Anomalous Dimensions of High-Spin Operators Beyond the Leading Order

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Twist-Two Anomalous Dimensions

- For large Q^2, the scaling of the deeply inelastic structure functions is related to the scale dependence of some forward hadronic matrix elements of twist-two Wilson operators $\langle O_N^{(a)}(x) \rangle$
- Twist $\equiv$ Canonical dimension $-$ Lorentz spin N
- Feature of their scale dependance : they mix under renormalization according to

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s}\right] \langle O_N^{(a)}(x) \rangle = -\gamma^{ab}(N, \alpha_s) \langle O_N^{(b)}(x) \rangle$$

where $\gamma(N) \equiv \gamma(N, \alpha_s)$ is the anomalous dimensions matrix

In the following, we will be concerned by

- quark flavour non-singlet anomalous dimension

$$\langle \bar{q}_i \mathrm{T}^a_{ij} \gamma_+ D_+^{N-1} q_j(x) \rangle \leftrightarrow \gamma^{ns}(N)$$

- flavour singlet two-by-two anomalous dimensions matrix

$$\begin{pmatrix} \langle \bar{q} \gamma_+ D_+^{N-1} q(x) \rangle \\ \langle \mathrm{Tr}\left[F_{+\mu} D_+^{N-2} F_+^{\mu}(x)\right] \rangle \end{pmatrix} \leftrightarrow \gamma^s(N) = \begin{pmatrix} \gamma^{qq}(N) & \gamma^{qg}(N) \\ \gamma^{gq}(N) & \gamma^{gg}(N) \end{pmatrix}$$

computed in massless perturbative QCD within MS-like scheme

Large Spin Expansion and Inheritance Structures

Beyond the leading order, the analytic expressions for anomalous dimensions are cumbersome.

Their large spin expansions are simpler and exhibit interesting features :

For both non-singlet and diagonal singlet anomalous dimensions :

- The logarithmic scaling

$$\gamma(N) = A(\alpha_s) \ln N + \mathcal{O}\left(N^0\right)$$

  is universal :

  the cusp anomalous dimension $A(\alpha_s)$ only depends on the colour charge

- Properties of subleading $1/N$ suppressed corrections ?

Large Spin Expansion and Inheritance Structures

For both non-singlet and diagonal singlet anomalous dimensions, we can parameterize the asymptotic expansion as follows

$$\gamma(N) = \frac{1}{2}A(\alpha_s)\ln\left[N(N+1)e^{2\gamma_E}\right] + B(\alpha_s)$$

$$+\frac{1}{2}C(\alpha_s)N^{-1}\ln\left[N(N+1)e^{2\gamma_E}\right] + D(\alpha_s)N^{-1} + \mathcal{O}(N^{-2}\ln^p N)$$

Inheritance structures among the expansion coefficients have been observed up to three loops :

Moch, Vermaseren, Vogt relation

$$C(\alpha_s) = \frac{1}{2}A^2(\alpha_s)$$

Dokshitzer, Marchesini, Salam relation

$$D(\alpha_s) = \frac{1}{2}A(\alpha_s)B(\alpha_s) - \frac{1}{2}A(\alpha_s)\beta(\alpha_s)/\alpha_s$$

All the expansion coefficients are not independent!
Leading coefficients A and B to order $\mathcal{O}(\alpha_s^n)$ determine subleading coefficients C and D to order $\mathcal{O}\left(\alpha_s^{n+1}\right)$

Behind Inheritance Structures

Following Dokshitzer, Marchesini, Salam, we define a (scaling) function f with the following functional relation

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N)\right)$$

Assumption : the large spin expansion of $f(N)$ takes the form

$$f(N) = A(\alpha_s)\ln\left[N(N+1)e^{2\gamma_E}\right] + B(\alpha_s) + 0.N^{-1} + \mathcal{O}\left(N^{-2}\ln^p N\right)$$

we constrain the anomalous dimension $\gamma(N)$ to satisfy the inheritance structures

$$C(\alpha_s) = \frac{1}{2}A^2(\alpha_s) \qquad D(\alpha_s) = \frac{1}{2}A(\alpha_s)B(\alpha_s) - \frac{1}{2}A(\alpha_s)\beta(\alpha_s)/\alpha_s$$

except for the beta-function contribution

Questions

- *How to incorporate the beta-function contribution?*
- *Extension of the inheritance structures to more subleading coefficients?*

