

# QCD factorizations in

$$\gamma^* \gamma^* \rightarrow \rho_L^0 \rho_L^0$$

M. Segond,  
LPT Orsay

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in a collaboration with :  
B.Pire ,  
L.Szymanowski and  
S.Wallon

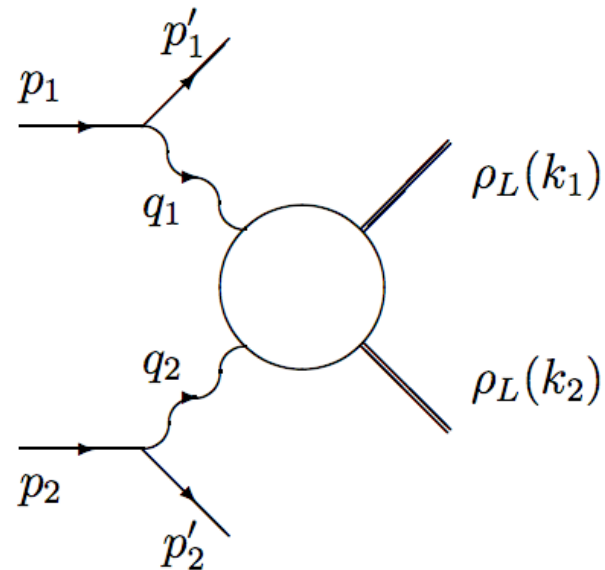
## Introduction

The exclusive reaction of  $\rho$  mesons electroproduction in  $\gamma^*\gamma^*$  collisions is a beautiful laboratory to study different dynamics and factorization properties in HE QCD.

It seems to be a promising probe of the BFKL effects which could be studied in the next generation of  $e^+e^-$  colliders (ILC) and at lower energy of other kind of QCD factorizations involving GDA and TDA, which could be observed at Babar or Belle.

We consider the following process :

$$e^+e^- \rightarrow e^+e^- \rho_L \rho_L$$



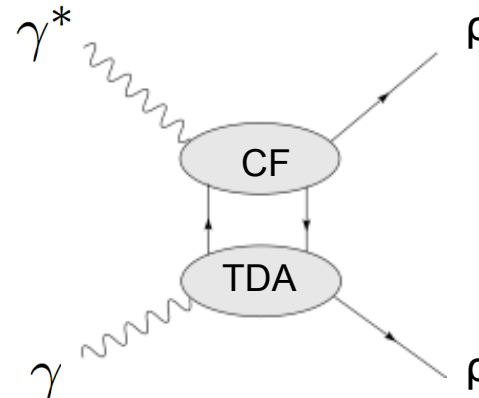
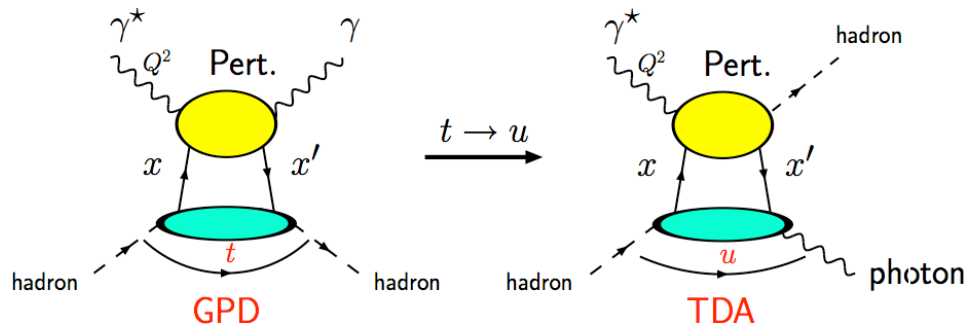
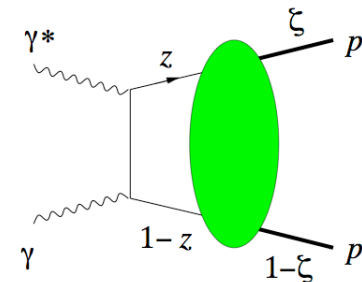
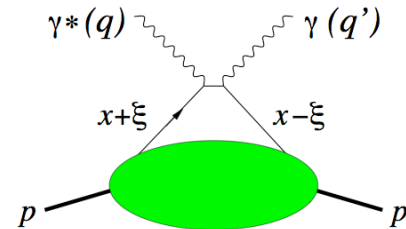
## First part: Motivation and origins

