



Dimensional Reduction applied to QCD at three loops

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work in collaboration with

D.R.T. Jones, P. Kant, L. Mihaila, M. Steinhauser

Motivation

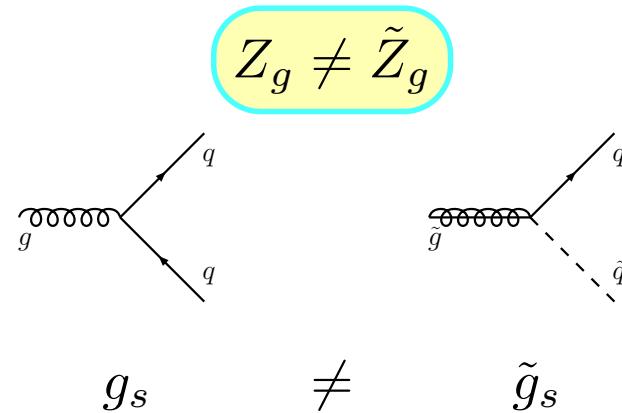
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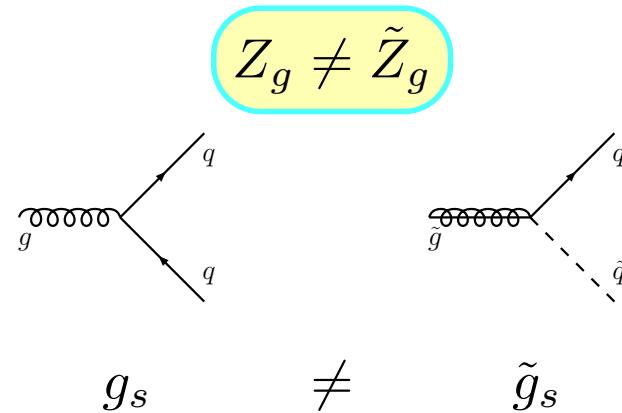
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- reason:

$$N_{\text{spin}1} = D, \quad N_{\text{spin}1/2} = 2^{D/2}$$

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i.e.

$$Z_g = \tilde{Z}_g$$

- but: restricted algebraic operations (inconsistencies with $\epsilon_{\mu\nu\rho\sigma}$)
[Siegel 80][Stöckinger 05]
→ no Fierz transformation

Dimensional Reduction

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- 4-vector v_μ :

$$\hat{v}_\mu = \hat{g}_{\mu\nu} v^\nu , \quad \tilde{v}_\mu = \tilde{g}_{\mu\nu} v^\nu ,$$

$$v_\mu = \hat{v}_\mu + \tilde{v}_\mu$$

Dimensional Reduction

$$A_\mu(x) = \hat{A}_\mu(x) + \tilde{A}_\mu(x)$$

$$\mathcal{L}(A_\mu, \psi, \dots) = \hat{\mathcal{L}}(\hat{A}, \psi, \dots) + \tilde{\mathcal{L}}(\hat{A}_\mu, \tilde{A}_\mu, \psi, \dots)$$

- $\tilde{A}_\mu(x)$: “epsilon scalar”

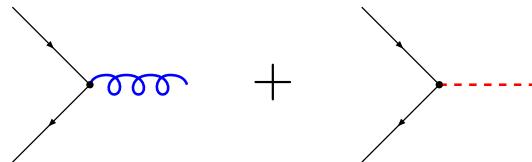
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- $\tilde{A}_\mu(x)$: “epsilon scalar”
- example:

$$A_\mu \bar{\psi} \psi = \hat{A}_\mu \bar{\psi} \psi + \tilde{A}_\mu \bar{\psi} \psi$$



→ additional Feynman rules for epsilon scalars

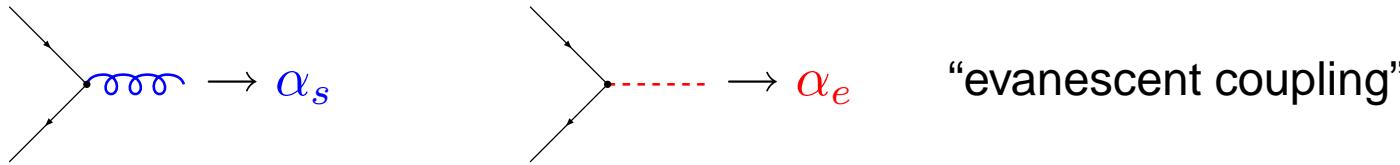
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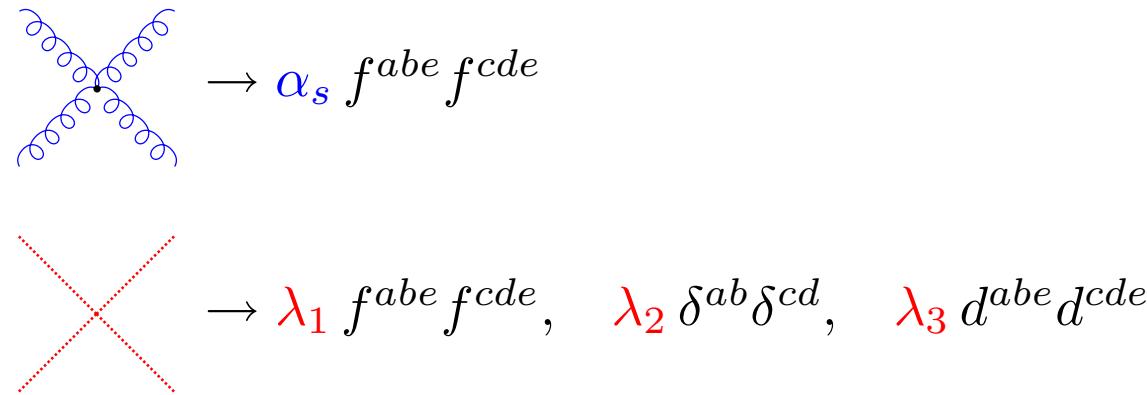
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“evanescent coupling”

even worse:

