Three-jet event-shapes: first NLL+NLO+$1/Q$ results

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In collaboration with Giulia Zanderighi (CERN)

Event-shape variables

- Event-shape variables $V(p_1, \ldots, p_n)$ are continuous measures of the geometrical properties of hadron energy-momentum flow.
- Thrust: longitudinal particle alignment

$$T \equiv \frac{1}{Q} \max_{\vec{n}_T} \sum_i |\vec{p}_i \cdot \vec{n}_T|$$

Pencil-like event: $T \lesssim 1$ Planar event: $T \gtrsim 2/3$

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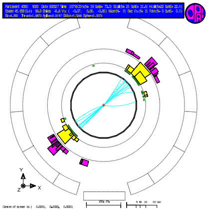

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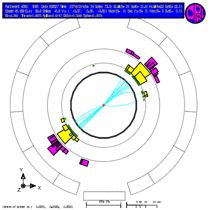

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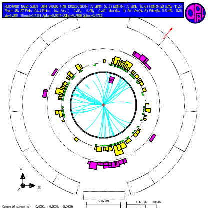
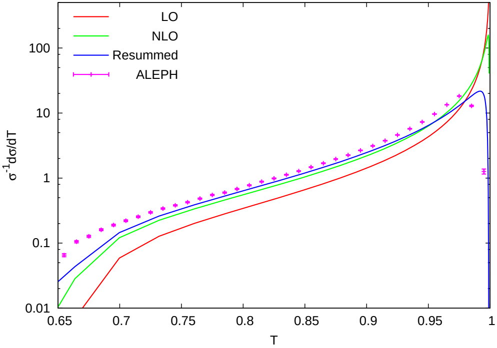


Sensitive to QCD radiation $\Rightarrow$ measure α_s

Large non-perturbative contributions



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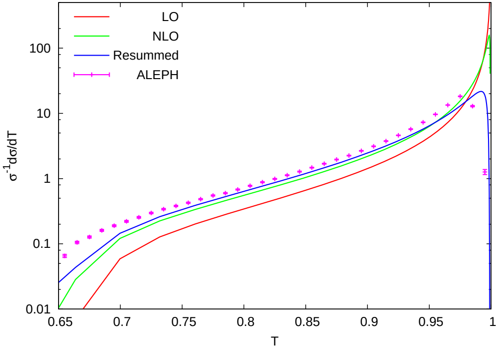

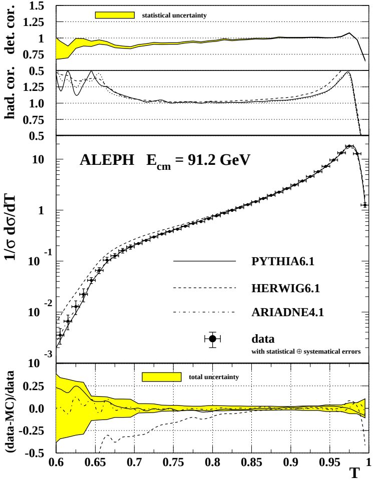

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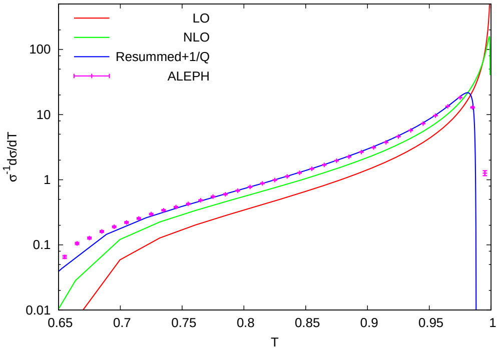

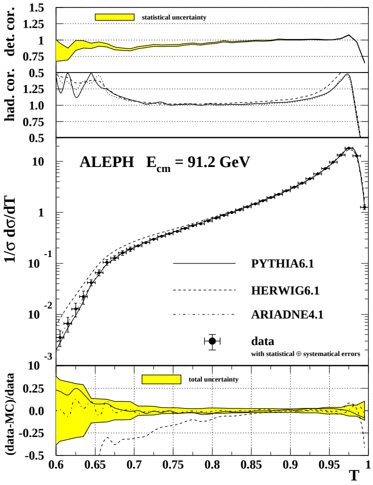

Consider $\Sigma(V) = \text{Prob}(V(\{k\}) < V)$. In the region $\langle V\rangle_{\text{NP}}\, Q \ll VQ \ll Q$

