

The ratio of σ_L/σ_T in DIS
at low x

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DIS 2007, April 2007

Dipole Picture-Bound on $\sigma_{\gamma_L^* p}(W^2, Q^2)/\sigma_{\gamma_T^* p}(W^2, Q^2)$.

$$\sigma_{\gamma_{L,T}^* p}(W^2, Q^2) = \sum_q \int d^2 r_\perp \omega_{L,T}^{(q)}(Q r_\perp, Q^2, m_q^2) \sigma_{(q\bar{q})p}(r_\perp^2, W^2).$$

Nikolaev and Zakharov (1994)

$$\omega_L^{(q)}(Q r_\perp, Q^2, m_q^2) \leq \omega_T^{(q)}(Q r_\perp, Q^2, m_q^2) \cdot \underbrace{\max_{r_\perp,q} \frac{\omega_L^{(q)}(Q r_\perp, Q^2, m_q^2)}{\omega_T^{(q)}(Q r_\perp, Q^2, m_q^2)}}_{0.37}$$

$$R(W^2, Q^2) = \frac{\sigma_{\gamma_L^* p}(W^2, Q^2)}{\sigma_{\gamma_T^* p}(W^2, Q^2)} \leq \max_{r_\perp,q} \frac{\omega_L^{(q)}(Q r_\perp, Q^2, m_q^2)}{\omega_T^{(q)}(Q r_\perp, Q^2, m_q^2)}$$

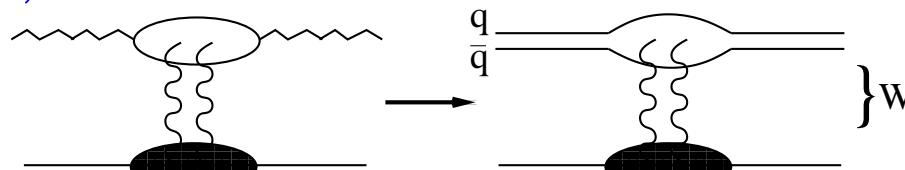
Ewerz and Nachtmann (2006)

$$R(W^2, Q^2) \leq 0.37$$

$$\frac{\sigma_{\gamma_L^* p}}{\sigma_{\gamma_L^* p} + \sigma_{\gamma_T^* p}} = \frac{F_L(W^2, Q^2)}{F_2(W^2, Q^2)} \leq 0.27$$

model-independent bound

The energy variable, W



$$\gamma^* p \rightarrow \gamma^* p$$



$$(q\bar{q})p \rightarrow (q\bar{q})p$$

on-shell $q\bar{q}$ scattering

$$M_{q\bar{q}}^2 = \frac{k_\perp^2}{z(1-z)}, \quad 0 \leq z \leq 1;$$

Vector Meson Dominance,
Generalized Vector dominance,
Sakurai + Schildknecht,
Gribov, Joffe (1970ies)

Restriction: $M_{q\bar{q}}^2 \leq m_1^2 = \bar{Q}^2 \ll W^2$

Ewerz and Nachtmann (2006)

$$k_\perp^2 \leq z(1-z)\bar{Q}^2$$

e.g. at HERA: $m_1 \approx 25$ GeV

Effect on effective photon wave function:

$$\int dk_\perp^2 \longrightarrow \int^{z(1-z)\bar{Q}^2} dk_\perp^2$$

$$\omega_{L,T}^{(q)} = \omega_{L,T}^{(q)}(Qr_\perp, Q^2, m_q^2, \bar{Q}^2)$$

$$\sigma_{(q\bar{q})p}(r_\perp^2, W^2) \text{ versus } \sigma_{q\bar{q})p} \left(r_\perp^2, x \simeq \frac{Q^2}{W^2} \right).$$

Employing x yields Q^2 dependence for on-shell $(q\bar{q})p \rightarrow (q\bar{q})p$ scattering. The (forbidden) Q^2 dependence means that employing x can at best be an approximation.

Usual argument: For $r_\perp^2 \rightarrow 0$:

$$\sigma_{(q\bar{q})p}(r_\perp^2, W^2) \sim r_\perp^2 f \left(\frac{1}{W^2 r_\perp^2} \right) \rightarrow r_\perp^2 f \left(\frac{Q^2}{\text{const } W^2} \right) = r_\perp^2 f \left(\frac{x}{\text{const}} \right),$$

$$\text{since } r_\perp^2 \Big|_{\max} \simeq \frac{\text{const}}{Q^2}$$

Not generally applicable, consider e.g.

$$\sigma_{q\bar{q})p}(r_\perp^2, W^2) \sim r_\perp^2 \sigma^{(\infty)} \Lambda_{\text{sat}}^2(W^2).$$

Effect of finite \bar{Q}^2 on $R(W^2, Q^2)$.

$$r^{(q)} \left(Qr_\perp, \frac{Q^2}{\bar{Q}^2} \right) = \frac{\omega_L^{(q)}(Qr_\perp, Q^2, \bar{Q}^2)}{\omega_T^{(q)}(Qr_\perp, Q^2, \bar{Q}^2)} \underset{r_\perp \rightarrow 0}{\sim} \begin{cases} Q^2 r_\perp^2 \rightarrow 0, & \text{for } \bar{Q}^2 \rightarrow \infty \\ \frac{1}{r_\perp^2 \bar{Q}^2} \rightarrow \infty, & \text{for } \bar{Q}^2 \text{ finite} \end{cases}$$

For finite \bar{Q}^2 , no upper bound:

