Beyond Collins and Sivers: further measurements of the target transverse spin-dependent azimuthal asymmetries in semi-inclusive DIS from COMPASS

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- General expression for polarized SIDIS cross-section
- Target transverse spin dependent azimuthal asymmetries
- COMPASS results
- Parton model for SIDIS in CFR
- Conclusions


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## General expression of polarized SIDIS cross-section



One photon exchange approximation:

$$
M \propto J_{\mu}^{l e p t} \frac{1}{q^{2}} J_{h a d r}^{\mu}
$$

- 1) New quark distributions and semi-inclusive electroproduction on polarized nucleons. A.K. NP B441 (1995) 234
* General expression is derived
* Parton model: 6 twist-two TMD DFs \& FFs $+\sim \mathrm{k}_{\mathrm{T}} / \mathrm{Q}$ kinematical (Cahn) corrections
* All possible (except Sivers and Boer-Mulders) azimuthal asymmetries appear in this approximation
- 2) Semi-inclusive deep inelastic scattering at small transverse momentum. Bacchetta, Diehl, Goeke, Metz, Mulders and Schlegel JHEP 0702:093,2007
* General expression: new notations
* Parton model: all twist-two and twist-three tree level contributions are considered


## General expression of polarized SIDIS cross-section (2)



Using current conservation + parity conservation + hermiticity one can show that 18 independent Structure Functions describe one particle SIDIS.

Moreover, the dependences on azimuthal angle of produced hadron and of the target nucleon polarization were calculated explicitly and factorized

## General expression of polarized SIDIS cross-section (3)

$$
\begin{aligned}
& \frac{d \sigma}{d x d y d \psi d z d \phi_{h} d P_{h \perp}^{2}}= \\
& \frac{a^{2}}{x y Q^{2}} \frac{y^{2}}{2(1-\varepsilon)}\left(1+\frac{\gamma^{2}}{2 x}\right)\left\{F_{U U, T}+\varepsilon F_{U U, L}+\underline{\underline{\sqrt{2 \varepsilon(1+\varepsilon)}} \cos \phi_{h} F_{U U}^{\cos \phi_{h}}}\right. \\
& +\varepsilon \cos \left(2 \phi_{h}\right) F_{U U}^{\cos 2 \phi_{h}}+P_{\text {beam }} \sqrt{2 \varepsilon(1-\varepsilon)} \sin \phi_{h} F_{L U}^{\text {sin } \phi_{h}} \\
& +P_{L}\left[\underline{\sqrt{2 \varepsilon(1+\varepsilon)} \sin \phi_{h} F_{U L}^{\sin \phi_{h}}+\varepsilon \sin \left(2 \phi_{h}\right) F_{U L}^{\sin 2 \phi_{h}}}\right] \\
& +P_{L} P_{\text {beam }}\left[\sqrt{1-\varepsilon^{2}} F_{L L}+\underline{\underline{\sqrt{2 \varepsilon(1-\varepsilon)}} \cos \phi_{h} F_{L L}^{\cos \phi_{h}}}\right] \\
& +\mid P_{T}\left[\underline{\underline{\sin \left(\phi_{h}-\phi_{S}\right)\left(F_{U T, T}^{\sin \left(\phi_{h}-\phi_{S}\right)}+\varepsilon F_{U T, L}^{\sin \left(\phi_{h}-\phi_{S}\right)}\right)}}\right. \\
& +\varepsilon \sin \left(\phi_{h}+\phi_{S}\right) F_{U T}^{\sin \left(\phi_{h}+\phi_{S}\right)}+\varepsilon \sin \left(3 \phi_{h}-\phi_{S}\right) F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)} \\
& \left.+\underline{\sqrt{2 \varepsilon(1+\varepsilon)} \sin \phi_{S} F_{U T}^{\sin \phi_{S}}}+\underline{\underline{\sqrt{2 \varepsilon(1+\varepsilon)} \sin \left(2 \phi_{h}-\phi_{S}\right) F_{U T}^{\sin \left(2 \phi_{h}-\phi_{s}\right)}}}\right] \\
& \begin{array}{l}
+\left|P_{T}\right| P_{\text {beam }}\left[\underline{\sqrt{1-\varepsilon^{2}} \cos \left(\phi_{h}-\phi_{S}\right) F_{L T}^{\cos \left(\phi_{h}-\phi_{S}\right)}}+\underline{\sqrt{2 \varepsilon(1-\varepsilon)} \cos \phi_{S} F_{L T}^{\cos \phi_{S}}}\right. \\
\left.\left.+\quad+\sqrt{2 \varepsilon(1-\varepsilon)} \cos \left(2 \phi_{h}-\phi_{S}\right) F_{L T}^{\cos \left(2 \phi_{h}-\phi_{S}\right)}\right]\right\},
\end{array}
\end{aligned}
$$