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For two-jet event shapes

$$V(k) \simeq \frac{k_t}{Q} f_V(\eta) \quad \Rightarrow \quad \langle V\rangle_{\text{NP}} = \langle k_t\rangle_{\text{NP}}\, c_V \qquad c_V = \int_0^\infty d\eta\, f_V(\eta)$$

- $\langle V\rangle_{\text{NP}}$ gives the power correction both to mean values and distributions
- $\langle k_t\rangle_{\text{NP}}$ is observable independent and parameterised in terms of α_0

$$\langle k_t\rangle_{\text{NP}} = \frac{4C_F}{\pi^2}\mu_I \mathcal{M}\, \alpha_0(\mu_I) \qquad \alpha_0(\mu_I) = \int_0^{\mu_I} \frac{dk}{\mu_I} \alpha_s^{\text{CMW}}(k)$$

$\mathcal{M} \simeq 1.49$ is the Milan factor due to non-inclusiveness of event shapes

[Y. Dokshitzer, A. Lucenti, G. Marchesini, G. Salam hep-ph/9802381]

$1/Q$ corrections to two-jet event shapes

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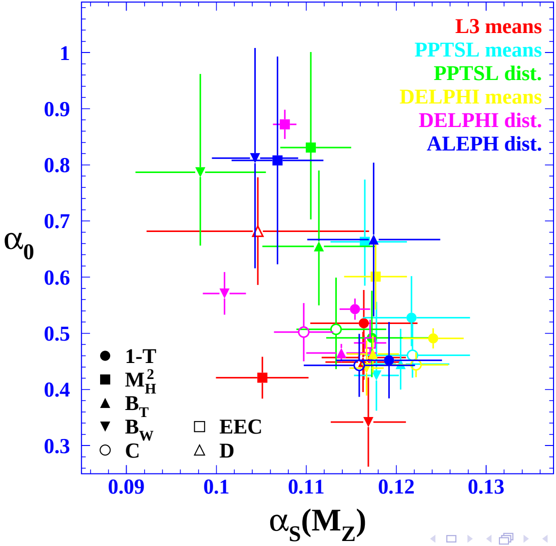
Unversality of NP parameter α_0 tested through α_s-α_0 fits

[S. Kluth hep-ex/0606046]



In multi-jet events no natural defi nition of k_t $\Rightarrow$ PQCD inspiration

- Soft dressed gluon emission from a $q\bar{q}$ dipole

$$dw = 2C_F \frac{dk_t}{k_t} d\eta \frac{\alpha_s^{\mathrm{CMW}}(k_t)}{\pi}$$

$$k_t^2 = \frac{(2pk)(2k\bar{p})}{2p\bar{p}} \qquad \eta = \frac{1}{2}\ln\frac{\bar{p}k}{pk}$$

- Soft dressed gluon emission from more dipoles

$$dw = \sum_{i<j}(-2\vec{T}_i \cdot \vec{T}_j)\frac{d\kappa_{ij}}{\kappa_{ij}} d\eta_{ij} \frac{\alpha_s^{\mathrm{CMW}}(\kappa_{ij})}{\pi}$$

$$\kappa_{ij}^2 = \frac{(2p_ik)(2kp_j)}{2p_ip_j} \qquad \eta_{ij} = \frac{1}{2}\ln\frac{p_jk}{p_ik}$$

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$\Rightarrow$ PC depend on event geometry and colour correlations

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Two-jet event shapes $\Rightarrow$ hadron distribution is uniform in η

Multi-jet event shapes $\Rightarrow$ test PQCD-inspired power corrections

$$\langle V \rangle_{\mathrm{NP}} = \frac{4}{\pi^2}\frac{\mu_I}{Q}\,\mathcal{M}\,\alpha_0(\mu_I)\sum_{i<j}(-2\vec{T}_i\cdot\vec{T}_j)\,c_V^{(ij)}$$