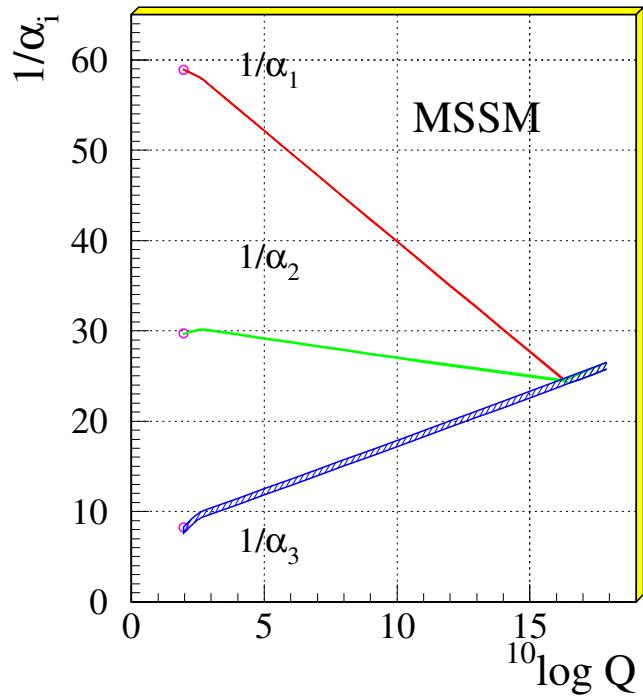


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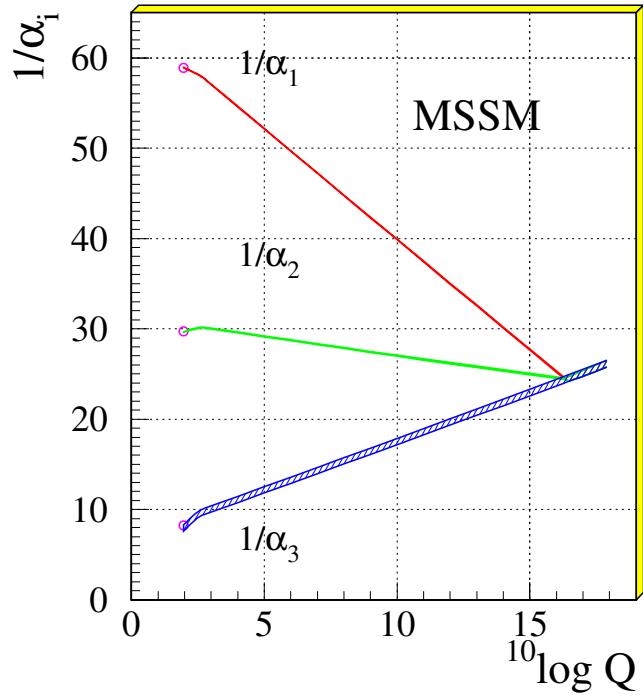
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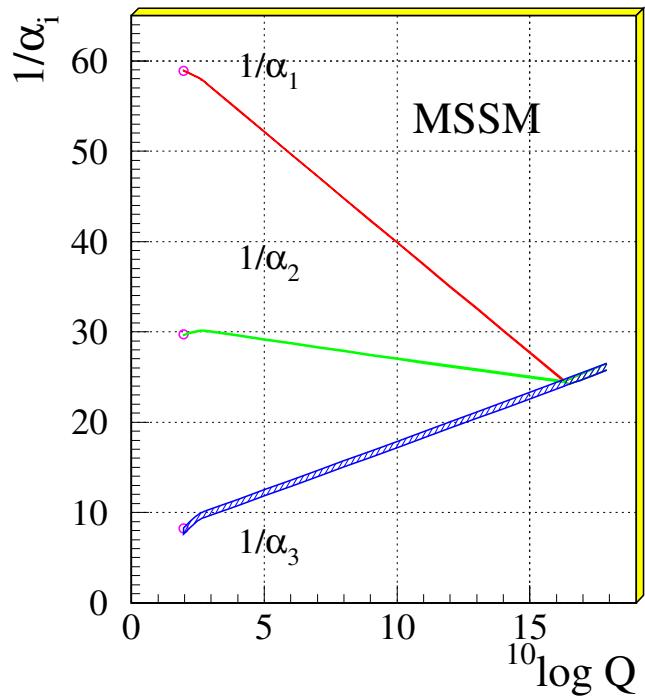
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$$\alpha_s(M_{\text{GUT}}) \equiv \alpha_s^{(\text{full}), \overline{\text{DR}}}(M_{\text{GUT}})$$

SUSY theory

$$\alpha_s^{(5)}(M_Z) \rightarrow \alpha_s(M_{\text{GUT}})$$

- Example:

$$\alpha_s^{(5),\overline{\text{MS}}}(M_Z) \rightarrow \alpha_s^{(5),\overline{\text{MS}}}(M_{\text{SUSY}})$$

— QCD running in $\overline{\text{MS}}$

$$\rightarrow \alpha_s^{(5),\overline{\text{DR}}}(M_{\text{SUSY}})$$

— $\overline{\text{MS}} - \overline{\text{DR}}$ conversion

$$\rightarrow \alpha_s^{(\text{full}),\overline{\text{DR}}}(M_{\text{SUSY}})$$

— matching (scenario D)

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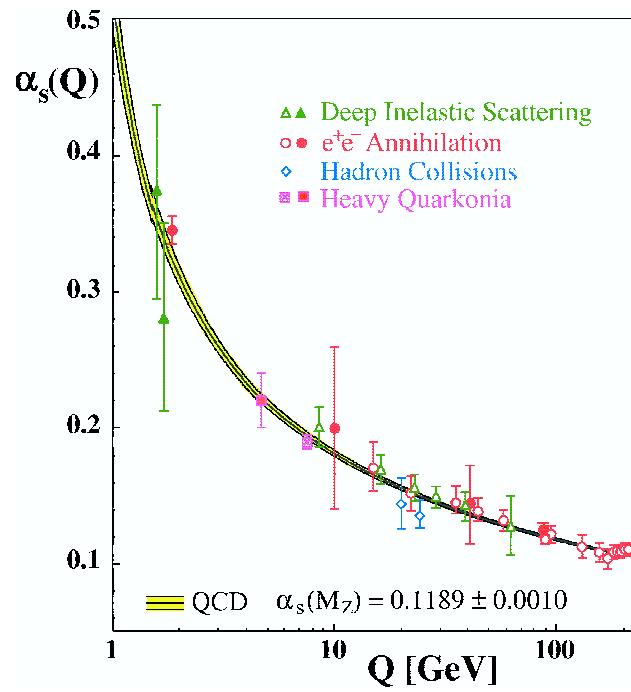
$\overline{\text{MS}}$ Running in QCD

$$\alpha_s^{(5),\overline{\text{MS}}}(M_Z) \rightarrow \alpha_s^{(5),\overline{\text{MS}}}(M_{\text{SUSY}})$$

$$\mu^2 \frac{d}{d\mu^2} \alpha_s^{\overline{\text{MS}}} = \beta_s^{\overline{\text{MS}}}(\alpha_s^{\overline{\text{MS}}})$$

