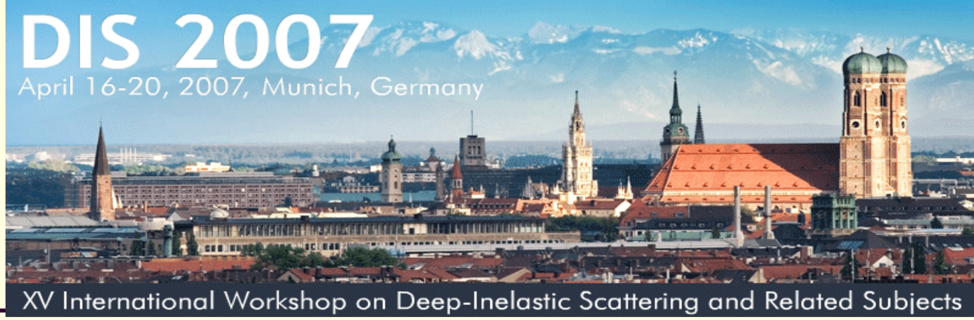

Evolution equations for Di-hadron Fragmentation Functions

Marco Radici

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A. Bacchetta – DESY F.A. Ceccopieri – Univ. Parma

Outline

1. DiFF $D(q\to h_1h_2)$ necessary to NLO calculations of
   $e+e- \to h_1\ h_2\ X$
2. Experimental info mostly on nontrivial dependence of
   $D(M)$ upon invariant mass $M(\ h_1h_2\ )$
   DiFF $D(z_1,z_2) \to$ extended DiFF (extDiFF) $D(z_1,z_2,M)$
3. "soft" scale $M \to$ new evolution equations for extDiFF
4. polarized extDiFF are spin analyzers of the fragmenting q:
   interesting scenarios in (transverse spin) azimuthal asymmetries (SSA)

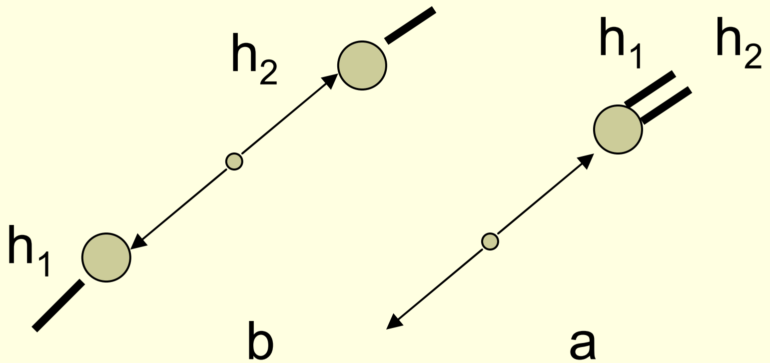
1. **Need for DiFF**
2. Exp. inv. mass M_h distribution
3. Th. M_h dependence: extDiFF
4. pol. extDiFF as spin analyzers

1. e+e- $\rightarrow h_1\, h_2 + X$ a NLO

$$\frac{d\sigma^{h_1\, h_2}}{dz_1\, dz_2\, dQ^2} = \sum_i \hat{\sigma}_i(Q^2) \otimes D^{i\to h_1 h_2}(z_1, z_2, Q^2) \quad \text{A}$$

$$+ \sum_{kl} \hat{\sigma}_{kl}(Q^2) \otimes D^{k\to h_1}(z_1, Q^2) \otimes D^{l\to h_2}(z_2, Q^2) \quad \text{B}$$

e+e- $\rightarrow h_1 h_2$ X
Konishi, Ukawa, Veneziano
P.L. **B78** (78) 243



	a	b
$O(\alpha_s^0)$	A	B
$O(\alpha_s)$	A+B	B

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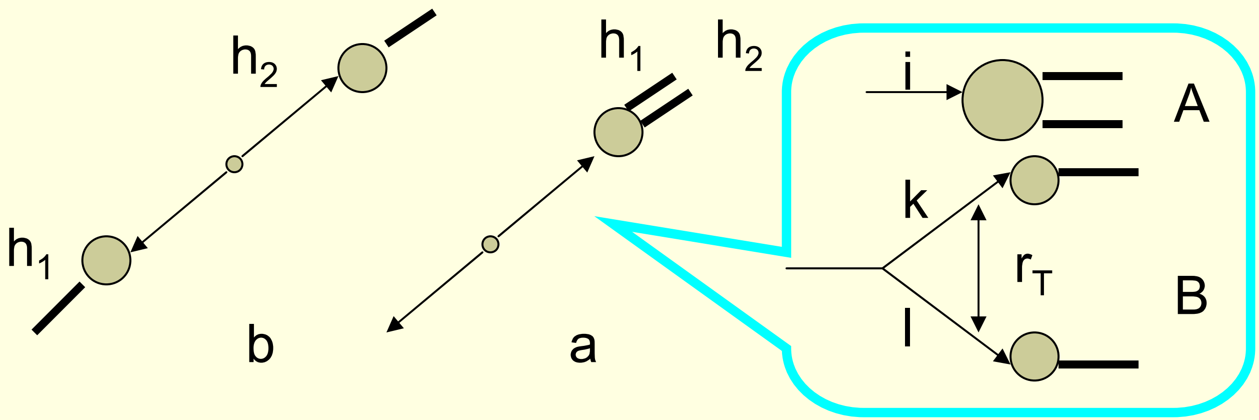
$$\frac{d\sigma^{h_1 h_2}}{dz_1\, dz_2\, dQ^2} = \sum_i \hat{\sigma}_i(Q^2) \otimes D^{i\to h_1 h_2}(z_1, z_2, Q^2) \quad \text{A}$$
$$+ \sum_{kl} \hat{\sigma}_{kl}(Q^2) \otimes D^{k\to h_1}(z_1, Q^2) \otimes D^{l\to h_2}(z_2, Q^2) \quad \text{B}$$

e+e- $\rightarrow$ $h_1 h_2$ X
Konishi, Ukawa, Veneziano
P.L. **B78** (78) 243



	a	b
$O(\alpha_s^0)$	A	B
$O(\alpha_s)$	A+B	B

de Florian and Vanni,
P.L. **B578** (04) 139

DiFF evolution equations

$1/r_T$ collinear singularities $\rightarrow$ $D(i\rightarrow h_1 h_2)$

$$\frac{d}{d\log Q^2} D^{i\to h_1 h_2} = \frac{\alpha_s}{2\pi}\Big[D^{k\to h_1 h_2} \otimes P_{ki} \quad \text{A}$$
$$+ D^{k\to h_1} \otimes D^{l\to h_2} \otimes \hat{P}^i_{kl} \Big] \quad \text{B}$$

time-like analogue of
Fracture Functions in SIDIS
Trentadue and Veneziano,
P.L. **B323** (94) 201

1. Need for DiFF
2. **Exp. inv. mass M_h distribution**
3. Th. M_h dependence: extDiFF
4. pol. extDiFF as spin analyzers

1. **e+e- semi-inclusive annihilation**
2. pp semi-inclusive collision
3. ep semi-inclusive DIS