- α_0 same as in two-jet event shapes $\Rightarrow$ Universality
- Dependence on the event geometry through $c_V^{(ij)}$
- Dependence on colour correlations $\vec{T}_i \cdot \vec{T}_j$

Full phenomenological study feasible only for three-jet shapes

- No matrix structure of colour correlations

$$\vec{T}_1+\vec{T}_2+\vec{T}_3=0 \qquad \Rightarrow \qquad -2\vec{T}_i\cdot\vec{T}_j=T_i^2+T_j^2-T_k^2$$

- No need to hack NLO programs to obtain flavour information

If $V(k)$ has the same behaviour for all legs in the SC region

$$V(k)\sim\left(\frac{k_t}{Q}\right)^a e^{-b|\eta|} \quad \Rightarrow \quad \Sigma(V)\sim e^{-(2C_F+C_A)\frac{\alpha_s}{(a+b)\pi}\ln^2 V}$$

Averaging over multiple hard configurations

$$\left\langle\left(1+C_1\frac{\alpha_s}{2\pi}\right)\Sigma(V)\right\rangle=\left(1+\langle C_1\rangle\frac{\alpha_s}{2\pi}\right)\langle\Sigma(V)\rangle+\mathrm{NNLL}$$

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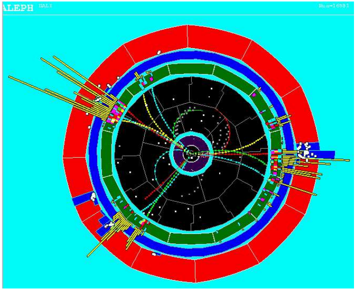
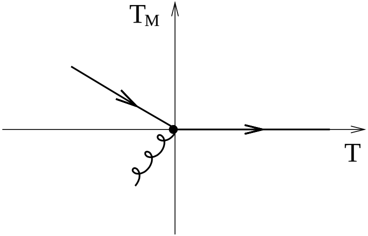




- Thrust-minor: sum of the momenta out of the event plane (T, T_M)

[AB, Y. Dokshitzer, G. Marchesini, G. Zanderighi hep-ph/0101205]

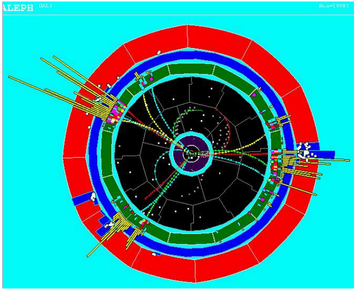
$$T_m Q \equiv \sum_h |\vec{p}_{th} \times \vec{n}_M| = \sum_h |p_h^{\rm out}| \qquad T_M\, Q \equiv \max_{\vec{n}_M \cdot \vec{n}_T = 0} \sum_i |\vec{p}_i \cdot \vec{n}_M|$$

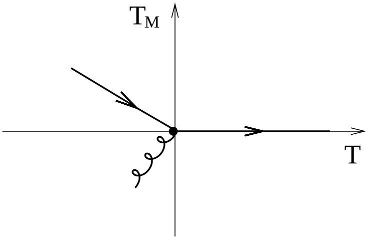
- D-parameter: determinant of the momentum tensor $\theta_{\alpha\beta}$

[AB, Y. Dokshitzer, G. Marchesini, G. Zanderighi hep-ph/0104162]

$$D \equiv 27 \det \theta \sim \sum_h \frac{(p_h^{\rm out})^2}{E_h Q} \qquad \theta_{\alpha\beta}\, Q \equiv \sum_h \frac{p_h^\alpha p_h^\beta}{|\vec{p}_h|}$$

D-parameter and T_m in e^+e^- annihilation





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- Select 3-jet events with $y_3 > y_{\rm cut}$
- Differential distributions obtained with CAESAR at $Q = 91.2\,\mathrm{GeV}$

[AB, G. Salam, G. Zanderighi hep-ph/0407286]

Tests of power corrections for D and T_m

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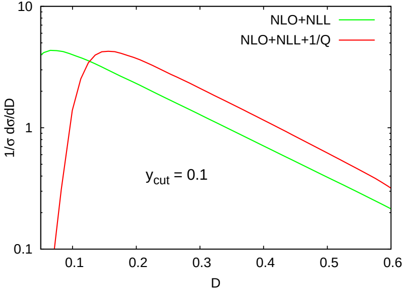