$$0 \leq R(W^2, Q^2) < \infty$$

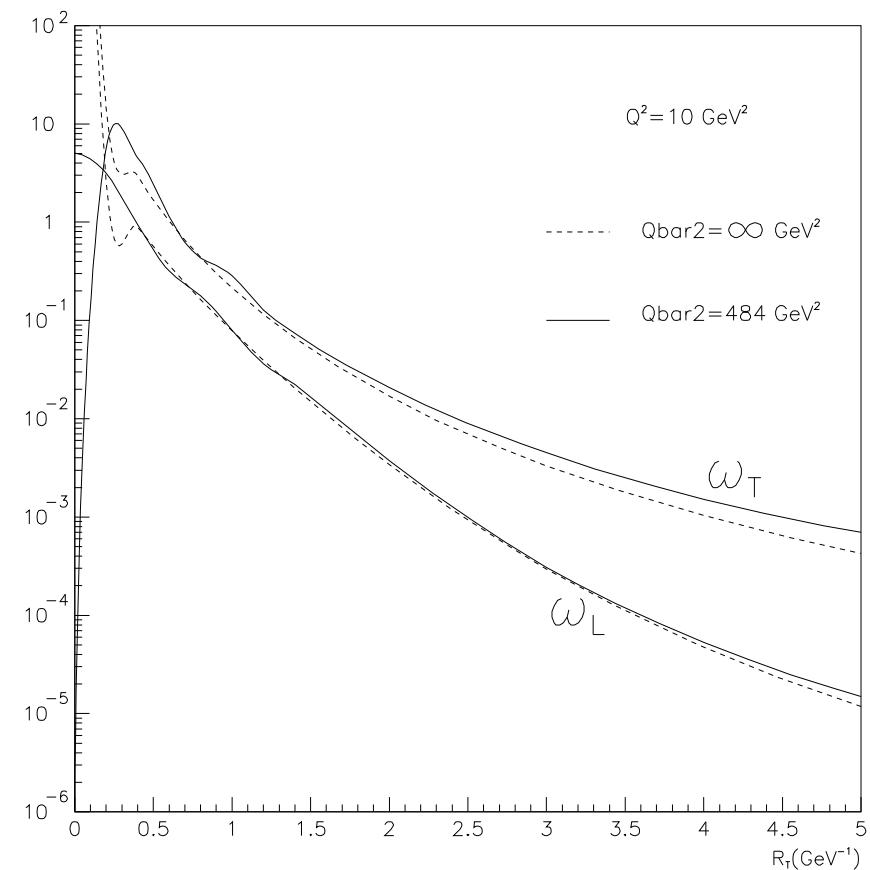
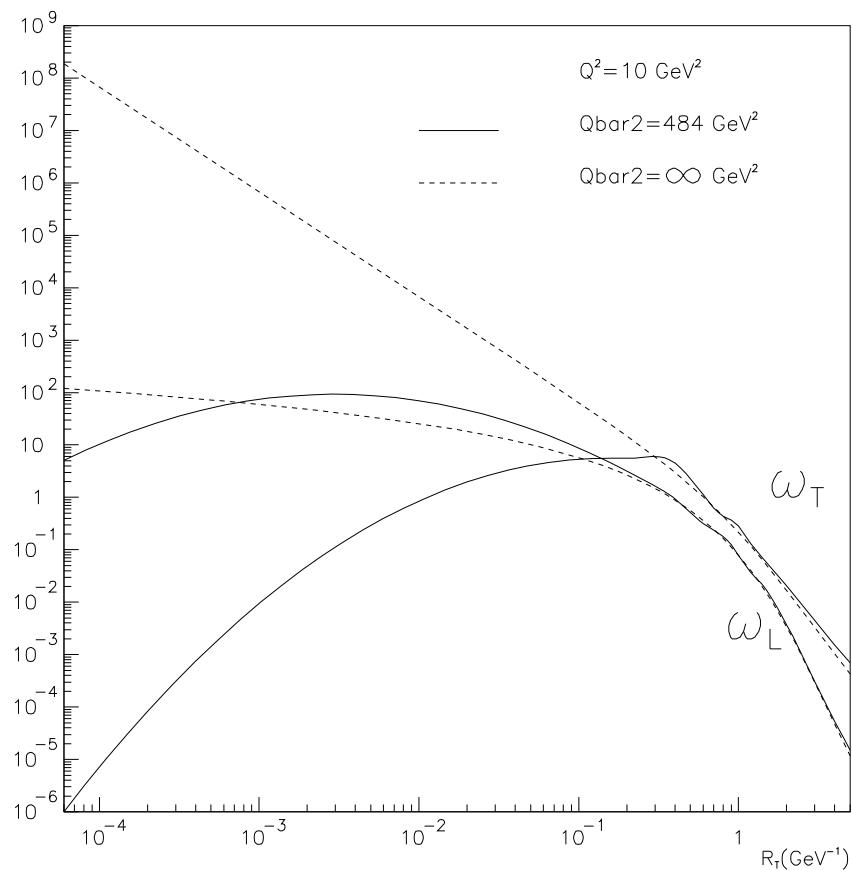
Kuroda, Schildknecht in preparation

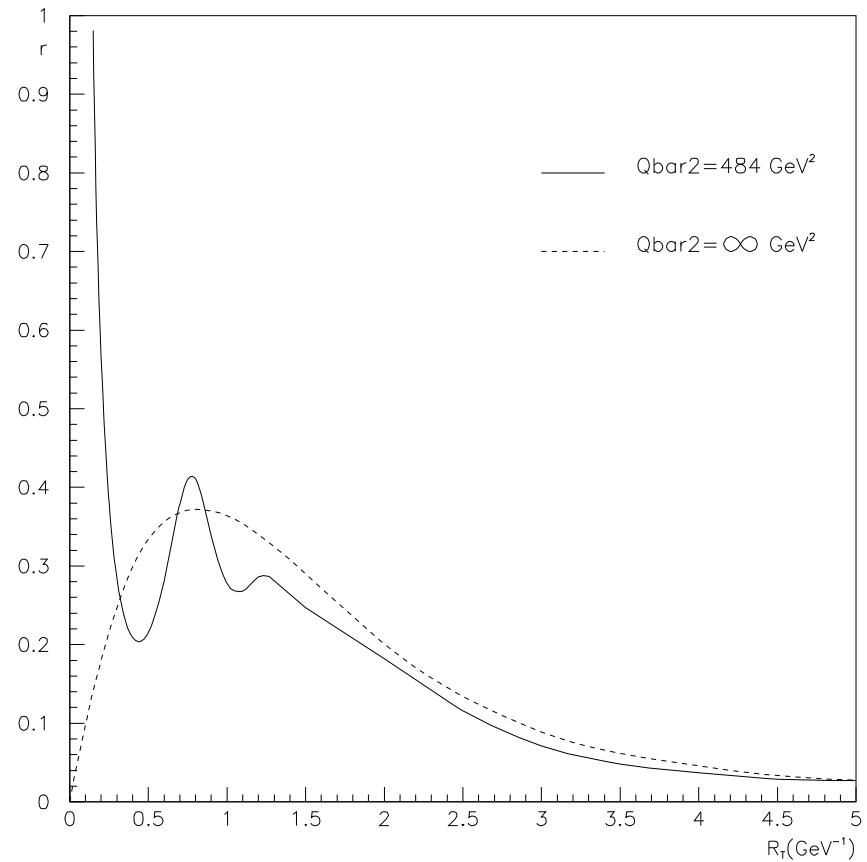
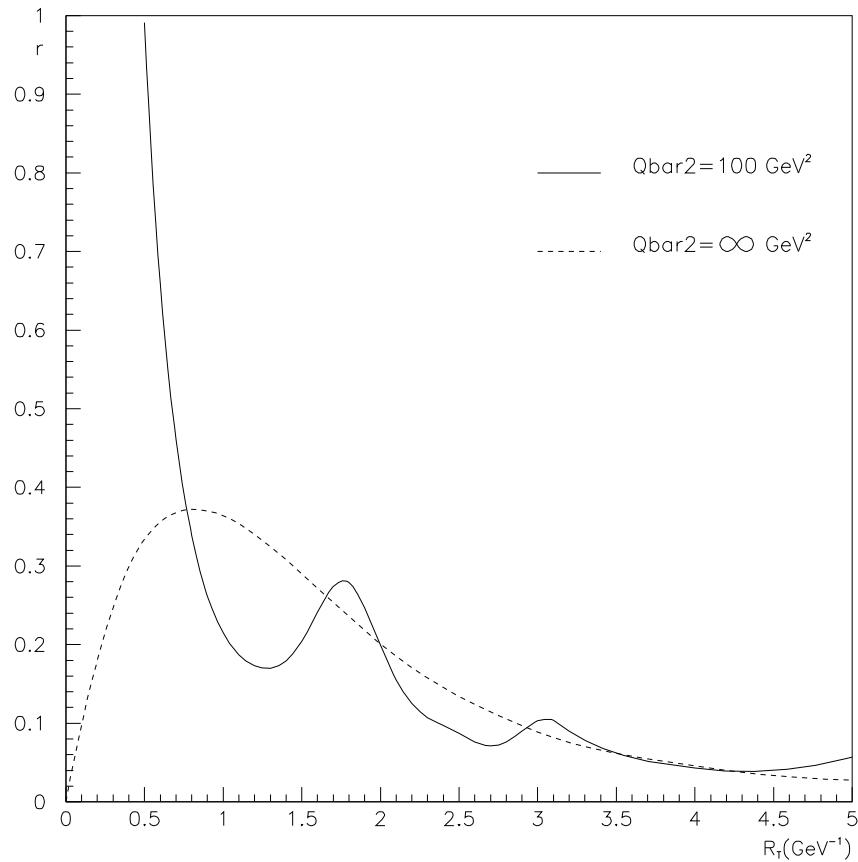
Reason: Transverse probability density:

$$\omega_T^{(q)}(Qr_\perp, Q^2, \bar{Q}^2) \underset{r_\perp \rightarrow 0}{\sim} \begin{cases} \frac{1}{r_\perp^2}, & \text{for } \bar{Q}^2 \rightarrow \infty \\ r_\perp^2 \bar{Q}^4, & \text{for } \bar{Q}^2 \text{ finite} \end{cases}$$

Rise as $\frac{1}{r_\perp^2}$ due to $M_{q\bar{q}}^2 \rightarrow \infty$.