$$
\varepsilon=\frac{d \psi \approx d \phi_{S}}{1-y-\frac{1}{4} y^{2} \gamma^{2}} \begin{gathered}
1-y+\frac{1}{2} y^{2}+\frac{1}{4} y^{2} \gamma^{2} \\
\gamma=2 x_{\mathrm{B}} M_{p} / Q
\end{gathered}
$$

This is a general expression which is also valid for exclusive reactions and for entire phase space of SIDIS (TFR, CFR)

## Azimuthal modulations:

2 polarization independent
1 single beam polarization dependent 2 single target longitudinal polarization dependent
1 double beam + target longitudinal polarization
dependent
5 single target transverse polarization dependent
3 double beam + target transverse polarization dependent

## Target transverse spin dependent azimuthal asymmetries

$$
\begin{aligned}
d \sigma\left(\phi_{h}, \phi_{s}, \ldots\right) & \propto\left(1+\left|\boldsymbol{S}_{T}\right| \sum_{i=1}^{5} D^{w_{i}\left(\phi_{h}, \phi_{s}\right)} A_{U T}^{w_{i}\left(\phi_{h}, \phi_{s}\right)} w_{i}\left(\phi_{h}, \phi_{s}\right)\right. \\
& \left.+P_{\text {beam }}\left|\boldsymbol{S}_{T}\right| \sum_{i=6}^{8} D^{w_{i}\left(\phi_{h}, \phi_{s}\right)} A_{L T}^{w_{i}\left(\phi_{h}, \phi_{s}\right)} w_{i}\left(\phi_{h}, \phi_{s}\right)+\ldots\right)
\end{aligned}
$$

$$
A_{B T}^{w_{i}\left(\phi_{h}, \phi_{s}\right)} \equiv \frac{F_{B T}^{w_{i}\left(\phi_{h}, \phi_{s}\right)}}{F_{U U, T}}
$$

Collins

| $w_{1}\left(\phi_{h}, \phi_{s}\right) \sin \left(\phi_{h}-\phi_{s}\right)$, |
| :--- |
| $w_{2}\left(\phi_{h}, \phi_{s}\right)=\sin \left(\phi_{h}+\phi_{s}\right)$, |
| $w_{3}\left(\phi_{h}, \phi_{s}\right)=\sin \left(3 \phi_{h}-\phi_{s}\right)$ |
| $w_{4}\left(\phi_{h}, \phi_{s}\right)=\sin \left(\phi_{s}\right)$, |
| $w_{5}\left(\phi_{h}, \phi_{s}\right)=\sin \left(2 \phi_{h}-\phi_{s}\right)$ |
| $w_{6}\left(\phi_{h}, \phi_{s}\right)=\cos \left(\phi_{h}-\phi_{s}\right)$, |
| $w_{7}\left(\phi_{h}, \phi_{s}\right)=\cos \left(\phi_{s}\right)$, |
| $w_{8}\left(\phi_{h}, \phi_{s}\right)=\cos \left(2 \phi_{h}-\phi_{s}\right)$ |