$\beta_s^{\overline{\text{MS}}}$ known to 4 loops

[v. Ritbergen, Larin, Vermaseren 97]
[Czakon 04]



[from

Bethke '06]

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$\overline{\text{DR}}$ *Running in SUSY*

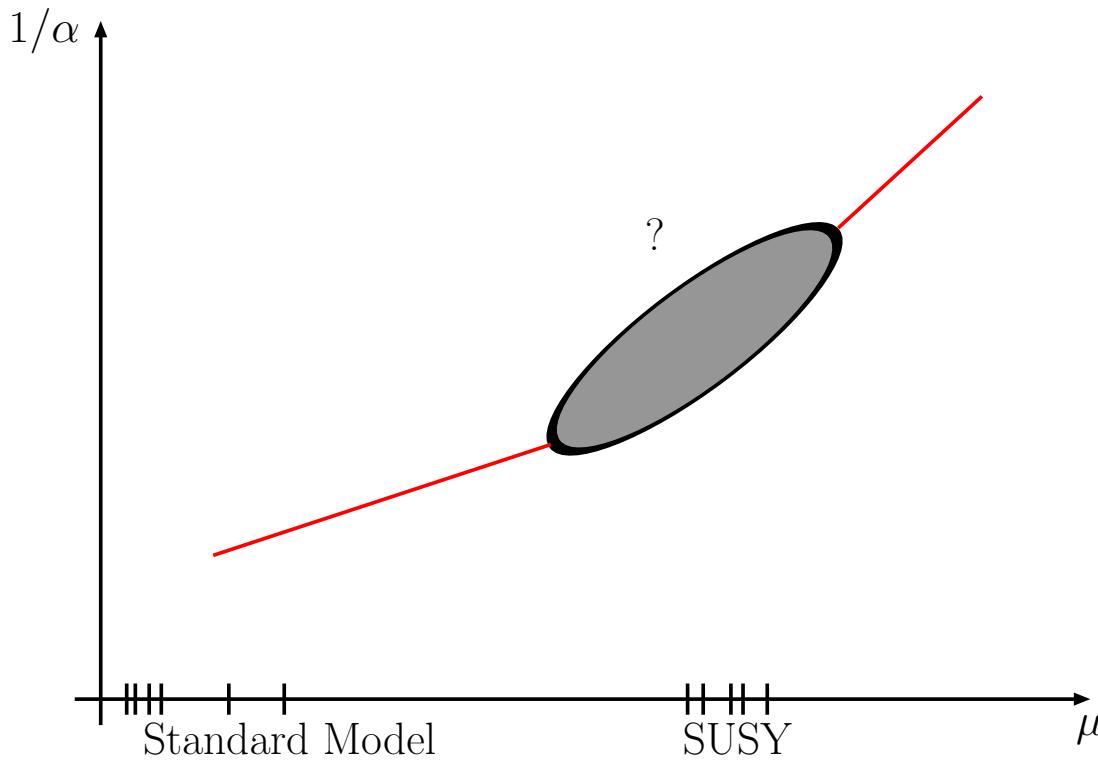
$$\alpha_s^{(\text{full}), \overline{\text{DR}}} (M_{\text{SUSY}}) \rightarrow \alpha_s^{(\text{full}), \overline{\text{DR}}} (M_{\text{GUT}})$$

$$\mu^2 \frac{d}{d\mu^2} \alpha_s^{\overline{\text{DR}}} = \beta_s^{\overline{\text{DR}}} (\alpha_s^{\overline{\text{DR}}})$$

$\beta_s^{\overline{\text{DR}}}$ known to 3 loops

[Jack, Jones, North 96]

