Diffractive Photoproduction of π^0 and the Odderon

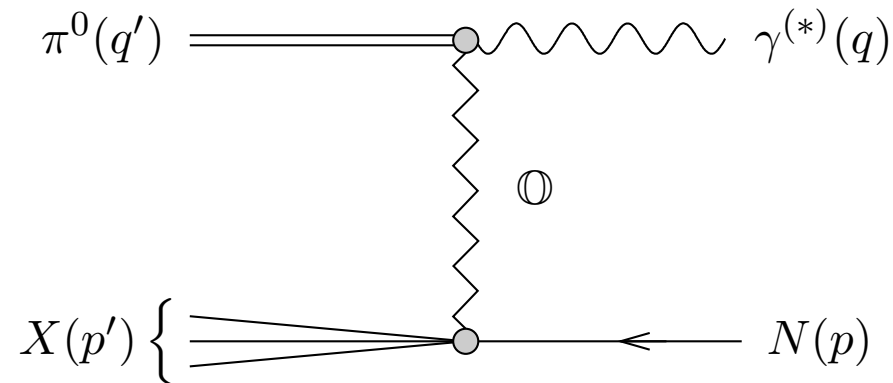**Carlo Ewerz**

ECT* Trento

Diffractive photo- and electroproduction of π^0

- Consider

$$\gamma^{(*)}(q) + N(p) \longrightarrow \pi^0(q') + X(p')$$

  at high energy $\sqrt{s} \gg m_p$.

- Photon has $C = -1$, pion has $C = +1$. Hence at high energies the reaction is due to exchange of an Odderon:

The Odderon

- Consider

$$a + b \longrightarrow a + b$$

and

$$a + \bar{b} \longrightarrow a + \bar{b}$$

with the latter obtained from the former by crossing from s- to u-channel.

Define

$$A_{\pm}(s,t) = \frac{1}{2}\left(A^{ab}(s,t) \pm A^{a\bar{b}}(s,t)\right)$$

A_{+} is unchanged under crossing & has positive C-parity, dominated by Pomeron.

A_{-} changes sign under crossing & has negative C-parity.

Odderon ($\mathbb{O}$) is contribution to A_{-} which does not vanish rapidly at high energy.

Lukaszuk, Nicolescu

Experimental evidence for the Odderon

- QCD (or at least pQCD) predicts existence of the Odderon: 3-gluon exchange

- only experimental evidence for Odderon so far is difference in differential cross sections for pp and $p\bar{p}$ elastic scattering at ISR:
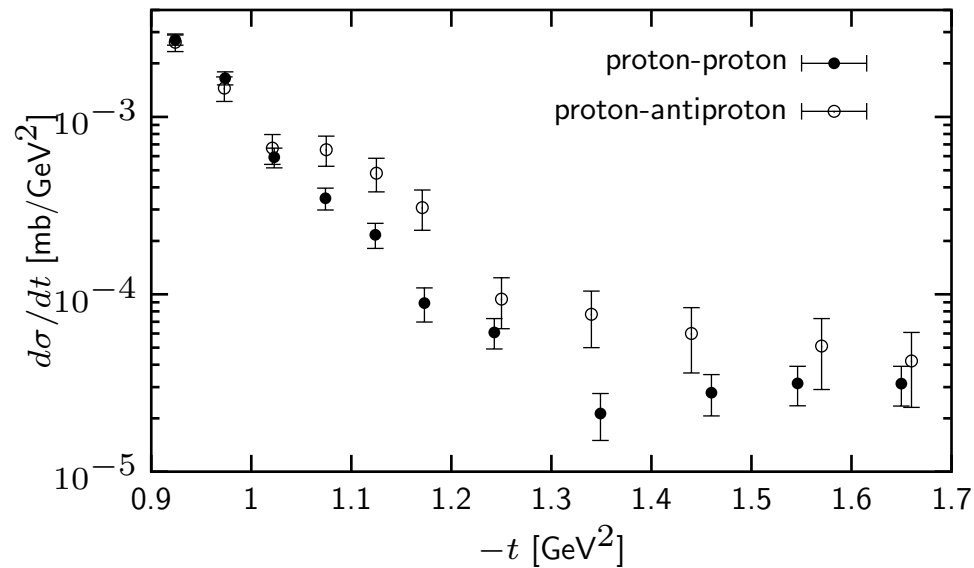
Odderon search in exclusive processes

- Problem: in elastic pp and $p\bar{p}$ scattering Odderon is only one of many contributions, difficult to pin down

- Better strategy: look for processes in which Odderon is only contribution (maybe besides photon)
  $\longrightarrow$ exclusive processes!

