QCD and MC Generators

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Born Level Calculation

$$\sigma[F_J] = \int_m d\Gamma^{(m)}(\{p\}_m)|\mathcal{M}(\{p\}_m)|^2 F_J(\{p\}_m)$$

- ✓ Easy to calculate, no IR singularities and several matrix element generators are available (Alpgen, Helac, MadGraph, Sherpa)
- ✗ Strong dependence on the unphysical scales (renormalization and factorization scales)
- ✗ Exclusive quantities suffer on the large logarithms
- ✗ Every jet is represented by a single parton
- ✗ No quantum corrections
- ✗ No hadronization

NLO Level Calculation

$$\sigma_{\mathrm{NLO}} = \int_N d\sigma^B + \int_{N+1} \left[d\sigma^R - d\sigma^A\right]_{\epsilon=0} + \int_N \left[d\sigma^V + \int_1 d\sigma^A\right]_{\epsilon=0}$$

$$d\sigma^A \sim d\Gamma(\{p\}_{N+1})\ \underbrace{S \otimes |\mathcal{M}\{\tilde{p}\}_N|^2}_{\text{Based on soft collinear factorization}} F_J(\{\tilde{p}\}_N)$$

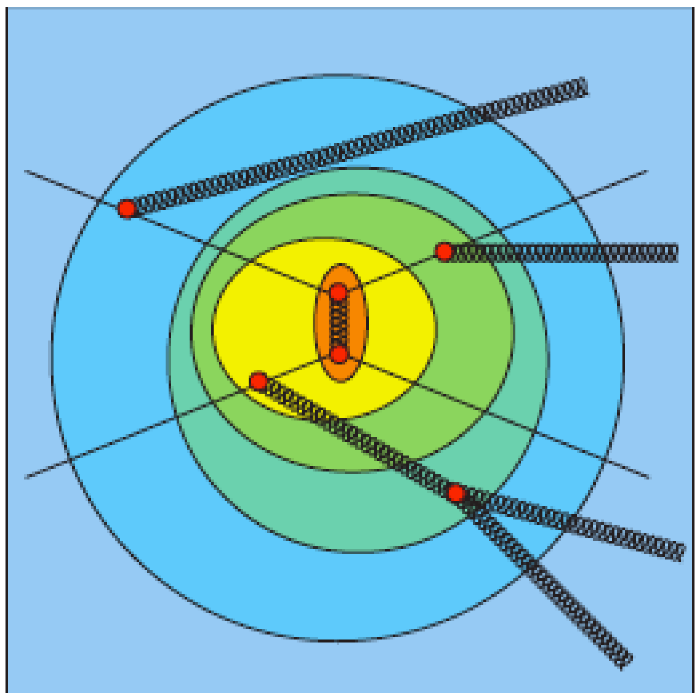
- ✓ Includes quantum corrections, in most of the cases it significantly reduces the unphysical scale dependences
- ✓ One of the jets consists of two partons (still very poor)
- ✓ Hard to calculate, the most complicated available processes are 2 -> 3 (NLOJET++, MCFM, PHOX,...)
- ✗ Exclusive quantities suffers on large logarithms
- ✗ No hadronization

LO Parton Shower

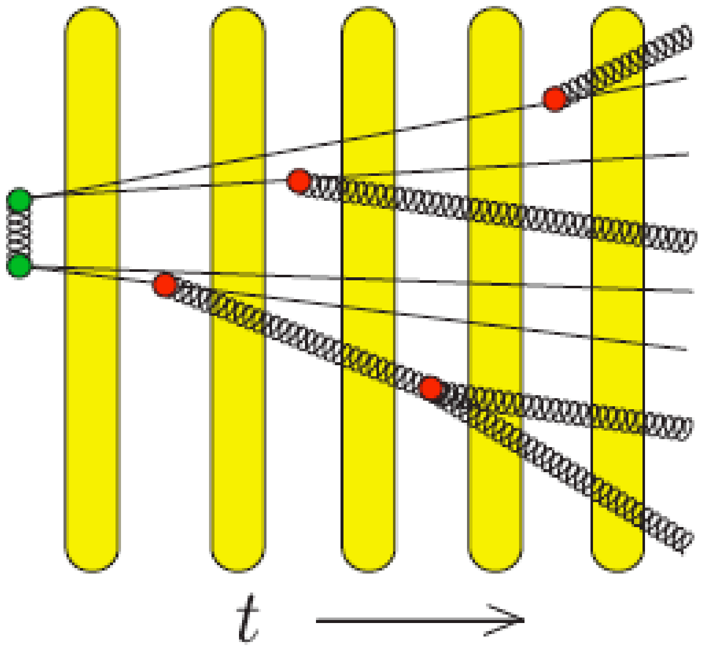
- ✓ It is an iterative algorithm. Arbitrary number of partons.
- ✓ Based on the universal soft and collinear factorization property of the QCD matrix elements. (This is the basic approximation and should be the only.)
- ✓ Matched to the hadronization models (which is universal effect).
- ✓ In the best cases it resumes the leading large logarithms properly.
- ✗ Needs more, rather non universal approximations. *(See next slides!)*
- ✗ Only leading order splitting kernels are involved, we can expect large dependence on the unphysical scales.
- ✗ The only exact matrix element in the calculations is 2->2 like at Born level.
- ✗ Positive unweighted events. I think it is a misleading concept.

Shower from Inside Out

Think of shower branching as developing in a "time" that goes from most virtual to least virtual.