How large is $R(W^2, Q^2)$? Experiment will decide.





Color Transparency

$$\begin{aligned}\sigma_{(q\bar{q})p}(r_\perp^2, W^2) &= \int d^2\vec{l}_\perp \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, W^2)(1 - e^{-i\vec{l}_\perp \cdot \vec{r}_\perp}) \\ &\simeq r_\perp^2 \frac{\pi}{4} \int d\vec{l}_\perp^2 \tilde{\sigma}_{(q\bar{q})p}(\vec{l}_\perp^2, W^2), \quad \text{for } \vec{r}_\perp^2 \rightarrow 0.\end{aligned}$$

Restriction:

$$M'_{q\bar{q}} = \frac{(\vec{k}_\perp + \vec{l}_\perp)^2}{z(1-z)} < m_1^2 = \bar{Q}^2 \ll W^2$$

i. e.

$$\begin{aligned}\vec{l}_\perp^2 &= \frac{\vec{l}_\perp^2}{z(1-z)} \ll W^2 \\ \int_0^\infty d\vec{l}_\perp^2 &\rightarrow \int_0^{z(1-z)\bar{Q}^2} d\vec{l}_\perp^2\end{aligned}$$

Dipole picture in $r'_\perp = \sqrt{z(1-z)}r$ representation

Factorize $z(1-z)$ dependence:

$$\sigma_{\gamma_{L,T}^* p}(W^2, Q^2) = \int d^2 r'_\perp \bar{\omega}_{T,L} \left(r'_\perp Q, Q^2, \frac{Q^2}{\bar{Q}^2} \right) \bar{\sigma}_{(q\bar{q})_{L,T}^{J=1} p}(r'_\perp, W^2)$$

$$\bar{\sigma}_{(q\bar{q})_{L,T}^{J=1} p}(r'^2_\perp, W^2) = \int d^2 l'_\perp \bar{\sigma}_{(q\bar{q})_{L,T}^{J=1} p}(l'^2_\perp, W^2) (1 - e^{-i \vec{l}'_\perp \cdot \vec{r}'_\perp})$$

$$\simeq r'^2_\perp \frac{\pi}{4} \int d\vec{l}'_\perp \vec{l}'_\perp \cdot \vec{l}'_\perp \bar{\sigma}_{(q\bar{q})_{L,T}^{J=1} p}(\vec{l}'_\perp \cdot \vec{l}'_\perp, W^2), \quad \text{for } r'_\perp \rightarrow 0.$$

$$\langle \vec{l}'_\perp \cdot \vec{l}'_\perp \rangle_{eff} \ll W^2$$

$$L : (q\bar{q})_{L,T}^{J=1} = (q\bar{q})_0^{J=1}$$

$$T : (q\bar{q})_{T,+}^{J=1} = (q\bar{q})_+^{J=1} = (q\bar{q})_{T,-}^{J=1} = (q\bar{q})_-^{J=1} = \frac{1}{2} ((q\bar{q})_+^{J=1} + (q\bar{q})_-^{J=1})$$

For $\langle \vec{l}'^2_{\perp} \rangle_{eff} \ll W^2$,

$$Q^2 \gg \langle l'^2_{\perp} \rangle_{eff} = \Lambda_{\text{sat}}^2(W^2)$$

one obtains (with $\bar{Q}^2 \rightarrow \infty$)

$$\begin{aligned} R(W^2, Q^2) &= \frac{\int dy y^3 K_0^2(y)}{\underbrace{\int dy y^3 K_1^2(y)}} \cdot \frac{\int d\vec{l}'^2 \vec{l}'^2_{\perp} \sigma_{(q\bar{q})_L^{J=1} p}(\vec{l}'^2_{\perp}, W^2)}{\int d\vec{l}'^2 \vec{l}'^2_{\perp} \sigma_{(q\bar{q})_T^{J=1} p}(\vec{l}'^2_{\perp}, W^2)} \\ &= \frac{1}{2} \end{aligned}$$

Assume helicity independence: $[(q\bar{q})_L^{J=1}] = [(q\bar{q})_T^{J=1}]$

then: $R(W^2, Q^2) = 0.5 > 0.37$

Thus: **Color transparency** $\langle \vec{l}'^2_{\perp} \rangle_{eff} \ll W^2$

and helicity independence

yields $R(W^2, Q^2) = 0.5$

Experimental Evidence for $m_1^2 \equiv \bar{Q}^2 = 484$ GeV, from $\sigma_{\gamma^* p}(W^2, Q^2)$

Constraint: $M_{q\bar{q}}^2 < m_1^2$;

Cvetic, Schildknecht, Surrow, Tentyukov (2000/2001)

$$\langle \vec{l}_\perp'^2 \rangle = \Lambda_{\text{sat}}^2(W^2) \ll W^2$$

Helicity independence: $\bar{\sigma}_{(q\bar{q})_L^{J=1}} = \bar{\sigma}_{(q\bar{q})_T^{J=1}} = \frac{\sigma^{(\infty)}}{\pi} \delta(\vec{l}_\perp'^2 - \Lambda_{\text{sat}}^2(W^2))$ i.e. $R(W^2, Q^2) = \frac{1}{2}$

$$\Lambda_{\text{sat}}^2(W^2) = B' \left(\frac{W^2}{1\text{GeV}^2} \right)^{C_2^{\text{exp}}}$$