$$\alpha_s(M_Z) \rightarrow \alpha_s(M_{\text{GUT}})$$

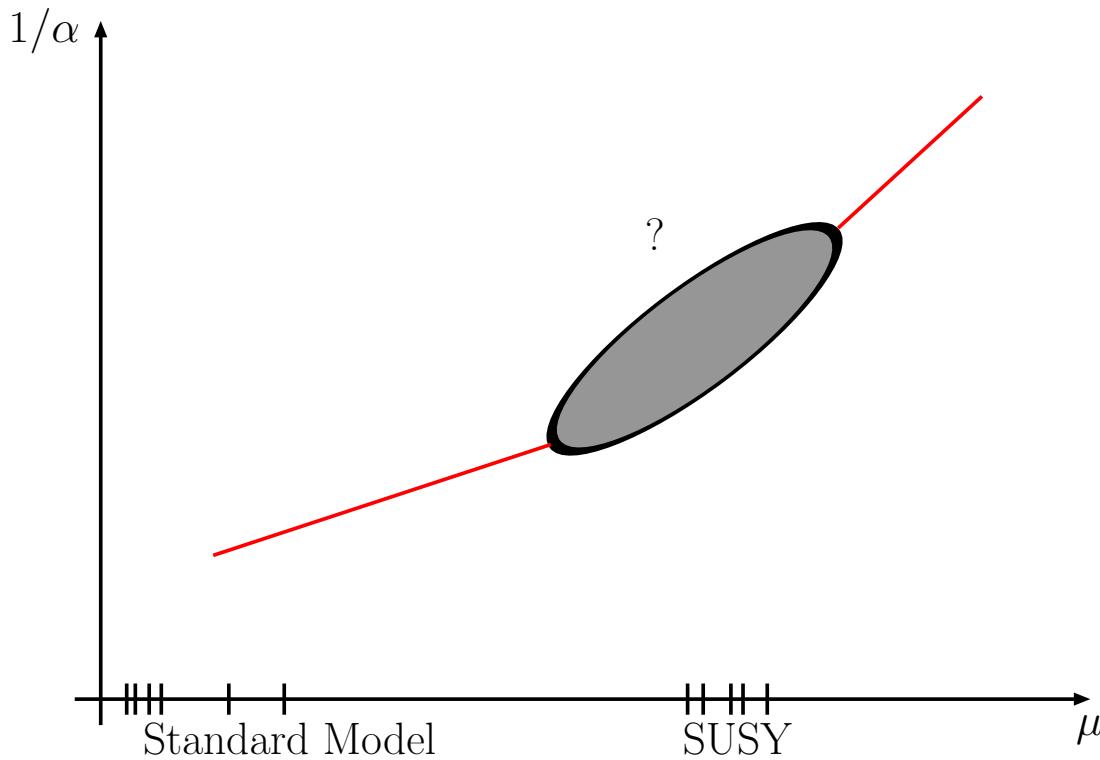


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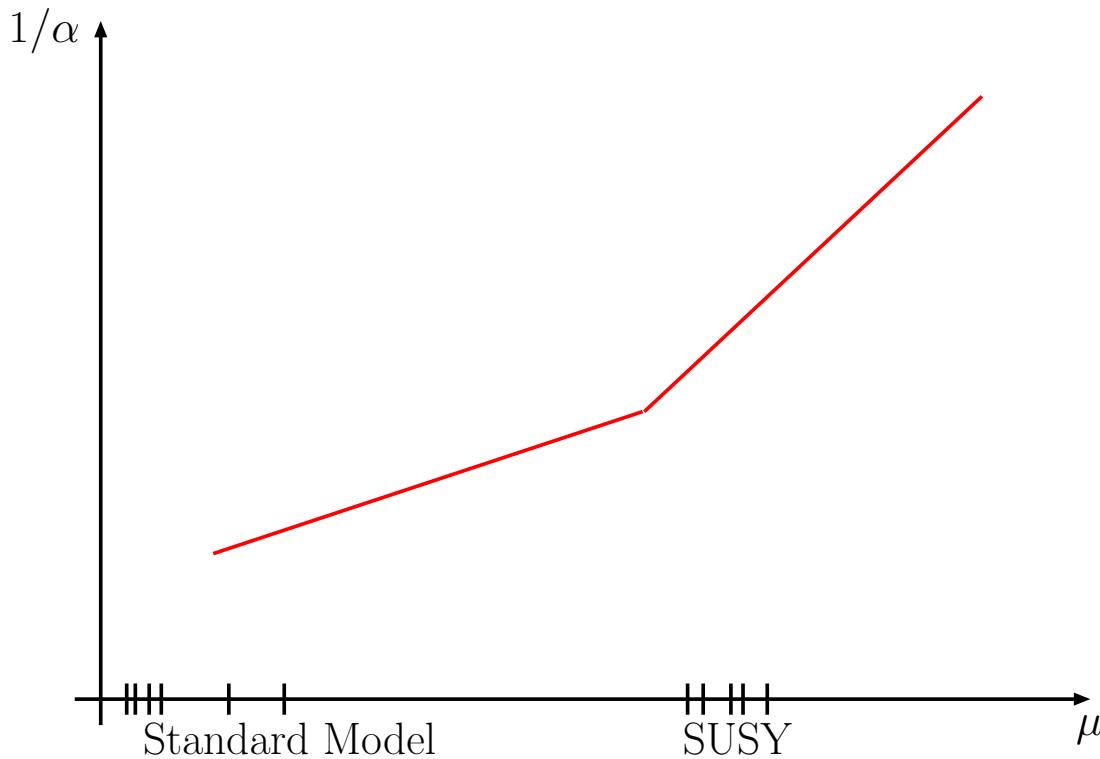
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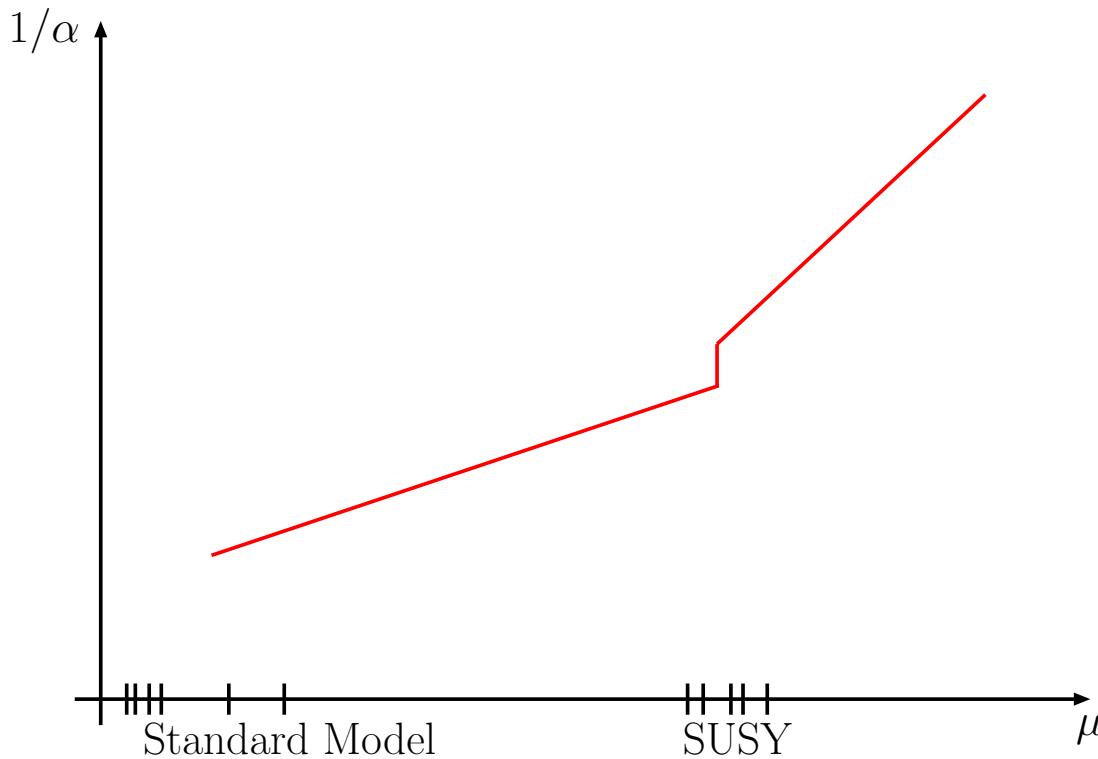
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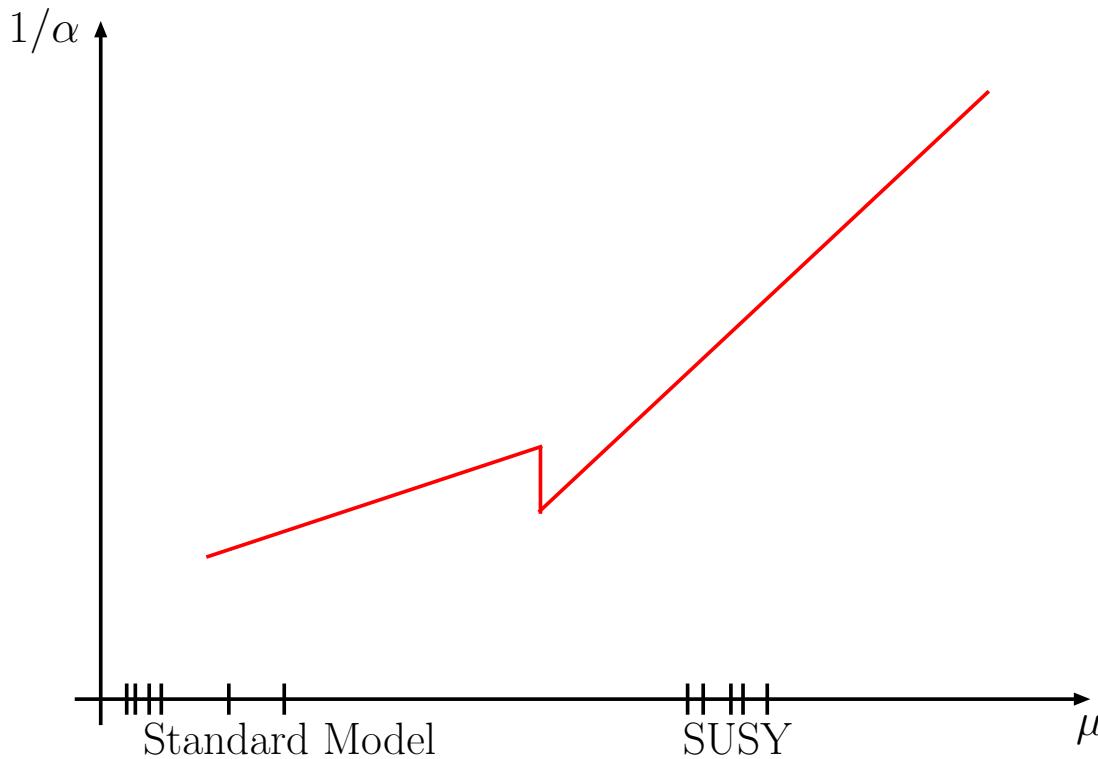