Exp. info mostly on nontrivial spectrum in pair invariant mass M_h

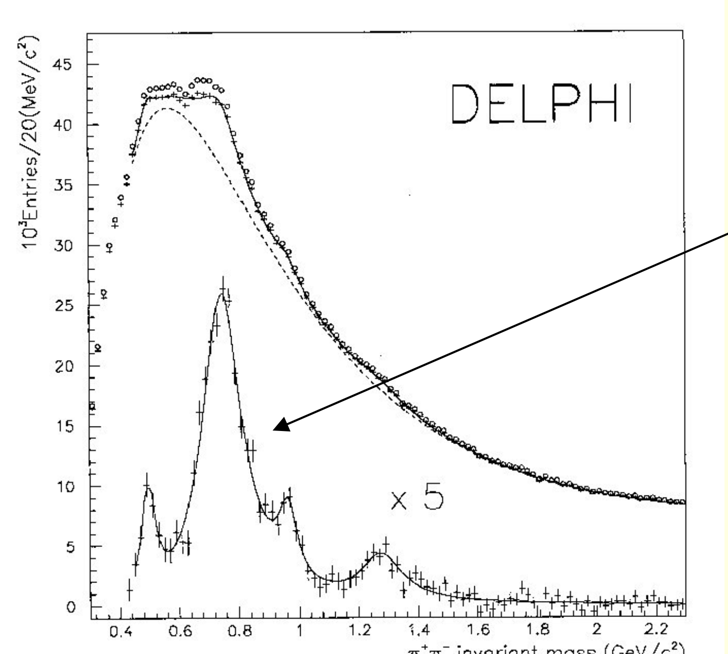


fig. 1

e+e- $\rightarrow \pi^+\pi^-$ $\sqrt{s}$=91.2 GeV z>0.1
Abreu *et al.* (DELPHI), P.L. **B298** (93) 236
background subtracted

1. Need for DiFF
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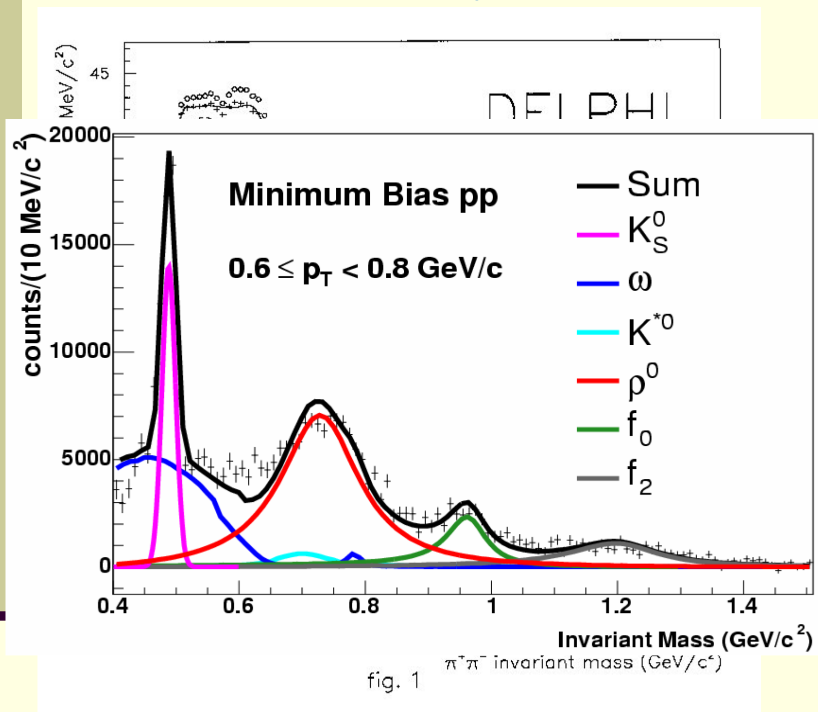

e+e- → π+π- √s=91.2 GeV z>0.1
Abreu *et al.* (DELPHI), P.L. **B298** (93) 236
background subtracted

pp → π+π- X √s=200 GeV
Adams *et al*. (STAR), P.R.L. **92** (04) 092301

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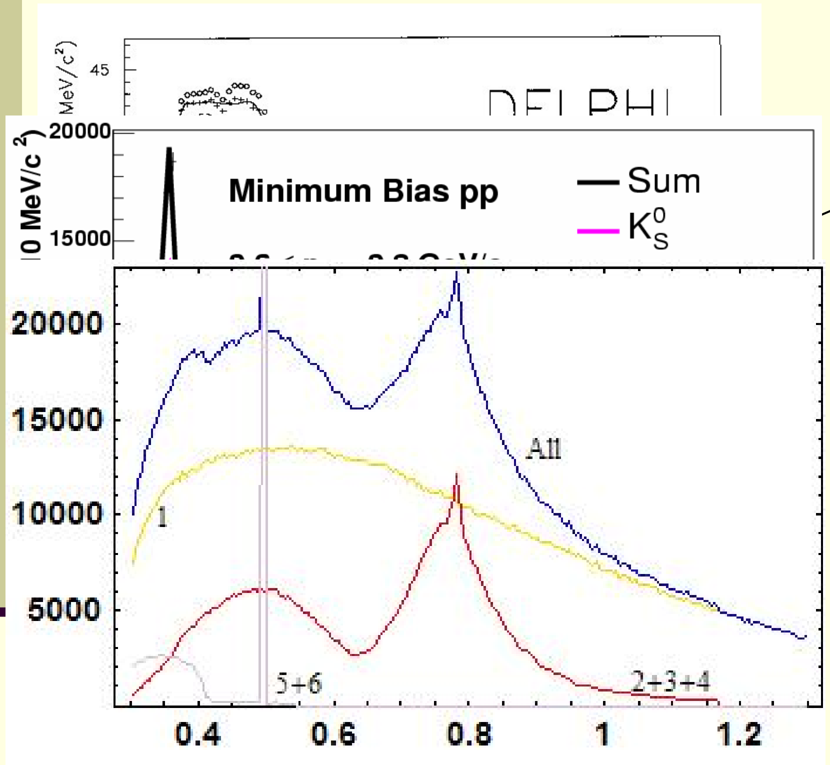

M_h

e+e- $\to \pi^+\pi^-$ $\sqrt{s}$=91.2 GeV z>0.1
Abreu *et al.* (DELPHI), P.L. **B298** (93) 236
background subtracted

pp $\to \pi^+\pi^-$ X $\sqrt{s}$=200 GeV
Adams *et al*. (STAR), P.R.L. **92** (04) 092301

ep $\to$ e' $(\pi^+\pi^-)$ X $\sqrt{s}$=7.5 GeV
PYTHIA output a kin. HERMES
$Q^2 > 1$ GeV^2 $W^2 > 4$ GeV^2
no elastic and diffractive events

1. Need for DiFF
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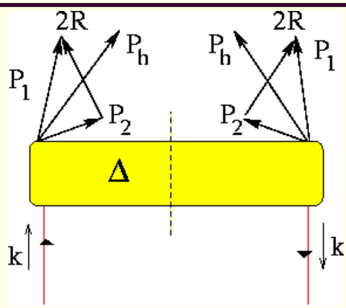
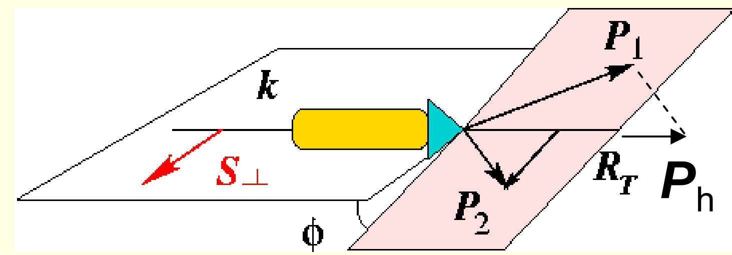


$$\Delta(k, P_h, R) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle 0|\psi(\xi)|P_1, P_2, X\rangle\langle P_1, P_2, X|\overline{\psi}(0)|0\rangle$$

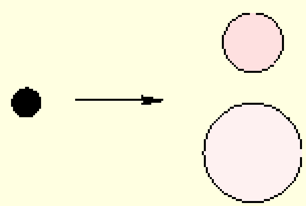
$$\Delta^{[\Gamma]} = \frac{1}{4z} \int d\mathbf{k}_T \int dk^+ \, \mathrm{Tr}\left[\Gamma\Delta(k, P_h, R)\right]\Big|_{k^- = \frac{P_h^-}{z}}$$