$$
\begin{aligned}
& D^{\sin \left(\phi_{h}-\phi_{s}\right)}(y)=1, \\
& D^{\sin \left(\phi_{h}+\phi_{s}\right)}(y)=D^{\sin \left(3 \phi_{h}+\phi_{s}\right)}(y)=D_{N N}(y)=\frac{2(1-y)}{1+(1-y)^{2}}, \\
& D^{\sin \left(2 \phi_{h}-\phi_{s}\right)}(y)=D^{\sin \left(\phi_{s}\right)}(y)=\frac{2(2-y) \sqrt{1-y}}{1+(1-y)^{2}}, \\
& D^{\cos \left(\phi_{h}-\phi_{s}\right)}(y)=D(y)=\frac{y(2-y)}{1+(1-y)^{2}}, \\
& D^{\cos \left(2 \phi_{h}-\phi_{s}\right)}(y)=D^{\cos \left(\phi_{s}\right)}(y)=\frac{2 y \sqrt{1-y}}{1+(1-y)^{2}} .
\end{aligned}
$$

## The COMPASS experiment @ CERN

hep-ex/0703049

- high energy beam
- large angular acceptance
- broad kinematical range
beam: $160 \mathrm{GeV} / \mathrm{c}$ longitudinal polarisation -76\% intensity $\quad 2 \cdot 10^{8} \mu^{+} /$spill (4.8s/16.2s)
two stages spectrometer
Large Angle Spectrometer (SM1), Small Angle Spectrometer (SM2)
tracking, calorimetry, PID
NEW TECHNOLOGIES



## COMPASS target system (2002-2004)

## solid state target operated in frozen spin mode

$$
{ }^{3} \mathrm{He}-{ }^{4} \mathrm{He} \text { Dilution }
$$ refrigerator (T~50mK)

superconductive
Solenoid (2.5 T) Dipole (0.5 T)
two 60 cm long cells
with opposite polarisation (systematics)

during data taking with transverse polarization polarization reversal in the 2 cells after $\sim 5$ days

## EVENT SELECTION



## Asymmetry extraction

The number-of-events $\quad A_{U T, \text { raw }}^{w_{i}\left(\phi_{h}, \phi_{s}\right)}=D^{w_{i}\left(\phi_{h}, \phi_{s}\right)}(y) f\left|S_{T}\right| A_{U T}^{w\left(\phi_{h}, \phi_{s}\right)}, \quad(i=1,5)$, asymmetries $\quad A_{L T, \text { raw }}^{w\left(\phi_{h}, \phi_{s}\right)}=D^{w\left(\phi_{h}, \phi_{s}\right)}(y) f P_{\text {beam }}\left|S_{T}\right| A_{L T}^{w\left(\phi_{h}, \phi_{s}\right)}, \quad(i=6,8)$

|  | $\Phi_{1}=\phi_{h}-\phi_{s}$ |
| :--- | :--- |
|  | $\Phi_{2}=\phi_{h}+\phi_{s}$ |
| Independent angles | $\Phi_{3}=3 \phi_{h}-\phi_{s}$ |
|  | $\Phi_{4}=\phi_{s}$ |
|  | $\Phi_{5}=2 \phi_{h}-\phi_{s}$ |

$$
\begin{aligned}
& W_{1}\left(\Phi_{1}\right)=A_{\text {raw }}^{w_{1}\left(\phi_{h}, \phi_{s}\right)} \sin \left(\Phi_{1}\right)+A_{\text {raw }}^{w_{6}\left(\phi_{h}, \phi_{s}\right)} \cos \left(\Phi_{1}\right) \\
& W_{2}\left(\Phi_{2}\right)=A_{\text {raw }}^{w_{2}\left(\phi_{h}, \phi_{s}\right)} \sin \left(\Phi_{2}\right) \\
& W_{3}\left(\Phi_{3}\right)=A_{\text {raw }}^{w_{3}\left(\phi_{h}, \phi_{s}\right)} \sin \left(\Phi_{3}\right) \\
& W_{4}\left(\Phi_{4}\right)=A_{\text {raw }}^{w_{4}\left(\phi_{h}, \phi_{s}\right)} \sin \left(\Phi_{4}\right)+A_{\text {raw }}^{w_{7}\left(\phi_{h}, \phi_{s}\right)} \cos \left(\Phi_{4}\right) \\
& W_{5}\left(\Phi_{5}\right)=A_{\text {raw }}^{w_{5}\left(\phi_{h}, \phi_{s}\right)} \sin \left(\Phi_{5}\right)+A_{\text {raw }}^{w_{8}\left(\phi_{h}, \phi_{s}\right)} \cos \left(\Phi_{5}\right)
\end{aligned}
$$