Real time picture

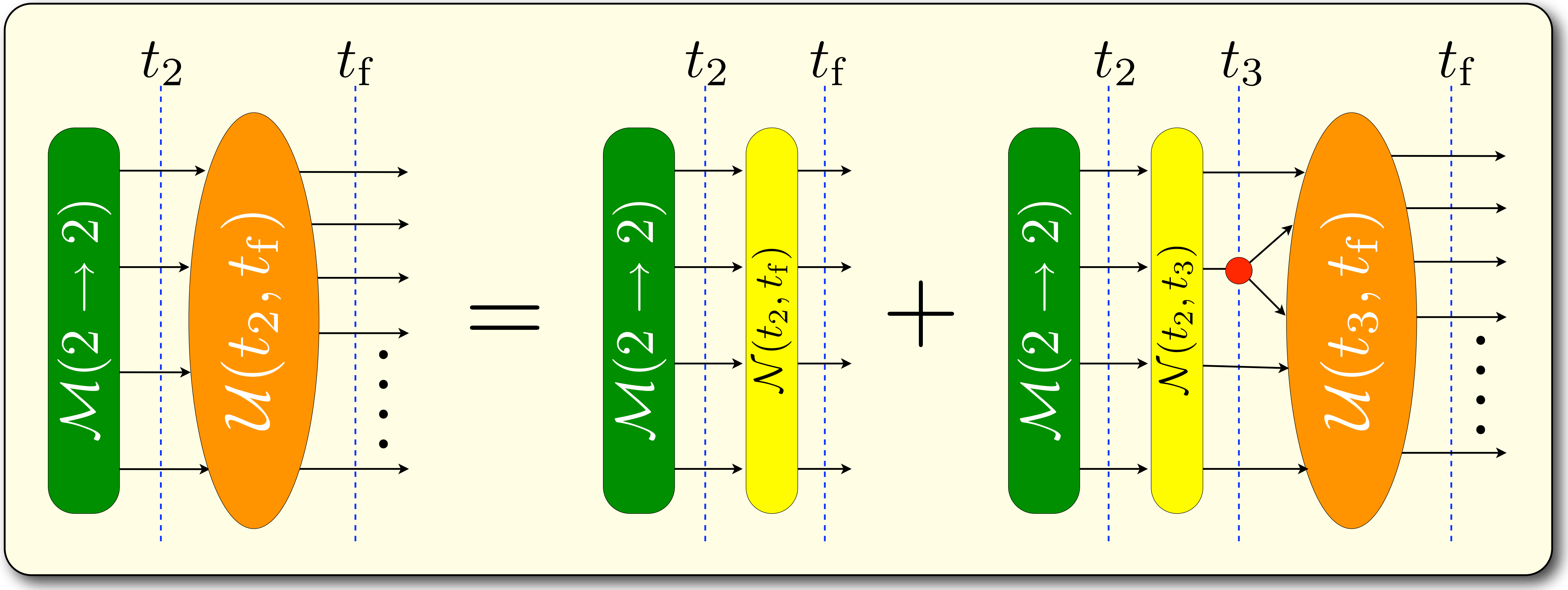


Shower time picture

Thus shower time proceeds backward in physical time for initial state radiation.

Iterative Algorithm

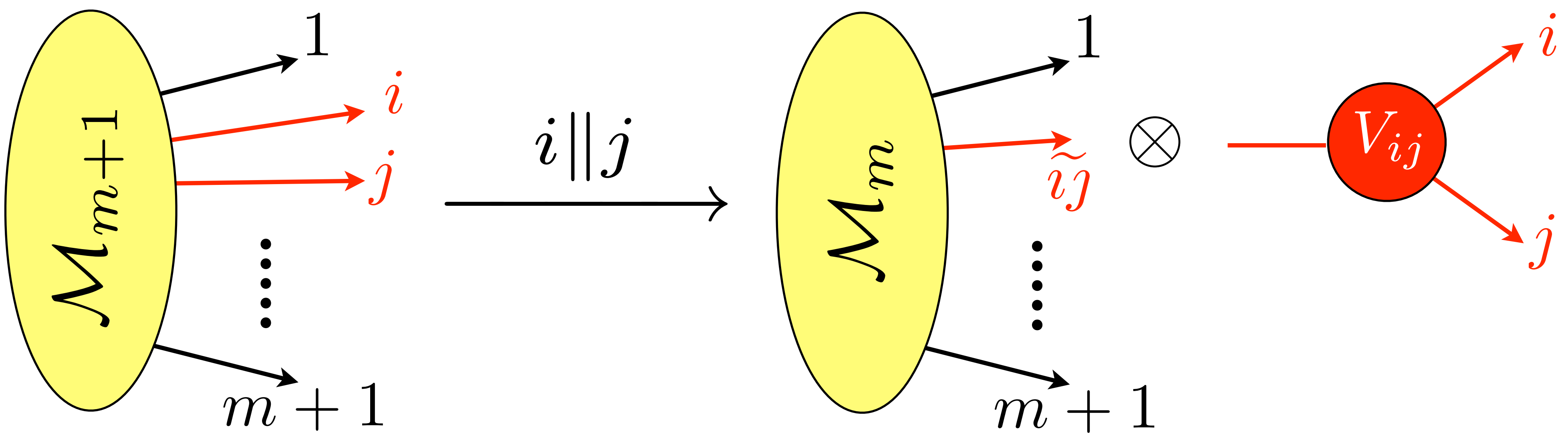
The parton shower evolution starts from the simplest hard configuration, that is usually 2->2 like.



$$\mathcal{U}(t_\mathrm{f}, t_2)\big|\mathcal{M}_2\big) = \underbrace{\mathcal{N}(t_\mathrm{f}, t_2)\big|\mathcal{M}_2\big)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\, \mathcal{U}(t_\mathrm{f}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)\big|\mathcal{M}_2\big)}^{\text{“Something happens”}}$$