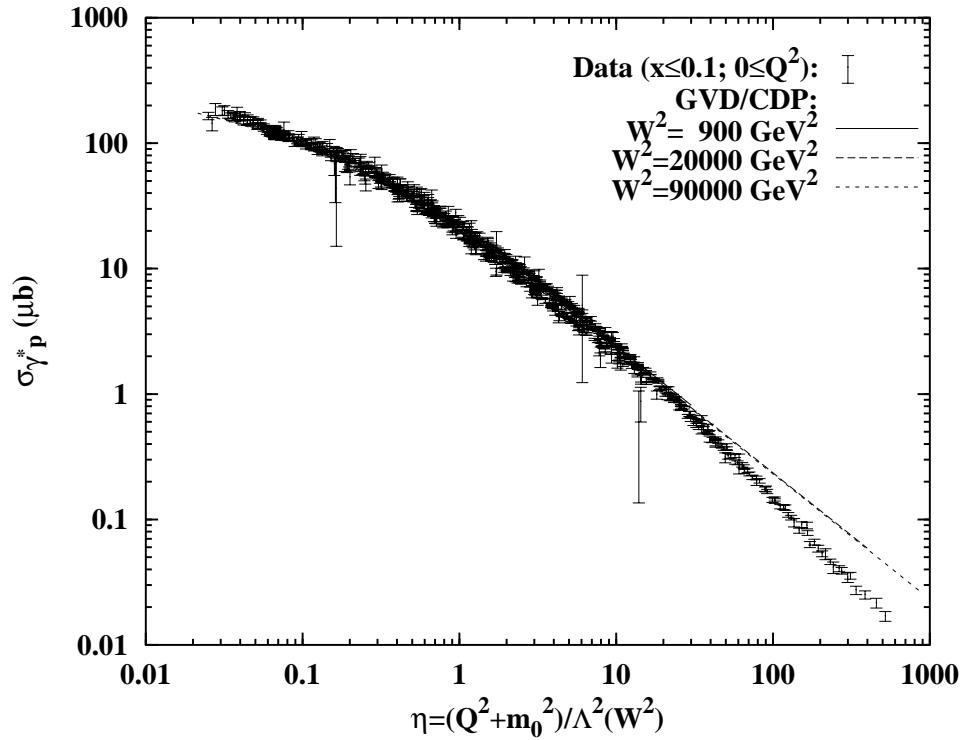
where $\sigma^{(\infty)} = 48\text{GeV}^{-2}$ (four flavors)

$$B' = 0.34 \pm 0.06 \text{GeV}^2$$

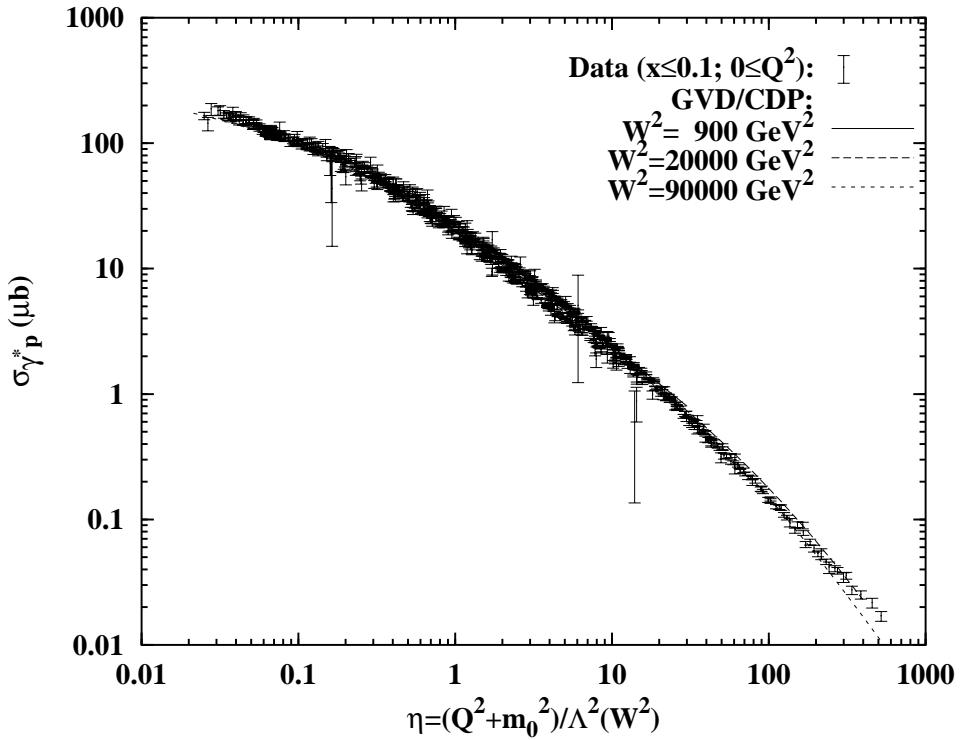
$$C_2^{\text{exp}} = 0.27 \pm 0.01$$

at HERA: $2\text{GeV}^2 \leq \Lambda_{\text{sat}}^2(W^2) \leq 7\text{GeV}^2$

$$\sigma_{\gamma^* p} = \sigma_{\gamma^* p} \left(\eta \left(\frac{Q^2 + m_0^2}{\Lambda_{\text{sat}}^2(W^2)} \right), m_1^2 \right), \quad \begin{aligned} m_0^2 &= 0.15 \pm 0.04 \text{ GeV}^2 \\ m_1^2 &= 484 \text{ GeV}^2 \end{aligned}$$