We have used the double ratio method (as for Sivers and Collins asymmetries extraction)

## Correlations

2D fit: correlations between different modulations are small


## Results (talks by Bressan \& D'Alesio)



## Results



## Results



## Results



## Parton model for SIDIS in CFR

## $d \sigma^{l+N \rightarrow l^{\prime}+h+X} \propto D F \otimes d \sigma^{l+q \rightarrow l^{\prime}+q^{\prime}} \otimes F F$

Factorization theorem for TMD SIDIS is proven only at twist-two

## At twist-two

## Sivers

$$
\begin{aligned}
& \mathcal{P}_{N}^{q}\left(x, \mathbf{k}_{T}\right)=f_{1}^{q}\left(x, k_{T}^{2}\right)+f_{1 T}^{\perp q}\left(x, k_{T}^{2}\right) \frac{\left[\mathbf{k}_{T} \times \hat{\mathbf{P}}_{N}\right] \cdot S_{T}^{N}}{M}, \\
& f_{1}^{q}\left(x, k_{T}^{2}\right) s_{L}^{q}\left(x, \mathbf{k}_{T}\right)=g_{1 L}^{q}\left(x, k_{T}^{2}\right) \lambda_{N}+g_{1 T}^{\perp q}\left(x, k_{T}^{2}\right) \frac{\mathbf{k}_{T} \cdot \mathbf{S}_{T}^{N}}{M}, \\
& f_{1}^{q}\left(x, k_{T}^{2}\right) \mathbf{s}_{T}^{q}\left(x, \mathbf{k}_{T}\right)=h_{1 T}^{q}\left(x, k_{T}^{2}\right) \mathbf{S}_{T}^{N}+\left[h_{1 L}^{\perp q}\left(x, k_{T}^{2}\right) \lambda_{N}+h_{1 T}^{\perp q}\left(x, k_{T}^{2}\right) \frac{\mathbf{k}_{T} \cdot \mathbf{S}_{T}^{N}}{M}\right] \frac{\mathbf{k}_{T}}{M}+h_{1}^{\perp q}\left(x, k_{T}^{2}\right) \frac{\left[\mathbf{k}_{T} \times \hat{\mathbf{P}}_{N}\right]}{M} \\
& \text { Often used: } \quad h_{1}^{q}\left(x, k_{T}^{2}\right)=h_{1 T}^{q}\left(x, k_{T}^{2}\right)+\frac{k_{T}^{2}}{2 M^{2}} h_{1 T}^{\perp q}\left(x, k_{T}^{2}\right) \\
& \mathcal{P}_{q \uparrow}^{h}\left(z, \mathbf{P}_{T q}^{h}\right)=D_{q}^{h}\left(z, P_{T q}^{h}\right)+H_{1 q}^{\perp h}\left(z, P_{T q}^{h}\right) \frac{\left[\mathbf{P}_{T q}^{h} \times \hat{\mathbf{k}}^{\prime}\right] \cdot \mathbf{s}_{T}^{\prime}}{M}=D_{q}^{h}\left(z, P_{T q}^{h}\right)+s_{T}^{\prime} \frac{P_{T q}^{h}}{M} H_{1 q}^{\perp h}\left(z, P_{T q}^{h}\right) \sin \left(\phi_{\text {Collins }}\right)
\end{aligned}
$$

## Twist-two contributions



## Interpretation of target transverse spin asymmetries

$$
\begin{array}{ll}
A_{U T}^{\sin \left(\varphi_{h}-\varphi_{s}\right)} \propto f_{1 T}^{\perp q} \otimes D_{1 q}^{h} & A_{U T}^{\sin \left(\varphi_{h}+\varphi_{s}\right)} \propto h_{1}^{q} \otimes H_{1 q}^{\perp h} \\
A_{L T}^{\cos \left(\varphi_{h}-\varphi_{s}\right)} \propto g_{1 T}^{q} \otimes D_{1 q}^{h} & A_{U T}^{\sin \left(3 \varphi_{h}-\varphi_{s}\right)} \propto h_{1 T}^{\perp q} \otimes H_{1 q}^{\perp h}
\end{array}
$$

BDGMMSch: Whether and how the tree-level factorization used in the present paper extends to subleading level in $1 / Q$ is presently not known.