- Good process for Odderon search should be diffractive pion production.

  Schäfer, Mankiewicz, Nachtmann
  Barakhovsky, Zhitnitsky, Shelkovenko
  Kilian, Nachtmann

(Early) Expectation for diffractive neutral pion production...

- Note that single-pion production has largest kinematical phase space of all reactions in which hadrons are diffractively produced.

- (naive) prediction based on model of nonperturbative QCD dynamics:
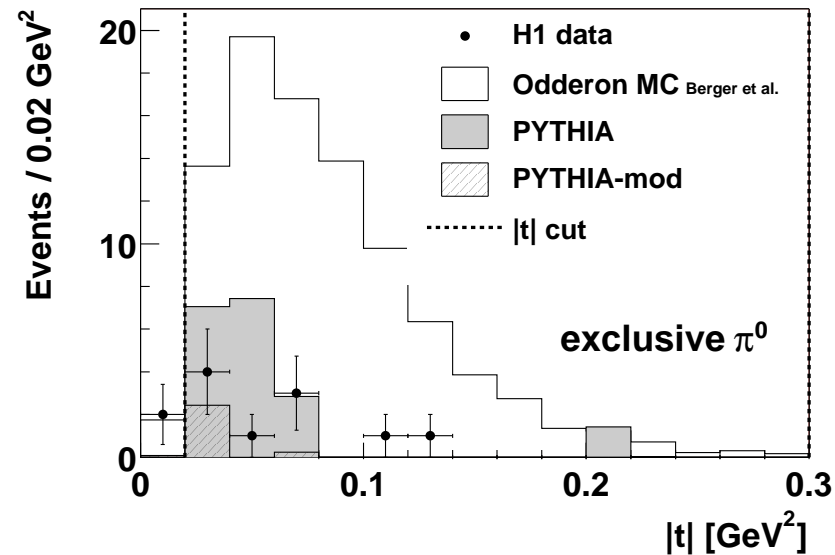
$$\sigma(\gamma p \to \pi^0 X) \approx 300 \, \text{nb}$$

at $\sqrt{s} = 20$ GeV, with weak energy dependence expected.
Large uncertainty due to nonpert. model: $\sim$ factor 2

Berger, Donnachie, Dosch, Kilian, Nachtmann, Rueter

... and experimental nonobservation

- H1 does not see a signal

- Resulting upper bound at 95 % CL: $\sigma(\gamma p \to \pi^0 N^*) < 49\,\mathrm{nb}$

- Striking result in view of large phase space! There must be a dynamical mechanism for this.

Possible reasons for failure of prediction

Dosch, Donnachie, Nachtmann

- low nonperturbative Odderon intercept

- small coupling of Odderon to proton (sensitive to proton structure!) - but this should not apply in break-up reaction

- nonperturbative model good for Pomeron but not for Odderon?!

- small coupling of Odderon in $\gamma \rightarrow \pi^0$ transition due to chiral symmetry

CE, Nachtmann
  $\rightarrow$ this talk!

$$\gamma^{(*)}p \to \pi^0 p \quad \textbf{and} \quad \gamma^{(*)}p \to A^3 p$$

- Amplitude $\mathcal{M}^{\nu}_{s's}(\pi^0; q', p, q)$ for

$$\gamma^{(*)}(q, \nu) + p(p, s) \longrightarrow \pi^0(q') + p(p', s')$$

satisfies with LSZ reduction formula

$$(2\pi)^4 \delta^{(4)}(p' + q' - p - q) \, \mathcal{M}^{\nu}_{s's}(\pi^0; q', p, q)$$

$$= -i \int d^4x' \, d^4x \, e^{iq'x'} e^{-iqx} \left(\Box_{x'} + m_\pi^2 \right) \langle p(p', s') | \mathrm{T}^* \phi^3(x') J^\nu(x) | p(p, s) \rangle$$

where ϕ^3 is neutral pion interpolating field operator,
$J^\nu(x) = \bar\psi(x) \gamma^\nu \mathbf{Q} \psi(x)$ is hadronic part of e. m. current,
ψ is quark field operator, and $\mathbf{Q}$ quark charge matrix.