Bianconi *et al*,
P.R. D**62** (00) 034008

t=2

$$\Delta^{[\gamma^-]} = D_1 =$$



$$\begin{aligned} P_h^2 &= (P_1 + P_2)^2 = M_h^2 \\ R^2 &= \frac{(P_1 - P_2)^2}{4} = \frac{M_1^2 + M_2^2}{2} - \frac{M_h^2}{4} \\ R_T^2 &= \frac{z_1 z_2}{z_1 + z_2}\left[\frac{M_h^2}{z_1 + z_2} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2}\right] \end{aligned}$$

Bacchetta and Radici,
P.R. D**69** (04) 074026

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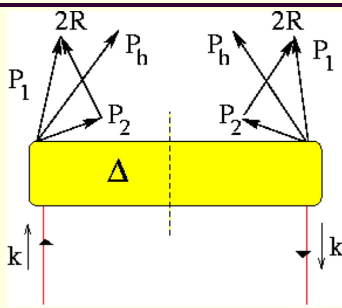

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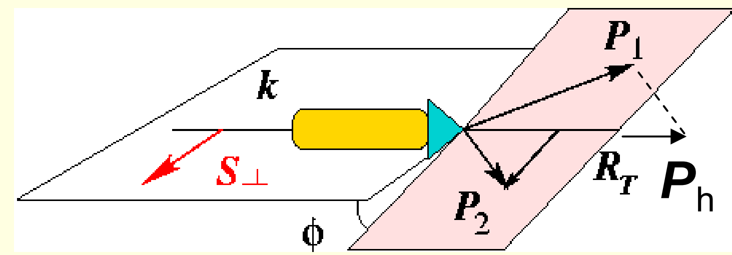

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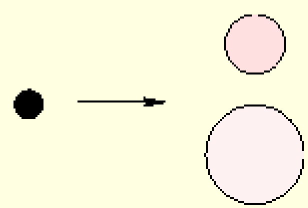

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$$\boxed{R_T^2 = \frac{z_1 z_2}{z_1 + z_2}\left[\frac{M_h^2}{z_1 + z_2} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2}\right]}$$

$$\Delta^{[\Gamma]}\,(z_1, z_2, \mathbf{R}_T) \equiv \Delta^{[\Gamma]}\,(z_1, z_2, M_h^2, \phi)$$

Bacchetta and Radici,
P.R. D**69** (04) 074026

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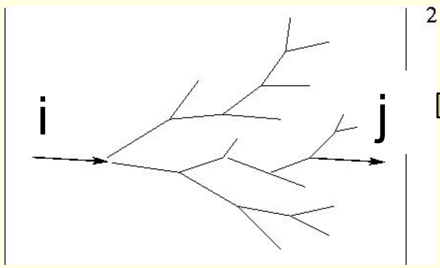
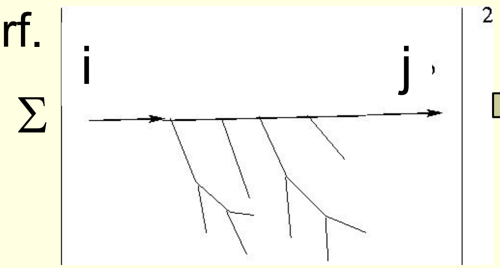
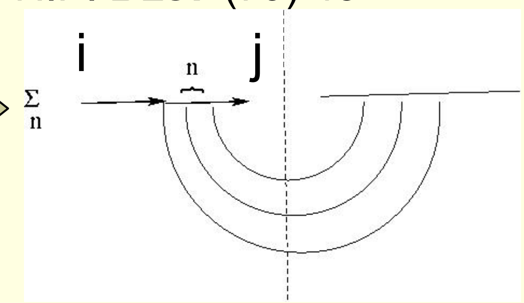
1-hadron fragmentation in Jet Calculus

Konishi, Ukawa, Veneziano
N.P. **B157** (79) 45



no interf.





branching $\leftrightarrow$ $P^{(0)}$
Leading Log (LL)

strong ordering $i \to j+k$
$q_j^2,\ q_k^2 << \varepsilon q_i^2 \quad \varepsilon <<1$

ladder diagram $\to$ exp $[P^{(0)}]$

$\equiv E_i^j$ time-like perturbative q-q fragmentation function
[at LL, resums all $\alpha_s^n \log^n(Q^2/Q_0^2)$]

$$\frac{dE_i^j}{dy} = \sum_k E_k^j \otimes P_{ki}^{(0)}$$

$$y = \frac{1}{2\pi\beta_0} \log \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \quad (dy \leftrightarrow d\log Q^2)$$

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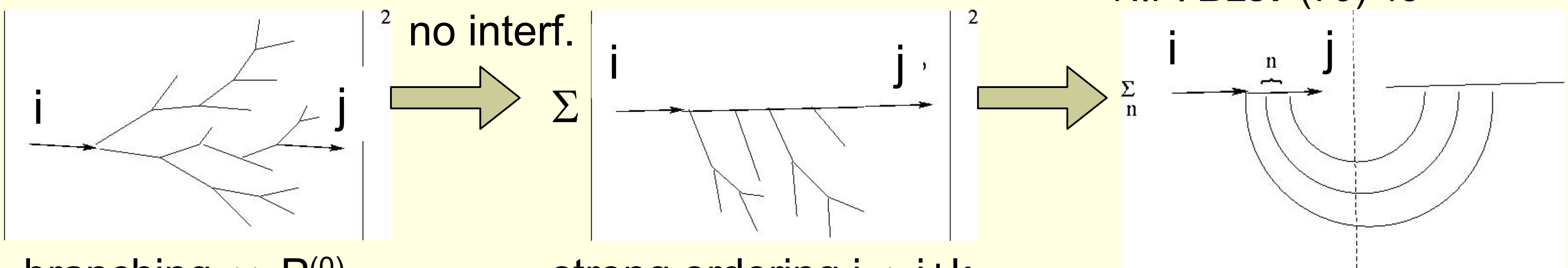

branching ↔ $P^{(0)}$
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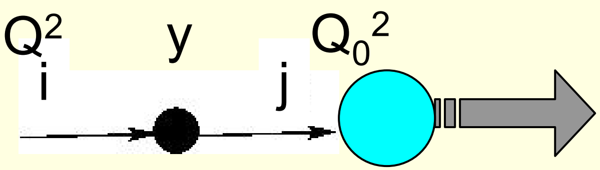

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$$y = \frac{1}{2\pi\beta_0} \log \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \quad (dy \leftrightarrow d\log Q^2)$$

$$D^{i\to h}(z,Q^2) = E_i^j(Q^2,Q_0^2) \otimes D^{j\to h}(z,Q_0^2)$$

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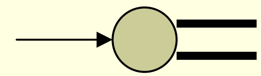
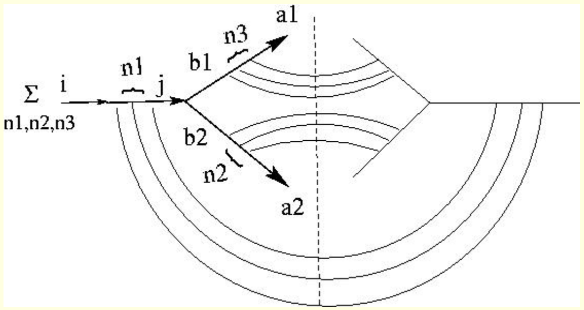
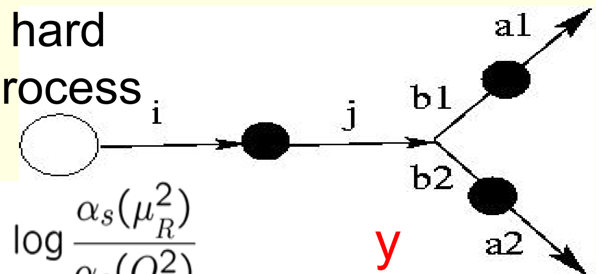
2-h fragmentation in Jet Calculus

zone "a" 