Collinear Approximation

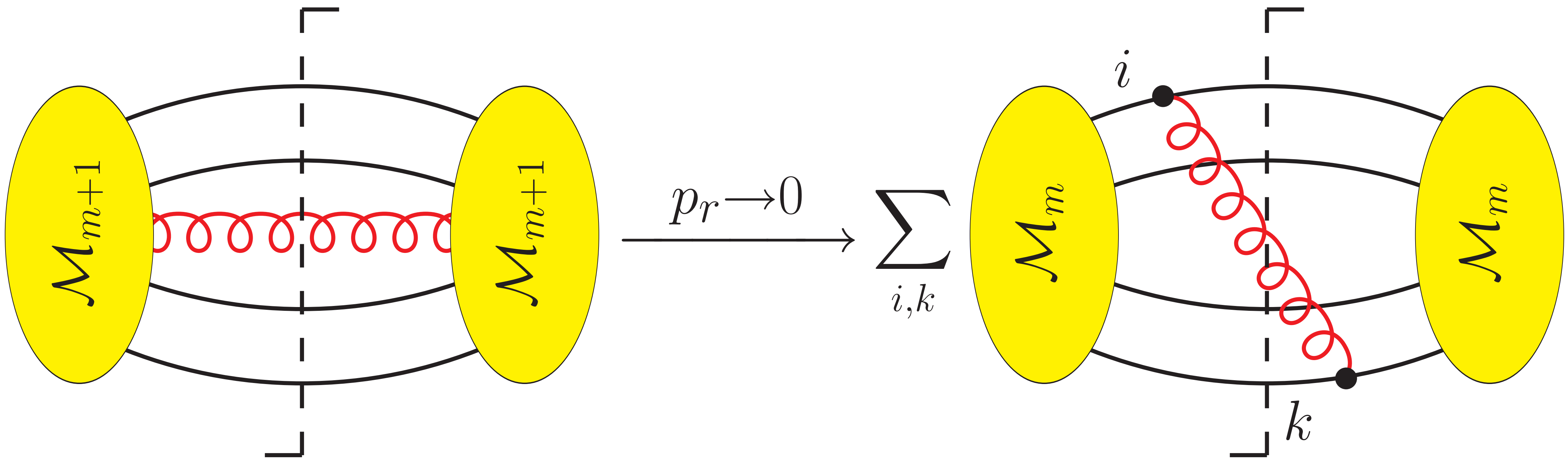
The QCD matrix elements have universal factorization property when two external partons become collinear



- Produces leading and next-to-leading logarithms.
- It is diagonal color, no color correlations.
- The gluon splitting is not diagonal in spin.
- The spin correlations are not really complicated but one can use average spin *as extra approximation*.

Soft Approximation

The QCD matrix elements have universal factorization property when an external gluon becomes soft



- Soft contributions produce next-to-leading logarithms.
- No spin correlation.
- Soft gluon connects everywhere and the color structure is not diagonal; *quantum interferences*.
- Does it spoil the independent evolution picture? Yes, it does, but ...

Color Coherence

There are three way to deal with the soft gluon color interferences:

1. The soft gluon contributions are cancelled in the wide angle region. One can apply angular ordering (Herwig/Herwig++) or impose angular ordering by angular veto (old Phytia). This is an extra approximation, especially for massive quarks. In the massive quark case the color coherence breaks down.

2. One can do leading color approximation. In the large Nc limit the soft gluon is radiated from a color dipole. The leading color contributions are diagonal in color space, thus no technical complication with colors. (Ariadne, new Phytia)

3. No extra approximation, treat the soft gluon as it is. Suppression of the wide angle radiations is a result. This is not the popular way, leads to negative weights.

Facts you should beware of

The shower is derived from QCD but you cannot use the shower cross sections as QCD prediction.

- ✗ Very crude approximation in the phase space. Angular ordered shower doesn't cover the whole phase space (dead cone).
  - ➡ In every step of the shower the phase space should be exact, every parton should be onshell.
- ✗ The independent emission picture is valid only in the strict collinear limit. The color correlations are not considered properly even at leading color level.
  - ➡ Color and spin correlation must be considered systematically. We should work with exact color and spin correlations.
- ✗ The parton shower algorithms use several technical parameters.
  - ➡ Since the QCD matrix elements don't have technical parameters, the parton shower should be free of them.

Facts you should beware of

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Cross sections at $\sqrt{s} = 1960$ GeV, with structure functions, in nanobarns, $p_T > 10\,\mathrm{GeV}$ $|\eta| < 2.0$.

Process	σ_0 : Normal	σ_1 : Large Nc component	$\frac{\sigma_1 - \sigma_0}{\sigma_0}$
ud→W+g	0.1029(5)D+01	0.1158(5)D+01	13%
ud→W+gg	0.1018(8)D+00	0.1283(10)D+00	26%
ud→W+ggg	0.1119(17)D-01	0.1564(22)D-01	40%
ud→W+gggg	0.1339(36)D-02	0.2838(71)D-02	120%

Results were calculated by HELAC

- ✗ The parton shower algorithms use several technical parameters.
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More Questions

- ✗ They are not defined systematically *e.g.*: angular ordering at NLO level??? Even the kinematics of the color dipole model is inconsistent at higher order.
  - ➡ The core algorithm shouldn't depend on the level of the calculation.
- ✗ The only exact matrix element in the calculation is $2 \to 2$ like.
  - ➡ In a 3,4,..-jet calculation we should use the $2 \to 3, 4, 5, \ldots$ parton exact matrix elements at least at tree level.
- ✗ Since the strong coupling is large even the exact tree level matrix elements are not enough.
  - ➡ The shower should be matched to the NLO fix order calculation.
- ✗ How to go beyond the LO shower?
- ✗ Hadronization model, Underlying event
- ✗ ...

