

Universality of QCD traveling waves with running coupling

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- Motivation

Stability of Saturation beyond leading QCD logarithms

- Saturation and QCD Traveling waves

Balitsky-Kovchegov with running coupling

- Higher-order QCD effects

Nonlinear vs. Linear effects

- From Theory to Phenomenology

Geometric Scaling from running coupling

^ahep-ph/0610354, hep-ph/070213. For the “Quality factors”: R.P. with F.Gélis, L.Schoeffel, G.Soyez hep-ph/0610435.

Motivation

Stability of Saturation beyond leading QCD logarithms

- *Observable dependence:*

Different rapidity dependence in the linear regime

Dipole amplitude vs. unintegrated gluon distribution:

cf. Kovchegov, Weigert, hep-ph/0612071.

- *RG-improved scheme dependence:*

Improved NLL kernels are scheme-dependent

cf. Ciafaloni, Colferai, hep-ph/9812366, Salam, hep-ph/9806482, etc...

- *Coupling/Kernel Dependence:*

Different Schemes at higher orders

cf. Parent-Dipole scheme, Balitsky scheme hep-ph/0609105,
Kovchegov-Weigert scheme hep-ph/0609090

- *Infra-red Regularization dependence:*

Landau poles need to be regularized

cf Renormalons at small- x , Levin hep-ph/9412345

- *Position vs. momentum-space dependence:*

Running coupling from dipoles vs. gluons

The Balitskiĭ-Kovchegov Equation with running coupling

- The Non-Linear BK Equation for the amplitude \mathcal{T} :

$$[b \text{Log} k^2] \partial_Y \mathcal{T} = \chi_{BFKL} (-\partial_{\text{Log} k^2}) \mathcal{T} - \mathcal{T}^2$$

- Equation BK \Rightarrow [modified] F-KPP

S.Munier, R.P., 2003,2004

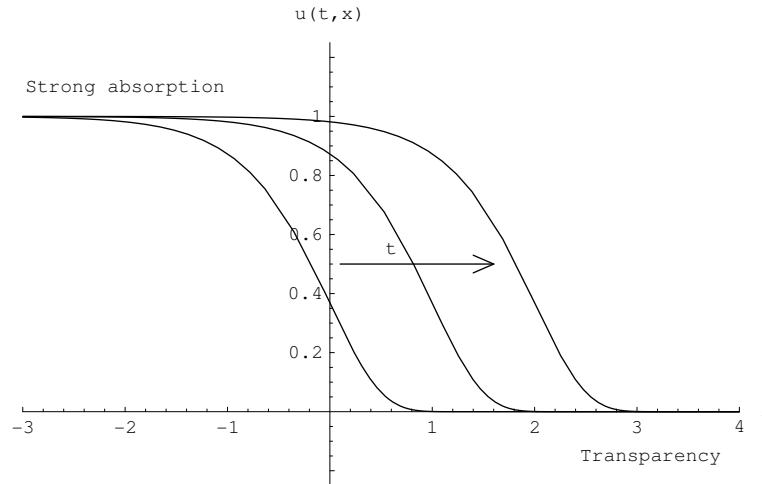
$$[x] \partial_t u(t, x) = \partial_x^2 u(t, x) + u(t, x)(1 - u(t, x))$$

- “Dictionary”

$$\begin{aligned} Time &= t \sim \sqrt{Y} \\ Space &= x \sim \text{Log} k^2 \\ Traveling Wave &= u(t, x) \sim u(t - v_c x) \propto \mathcal{T} \left(\frac{k}{Q_s(Y)} \right) \\ Non universal &= t_0 \sim \sqrt{Y_0} \end{aligned}$$

$$\log Q_s^2(\text{run}) = \sqrt{\frac{2\chi(\gamma_c)}{b\gamma_c}(Y + Y_0)} + \frac{3}{4} \left(\frac{\chi''(\gamma_c)}{\sqrt{2b\gamma_c\chi(\gamma_c)}} \right)^{1/3} \xi_1 (Y + Y_0)^{1/6} + \dots,$$