Twist-2 $+\mathrm{k}_{\mathrm{T}} / \mathrm{Q}$ kinematical corrections:

$$
A_{L T}^{\cos \left(\varphi_{s}\right)} \propto \frac{M}{Q} g_{1 T}^{q} \otimes D_{1 q}^{h}
$$

$$
A_{L T}^{\cos \left(2 \varphi_{n}-\varphi_{s}\right)} \propto \frac{M}{Q} g_{I T}^{q} \otimes D_{1 q}^{h}
$$

$$
A_{U T}^{\sin \left(\varphi_{s}\right)} \propto \frac{M}{Q}\left(h_{1}^{q} \otimes H_{1 q}^{\perp h}+f_{1 T}^{\perp q} \otimes D_{1 q}^{h}\right)
$$

$$
A_{U T}^{\sin \left(2 \varphi_{h}-\varphi_{s}\right)} \propto \frac{M}{Q}\left(h_{1 T}^{\perp q} \otimes H_{1 q}^{\perp h}+f_{1 T}^{\perp q} \otimes D_{1 q}^{h}\right)
$$

## Double spin $\cos \left(\varphi_{\mathrm{h}}-\varphi_{\mathrm{s}}\right)$ asymmetry



Predictions by
$g_{1 T}^{q(1)}(x) \approx x \int_{x}^{1} d y \frac{g_{1}^{q}(y)}{y}$

## Estimation of $\sin \left(3 \varphi_{\mathrm{h}}-\varphi_{\mathrm{s}}\right)$ asymmetry

Quark-Diquark model
R. Jakob, P. Mulders \& Rodrigues

NP A626, 937 (1997)

$$
R=\frac{A_{U T}^{\sin \left(3 \rho_{h}-\varphi_{s}\right)}}{A_{U T}^{\sin \left(\varphi_{h}+\varphi_{s}\right)}}
$$



$$
\mathrm{R}\left(\mathrm{x}, \mathrm{z}, P_{T}^{h}\right) \approx \frac{\left\langle k_{T}^{2}\right\rangle^{2} z^{2}}{\left(\left\langle p_{T}^{2}\right\rangle+z^{2}\left\langle k_{T}^{2}\right\rangle\right)^{2}} \frac{\left(P_{T}^{h}\right)^{2}}{2 M^{2}} \frac{\sum_{q} h_{1 T}^{\perp q}(x) H_{q}^{h}(z)}{\sum_{q} h_{1}^{q}(x) H_{q}^{h}(z)}
$$

## At COMPASS

$\langle\mathrm{z}\rangle \approx 0.4,\left\langle P_{T}^{h}\right\rangle \approx 0.5 \mathrm{GeV} / \mathrm{c}$

$$
\langle\mathrm{R}(\mathrm{x})\rangle \approx 0.02 \frac{\sum_{q} h_{1 T}^{\perp q}(x) H_{q}^{h}(0.4)}{\sum_{q} h_{1}^{q}(x) H_{q}^{h}(0.4)} \ll 1
$$

## Conclusions

- There are 8 target transverse spin dependent azimuthal modulations in one particle SIDIS
* HERMES \& COMPASS have already published the results on Collins and Sivers asymmetries
* Here we presented the remaining 6 physical asymmetries extracted from COMPASS data with transversely polarized deuteron target *Two twist-2 asymmetries can be interpreted in QCD parton model and will allow to extract unexplored DFs $g_{1 T}^{q}$ and $h_{1 T}^{\perp q}$蔂Remaining four can be interpreted as twist-3 contributions
- There are more data to come soon from COMPASS and other experiments that will yield further understanding of TMD formulation of SIDIS and other processes in QCD


## Additional slides

## Collins



## Analysis by

M. Anselmino, M. Boglione,
U. D'Alesio, F. Murgia, A.K., A. Prokudin, C. Türk

Phys.Rev.D75:054032,2007.