LO Matching Schemes

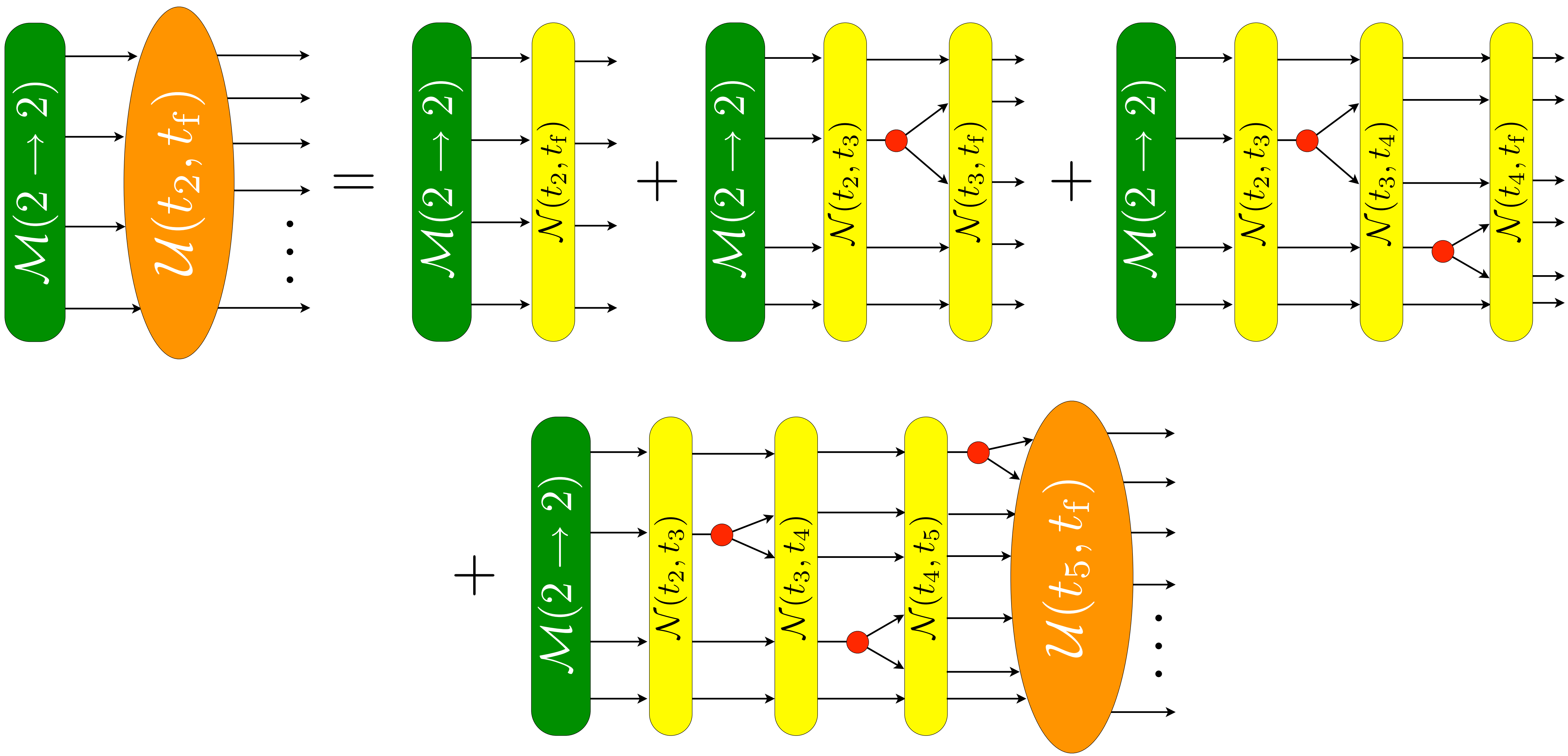
There are two algorithm available in the literature for LO matching:

- CKKW-L algorithm: Reweighting Born matrix elements with Sudakov factors
  S. Catani, R. Kuhn, F. Krauss, B. Webber: **JHEP** 0111:063,2001
  L. Lönnblad: **JHEP** 0205:046,2002

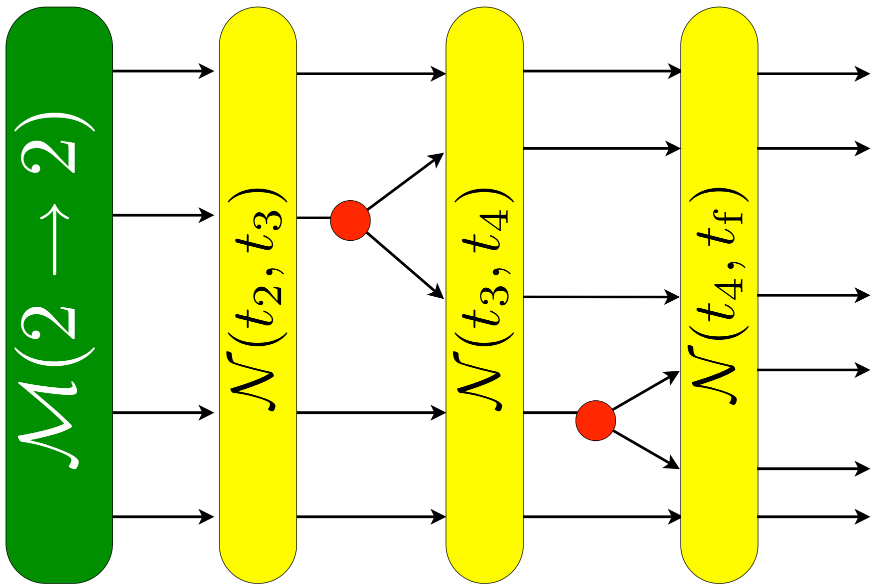
- MLM algorithm: Reweighting shower contributions with Born level matrix elements
  M. Mangano
  M. Mangano , M. Moretti, F. Piccinini, M. Treccani: **JHEP** 0701:013,2007

Shower Cross Section

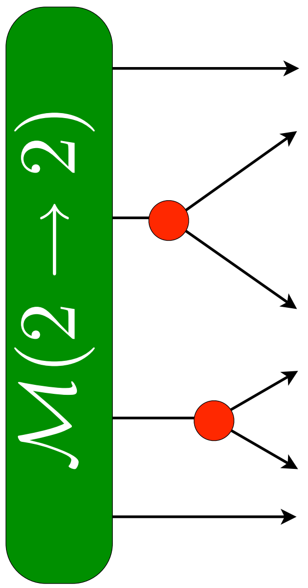
Iterating the evolution twice, then we have