$\equiv E_i^{a1,a2}$

hard process



y_0 ($\equiv 0$)

$$Y = \frac{1}{2\pi\beta_0} \log \frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)}$$

$$E_i^{a_1,a_2}(Y) = \int_{y_0}^{Y} dy\, E_{b_1}^{a_1}(y) \otimes E_{b_2}^{a_2}(y) \otimes \hat{P}_{b_1 b_2}^{j} \otimes E_i^j(Y-y)$$

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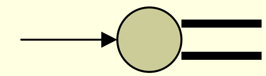
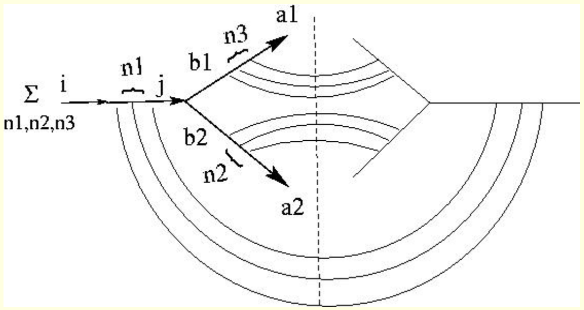
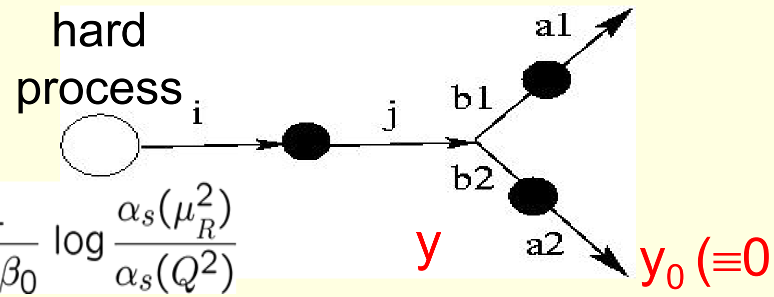
zone "a" 



$\equiv E_i^{a1,a2}$



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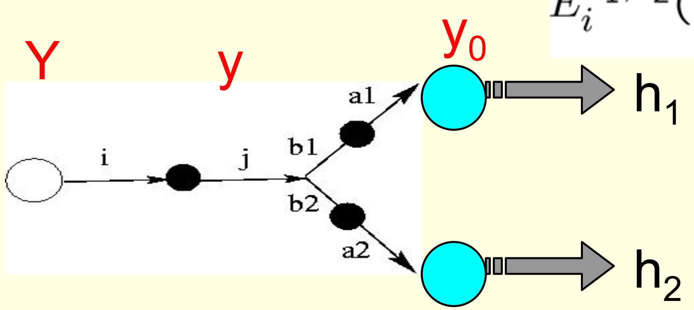
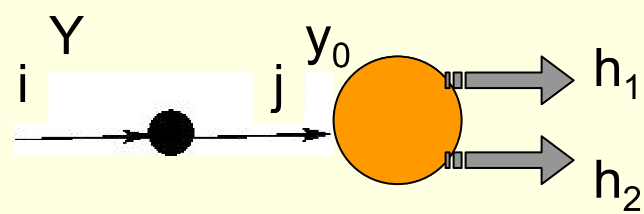
B : 

+ A : 

$$D^{i\to h_1 h_2} = E_i^{a_1,a_2}(Y) \otimes D^{a_1\to h_1} \otimes D^{a_2\to h_2} + E_i^{j}(Y-y_0) \otimes D_A^{j\to h_1 h_2}$$

$$\frac{d}{dY} D^{i\to h_1 h_2} = \frac{\alpha_s}{2\pi} \left[D^{b_1\to h_1} \otimes D^{b_2\to h_2} \otimes \hat{P}_{b_1 b_2}^{i} + D^{j\to h_1 h_2} \otimes P_{ji} \right]$$

☑

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4. pol. extDiFF as spin analyzers

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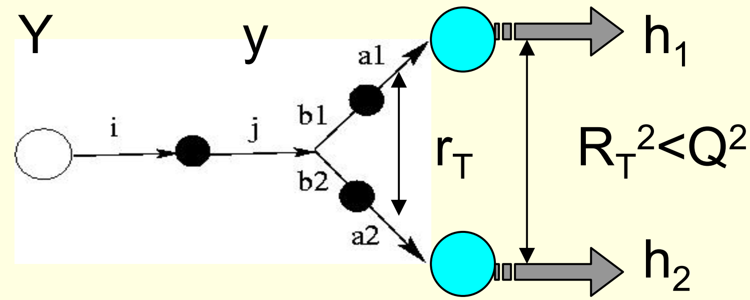
introducing the scale R_T

B:



fixing $R_T \rightarrow$ scale k_j^2 ($\Leftrightarrow$ y) no longer arbitrary

$$k_j^2 = \frac{k_{b_1}^2}{u} + \frac{k_{b_2}^2}{1-u} + \frac{r_T^2}{4u(1-u)} \overset{\text{LL}}{\approx} r_T^2 \approx R_T^2$$

$$y_T = \frac{1}{2\pi b_0} \log\left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(R_T^2)}\right] \qquad \frac{d}{dR_T^2} = \frac{\alpha_s(R_T^2)}{2\pi R_T^2}\frac{d}{dy_T}$$

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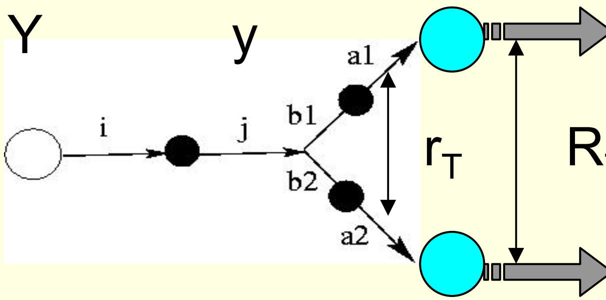
$$E_i^{a_1,a_2} = \int_{y_0}^{Y} dy\, E_{b_1}^{a_1}(y) \otimes E_{b_2}^{a_2}(y) \otimes \hat{P}_{b_1 b_2}^{j} \otimes E_i^{j}(Y-y)$$

B:



$$D_B^{i\to h_1 h_2} = E_i^{a_1,a_2}(Y) \otimes D^{a_1\to h_1} \otimes D^{a_2\to h_2}$$

fixing $R_T \to$ scale k_j^2 ($\Leftrightarrow$ y) no longer arbitrary

$$D_B^{i\to h_1 h_2}(R_T^2) = \frac{d}{dR_T^2} D_B^{i\to h_1 h_2}$$

$$\propto \frac{d}{dy_T} E_i^{a_1,a_2} \otimes \ldots$$

$$= D^{b_1\to h_1}(y_T) \otimes D^{b_2\to h_2}(y_T) \otimes \hat{P}_{b_1 b_2}^{j} \otimes E_i^{j}(Y-y_T)$$

$$k_j^2 = \frac{k_{b_1}^2}{u} + \frac{k_{b_2}^2}{1-u} + \frac{r_T^2}{4u(1-u)} \overset{\text{LL}}{\approx} r_T^2 \approx R_T^2$$