## Sivers



Analysis by
M. Anselmino, M. Boglione,
U. D'Alesio, F. Murgia, A.K.,
A. Prokudin

Phys.Rev.D72:094007,2005.

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## Asymmetry extraction (2)

Counting rates:

$$
N_{u / d}^{ \pm}\left(\Phi_{j}\right)=F_{u / d}^{ \pm} n_{u / d}^{ \pm} a_{u / d}^{ \pm}\left(\Phi_{j}\right) \sigma\left(1 \pm W_{j}\left(\Phi_{j}\right)\right)
$$

Double ratio method:

$$
F\left(\Phi_{j}\right)=\frac{N_{u}^{+}\left(\Phi_{j}\right) N_{d}^{+}\left(\Phi_{j}\right)}{N_{u}^{-}\left(\Phi_{j}\right) N_{d}^{-}\left(\Phi_{j}\right)},
$$

with

$$
\begin{gathered}
\sigma_{F}\left(\Phi_{j}\right)=\sqrt{\frac{1}{N_{u}^{+}\left(\Phi_{j}\right)}+\frac{1}{N_{d}^{+}\left(\Phi_{j}\right)}+\frac{1}{N_{u}^{-}\left(\Phi_{j}\right)}+\frac{1}{N_{d}^{-}\left(\Phi_{j}\right)}} \\
\text { Assuming for acceptance: } \frac{a_{u}^{+}\left(\Phi_{j}\right)}{a_{d}^{-}\left(\Phi_{j}\right)}=\frac{a_{u}^{-}\left(\Phi_{j}\right)}{a_{d}^{+}\left(\Phi_{j}\right)} \\
F\left(\Phi_{j}\right)=\operatorname{const} \frac{\left(1+W_{j}\left(\Phi_{j}\right)\right)\left(1+W_{j}\left(\Phi_{j}\right)\right)}{\left(1-W_{j}\left(\Phi_{j}\right)\right)\left(1-W_{j}\left(\Phi_{j}\right)\right)} \\
F\left(\Phi_{j}\right)=\operatorname{par}(0)\left(1+4 \operatorname{par}(1) \sin \Phi_{j}+4 \operatorname{par}(2) \cos \Phi_{j}\right)
\end{gathered}
$$