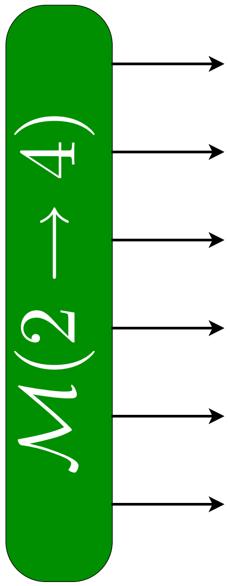
Deficiency of Shower



Standard shower contribution



Small pT approximation

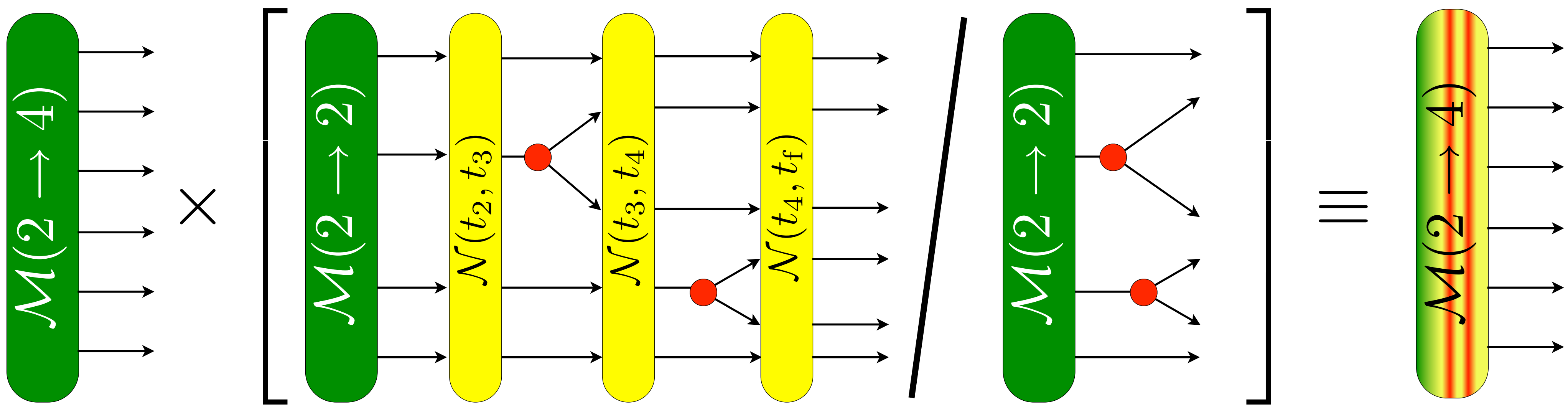


$|\mathcal{M}(2 \to 4)|^2$

- The shower approximation relies on the small pT splittings.
- May be the exact matrix element would be better.
- But that lacks the Sudakov exponents.