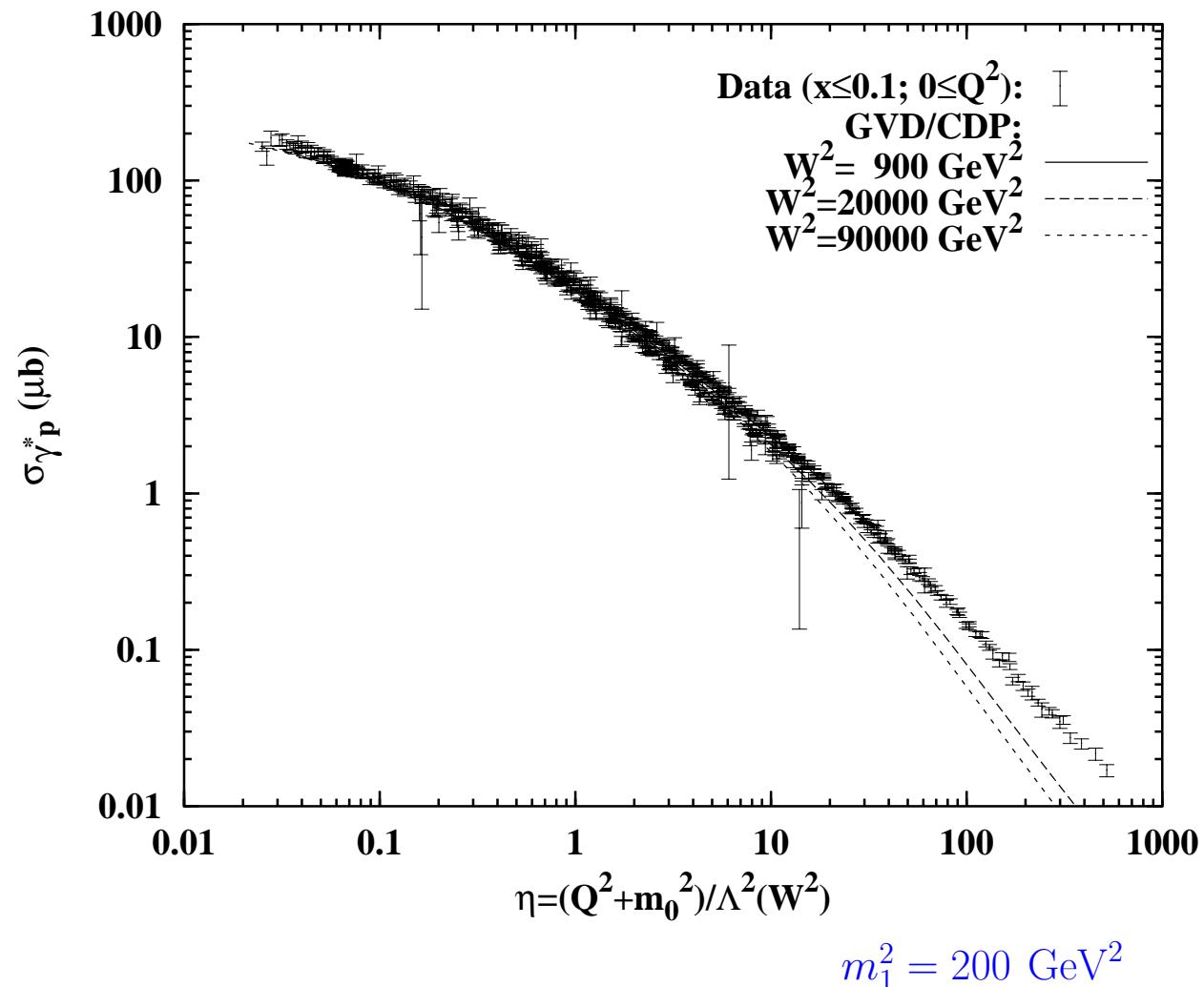


$$m_1^2 = \infty$$



$$m_1^2 = 484 \text{ GeV}^2$$

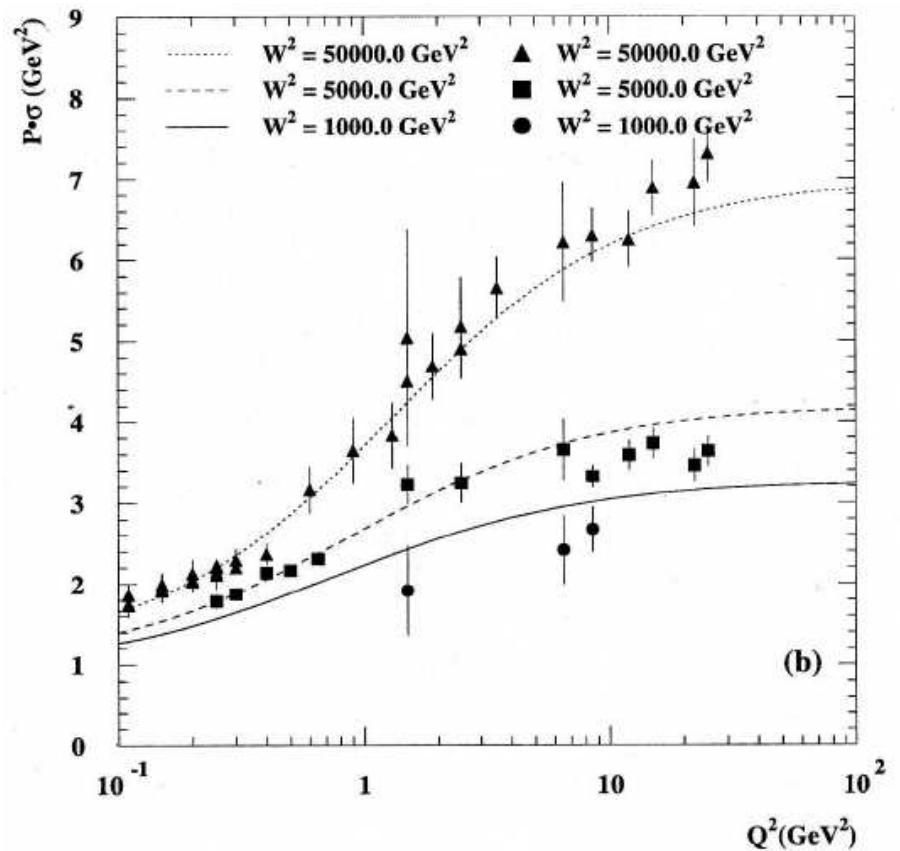
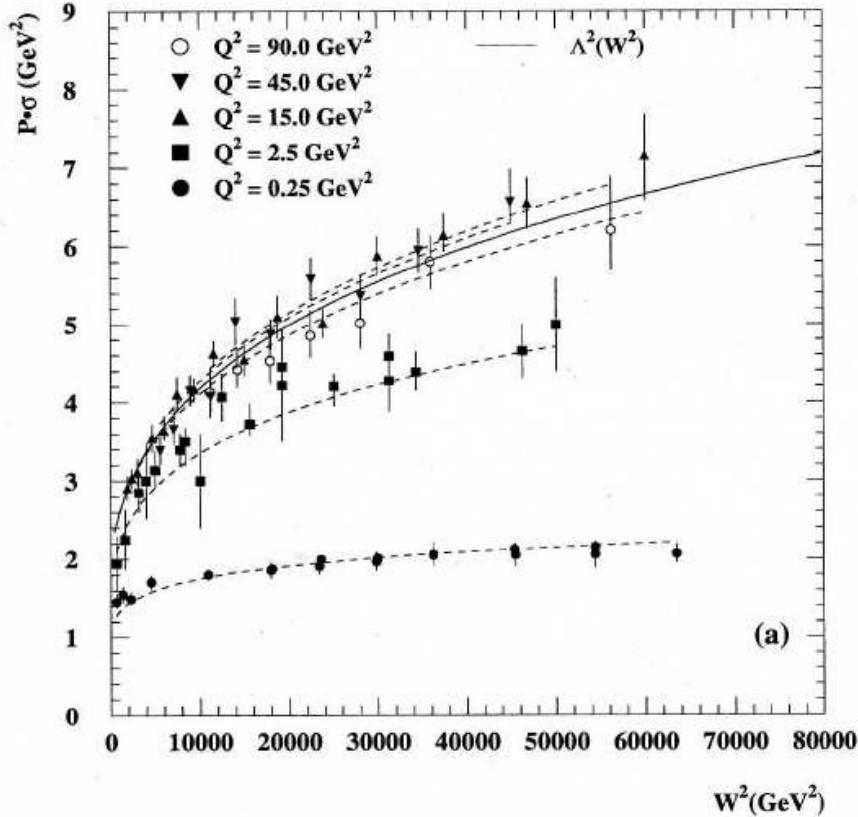
Kuroda and Schildknecht (2002)