- Consider further reaction

$$\gamma^{(*)}(q,\nu) + p(p,s) \longrightarrow A^3(q',\mu) + p(p',s')$$

with isotriplet of axial vector currents

$$A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\mathbf{T}_a\psi(x) \qquad (a=1,2,3)$$

where

$$\mathbf{T}_a = \left(\begin{array}{c|c} \frac{1}{2}\tau_a & 0 \\ \hline 0 & 0 \end{array}\right)$$

Amplitude $\mathcal{M}_{s's}^{\mu\nu}(A^3; q', p, q)$ is defined by

$$(2\pi)^4\delta^{(4)}(p'+q'-p-q)\,\mathcal{M}_{s's}^{\mu\nu}(A^3; q', p, q)$$

$$= \frac{i}{2\pi m_p}\int d^4x'\,d^4x\,e^{iq'x'}e^{-iqx}\langle p(p',s')|\,\mathrm{T}^*A^{3\mu}(x')J^\nu(x)|p(p,s)\rangle$$

PCAC

- Using PCAC (partially conserved axial vector current) relation

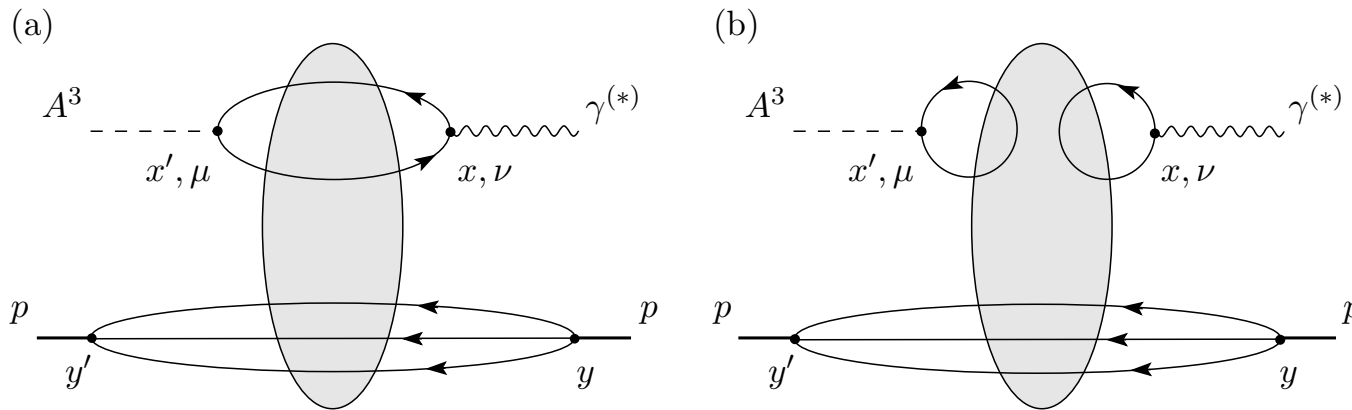$$\partial_\lambda A^{a\lambda}(x) = \frac{f_\pi m_\pi^2}{\sqrt{2}}\,\phi^a(x)$$

and integration by parts we can relate the amplitudes for the two reactions to each other:

$$\mathcal{M}_{s's}^\nu(\pi^0; q', p, q) = \frac{2\pi m_p \sqrt{2}}{f_\pi m_\pi^2}\left(-q'^2 + m_\pi^2\right) i q'_\mu \mathcal{M}_{s's}^{\mu\nu}(A^3; q', p, q)$$

Note divergence factor on r.h.s.

Leading diagrams at high energy

- At high energy the leading nonperturbative diagrams are pure gluon exchanges:

Blobs represent functional integration over gluon field configurations.

- We have

$$
\begin{aligned}
\mathcal{M}^{(a)\mu\nu}_{s's}(A^3; q', p, q) &= \langle U_{s's}(p', p)\, A^{\mu\nu}(q', q) \rangle_G \,, \\
\mathcal{M}^{(b)\mu\nu}_{s's}(A^3; q', p, q) &= \langle U_{s's}(p', p)\, \tilde{B}^\mu(q')\, B^\nu(q) \rangle_G \,.
\end{aligned}
$$

$U_{s's}$ represents scattering of proton on gluon field.