Rewieght the exact matrix elements with Sudakov exponents

Improved weighting

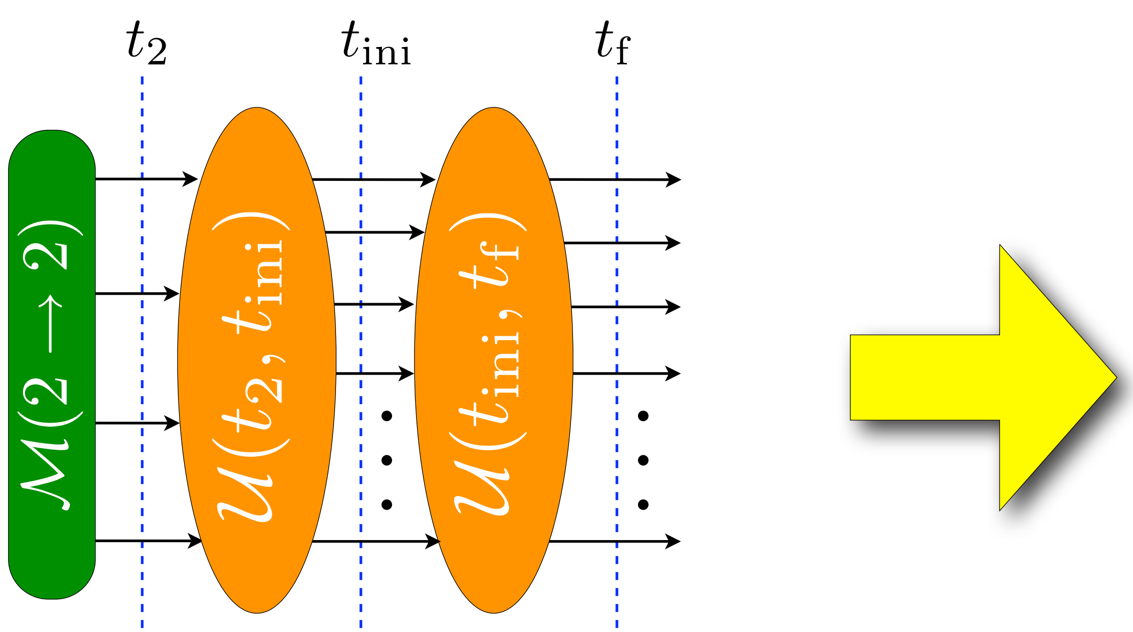


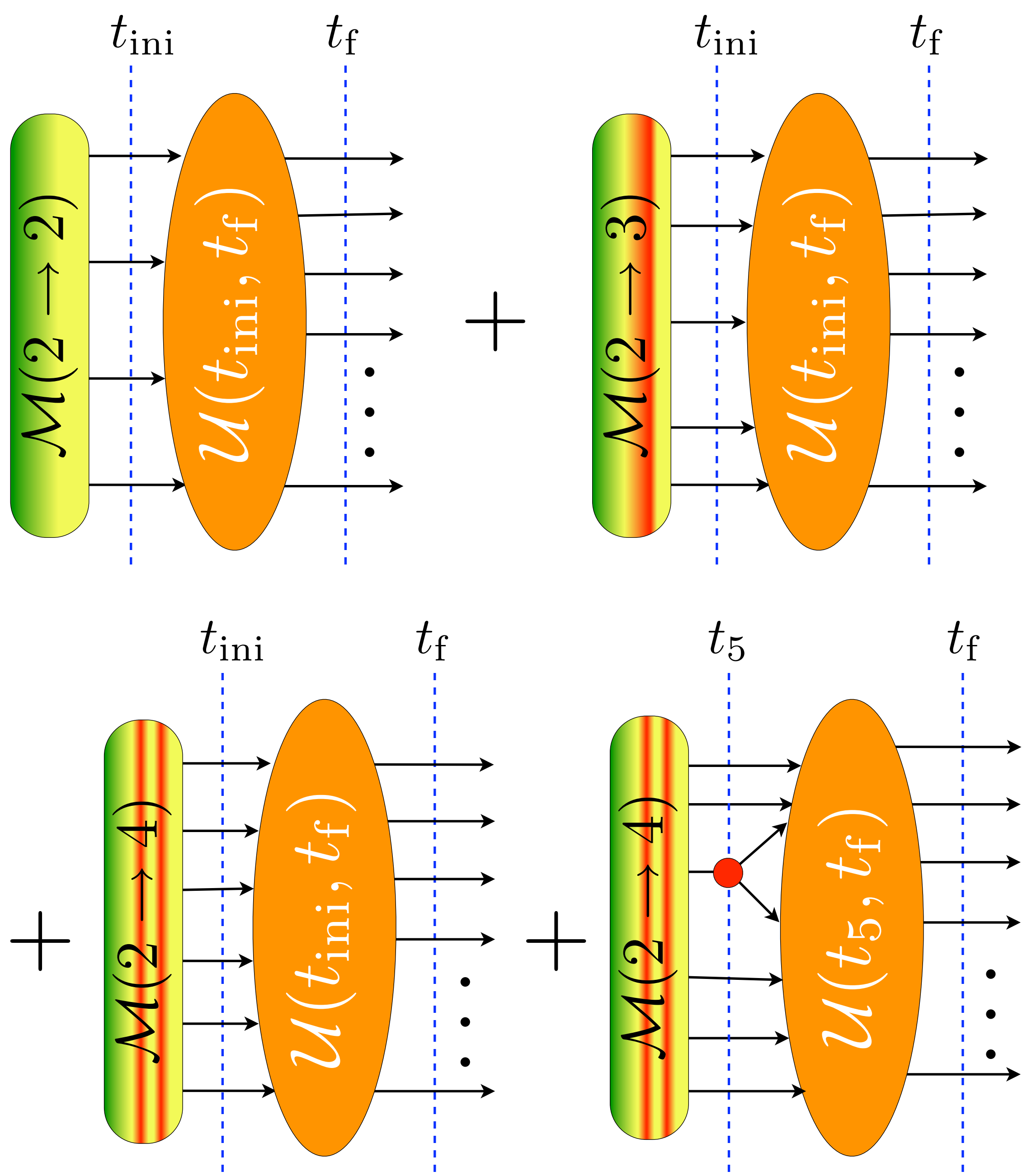
Rewieghting factor

- This is the essential part of the CKKW matching procedure.
- In general there are many ways to get from $2\to2$ configuration to $2\to m$ configuration.
- CKKW use the kT algorithm to find a unique history to define the Sudakov reweighting.
- The unique history requires to introduce matching scale.

CKKW Algorithm

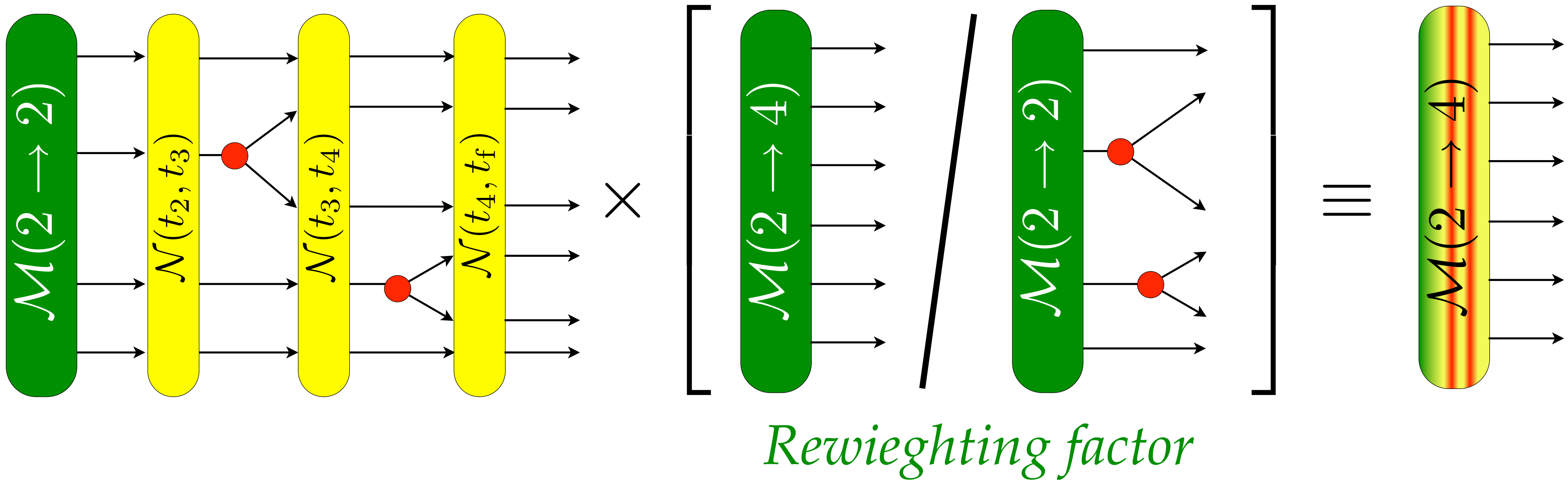
CKKW break the evolution
into $0 < t < t_{\rm ini}$ and $t_{\rm ini} < t < t_{\rm f}$





- CKKW use improve weighting for $0 < t < t_{\rm ini}$
- For $t_{\rm ini} < t < t_{\rm f}$ they have standard shower (in Herwig and old Phytia case transverse momentum veto is needed)
- They use the kT algorithm and NLL Sudakov factors to do the reweighting.