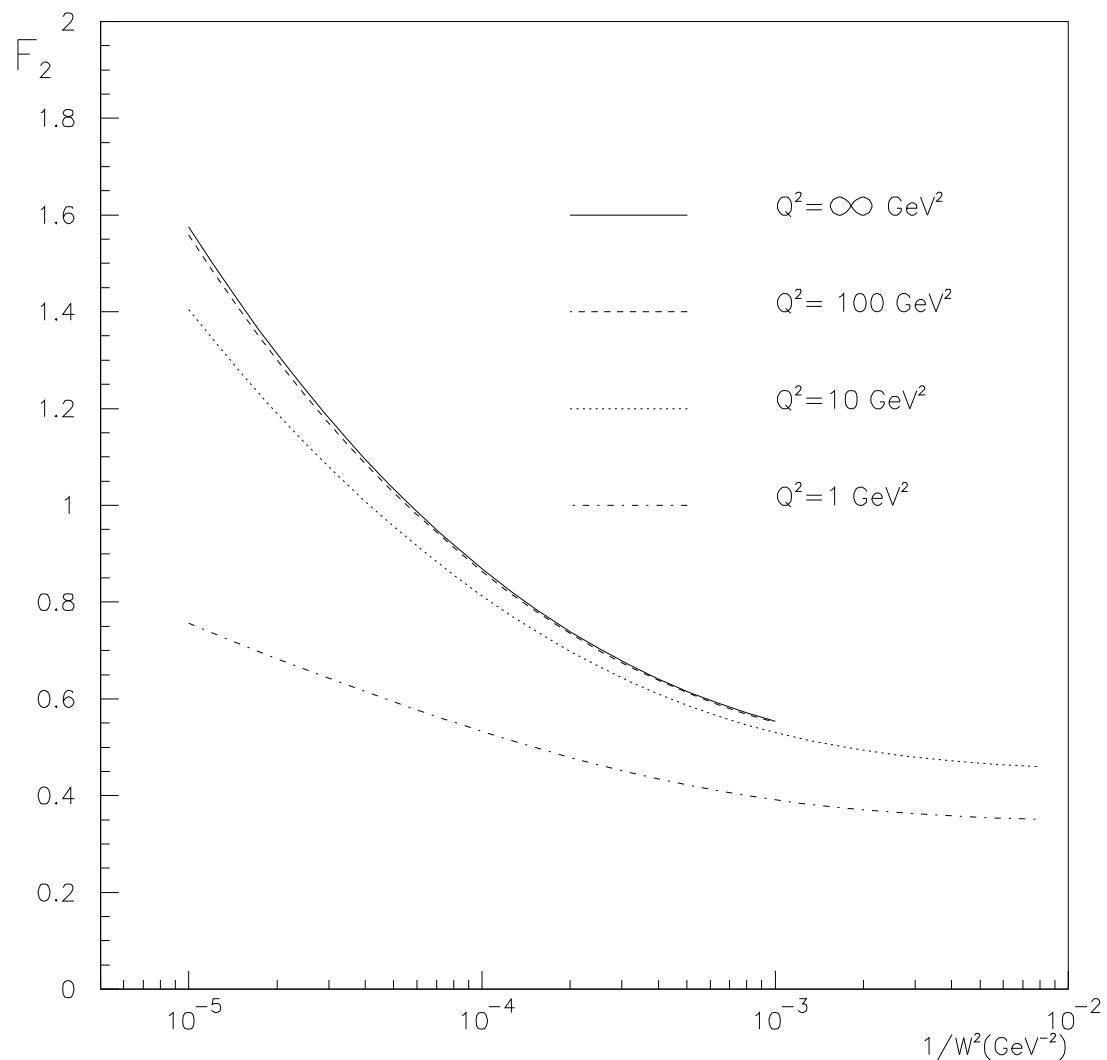
For $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$: $F_2 \sim \Lambda_{\text{sat}}^2(W^2)$

$F_2 \sim P \cdot \sigma$,

$$2 \text{ GeV}^2 \leq \Lambda_{\text{sat}}^2(W^2) \leq 7 \text{ GeV}^2$$



Cvetic et al. (2000/2001)



Generalisation of helicity independence:

$$(1) \quad \bar{\sigma}_{(q\bar{q})_T^{J=1}} = \rho \bar{\sigma}_{(q\bar{q})_L^{J=1}}, \quad 0 < \rho < \infty$$

implies

$$\sigma_{\gamma_T^* p} = 2\rho \sigma_{\gamma_L^* p}$$

For $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$ dual description in terms of

Sea-quark and gluon-distributions

(1) becomes

$$[q\bar{q} - \text{sea}] \quad \text{proportional to} \quad (2\rho + 1) \cdot [\alpha_s(Q^2) \cdot \text{gluon distribution}]$$

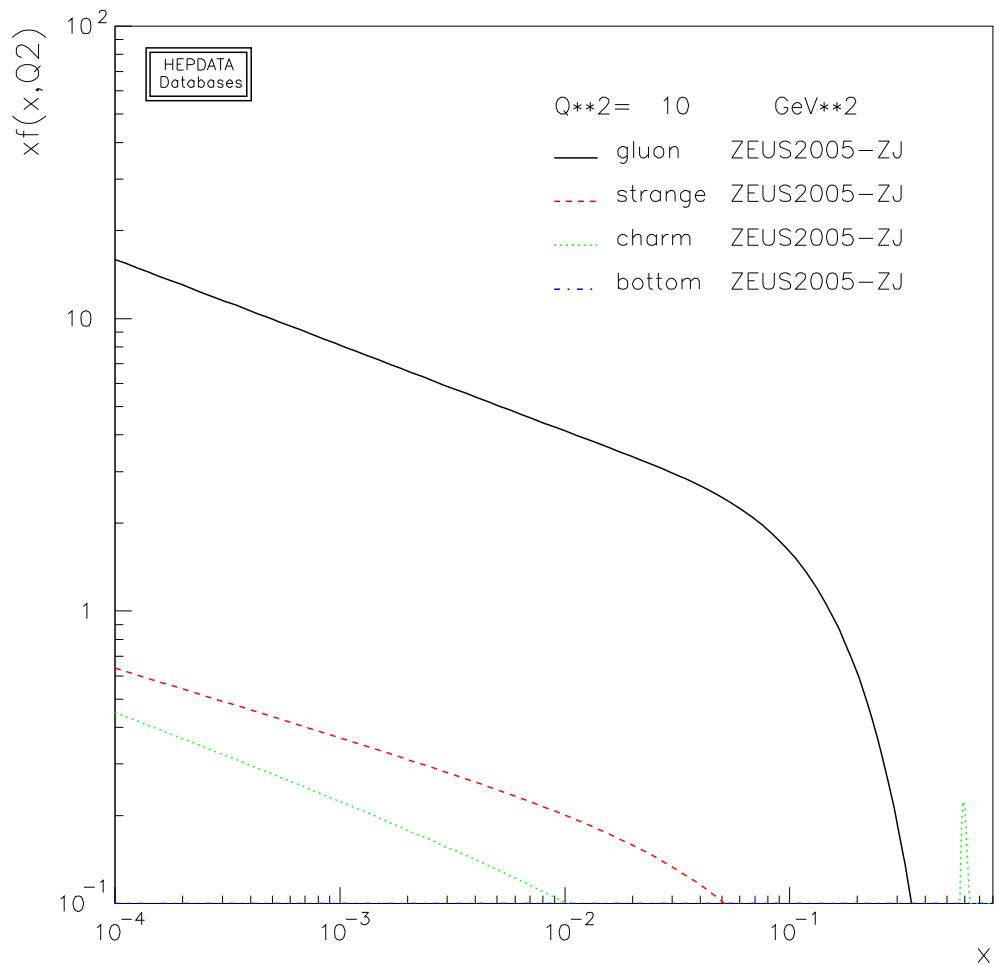
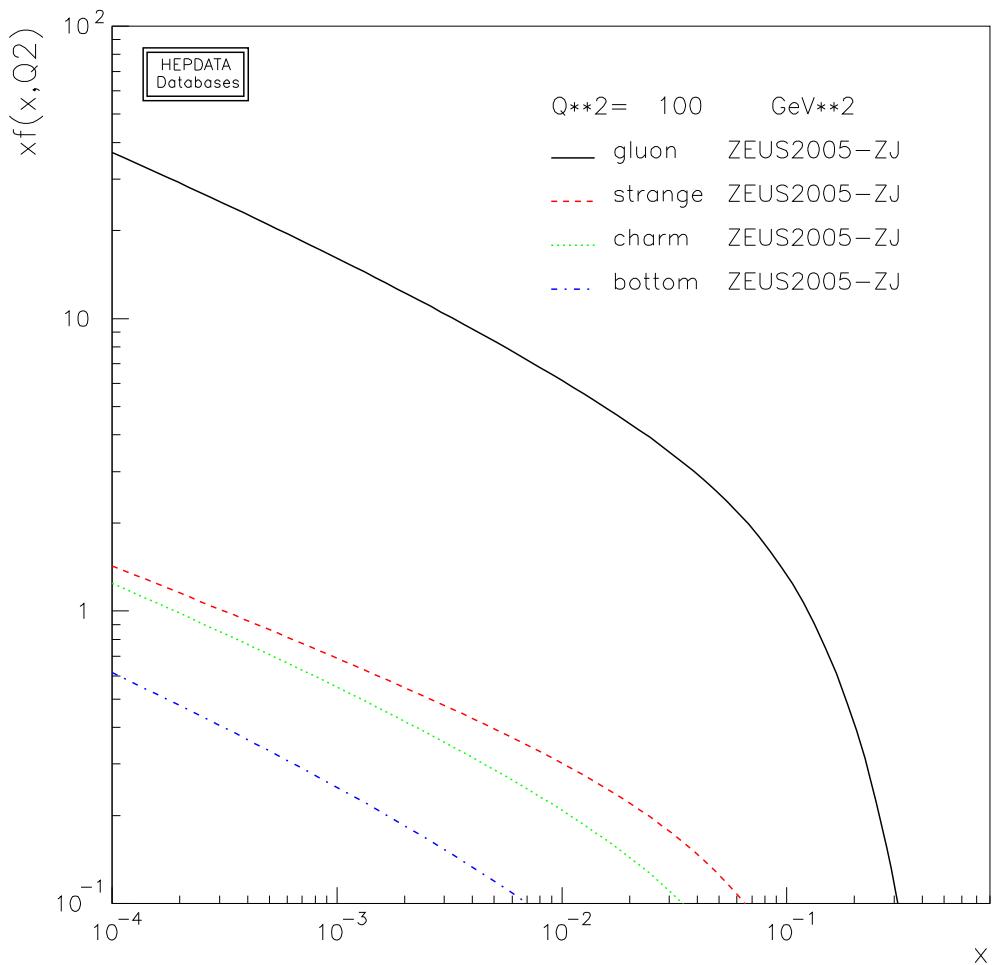
Evolution at low x :