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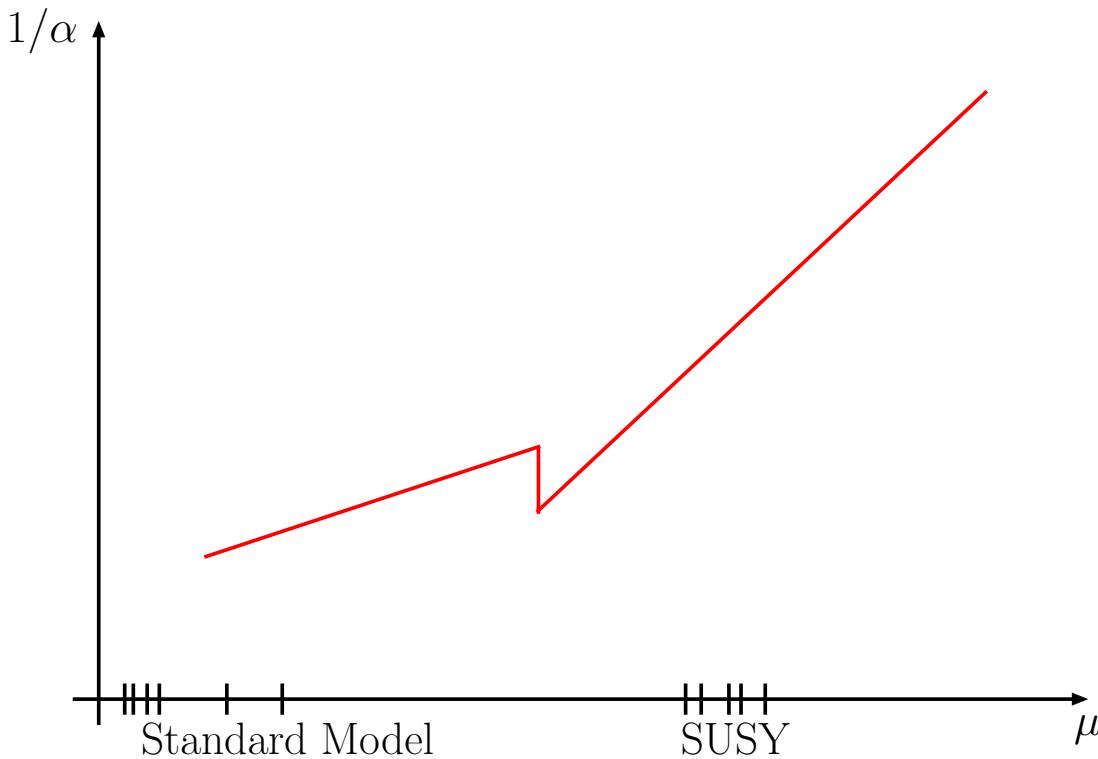
Decoupling



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Decoupling



- 3-loop running needs 2-loop decoupling
- for $\alpha_s^{\overline{\text{DR}}}$: [R.H., Mihaila, Steinhauser 05]