MLM Algorithm



- This is the essential part of the MLM matching procedure.
- MLM algorithm use the cone jet finding algorithm to define the ratio
- No analytic Sudakov factors, it use the native Sudakov of the underlying parton shower.
- Matching parameters: $p_{T_{\min}}, \eta_{\max}, R_{\min}$

LO Matching Schemes

✓ The CKKW-L algorithm is implemented in Sherpa and a slightly modified version in Ariadne. The Ariadne implementation gives better matching since it use the native Sudakov exponent of the underlaying shower algorithm.

✓ It is certainly a big improvement.

✗ Only normalized cross section can be calculated.

✗ The result could strongly depend on the matching scale.

➡ It would be nice NOT to use matching scale.

✗ Matching scale dependence cancelled at NLL level but only in e+e- annihilation.

✗ It is still LO order calculation thus the scale dependence is large.

➡ The algorithm can be generalized at NLO level. *ZN and D. Soper:* **JHEP** 0510:024,2005

NLO Matching Schemes

There are several algorithm available in the literature for NLO matching:

- MC@NLO: Avoiding double counting by introducing extra subtract terms.

  S. Frixione and B. Webber: **JHEP** 0206:029,2002
  S. Frixione, P. Nason and B. Webber: **JHEP** 0308:007,2003

- KS approach: The main idea is to include the first step of the shower in NLO calculation and then start the shower from this configuration.

  M. Krämer and D. Soper: **Phys.Rev.** D69:054019,2004
  ZN and D. Soper: **JHEP** 0510:024,2005
  P. Nason: **JHEP** 0411:040,2004

NLO Calculation

The NLO fix order calculations can be organized by the following way

$$\sigma_{\mathrm{NLO}} = \int_m \left[d\sigma^B + d\sigma^V + d\sigma^C + \int_1 d\sigma^A\right] F_{\mathrm{J}}^{(m)} + \int_{m+1} \left[d\sigma^{\mathrm{R}}_{m+1} F_{\mathrm{J}}^{(m+1)} - d\sigma^{\mathrm{A}}_{m+1} F_{\mathrm{J}}^{(m)}\right]$$

The born ($d\sigma^B$) and the real ($d\sigma^R$) are based on the *m* and *m+1* parton matrix elements, respectively and $d\sigma^V$ is the contribution of the virtual graphs. The universal collinear counterterm is $d\sigma^C$. The approximated *m+1* parton matrix element has universal structure

$$\boxed{d\sigma^{\mathrm{A}} \sim S \otimes |\mathcal{M}_m|^2}$$

It has the same singularity structure as $d\sigma^R$

MC@NLO

The naive way doesn't work when we want to match the shower to NLO calculation. It leads to double counting. Frixione and Webber managed in the following way:

$$\sigma_{\mathrm{MC}} = \int_m \left[d\sigma^B + d\sigma^V + d\sigma^C + \int_1 d\sigma^A\right] I_{\mathrm{MC}}^{(2\to m)}$$

$$+ \int_{m+1} \left[d\sigma^{\mathrm{R}}_{m+1} - d\sigma^{\mathrm{MC}}_{m+1}\right] I_{\mathrm{MC}}^{(2\to m+1)} + \int_{m+1} \left[d\sigma^{\mathrm{MC}}_{m+1} - d\sigma^{\mathrm{A}}_{m+1}\right] I_{\mathrm{MC}}^{(2\to m)}$$

here m=0,1,2 only!

The $d\sigma^{\mathrm{MC}}$ term is extracted from the underlaying shower algorithm and it is subtracted and added back in different way. The function $I_{\mathrm{MC}}^{(2\to m)}$ and $I_{\mathrm{MC}}^{(2\to m+1)}$ are the interface to the shower.

$$I_{\mathrm{MC}}^{(2\to m)} \sim \mathcal{U}(t_{\mathrm{f}}, t_2) \quad \text{and} \quad I_{\mathrm{MC}}^{(2\to m+1)} \sim \mathcal{U}(t_{\mathrm{f}}, t_3)\Delta(t_3, t_2)$$

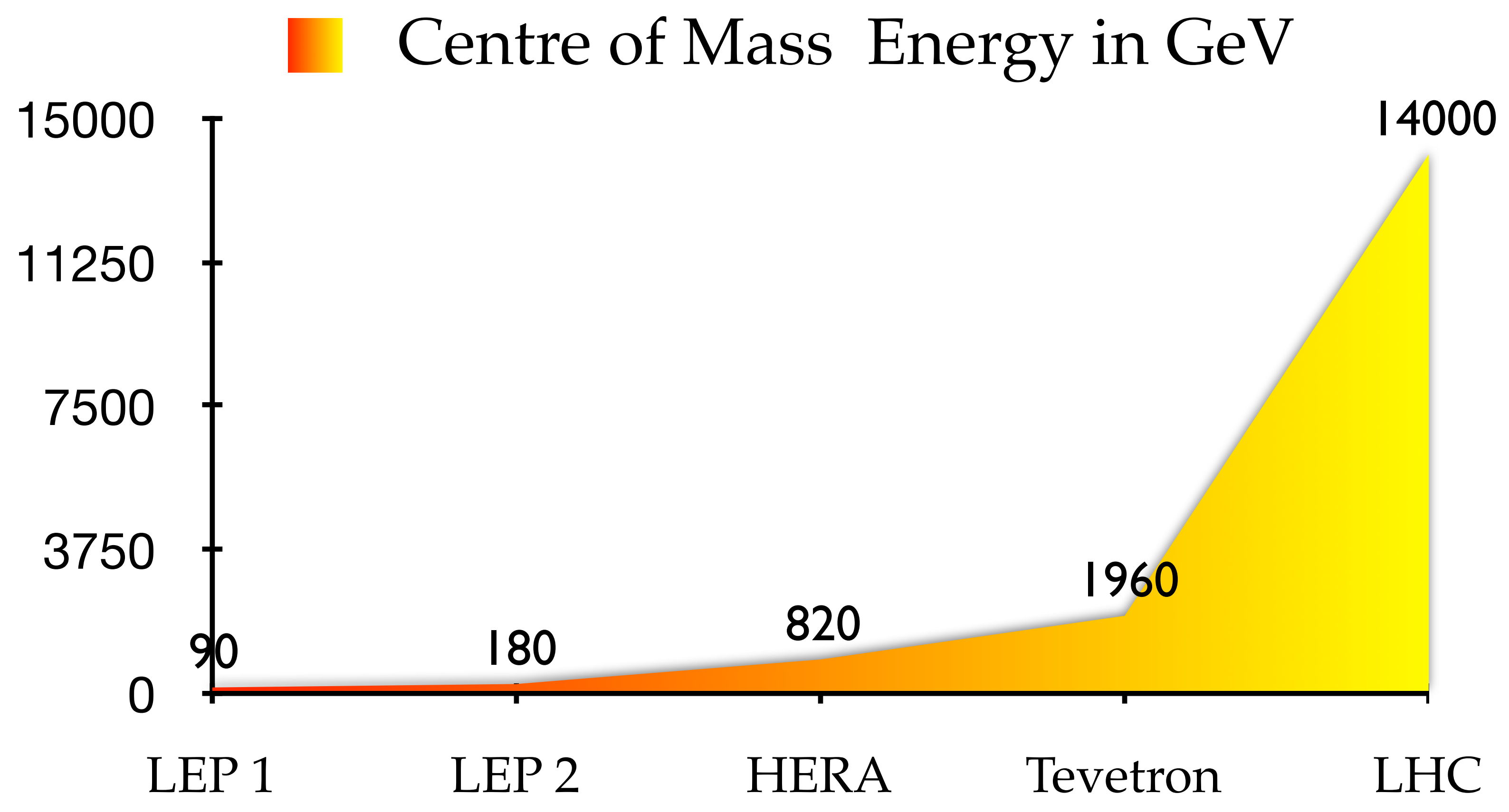
With these choices one can avoid double counting.