## Subleading twist (from paper (2))

$$
\begin{aligned}
& F_{U T, T}^{\sin \left(\phi_{h}-\phi_{S}\right)}=\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}}{M} f_{1 T}^{\perp} D_{1}\right], \\
& F_{U T, L}^{\sin \left(\phi_{h}-\phi_{S}\right)}=0, \\
& F_{U T}^{\sin \left(\phi_{\boldsymbol{h}}+\phi_{S}\right)}=\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}}{M_{h}} h_{1} H_{1}^{\perp}\right], \\
& F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)}=\mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)\left(\boldsymbol{p}_{T} \cdot \boldsymbol{k}_{T}\right)+\boldsymbol{p}_{T}^{2}\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}\right)-4\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)^{2}\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}\right)}{2 M^{2} M_{h}} h_{1 T}^{\perp} H_{1}^{\perp}\right], \\
& F_{U T}^{\sin \phi_{S}}=\frac{2 M}{Q} \mathcal{C}\left\{\left(x f_{T} D_{1}-\frac{M_{h}}{M} h_{1} \frac{\tilde{H}}{z}\right)\right. \\
& \left.-\frac{\boldsymbol{k}_{T} \cdot \boldsymbol{p}_{T}}{2 M M_{h}}\left[\left(x h_{T} H_{1}^{\perp}+\frac{M_{h}}{M} g_{1 T} \frac{\tilde{G}^{\perp}}{z}\right)-\left(x h_{T}^{\perp} H_{1}^{\perp}-\frac{M_{h}}{M} f_{1 T}^{\perp} \frac{\tilde{D}^{\perp}}{z}\right)\right]\right\} \\
& F_{U T}^{\sin \left(2 \phi_{h}-\phi_{S}\right)}=\frac{2 M}{Q} \mathcal{C}\left\{\frac{2\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)^{2}-\boldsymbol{p}_{T}^{2}}{2 M^{2}}\left(x f_{T}^{\perp} D_{1}-\frac{M_{h}}{M} h_{1 T}^{\perp} \frac{\tilde{H}}{z}\right)\right. \\
& -\frac{2\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}\right)\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)-\boldsymbol{k}_{T} \cdot \boldsymbol{p}_{T}}{2 M M_{h}}\left[\left(x h_{T} H_{1}^{\perp}+\frac{M_{h}}{M} g_{1 T} \frac{\tilde{G}^{\perp}}{z}\right)\right. \\
& \left.\left.+\left(x h_{T}^{\perp} H_{1}^{\perp}-\frac{M_{h}}{M} f_{1 T}^{\perp} \frac{\tilde{D}^{\perp}}{z}\right)\right]\right\}, \\
& F_{L T}^{\cos \left(\phi_{h}-\phi_{S}\right)}=\mathcal{C}\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}}{M} g_{1 T} D_{1}\right], \\
& F_{L T}^{\cos \phi_{S}}=\frac{2 M}{Q} \mathcal{C}\left\{-\left(x g_{T} D_{1}+\frac{M_{h}}{M} h_{1} \frac{\tilde{E}}{z}\right)\right. \\
& \left.+\frac{\boldsymbol{k}_{T} \cdot \boldsymbol{p}_{T}}{2 M M_{h}}\left[\left(x e_{T} H_{1}^{\perp}-\frac{M_{h}}{M} g_{1 T} \frac{\tilde{D}^{\perp}}{z}\right)+\left(x e_{T}^{\perp} H_{1}^{\perp}+\frac{M_{h}}{M} f_{1 T}^{\perp} \frac{\tilde{G}^{\perp}}{z}\right)\right]\right\} \\
& F_{L T}^{\cos \left(2 \phi_{h}-\phi_{S}\right)}=\frac{2 M}{Q} \mathcal{C}\left\{-\frac{2\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)^{2}-\boldsymbol{p}_{T}^{2}}{2 M^{2}}\left(x g_{T}^{\perp} D_{1}+\frac{M_{h}}{M} h_{1 T}^{\perp} \frac{\tilde{E}}{z}\right)\right. \\
& +\frac{2\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{k}_{T}\right)\left(\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_{T}\right)-\boldsymbol{k}_{T} \cdot \boldsymbol{p}_{T}}{2 M M_{h}}\left[\left(x e_{T} H_{1}^{\perp}-\frac{M_{h}}{M} g_{1 T} \frac{\tilde{D}^{\perp}}{z}\right)\right. \\
& \left.\left.-\left(x e_{T}^{\perp} H_{1}^{\perp}+\frac{M_{h}}{M} f_{1 T}^{\perp} \frac{\tilde{G}^{\perp}}{z}\right)\right]\right\} .
\end{aligned}
$$