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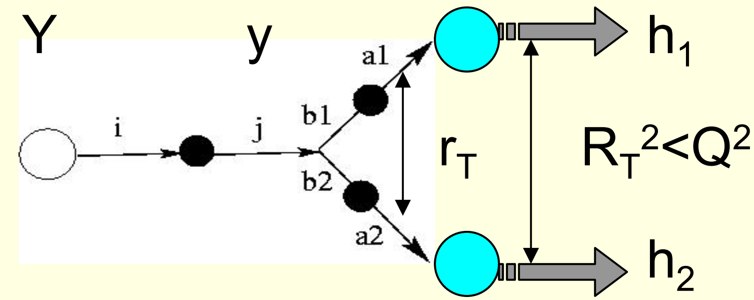
$$D_B^{i\to h_1 h_2} = E_i^{a_1,a_2}(Y) \otimes D^{a_1\to h_1} \otimes D^{a_2 \to h_2}$$

B:



fixing $R_T \to$ scale k_j^2 ($\Leftrightarrow$ y) no longer arbitrary

$$k_j^2 = \frac{k_{b_1}^2}{u} + \frac{k_{b_2}^2}{1-u} + \frac{r_T^2}{4u(1-u)} \overset{LL}{\approx} r_T^2 \approx R_T^2$$

$$y_T = \frac{1}{2\pi b_0} \log\left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(R_T^2)}\right] \qquad \frac{d}{dR_T^2} = \frac{\alpha_s(R_T^2)}{2\pi R_T^2}\frac{d}{dy_T}$$

$$D_B^{i\to h_1 h_2}(R_T^2) = \frac{d}{dR_T^2} D_B^{i\to h_1 h_2}$$

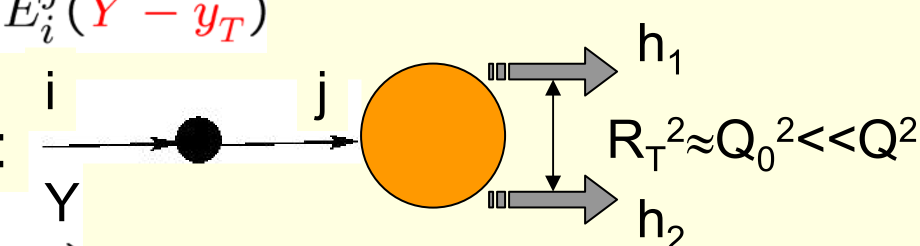
$$\propto \frac{d}{dy_T} E_i^{a_1,a_2} \otimes \ldots$$

$$= D^{b_1\to h_1}(y_T) \otimes D^{b_2\to h_2}(y_T) \otimes \hat{P}^j_{b_1 b_2} \otimes E_i^j(Y-y_T)$$

$$D_A^{i\to h_1 h_2} = D_A^{j\to h_1 h_2} \otimes E_i^j(Y-y_0)$$

$$D_A^{i\to h_1 h_2}(R_T^2) = D_A^{j\to h_1 h_2}(R_T^2) \otimes E_i^j(Y-y_0)$$

A:



scale k_j^2 arbitrary $\to$ set to Q_0^2 ($\Leftrightarrow$ y_0)

1. Need for DiFF
2. Exp. inv. mass M_h distribution
3. **Th. M_h dependence: extDiFF**
4. pol. extDiFF as spin analyzers

1. twist analysis of q-q correlator
2. Jet calculus: 1h fragmentation
3. Jet calculus: DiFF
4. Jet calculus: extDiFF
5. **evolution equations for extDiFF**

evolution equations for extDiFF

$$\begin{aligned} D^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) &= D_A^{i\to h_1 h_2} + D_B^{i\to h_1 h_2} \\ &= D_A^{j\to h_1 h_2} \otimes E_i^j(Y - y_0)\,\theta(y_0 - y_T) \\ &+ D^{b_1\to h_1} \otimes D^{b_2\to h_2} \otimes \hat{P}_{b_1 b_2}^j \otimes E_i^j(Y - y_T)\,\theta(y_T - y_0) \end{aligned}$$

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evolution equations for extDiFF

Ceccopieri, Radici, Bacchetta, hep-ph/0703265

$$\begin{aligned} D^{i\to h_1h_2}(z_1,z_2,R_T^2,Y) &= D_A^{i\to h_1h_2} + D_B^{i\to h_1h_2} \\ &= D_A^{j\to h_1h_2}\otimes E_i^j(Y-y_0)\,\theta(y_0-y_T) \\ &+ D^{b_1\to h_1}\otimes D^{b_2\to h_2}\otimes \hat{P}^j_{b_1b_2}\otimes E_i^j(Y-y_T)\,\theta(y_T-y_0) \end{aligned}$$

$$\frac{d}{dY}D^{i\to h_1h_2}(z_1,z_2,R_T^2,Y) = D^{k\to h_1h_2}(z_1,z_2,R_T^2,Y)\otimes P_{ki}$$

R_T^2 breaks degeneracy of terms A-B → homogeneous evolution

conjecture: at NLO & LL, factorization with same kernel as 1h-fragmentation

$$\frac{d\sigma}{dz_1\,dz_2\,dR_T^2} = \sum_i \hat{\sigma}_i^{NLO}(Q^2)\otimes D^{i\to h_1h_2}(z_1,z_2,R_T^2,Q^2)$$

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Ceccopieri, Radici, Bacchetta, hep-ph/0703265

$$
\begin{aligned}
D^{i\to h_1h_2}(z_1,z_2,R_T^2,Y) &= D_A^{i\to h_1h_2}+D_B^{i\to h_1h_2} \\
&= D_A^{j\to h_1h_2}\otimes E_i^j(Y-y_0)\,\theta(y_0-y_T) \\
&+ D^{b_1\to h_1}\otimes D^{b_2\to h_2}\otimes \hat{P}^j_{b_1b_2}\otimes E_i^j(Y-y_T)\,\theta(y_T-y_0)
\end{aligned}
$$

$$
\frac{d}{dY}D^{i\to h_1h_2}(z_1,z_2,R_T^2,Y) = D^{k\to h_1h_2}(z_1,z_2,R_T^2,Y)\otimes P_{ki}
$$

same for polarized extDiFF

R_T^2 breaks degeneracy of terms A-B $\to$ homogeneous evolution

conjecture: at NLO & LL, factorization with same kernel as 1h-fragmentation

$$
\frac{d\sigma}{dz_1\,dz_2\,dR_T^2} = \sum_i \hat{\sigma}_i^{NLO}(Q^2)\otimes D^{i\to h_1h_2}(z_1,z_2,R_T^2,Q^2)
$$