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$\overline{\text{MS}}$ – $\overline{\text{DR}}$ *conversion*

$$\alpha_s^{\overline{\text{DR}}} = \alpha_s^{\overline{\text{MS}}} \left[1 + \frac{\alpha_s^{\overline{\text{MS}}}}{4\pi} + \frac{11}{8} \left(\frac{\alpha_s^{\overline{\text{MS}}}}{\pi} \right)^2 - \frac{n_f}{12} \frac{\alpha_s^{\overline{\text{MS}}}}{\pi} \frac{\alpha_e}{\pi} + \dots \right]$$

[R.H., Kant, Mihaila, Steinhauser 06]

even 3-loop: [R.H., Jones, Kant, Mihaila, Steinhauser 06]

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what is α_e ?

in non-SUSY theory: renormalization scheme

- $R^{\text{exp}}(s) = R^{\overline{\text{MS}}}(\alpha_s^{\overline{\text{MS}}}(s)) = R^{\overline{\text{DR}}}(\alpha_s^{\overline{\text{DR}}}(s), \alpha_e(s))$
→ determines combination of $\alpha_s^{\overline{\text{DR}}}(s)$ and $\alpha_e(s)$
- other scale μ : RG equations for $\alpha_s^{\overline{\text{DR}}}$ and α_e

$\alpha_s^{\overline{\text{DR}}}$ in QCD

- coupled differential equations:

$$\mu^2 \frac{d}{d\mu^2} \alpha_s^{\overline{\text{DR}}} = \beta_s^{\overline{\text{DR}}}(\alpha_s^{\overline{\text{DR}}}, \alpha_e) ,$$

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- $\beta_s^{\overline{\text{DR}}}$ and β_e calculated to 3 loops

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→ consistency of DRED with SUSY!

α_e in SUSY

- assume softly broken SUSY where all SUSY masses $< M_{\text{SUSY}}$
- at high scale $\mu > M_{\text{SUSY}}$:

$$\alpha_e(\mu) = \alpha_s^{\overline{\text{DR}}}(\mu)$$

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- need α_e in QCD: decoupling for α_e

$$\alpha_e^{(5)}(M_{\text{SUSY}}) = \zeta_e \alpha_e^{(\text{full})}(M_{\text{SUSY}}) = \zeta_e \alpha_s^{\overline{\text{DR}},(\text{full})}(M_{\text{SUSY}})$$

Ready for 3-loop running...

- remark: SPA prescription: [hep-ph/0511344]
 - 1-loop running
 - 1-loop decoupling (resummed)
 - 1-loop $\overline{\text{MS}}$ — $\overline{\text{DR}}$ conversion (resummed)

resummed:

$$\alpha_s^{\overline{\text{DR}},(\text{full})} = \frac{\alpha_s^{\overline{\text{MS}},(5)}}{1 - \Delta\alpha_s}$$

→ leads to independence of decoupling scale!