$$\frac{\partial F\left(\frac{x}{2}, Q^2\right)}{\partial \ln Q^2} = \alpha_s(Q^2)x g(x, Q^2); \quad \text{Prytz (1993)}$$

$$(2\rho + 1) \frac{\partial}{\partial \ln W^2} \Lambda_{\text{sat}}^2(2W^2) = \Lambda_{\text{sat}}^2(W^2);$$

$$(2\rho + 1) C_2^{\text{theor.}} 2^{C_2^{\text{theor.}}} = 1$$

Kuroda, Schildknecht (2005)



Results for $C_2^{\text{theor.}}$ for different values of ρ

ρ	$C_2^{\text{theor.}}$	$\alpha_s \cdot \text{glue}$	$\sigma_{\gamma_L^*}/\sigma_{\gamma_T^*}$	$F_2\left(\frac{Q^2}{x}\right)$
$\rightarrow \infty$	0	$\ll \text{sea}$	0	$(Q^2/x)^0 = \text{const.}$
1	0.276	$\approx \text{sea}$	$\sim \frac{1}{2}$	$(Q^2/x)^{0.276}$
0	0.65	$> \text{sea}$	∞	$(Q^2/x)^{0.65}$

Thus:

- For $\rho = 1$, $C_2^{\text{theor.}} \simeq C_2^{\text{exp}} = 0.27 \pm 0.01$ supporting helicity independence
- Essentially only 2 parameters $\sigma^{(\infty)}, B'$
- Correlation between $R(W^2, Q^2)$, relative magnitude of sea to gluon, and Q^2, x dependence for $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$.

Measurement of $R(W^2, Q^2)$ allows one to directly investigate the limits of validity of assumed proportionality of sea and glue, i.e. helicity independence.

J/ψ Production

The r'_\perp representation allows direct transition to vector meson production.

Note: For $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$:

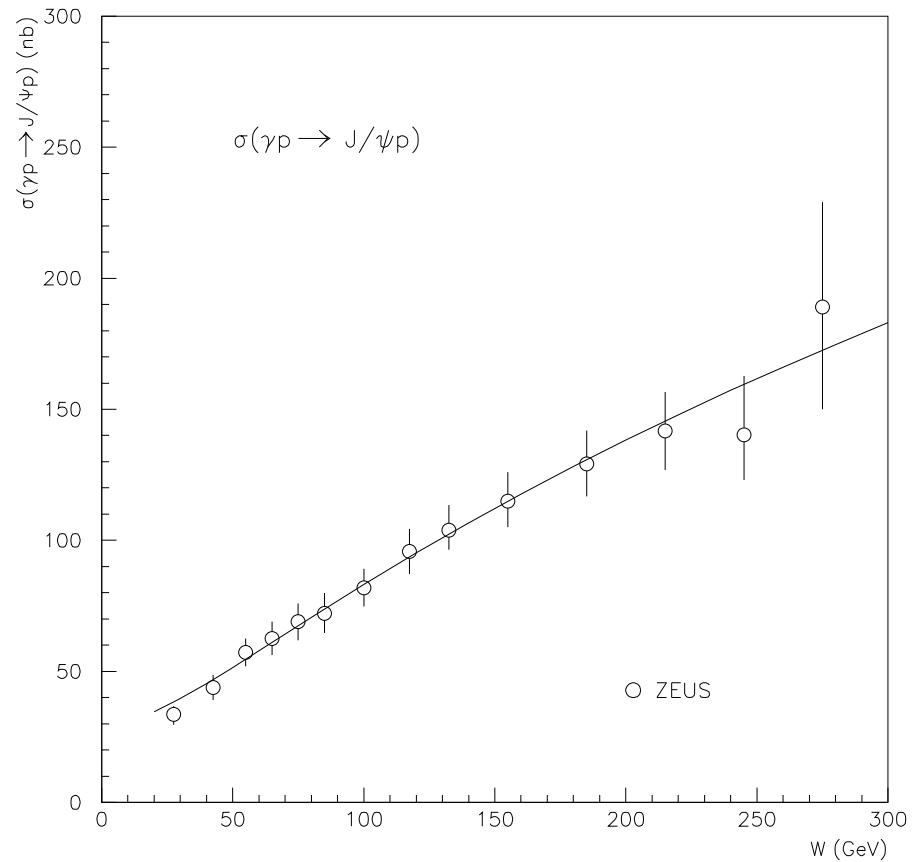
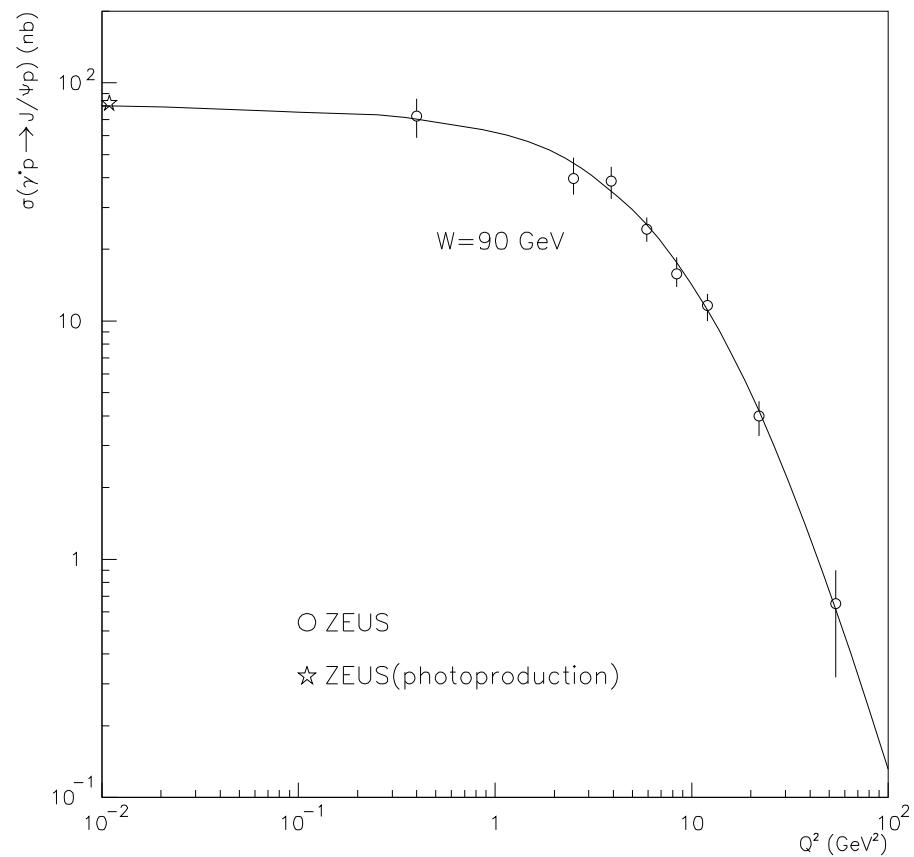
$$\alpha_s(Q^2) x g(x, Q^2) \Big|_{x=\frac{Q^2}{W^2}} = \frac{1}{8\pi^2} \sigma^{(\infty)} \Lambda_{\text{sat}}^2(W^2)$$