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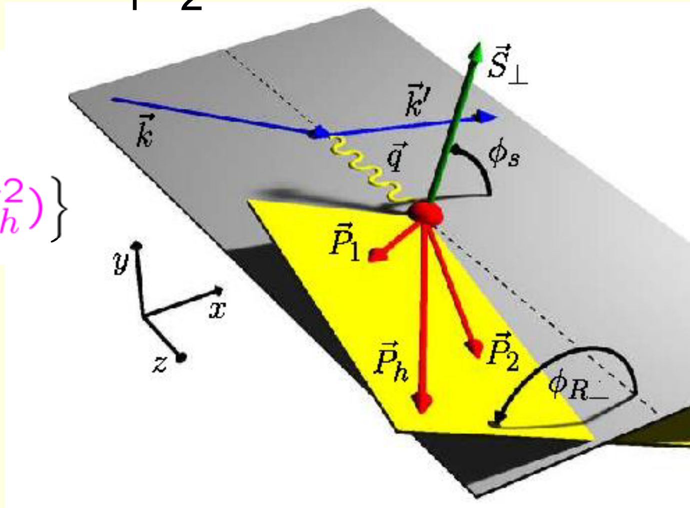
Single Spin Asymmetry (SSA) in $ep^\uparrow \to e' h_1 h_2 X$

$$d\sigma = K \sum_q e_q^2 \left\{ A(y)\, f_1^q(x)\, D_1^q(z_1, z_2, M_h^2) + B(y) \frac{|\mathbf{S}_T||\mathbf{R}_T|}{M_h} \sin(\phi_{R_T} + \phi_{S_T})\, h_1^q(x)\, H_1^{\sphericalangle q}(z_1, z_2, M_h^2) \right\}$$



$$A_{UT}^{\sin(\phi_{R_T}+\phi_{S_T})} \equiv \frac{1}{\sin(\phi_{R_T} + \phi_{S_T})} \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$$\propto \frac{B(y)}{A(y)} \frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle q}(z_1, z_2, M_h^2)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z_1, z_2, M_h^2)}$$

info on extDiFF from
$e^+e^- \to (h_1h_2)\,(h'_1h'_2) + X$
Boer, Jakob, Radici, P.R. D**67** (03) 094003 BELLE

or from models

Jaffe, Jin, Tang, P.R.L. **80** (98) 1166 $H_1^{\sphericalangle} \to \delta \hat{q}_I$

Radici, Jakob, Bianconi, P.R. D**65** (02) 074031

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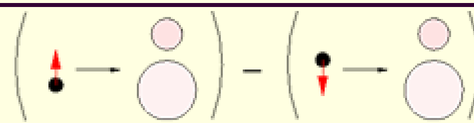
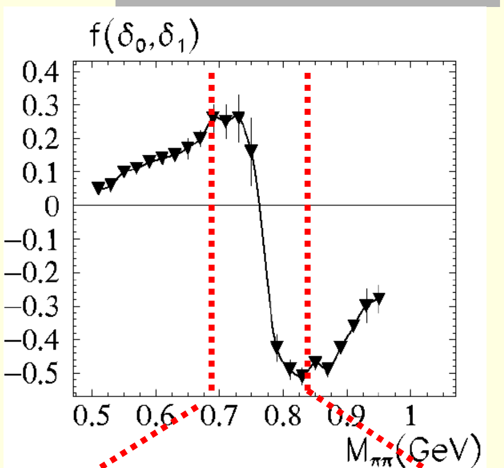
$H_1^{\sphericalangle} \longrightarrow \delta\hat{q}_I$



interference



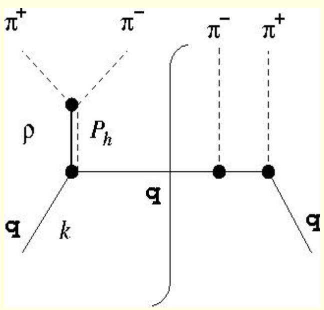
Collins *et al*., N.P. **B420** (94) 525

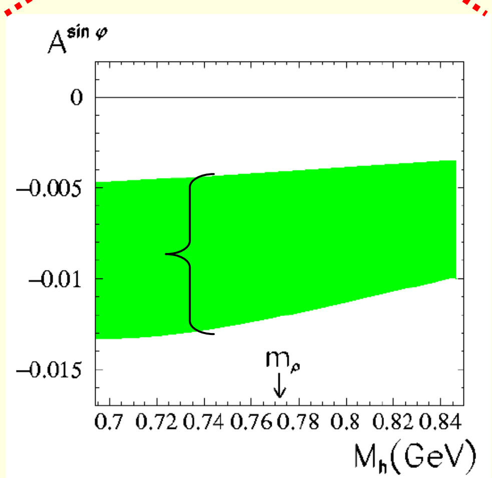
Jaffe, Jin, Tang, P.R.L. **80** (98) 1166

- ($L_{\pi\pi}$=0) – (L=1) interference from el. scatt. π-π phase shifts only; sign change from Re[ρ]

Radici, Jakob, Bianconi, P.R. D**65** (02) 074031

- spectator model
  interference ~ (L=0)–$\mathcal{I}$m[ρ]
- th. uncertainty band





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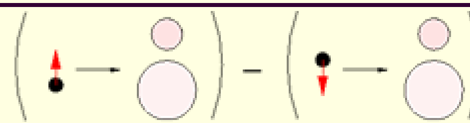
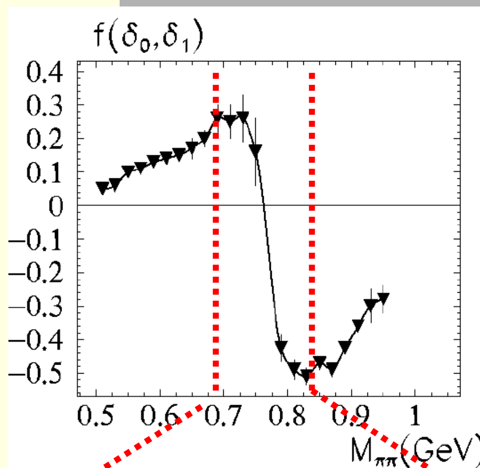


interference

Collins *et al*., N.P. **B420** (94) 525

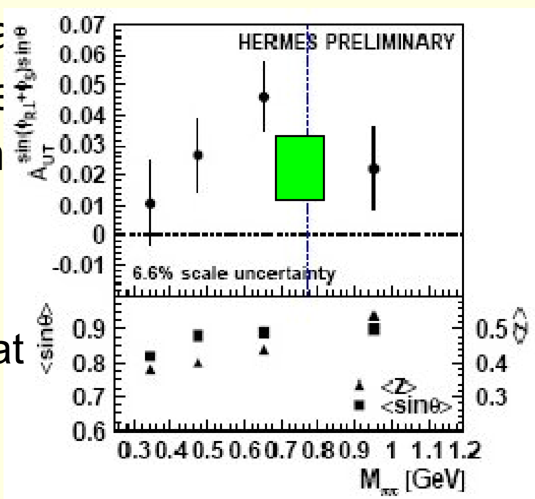
Jaffe, Jin, Tang, P.R.L. **80** (98) 1166

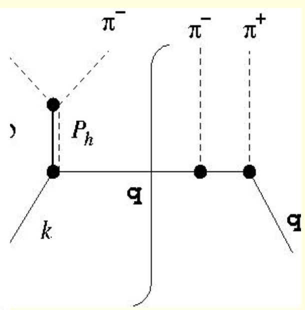


- ($L_{\pi\pi}$=0) – (L=1) interference from el. scatt. π-π phase shifts only; sign change from Re[ρ]

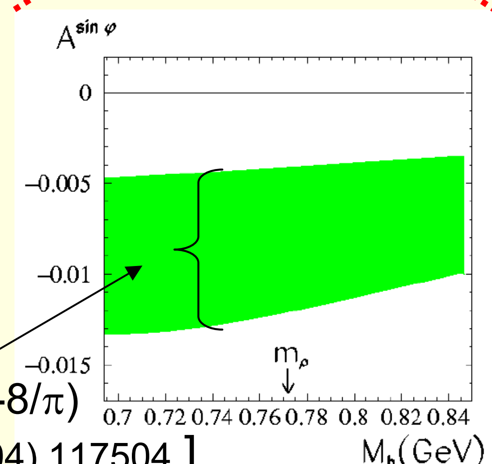
Radici, Jakob, Bianconi, P.R. D**65** (02) 074031

- specta
  interfe
- th. un





P. van der Nat
DIS2005



Trento Conventions → x (-8/π)

[Bacchetta *et al*., P.R. D**70** (04) 117504]

