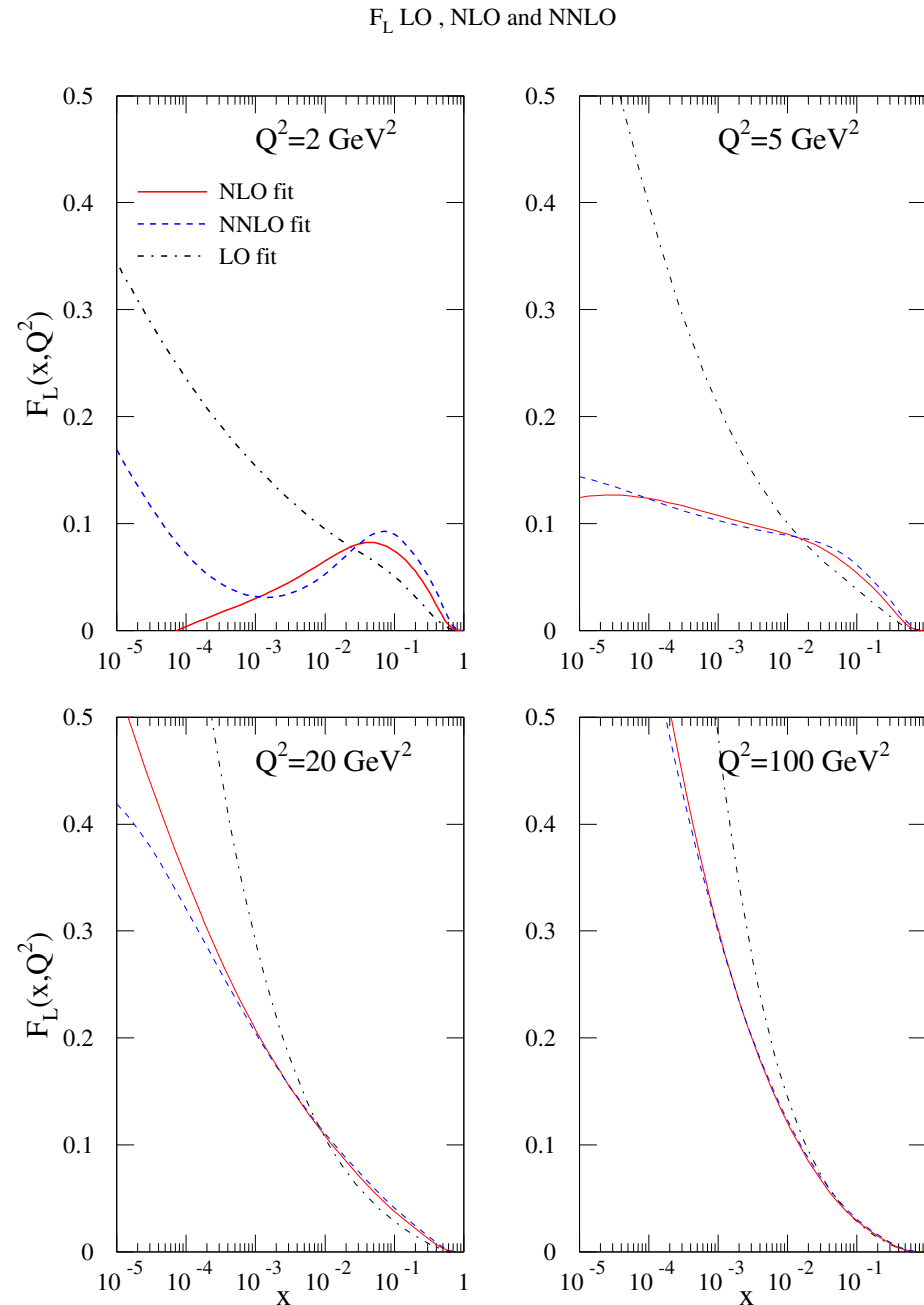
Clearly a significant positive contribution at small  $x$ .

Counters decrease in small- $x$  gluon.



$F_L(x, Q^2)$  predicted from the global fit at LO, NLO and NNLO.

NNLO coefficient function more than compensates decrease in NNLO gluon.



## Comparisons

Compare with only other NNLO partons on market – Alekhin2002.

Nothing from CTEQ?