$$
\begin{aligned}
x e & =x \tilde{e}+\frac{m}{M} f_{1}, \\
x f^{\perp} & =x \tilde{f}^{\perp}+f_{1}, \\
x g_{T}^{\prime} & =x \tilde{g}_{T}^{\prime}+\frac{m}{M} h_{1 T}, \\
x g_{T}^{\perp} & =x \tilde{g}_{T}^{\perp}+g_{1 T}+\frac{m}{M} h_{1 T}^{\perp}, \\
x g_{T} & =x \tilde{g}_{T}-\frac{p_{T}^{2}}{2 M^{2}} g_{1 T}+\frac{m}{M} h_{1}, \\
x g_{L}^{\perp} & =x \tilde{g}_{L}^{\perp}+g_{1 L}+\frac{m}{M} h_{L L}^{\perp}, \\
x h_{L} & =x \tilde{h}_{L}+\frac{p_{T}^{2}}{M^{2}} h_{1 L}^{\perp}+\frac{m}{M} g_{1 L}, \\
x h_{T} & =x \tilde{h}_{T}-h_{1}+\frac{p_{T}^{2}}{2 M^{2}} h_{1 T}^{\perp}+\frac{m}{M} g_{1 T}, \\
x h_{T}^{\perp} & =x \tilde{h}_{T}^{\perp}+h_{1}+\frac{p_{T}^{2}}{2 M^{2}} h_{1 T}^{\perp} . \\
x e_{L} & =x \tilde{e}_{L}, \\
x e_{T} & =x \tilde{e}_{T}, \\
x e_{T}^{\perp} & =x \tilde{e}_{T}^{\perp}+\frac{m}{M} f_{1 T}^{\perp}, \\
x f_{T}^{\prime} & =x \tilde{f}_{T}^{\prime}+\frac{p_{T}^{2}}{M^{2}} f_{\frac{1}{T}}^{\perp}, \\
x f_{T}^{\perp} & =x \tilde{f}_{T}^{\perp}+f_{1 T}^{\perp}, \\
x f_{T} & =x \tilde{f}_{T}+\frac{p_{T}^{2}}{2 M^{2}} f_{1 T}^{\perp}, \\
x f_{L}^{\perp} & =x \tilde{f}_{L}^{\perp}, \\
x g^{\perp} & =x \tilde{g}^{\perp}+\frac{m}{M} h_{1}^{\perp}, \\
x h & =x \tilde{h}+\frac{p_{T}^{2}}{M^{2}} h_{1}^{\perp} .
\end{aligned}
$$

## Asymmetry extraction (3)

Two dimensional fit

$$
\begin{aligned}
& N_{u / d}^{ \pm}\left(\phi_{h}, \phi_{s}\right)=F_{u / d}^{ \pm} n_{u / d}^{ \pm} a_{u / d}^{ \pm}\left(\phi_{h}, \phi_{s}\right) \sigma\left\{1 \pm \sum_{i=1}^{8} A_{\text {raw }}^{w_{i}\left(\phi_{h}, \phi_{s}\right)} w_{i}\left(\phi_{h}, \phi_{s}\right)\right\} \\
& F\left(\phi_{h}, \phi_{s}\right)=\frac{N_{u p}^{\dagger}\left(\phi_{h}, \phi_{s}\right) N_{\text {down }}^{\dagger}\left(\phi_{h}, \phi_{s}\right)}{N_{u p}^{\downarrow}\left(\phi_{h}, \phi_{s}\right) N_{\text {down }}^{\downarrow}\left(\phi_{h}, \phi_{s}\right)}, \\
& \sigma_{F}\left(\phi_{h}, \phi_{s}\right)=\sqrt{\frac{1}{N_{u p}^{\dagger}\left(\phi_{h}, \phi_{s}\right)}+\frac{1}{N_{\text {down }}^{\uparrow}\left(\phi_{h}, \phi_{s}\right)}+\frac{1}{N_{u p}^{\downarrow}\left(\phi_{h}, \phi_{s}\right)}+\frac{1}{N_{\text {down }}^{\downarrow}\left(\phi_{h}, \phi_{s}\right)}}
\end{aligned}
$$

Fitting function:

$$
\begin{array}{r}
F\left(\phi_{h}, \phi_{s}\right)=\operatorname{par}(0)\left[1+4\left[\operatorname{par}(1) \sin \left(\phi_{h}+\phi_{s}-\pi\right)+\operatorname{par}(2) \sin \left(3 \phi_{h}-\phi_{s}\right)+\right.\right. \\
\operatorname{par}(3) \sin \left(\phi_{h}-\phi_{s}\right)+\operatorname{par}(4) \cos \left(\phi_{h}-\phi_{s}\right)+\operatorname{par}(5) \sin \left(\phi_{s}\right)+ \\
\left.\left.\operatorname{par}(6) \sin \left(2 \phi_{h}-\phi_{s}\right)+\operatorname{par}(7) \cos \left(\phi_{s}\right)+\operatorname{par}(8) \cos \left(2 \phi_{h}-\phi_{s}\right)\right]\right]
\end{array}
$$

For each of 9 x -bins, 8 z -bins and $9 \mathrm{P}_{\mathrm{hT}}$-bins -- $64\left(\varphi_{\mathrm{h}}, \varphi_{\mathrm{S}}\right)$-bin