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upgrading the spectator model $|P_1, P_2, X\rangle \sim |(\pi^+\pi^-)_L, \tilde{q}\rangle$

fit M_h and z distributions from PYTHIA at HERMES

Bacchetta & Radici, P.R.D**74** (06) 114007

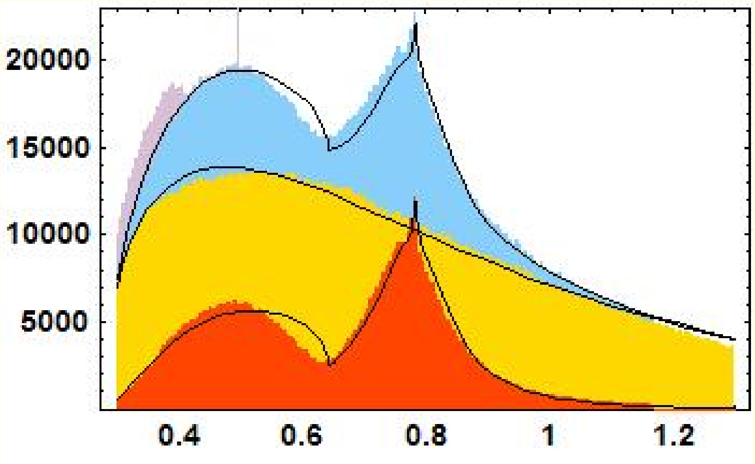
2 ($\rho\to\pi^+\pi^- X$) + 3 ($\omega\to\pi^+\pi^- X$) + 4 ($\omega\to\pi^+\pi^-[\pi^0 X]$) p-wave

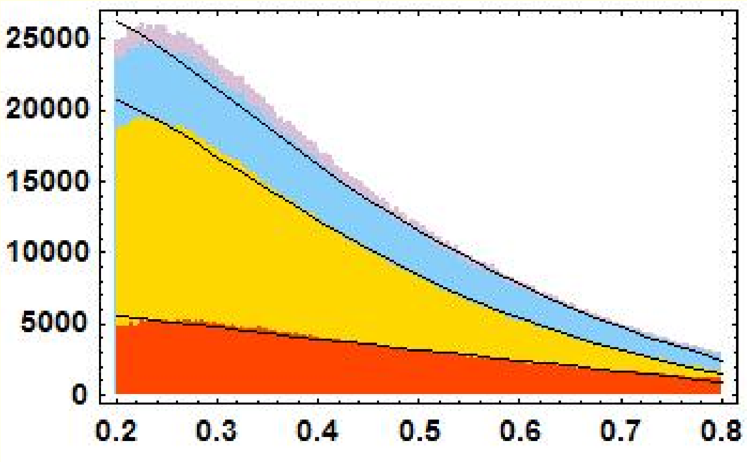
1. background s-wave

Total

Total + 5 ($\eta\to\pi^+\pi^- X$) + 6 ($K_S^0\to\pi^+\pi^- X$)

Warning: $\omega \to [(\pi\pi)_{L=1}\,\pi]_{J=1}$





M_h z

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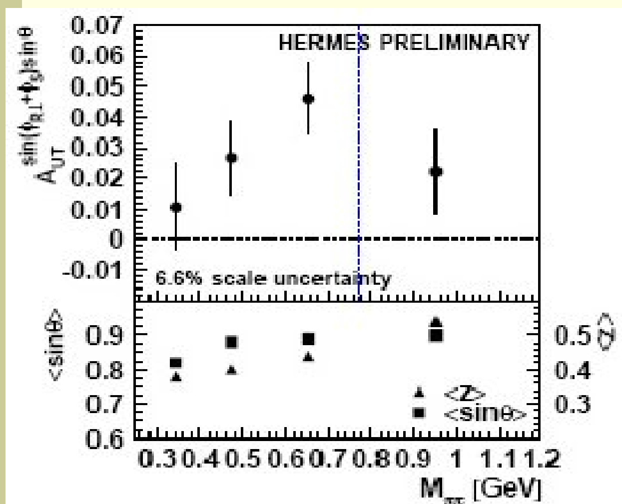
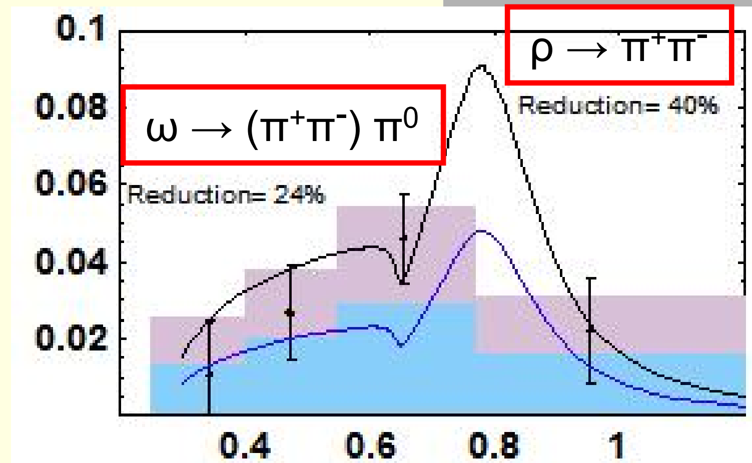
HERMES PRELIMINARY

6.6% scale uncertainty

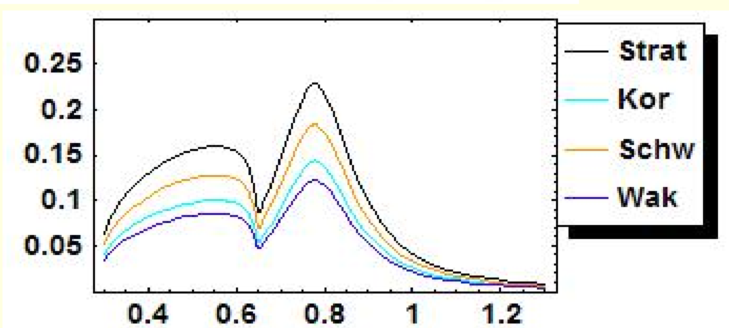
$Q^2>1$ GeV^2
$s=56.2$ GeV^2





bins 0.15 0.15 0.22 1.23

$$A_{UT}^{\sin(\phi_R+\phi_S)}(\text{bin}[M_h]) = \frac{\int_{\text{bin}} dM_h^2 \int_{0.2}^{0.8} dz\ldots}{\int_{\text{bin}} dM_h^2 \int_{0.2}^{0.8} dz\ldots}$$