Measurement of J/ψ photoproduction ($Q^2 = 0$) yields gluon structure function for

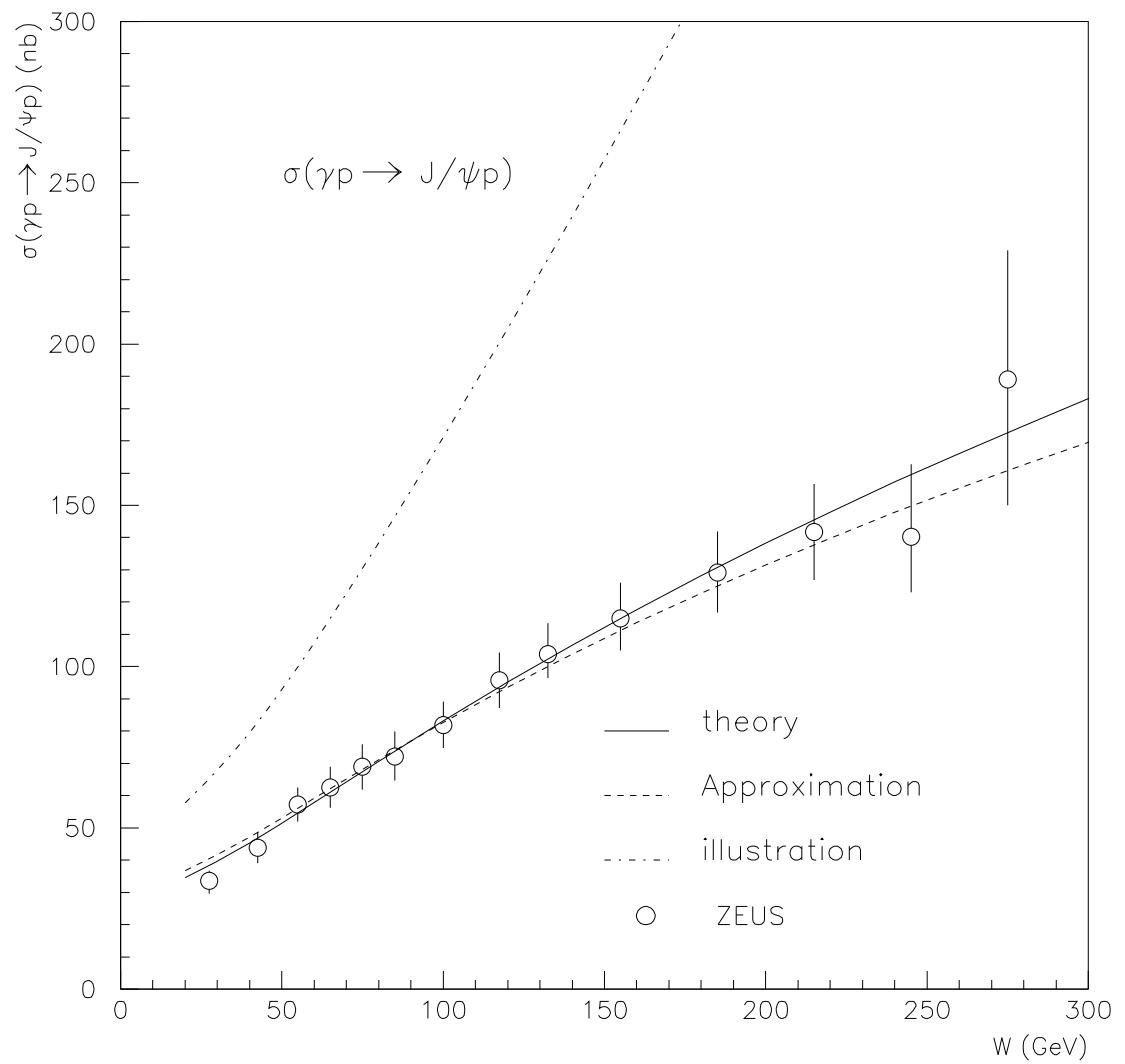
$$Q^2 \gg \Lambda_{\text{sat}}^2(W^2),$$

(HERA: $2 \leq \Lambda_{\text{sat}}^2(W^2) \leq 7 \text{GeV}^2$)

not: for $Q^2 < \Lambda_{\text{sat}}^2(W^2)$.



Kuroda and Schildknecht (2003/2005)



Conclusion

Prediction: $R(W^2, Q^2) = \frac{\sigma_{\gamma L^p}^*(W^2, Q^2)}{\sigma_{\gamma T^p}^*(W^2, Q^2)} \simeq \frac{1}{2}$ for $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$

i.e. $\frac{F_{2L}}{F_2} \simeq \frac{1}{3}$.

(assumes $\langle \vec{l}_\perp'^2 \rangle_{\text{eff}} = \Lambda_{\text{sat}}^2(W^2)$, but $\bar{Q}^2 = m_1^2 = \infty$, calculation for finite \bar{Q}^2 in preparation).

Measurement tests validity of underlying low x concepts:

On-shell $(q\bar{q})p \rightarrow (q\bar{q})p$ scattering,

Dipole cross section depending on W^2 ,

Helicity independence, or equivalently, proportionality of sea quark and gluon distribution

Experimentalists should plot $F_2(Q^2, W^2)$ against $\frac{1}{W^2}$ at fixed Q^2 , testing $F_2 \sim \Lambda_{\text{sat}}^2(W^2)$ for $Q^2 \gg \Lambda_{\text{sat}}^2(W^2)$.