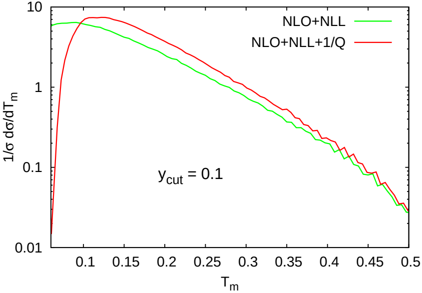

- D-parameter: large constant shift (see T and C)
- T_m: both shift and squeeze (see B_T and B_W)

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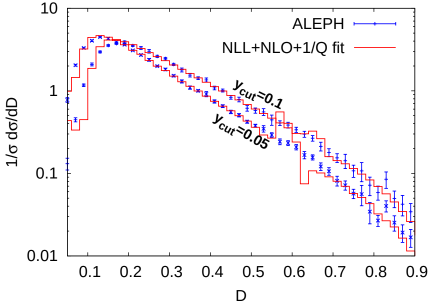

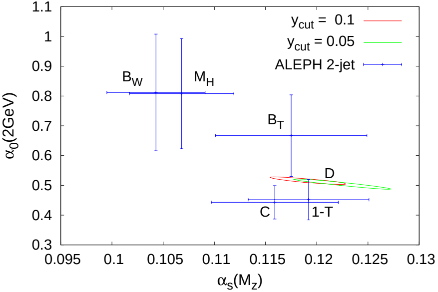

- D-parameter: first ever α_s-α_0 fits in a three-jet event shapes!
- Good fits only for $D > 0.2$: $\chi^2/{\rm d.o.f.}(y_{\rm cut} = 0.1) = 12/20$
  $\Rightarrow$ Small-D region: shape function or large subleading logs?

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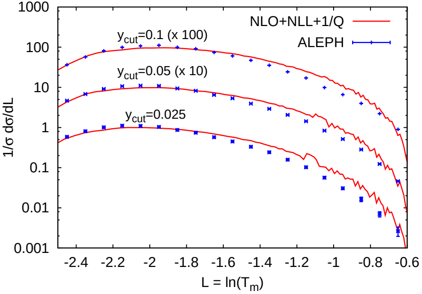
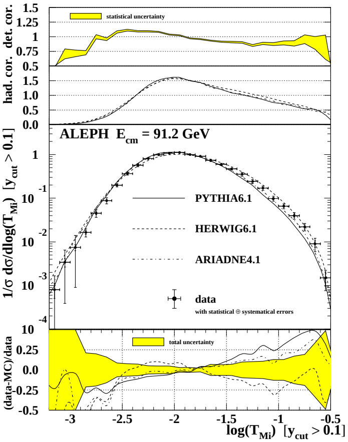




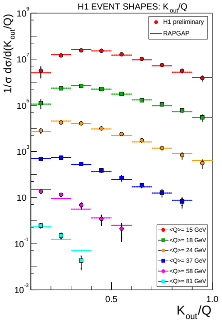
Thrust minor PC should be positive

MC say PC are negative at large T_m
$\Rightarrow$ PC from 4-jet configurations?

$K_{\rm out}$ and E_T-E_T correlation in DIS

Measurements in the Breit frame $q = (0,0,0,Q)$

[T. Kluge hep-ex/0606053]



- Out of plane momentum $K_{\rm out}$ (with $\eta_h < \eta_0$)
  [AB,G. Marchesini,G. Smye,G. Zanderighi hep-ph/0111157]

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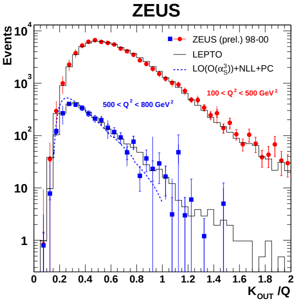