Strat
Soffer *et al.*,
P.R. **D65** (02) 114024

Kor
Korotkov *et al.*,
E.P.J. **C18** (01) 639

Schw
Schweitzer *et al.*
P.R. **D64** (01) 034013

Wak
Wakamatsu
P.L. B**509** (01) 59

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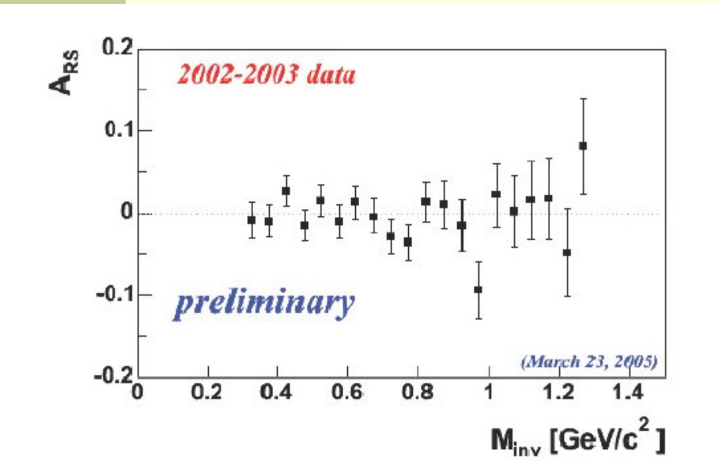
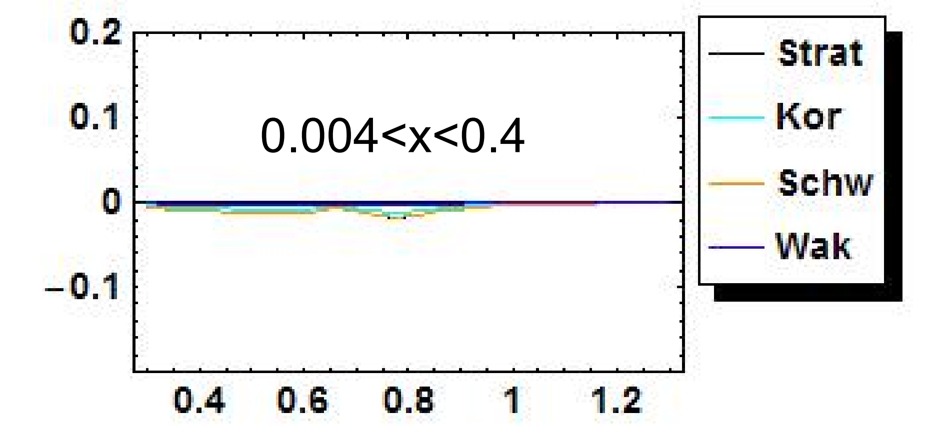
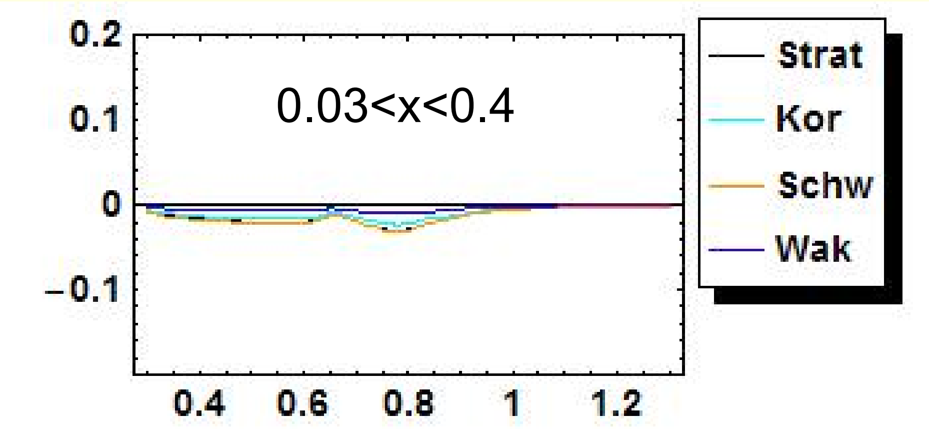
COMPASS deuteron Q^2>1 GeV^2 s=604 GeV^2



Joosten – DIS2005





isospin symmetry in d={p,n}

$$\frac{1}{9}[4u(x) - d(x) - \bar{u}(x) + \bar{d}(x) \\ +4d(x) - u(x) - \bar{d}(x) + \bar{u}(x)]$$

$$f_1^d \approx \frac{1}{2} f_1^u \qquad h_1^d \approx -\frac{1}{4} h_1^u$$

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Bacchetta, Radici, P.R. **D90** (04) 094032

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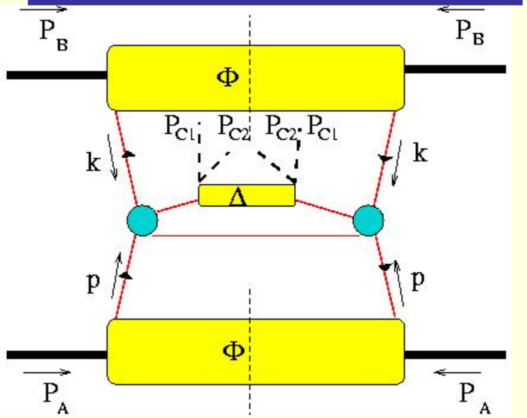
$\boxed{\pi(\bar{p})\, p^{(\uparrow)} \to (\pi\pi)_C\, X}$



$$d\sigma = d\sigma_{UU} + |\mathbf{S}_{BT}|\, \frac{|\mathbf{R}_C|}{M_C} \sin(\phi_{S_B} - \phi_{R_C})\, d\sigma_{UT}$$

$$f_1^b \otimes d\hat{\sigma}_{ab\to cd} \otimes D_1^c \otimes f_1^a$$

$$h_1^b \otimes d\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d} \otimes H_1^{\sphericalangle c} \otimes f_1^a$$

$$\text{SSA} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma_{UT}}{d\sigma_{UU}}$$

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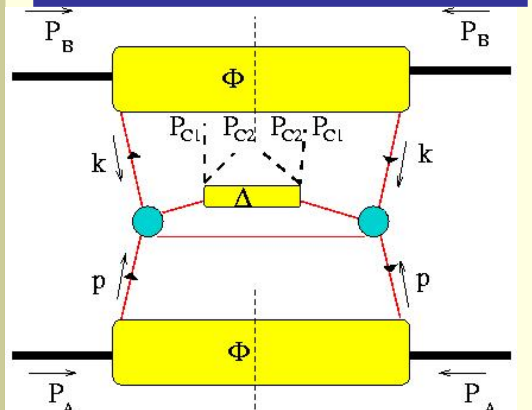
$\pi(\bar{p})\, p^{(\uparrow)} \to (\pi\pi)_C\, X$

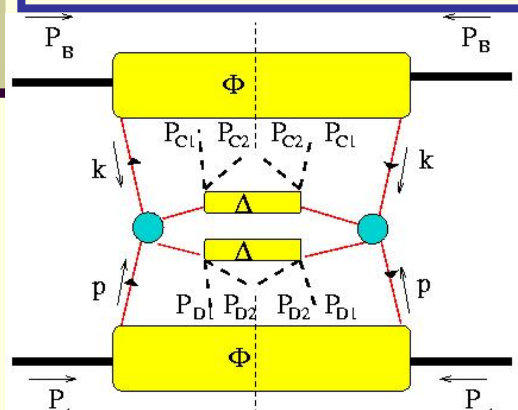


$$d\sigma = d\sigma_{UU} + |\mathbf{S}_{BT}|\, \frac{|\mathbf{R}_C|}{M_C} \sin(\phi_{S_B} - \phi_{R_C})\, d\sigma_{UT}$$

$$f_1^b \otimes d\hat{\sigma}_{ab\to cd} \otimes D_1^c \otimes f_1^a \qquad h_1^b \otimes d\Delta\hat{\sigma}_{ab^\uparrow\to c^\uparrow d} \otimes H_1^{\sphericalangle c} \otimes f_1^a$$

$$\text{SSA} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma_{UT}}{d\sigma_{UU}}$$

$\pi(\bar{p})\, p \to (\pi\pi)_C (\pi\pi)_D\, X$



$$d\sigma = \mathcal{A} + \frac{|\mathbf{R}_C|\,|\mathbf{R}_D|}{M_C\, M_D} \cos(\phi_{R_C} - \phi_{R_D})\, \mathcal{B} + \ldots.$$

$$f_1^b \otimes d\hat{\sigma}_{ab\to cd} \otimes D_1^c \otimes D_1^d \otimes f_1^a \qquad f_1^b \otimes d\Delta\hat{\sigma}_{ab\to c^\uparrow d^\downarrow} \otimes H_1^{\sphericalangle c} \otimes H_1^{\sphericalangle d} \otimes f_1^a$$

self-consistent extraction of

$$D_1,\ H_1^{\sphericalangle} \Rightarrow h_1$$