Much larger  $\alpha_S(M_Z^2)$  in this fit than that of Alekhin ( $\alpha_S(M_Z^2) = 0.119$  compared to 0.114).

Not much difference in high- $x$  valence quarks, except than explained by difference in  $\alpha_S(M_Z^2)$ . Very well-constrained.

Differences in low- $x$  sea quarks. Swamped by differences in flavour treatments –  $\bar{u} - \bar{d}$  and  $s(x, Q^2)$ .

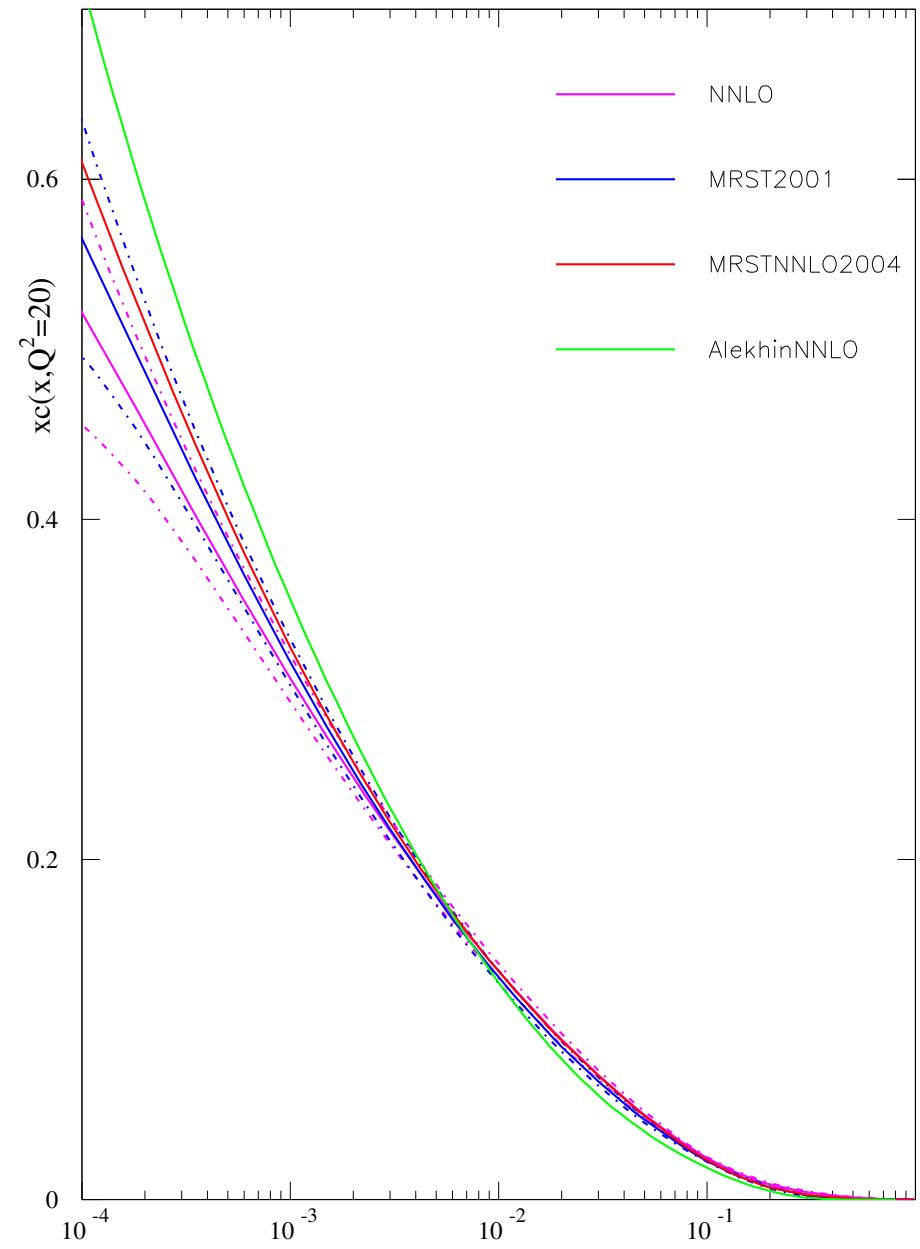
Main difference in gluon distribution.



Difference in gluon feeds through to charm.

Alekhin2002 much bigger at small  $x$ .