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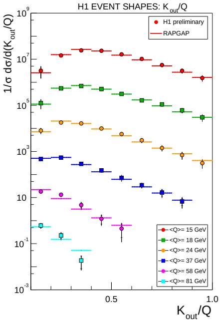
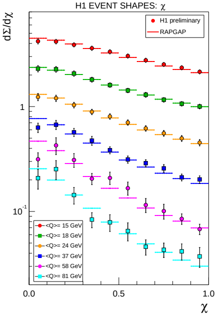
- Transverse energy-energy correlation
  [AB,G. Marchesini,G. Smye hep-ph/0203150]

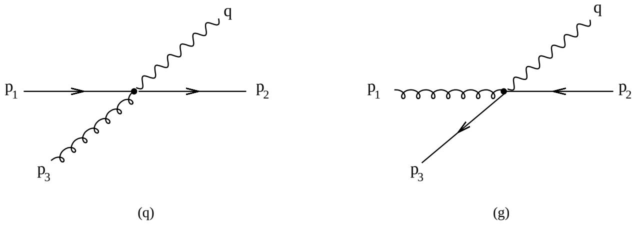
$$E_T E_T C(\chi) = \sum p_{ti} p_{tj} \delta(\chi - |\pi - \phi_{ij}|)$$

Probability of $\chi = 0$ is finite due to vectorial cancellations

$$\Sigma(\chi) = P_t \int_0^\infty db \cos(bP_t\chi) f_{\rm NP}(b) e^{-R(b)} \sim \text{const}$$







Out-of-event-plane momentum $K_{\rm out}$

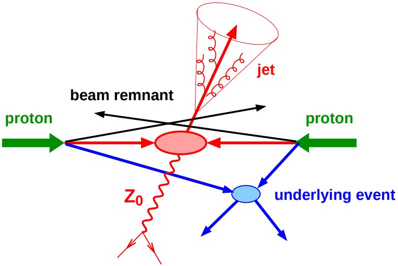
[A. Banfi , G. Marchesini, G. Smye hep-ph/0106278]

$$K_{\rm out} \equiv \sum_h \frac{|\vec{p}_{th} \times \vec{q}_t|}{p_t^{\rm jet}} \times \Theta(|\eta_h| - \eta_0)$$

rapidity cut around beam pipe

Normalisation is defi ned so as cancel systematic uncertainties in determination of jet transverse energy scale

K_{out} and beam remnant interactions



Extra contribution from underlying event $\Rightarrow$ $K_{\mathrm{out}} = K_{\mathrm{out}}^{\mathrm{primary}} + K_{\mathrm{out}}^{\mathrm{UE}}$

$$\Sigma(K_{\mathrm{out}}) = \int_0^{K_{\mathrm{out}}} dK_{\mathrm{out}}^{\mathrm{UE}} D^{\mathrm{UE}}(K_{\mathrm{out}}^{\mathrm{UE}}) \Sigma_{\mathrm{PT}} \left(K_{\mathrm{out}} - \langle K_{\mathrm{out}} \rangle_{\mathrm{NP}} - K_{\mathrm{out}}^{\mathrm{UE}} \right)$$

In case $\langle K_{\mathrm{out}}^{\mathrm{UE}} \rangle \ll K_{\mathrm{out}}$ we have

$$\Sigma(K_{\mathrm{out}}) = \Sigma_{\mathrm{PT}} \left(K_{\mathrm{out}} - \langle K_{\mathrm{out}} \rangle_{\mathrm{NP}} - \left\langle K_{\mathrm{out}}^{\mathrm{UE}} \right\rangle \right)$$

K_{out} can be used for tuning of MC models of underlying event

Conclusions

Three-jet event shapes benefits

- More stringent tests of universality of NP corrections
- $1/Q$ corrections depend on hard parton colour and geometry

e^+e^- annihilation (AB, G. Zanderighi)

- Only ALEPH data for distributions are available
- First NLO+NLL+PC fits are available for D-parameter
- NLO+NLL+PC predictions available for T_m

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- Work in progress to fit K_{out} H1 data (Thomas Kluge's PhD thesis)
- No progress for azimuthal correlation, man power needed (only AB)

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- Global event shape: can be used to tune UE models

Three-jet event shapes benefits

- More stringent tests of universality of NP corrections
- $1/Q$ corrections depend on hard parton colour and geometry

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