Traveling waves beyond leading logs



- General Method
 - i) solve the linear equation: $\rightarrow u(t, x) \sim \int d\gamma e^{\gamma[x - v(\gamma)t]}$
 - ii) find the critical (minimal) velocity $v_c = \min v(\gamma) = v(\gamma_c)$
 - iii) sharp initial conditions $\gamma_0 > \gamma_c$
- Mathematical properties

Universality classes: Independence from initial data, nonlinear damping, noncritical features. different equations \rightarrow same asymptotics

Validity: “leading edge-interior” of the wave and late times.
- Study the Stability of traveling waves

Universality classes vs. higher order effects

Higher-order QCD effects (1)

- The Higher-order BK Equation: Ex. “Triumvirate”

Levin hep-ph/9412345, Kovchegov-Weigert hep-ph/0609090

$$\frac{\partial \tilde{T}}{\partial Y}(k, Y) = \int_{\mathcal{R}} \frac{d^2 q}{2\pi} \frac{\bar{\alpha}(q^2)\bar{\alpha}((k-q)^2)}{\bar{\alpha}(k^2)} \left[\frac{1}{(k-q)^2} \tilde{T}(q, Y) + \frac{1}{q^2} \tilde{T}(|k-q|, Y) - \frac{k^2}{q^2(k-q)^2} \tilde{T}(k, Y) \right] + \text{nonlinear}$$

- Infra-red Regularized BK Equation

$$\frac{1}{\bar{\alpha}(L)} \partial_Y N(L, Y) = \chi_{\mathcal{R}}^{Tri}(-\partial_L, \bar{\alpha}(L)) N(L, Y) - N^{\otimes 2}(L, Y)$$

- General Method: linear solution

$$N(L, Y) = \int \frac{d\gamma}{2\pi i} \int \frac{d\omega}{2\pi i} N_0(\gamma, \omega) \exp \left(-\gamma L + \omega Y + \frac{1}{b\omega} \int_{\hat{\gamma}}^{\gamma} d\gamma' \kappa^{Tri}(\gamma', \omega) \right)$$

with modified kernel $\alpha \rightarrow \omega$ - expansion

Ciafaloni, Colferai hep-ph/9812366

$$\kappa^{Tri}(\gamma, \omega) = \frac{\chi(\gamma) + \sqrt{\chi(\gamma)^2 + 4b\omega \left[\frac{\chi(\gamma)^2}{2} + \frac{3}{2} (\Psi'(\gamma) - \Psi'(1-\gamma)) \right]}}{2} + \dots$$

Higher-order QCD effects (2)

- Other example: Balitsky scheme

Balitsky, hep-ph/0609105

$$\kappa^{Bal}(\gamma, \omega) = \frac{\chi(\gamma) + \sqrt{\chi(\gamma)^2 + 4b\omega \left[\frac{1}{2} (\chi(\gamma)^2 + \Psi'(\gamma) - \Psi'(1-\gamma)) - 2\frac{\chi(\gamma)}{\gamma} \right]}}{2} + \dots$$

- **General Method: critical velocity and slope**

$$v_g = \sqrt{\frac{2\kappa^{Tri,Bal}(\gamma_c, \omega=0)}{b\gamma_c}} + \text{subasym.} \quad \gamma_c = \frac{\kappa^{Tri,Bal}(\gamma_c, \omega=0)}{\partial_\gamma \kappa^{Tri}(\gamma_c, \omega=0)} + \text{subasym.}$$