Starts from zero as with MRST2004NNLO.

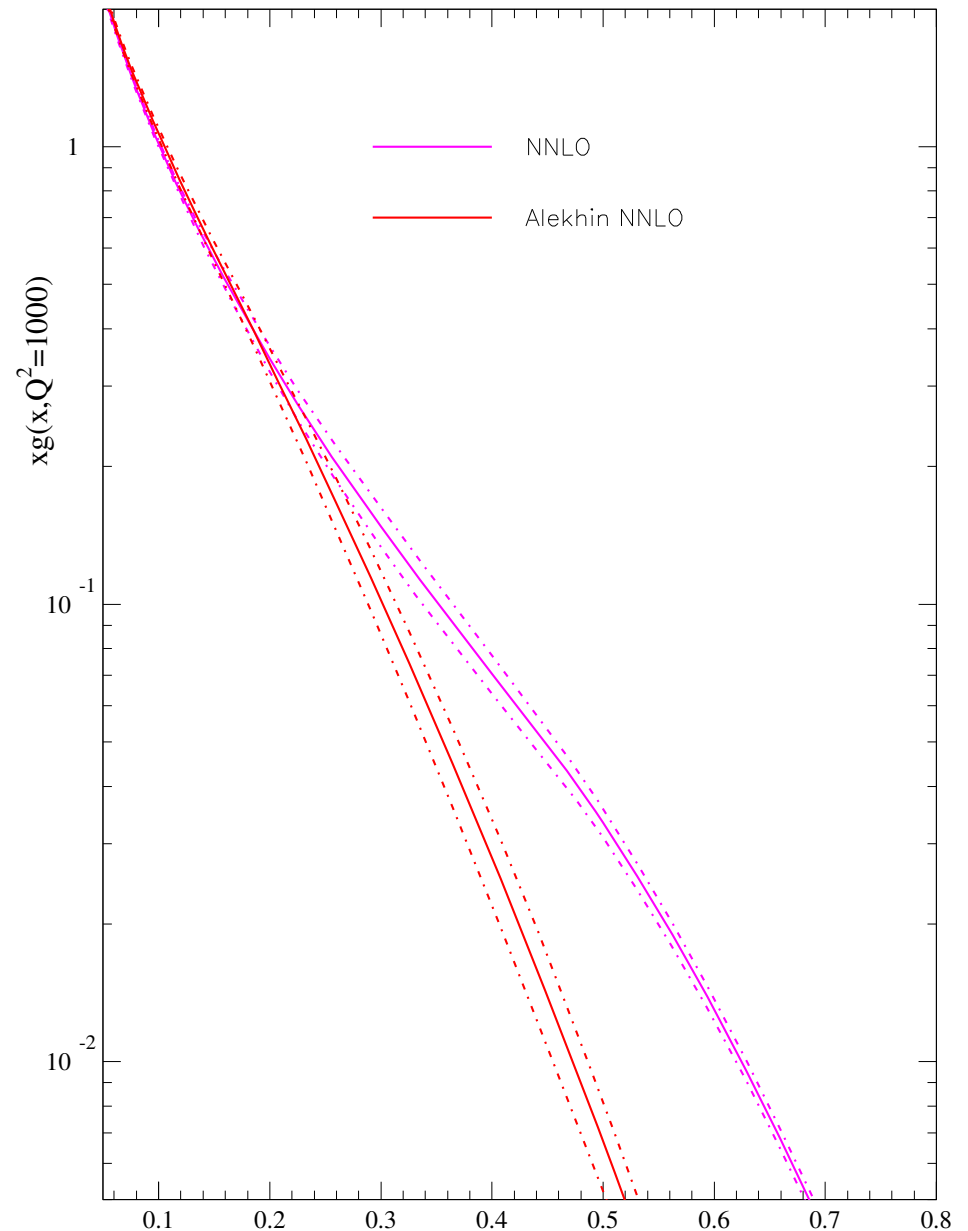


Big difference at high  $x$  and  $Q^2$ .

Determined by Tevatron jet data for MRST. Fit now excellent.

Divergences at  $x = 0.25$  corresponds to  $E_T \sim 225\text{GeV}$ .

In  $\overline{MS}$  scheme gluon more important for jets at high  $x$  at NNLO because high- $x$  quarks smaller.



x