- Consider only part (a) here: ((b) can be treated similarly)

$$A^{\mu\nu}(q',q) = \int d^4x'\,d^4x\,e^{iq'x'}e^{-iqx}\mathrm{Tr}\left[\gamma^\mu\gamma_5\mathbf{T}_3\mathbf{S}_F(x',x;G)\gamma^\nu\mathbf{Q}\,\mathbf{S}_F(x,x';G)\right]$$

with $\mathbf{S}_F(x,x';G) = \mathrm{diag}\left(S_F^{(u)}(x,x';G), S_F^{(d)}(x,x';G),\ldots, S_F^{(t)}(x,x';G)\right)$

- Divergence of this gives with Dirac equation for S_F

$$\begin{aligned}q'_\mu A^{\mu\nu}(q',q) &= -\frac{1}{3}\int d^4x'\,d^4x\,e^{iq'x'}e^{-iqx}\\ &\quad\left\{2\,m_u^{(0)}E^\nu(x,x';G,m_u^{(0)}) + m_d^{(0)}\,E^\nu(x,x';G,m_d^{(0)})\right\}\end{aligned}$$

with

$$E^\nu(x,x';G,m_q^{(0)}) = \mathrm{Tr}\left[S_F^{(q)}(x,x';G,m_q^{(0)})\gamma_5 S_F^{(q)}(x',x;G,m_q^{(0)})\gamma^\nu\right]$$

- With Lippmann-Schwinger equation for S_F one finds to all orders that the trace in $E^\nu(x, x'; G, m_q^{(0)})$ contains at least one factor of $m_q^{(0)}$,

$$E^\nu(x, x'; G, m_q^{(0)}) = m_q^{(0)} E'^\nu(x, x'; G, 0) + \mathcal{O}\left((m_q^{(0)})^2\right)$$

and E'^ν contains only massless propagators.

⋆ Similar observation made in perturbation theory for process $\gamma^{(*)} + p \rightarrow \eta_c + p$ for arbitrary number of gluons in t-channel: quark loop with γ_5 is $\sim m_c$.

Braunewell, CE

- With the quark mass factors from the Dirac equation hence:

$$
\begin{aligned}
q'_\mu A^{\mu\nu}(q', q) \;=\; & -\frac{1}{3} \int d^4x' \, d^4x \, e^{iq'x'} e^{-iqx} \\
& \left[2(m_u^{(0)})^2 E'^\nu(x, x'; G, m_u^{(0)}) + (m_d^{(0)})^2 E'^\nu(x, x'; G, m_d^{(0)}) \right] \\
& + \mathcal{O}\left((m_u^{(0)})^3, (m_d^{(0)})^3\right)
\end{aligned}
$$

No anomaly contribution

- Divergence of axial vector current in external gluon field in general gives anomalous contribution,

$$\partial_\mu \bar{q}(x)\gamma^\mu\gamma_5 q(x) = 2im_q^{(0)}\bar{q}(x)\gamma_5 q(x) + \frac{(g^{(0)})^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}G^{\mu\nu}(x)G^{\rho\sigma}(x)$$

But anomalous gluonic part is independent of quark mass.

- Hence anomalous piece cancels in divergence of A^3, because

$$A_\mu^3(x) = \frac{1}{2}\bar{u}(x)\gamma_\mu\gamma_5 u(x) - \frac{1}{2}\bar{d}(x)\gamma_\mu\gamma_5 d(x)$$

Renormalisation

- Going from bare to renormalised quantities we have

$$m_q^R = m_q^{(0)} Z_{mq}^{-1} \qquad (q = u, d)$$

and

$$
\begin{aligned}
(\bar{q}(x)q(x))^R &= Z_{mq}\,\bar{q}(x)q(x), \\
(\bar{q}(x)\gamma_5 q(x))^R &= Z_{mq}\,\bar{q}(x)\gamma_5 q(x) \qquad (q = u, d)\,.
\end{aligned}
$$

- After renormalisation we find similar dependencies on then renormalised quark masses, now for contributions (a)+ (b):

$$q'_\mu \mathcal{M}_{s's}^{(a+b)\mu\nu}(A^3; q', p, q) = (m_u^R)^2\, \mathcal{C}_{s's}^{(u)\nu}(q', p, q) - (m_d^R)^2\, \mathcal{C}_{s's}^{(d)\nu}(q', p, q)$$

where $\mathcal{C}^{(q)\nu}$ have pion pole but are otherwise finite.