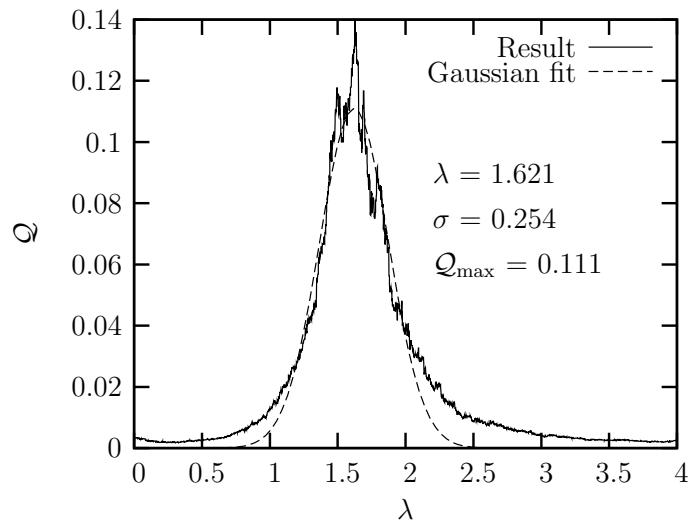
- **Universality class**

$$\boxed{\kappa^{Tri,Bal}(\gamma_c, \omega=0) \equiv \chi_{BFKL}(\gamma_c)}$$

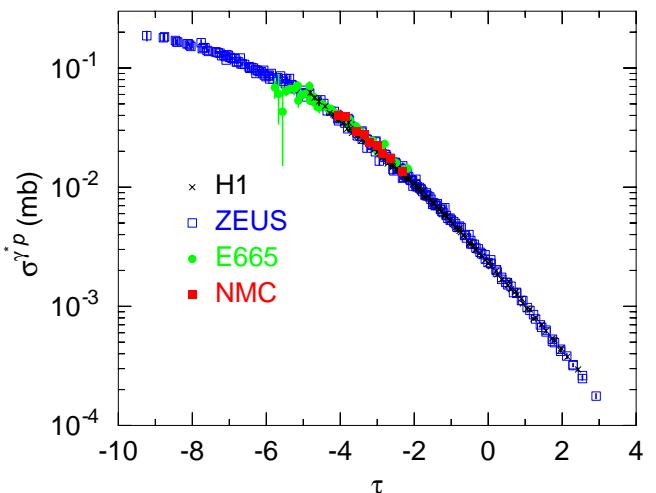
Same Universality class as BK with Parent-Dipole coupling