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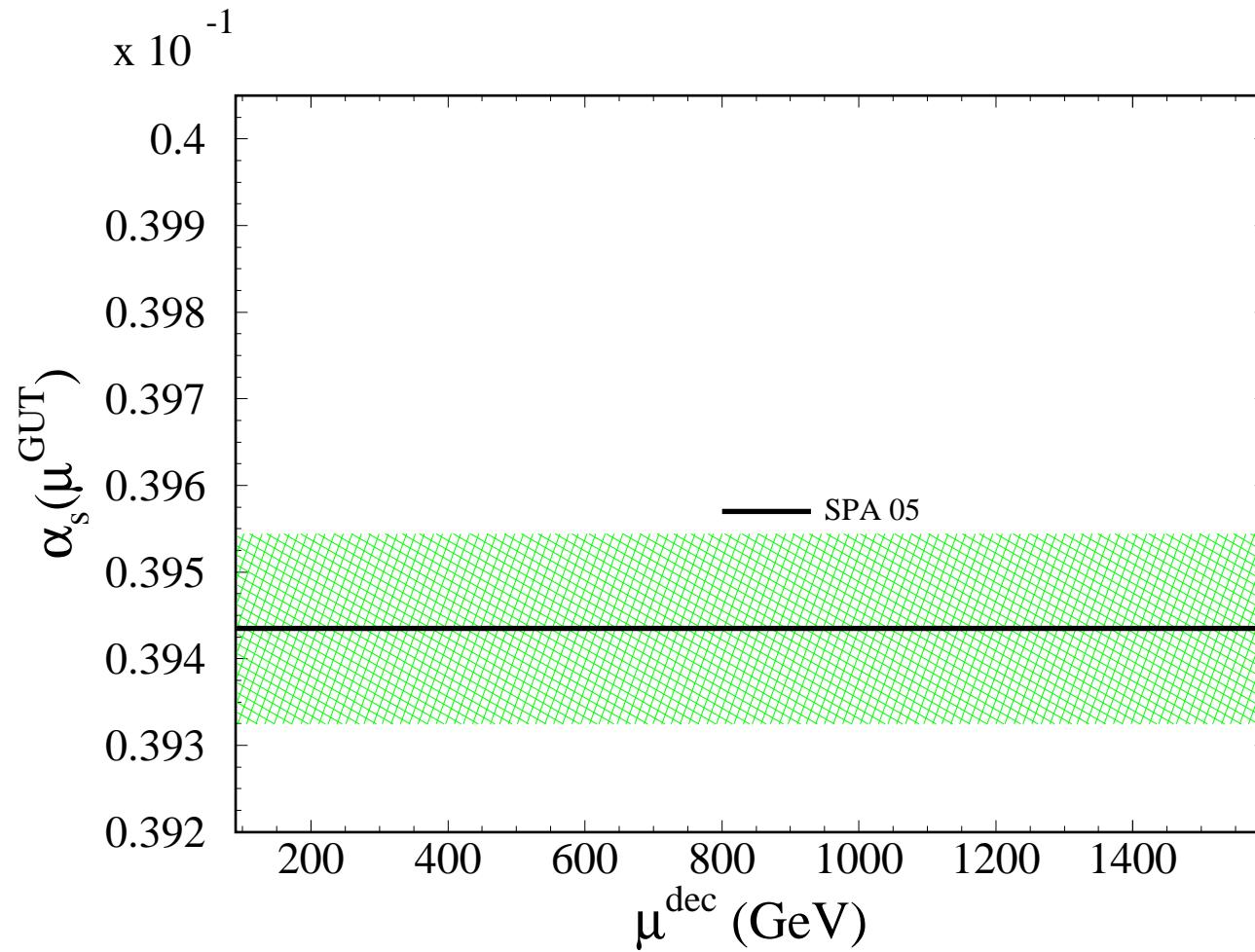
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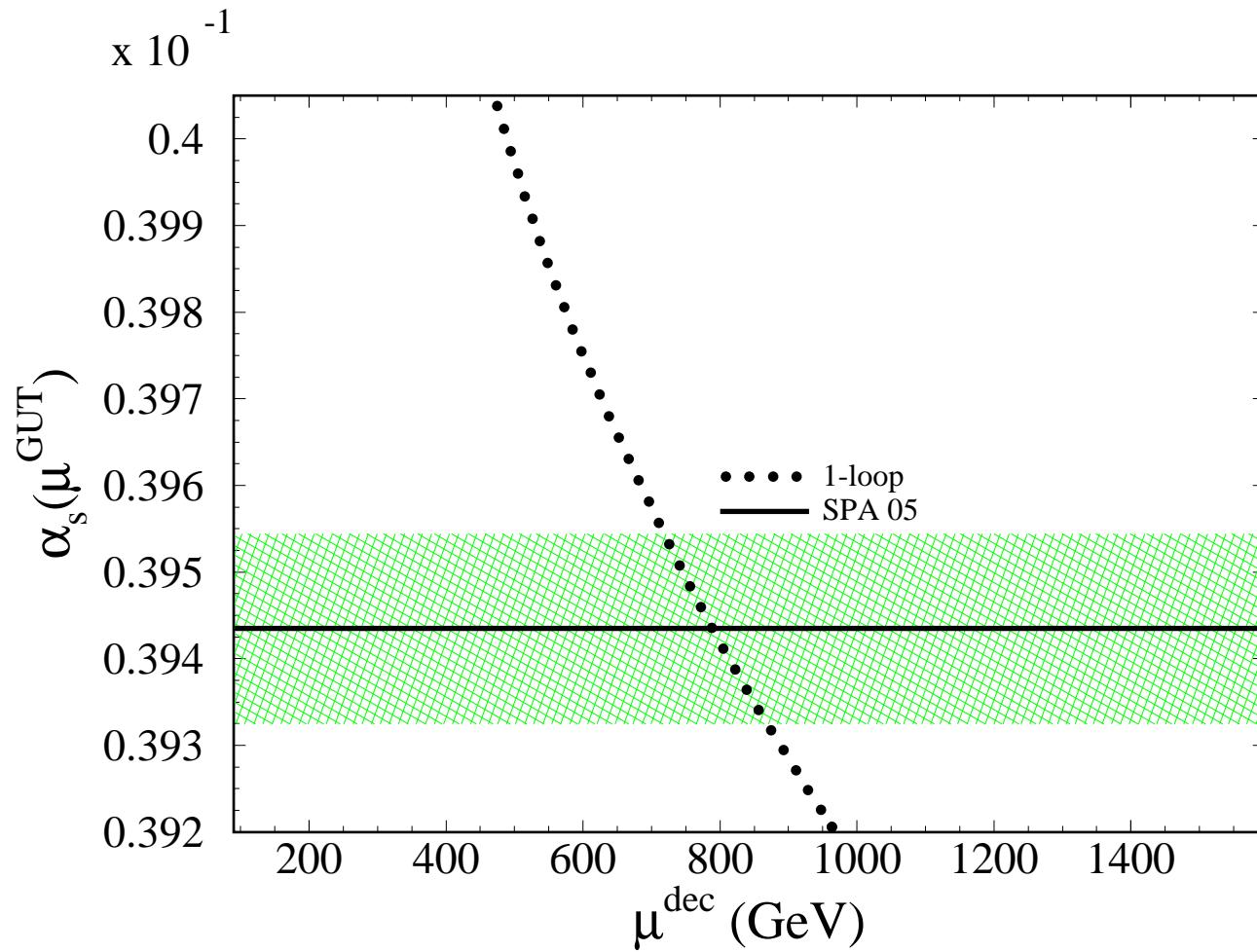
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- here:
 - 3-loop running
 - 2-loop matching
 - 2-loop $\overline{\text{MS}}$ — $\overline{\text{DR}}$ conversion