Main Results

To incorporate the beta-function contribution : we define the scaling function f with

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N) - \frac{\beta(\alpha_s)}{2\alpha_s}\right)$$

The property underlying the inheritance structures is :

The parity preserving relation : the whole large spin expansion of the scaling function $f(N)$ is invariant under the "parity" $N \to -N-1$

Large spin expansion of $f(N)$ only runs in the parameter $J^2 = N(N+1)$

$$f(N) = \frac{1}{2}A(\alpha_s)\ln\left[J^2 e^{2\gamma_E}\right] + B(\alpha_s) + f^{(1)}\left(\ln J^2\right)J^{-2} + f^{(2)}\left(\ln J^2\right)J^{-4} + \mathcal{O}\left(J^{-6}\right)$$

Extended inheritance structures in the large spin expansion of $\gamma(N)$:

Coefficients in front of odd powers of $1/J$ (non parity preserving contributions) can be expressed in terms of the coefficients accompanying smaller even powers of $1/J$ to less number of loops

Remark : All this holds for eigenvalues of the anomalous dimensions matrix

Symmetries of Anomalous Dimensions

Symmetries of anomalous dimensions reflect the symmetries of the theory

Operators belonging to the same multiplet have the same anomalous dimensions

Anomalous dimension depends on the quantum numbers of the multiplet

- Lorentz invariance $\rightarrow \gamma$ depends on the Lorentz spin N
- Conformal invariance $\rightarrow \gamma$ depends on the conformal spin j

What is the conformal spin of the multiplet that we consider?

- Bare $\mathcal{O}\left(\alpha_s^0\right)$ conformal spin $j_0 = N + 1$

The conformal spin is renormalized by the anomalous dimension :

- Full (all order) conformal spin $j = N + 1 + \frac{1}{2}\gamma(N)$

And therefore, conformal invariance ensures that the anomalous dimension is a function of the conformal spin

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N)\right)$$

Symmetries of Anomalous Dimensions

Symmetries of anomalous dimensions reflect the symmetries of the theory

Conformal invariance

- Define the scaling function f as

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N)\right)$$

Conformal invariance is broken by the running of the coupling constant

- Nevertheless, the beta-function contribution can be incorporated in the above consideration, within MS-like renormalization scheme. This leads to a slightly modified definition of the scaling function f

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N) - \frac{\beta(\alpha_s)}{2\alpha_s}\right)$$

Verification of the Parity Preserving Relation

We verified in QCD that the parity preserving relation holds for

- two-loop longitudinally polarized singlet anomalous dimensions,
- two-loop gluon linearly polarized anomalous dimension,
- two-loop quark transversity anomalous dimension,
- three-loop non-singlet unpolarized anomalous dimension,
- three-loop singlet unpolarized anomalous dimensions.

We also verified in QCD that the parity preserving relation holds to all loops in the large β_0 limit for both singlet and non-singlet anomalous dimensions

Conclusion

To summarize

- Inheritance structures are explained and extended with the parity preserving relation of the scaling function
- Parity preserving relation holds in QCD for twist-two anomalous dimensions, to the accuracy available in the literature

Other results

- Parity preserving relation also holds in many cases in SYM theories for twist-two anomalous dimensions
- Parity preserving relation is not a unique feature of gauge theories : it holds for the twist-two anomalous dimension in scalar ϕ^4 theory
- Extension to quasi-partonic operators of arbitrary twist L

$$\gamma(N) = f\left(N + \frac{1}{2}\gamma(N) - \frac{L}{4}\frac{\beta(\alpha_s)}{\alpha_s}\right)$$

- Verification of the parity preserving relation in the strong coupling limit of the planar $\mathcal{N} = 4$ SYM, for high-twist scalar operator dual to a folded string rotating on $AdS_3 \times S^1$ in the AdS/CFT correspondence