From Theory to Phenomenology

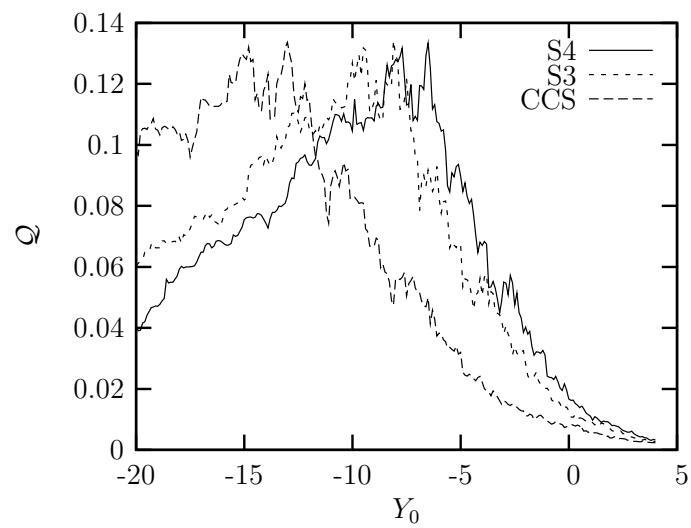
Quality Factor



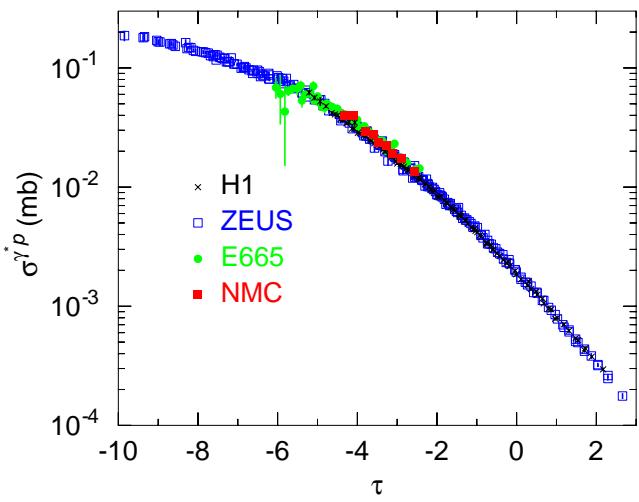
Geometric Scaling in \sqrt{Y}



Scheme dependence



QCD Scaling with subasymptotics



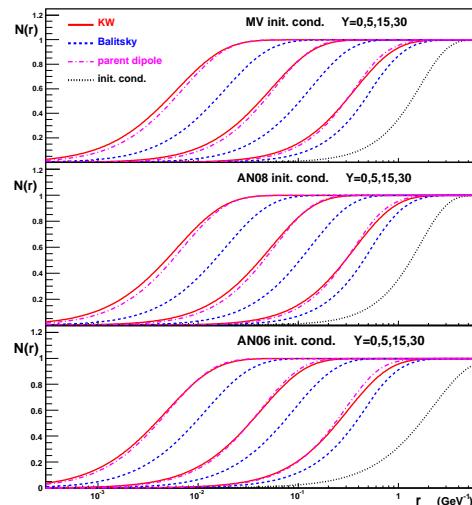
Conclusions

- *Stability of saturation:*
The BK equations with running coupling possess universal asymptotics
- *Characterisation of the universality class:*
The universality class is the original BK with a “factorized” running coupling
- *Phenomenological issues*
Geometric scaling in \sqrt{Y} verified
But need for nonuniversal terms
- *Prospects*
Subasymptotic Traveling Waves?
Universality with “Complete” Kernel?
Universality beyond BK?

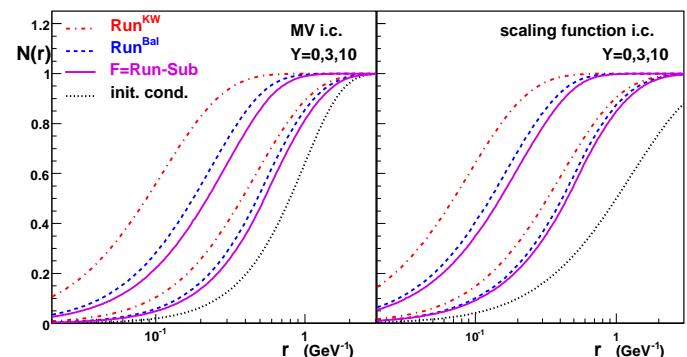
Note added in proof(s)

Albacete and Kovchegov, arXiv:0704.0612

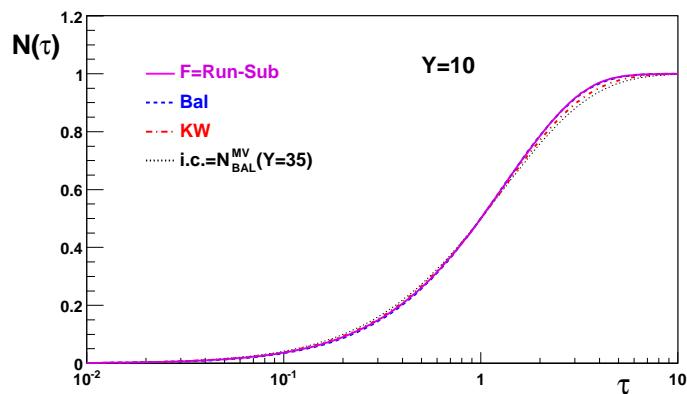
Numerics for BK



"Subtracted" Kernel



Scaling of Traveling Waves



Subasymptotic Waves?