$\alpha_s(M_{\text{GUT}})$ *from* $\alpha_s(M_Z)$



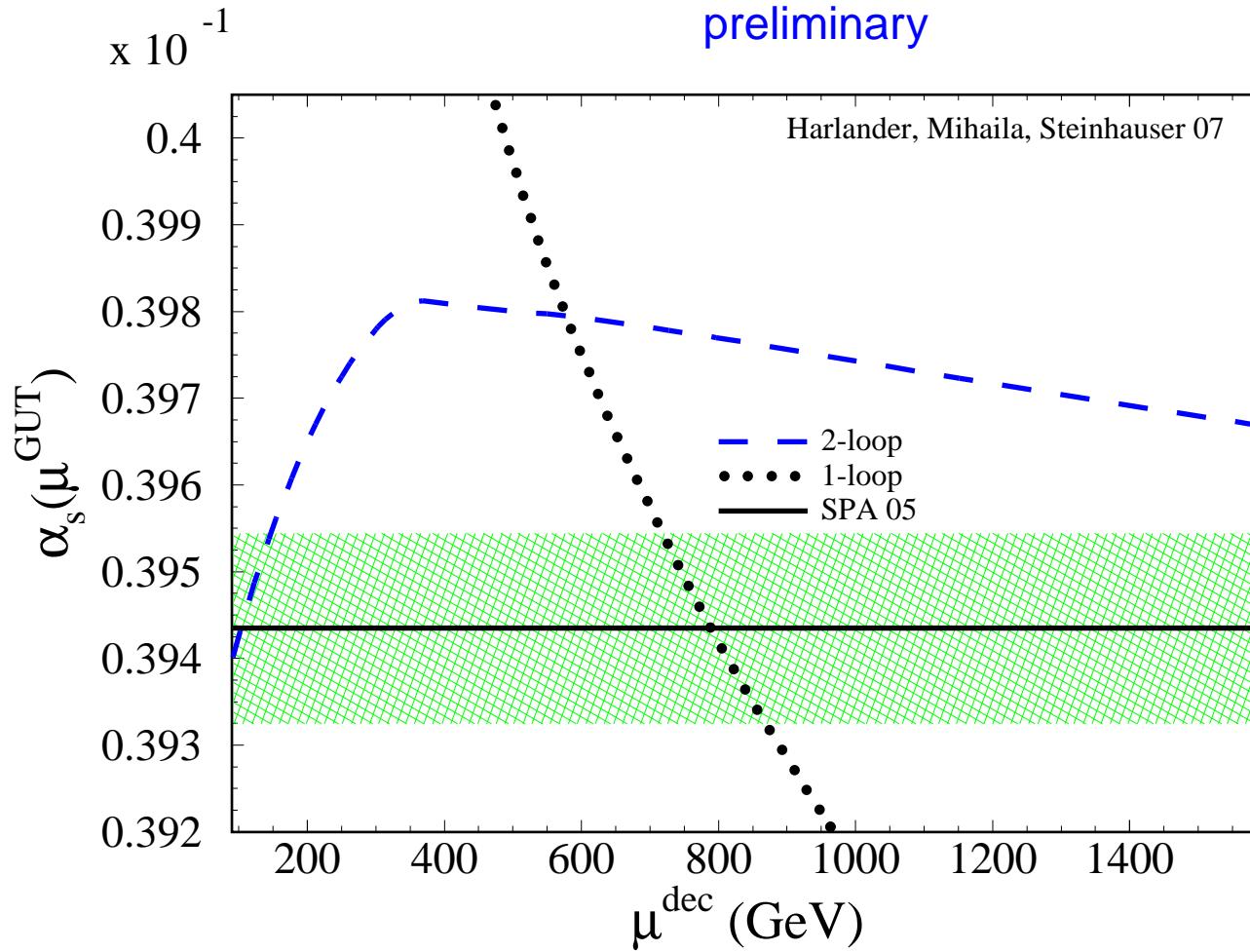
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[R.H., Mihaila, Steinhauser]

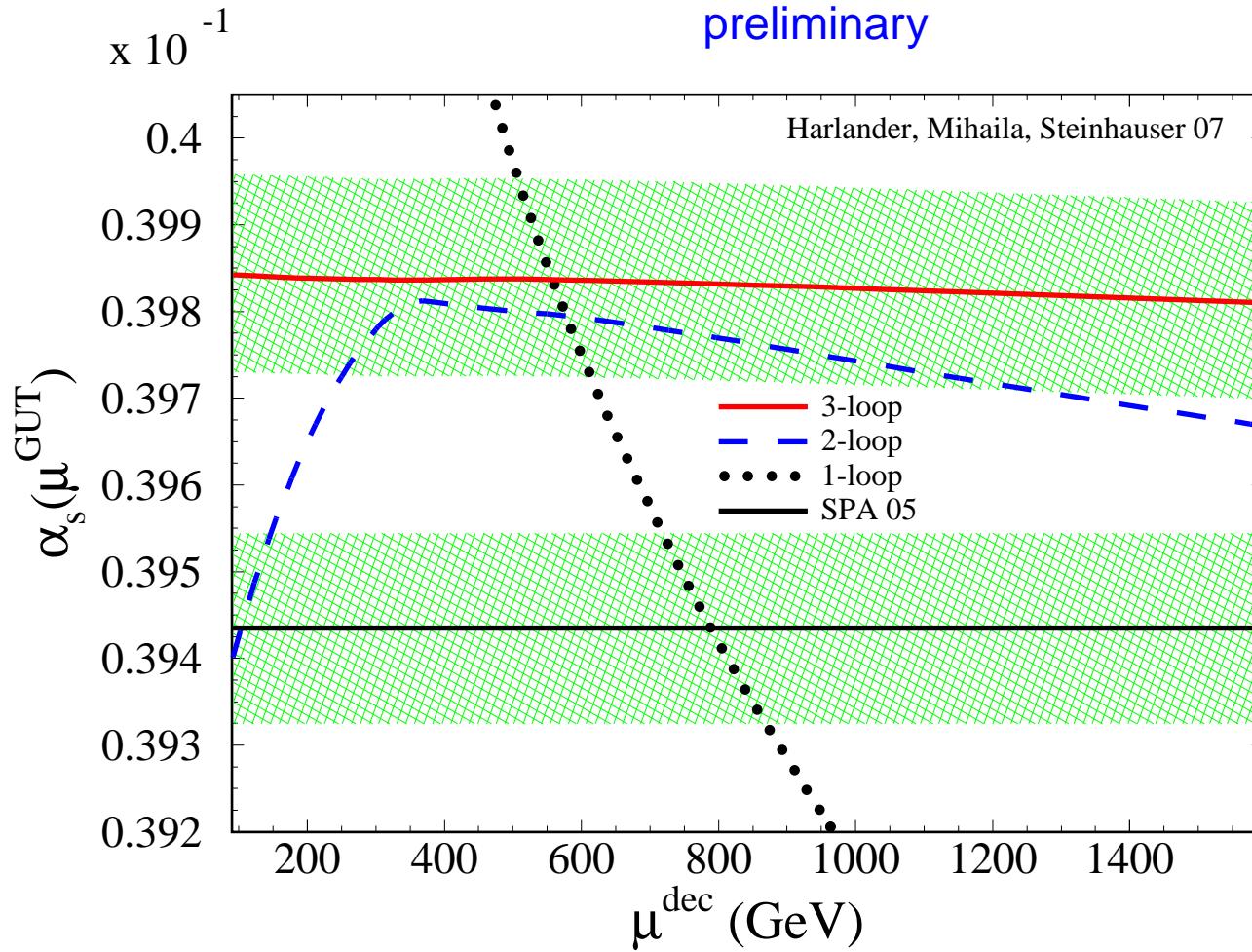
preliminary



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- **ToDo:**
 - quantify **validity range** of DRED in SUSY
 - combine running with electro-weak couplings