MC@NLO

- ✓ Several simple processes are implemented in the MC@NLO framework.
- ✓ It generates negative events.
- ✗ The MC@NLO is worked out for HERWIG. If you want to use it with PYTHIA you have to redo the MC subtraction.
- ✗ MC@NLO is defined only for the simplest processes like $2 \to 2$ processes. It is more messy if we want to calculate say 3-jet cross section.
- ✗ The double counting problem is not fully solved but it is probably invisible numerically because of the Sudakov suppression.
- ✗ The MC@NLO and the other NLO matching schemes are / could be inconsistent with the higher order contributions. The problem related to the NLO subtraction scheme.
- ✗ The MC@NLO and the Nason matching scheme wash out the spin and color correlations.

Conclusions

In this talk I had more ✗ than ✓. I addressed several questions and the current MCs don't give reasonable answers. In the parton shower program we have a lot rather *nonsystematic* approximation and "*tricks*". They could cause big problem at LHC.



There is a chance that our Monte Carlo tools will fail at LHC and that time it will be too late to do something about it.

Conclusions

- The parton shower relies on the universal soft and collinear factorization of the QCD matrix elements. It is universal property and true at all order. This should be the only approximation ...

... but we have some further approximations:

- ✗ Interference diagrams are treated approximately with the angular ordering
- ✗ Color treatment is valid in the $N_c \to \infty$ limit (correct only in e^+e^- annihilation)
- ✗ Spin treatment is usually approximated.
- ✗ Usually very crude approximation in the phase space
- ✗ "Hidden tricks"

⇒ Parton shower as classical statistical mechanics

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Parton shower as Quantum statistical mechanics

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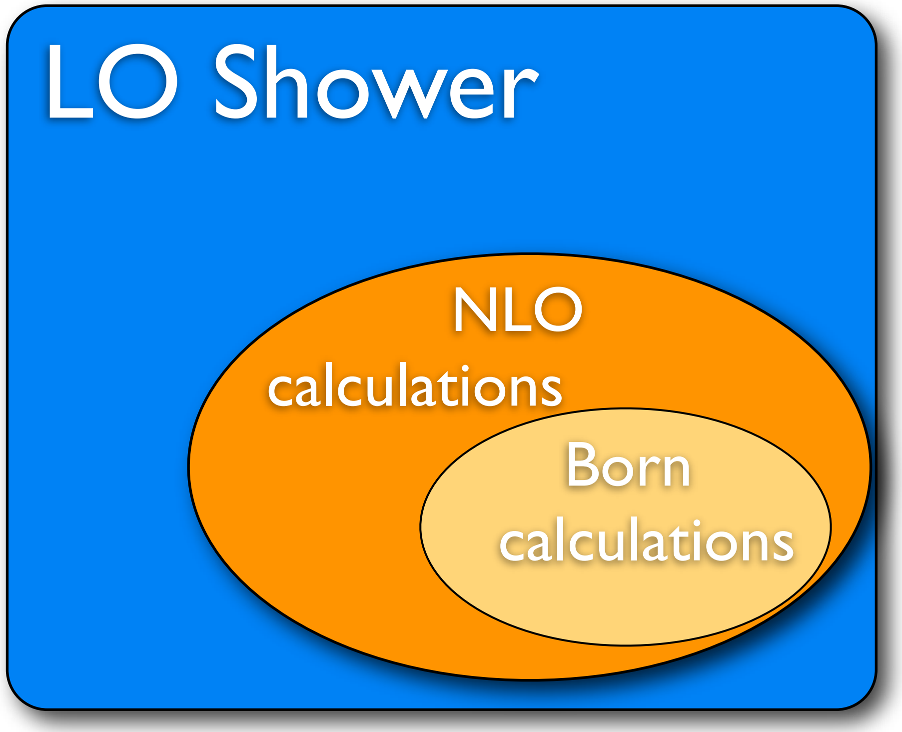
Parton shower as classical statistical mechanics

Conclusions

Instead of having defined LO, NLO and shower calculation separately and patching the gap between them by matching schemes

we should define a new shower concept that can naturally cooperate with NLO calculations



Conclusions

Or, one can be more ambitious and define this framework at NLO level.