Results

- Average quark masses $\hat{m}$ are related to pion mass via

$$\hat{m} \equiv \frac{1}{2}(m_u^R + m_d^R) = \frac{1}{2B}\, m_\pi^2$$

with

$$B = -\frac{2}{f_\pi^2}\langle 0|\, (\bar{u}(x)u(x))^R\, |0\rangle$$

Note that m_p, pion decay constant f_π, and B are finite in the chiral limit $m_{u,r}^R \to 0$.

- We further define

$$r_u = \frac{m_u^R}{\hat{m}} \cong 0.6\,, \qquad r_d = \frac{m_d^R}{\hat{m}} \cong 1.4$$
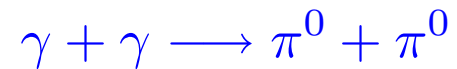
- In summary, we have

$$\mathcal{M}^{(a+b)\nu}_{s's}(\pi^0; q', p, q) = m_\pi^2 \frac{\pi m_p}{f_\pi B^2 \sqrt{2}} \left[-r_u^2\, i(q'^2 - m_\pi^2)\, \mathcal{C}^{(u)\nu}_{s's}(q', p, q) \right.$$

$$\left. + r_d^2\, i(q'^2 - m_\pi^2)\, \mathcal{C}^{(d)\nu}_{s's}(q', p, q) \right]$$

- The factors $(q'^2 - m_\pi^2)$ are cancelled by the pion poles in $\mathcal{C}^{(q)\nu}$.

- Our main result is that the
  amplitude for diffractive neutral pion production vanishes $\sim m_\pi^2$ in the chiral limit.

- We still expect strong suppression for approximate chiral symmetry as realised in Nature .

- A rough numerical estimate suggests that this implies a suppression by a factor ~ 50 from m_π^4 for the cross section, explaining the discrepancy of the early prediction with data.

- Similar considerations hold for breakup reaction

$$\gamma^{(*)} + p \to \pi^0 + n + \pi^+$$
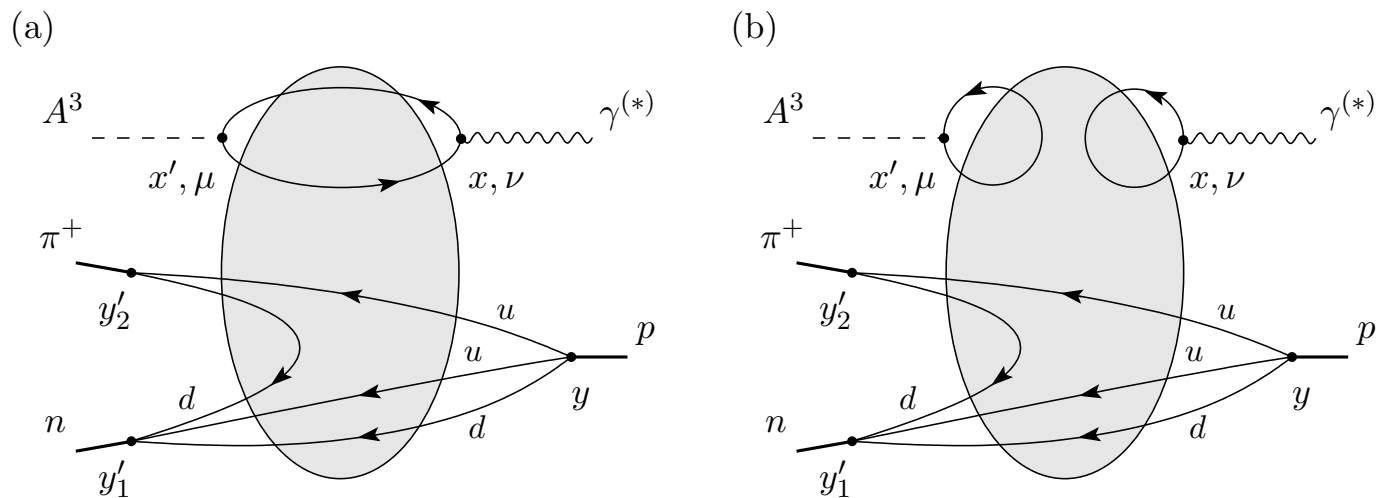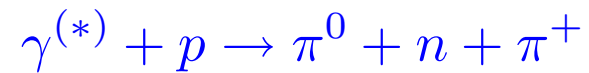
- An even stronger suppression, $\sigma \sim m_\pi^8$, is expected in the diffractive process

$$\gamma + \gamma \longrightarrow \pi^0 + \pi^0$$

at LHC or at a future ILC.