- DIS : **inclusive** process  $t = 0$   
Structure function = perturbative CF \* PDF

- DVCS : **exclusive** process at small  $t$   
Amplitude = perturbative CF \* GPD

Crossed process, small cm energy  
Amplitude = perturbative CF \* GDA

- Generalization of DVCS, small  $t$   
Amplitude = perturbative CF \* TDA

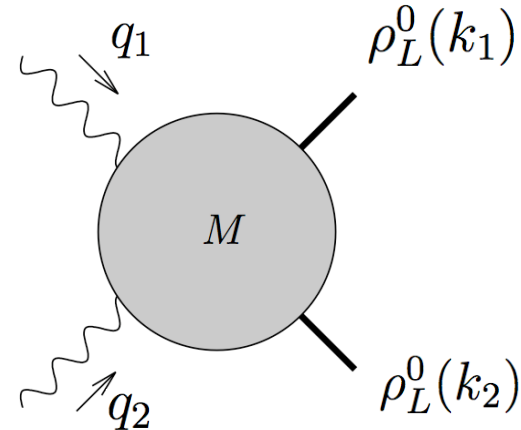


## outline

$$\gamma^*(Q_1)\gamma^*(Q_2) \rightarrow \rho_L^0(k_1)\rho_L^0(k_2)$$

Exclusive reaction at Born order  
quark exchange contribution

$Q_1, Q_2$  hard scales



- Direct calculation and factorization with two  $\rho$  DAs  
longitudinal mesons  $\longrightarrow$  leading twist

Brodsky-Lepage (1981)

- Factorization with pp GDA for **transverse**  $\gamma^*$
- Factorization with  $\gamma^* \rightarrow \rho$  TDA for **longitudinal**  $\gamma^*$

## Analytic expressions in collinear factorization

The virtualities  $Q_1$  and  $Q_2$  of the photons supply the **hard scale**

**Collinear approximation**  $\rightarrow$  we neglect transverse relative quark momenta in the rho mesons

and in the forward kinematics

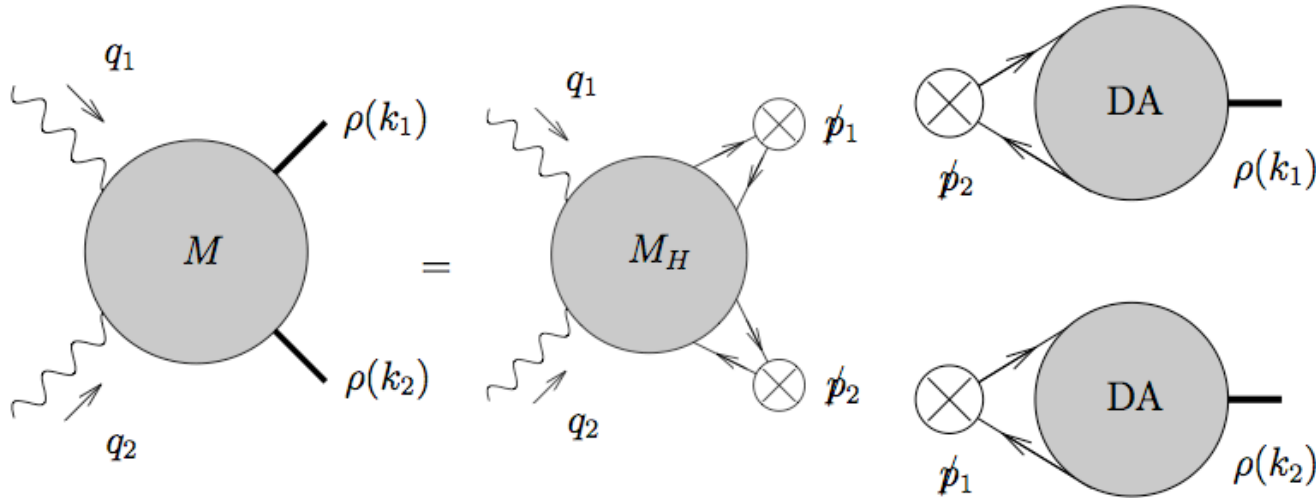
$$\begin{aligned} \ell_1 &\sim z_1 k_1, & \ell_2 &\sim z_2 k_2 \\ \tilde{\ell}_1 &\sim \bar{z}_1 k_1, & \tilde{\ell}_2 &\sim \bar{z}_2 k_2 \end{aligned}$$

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as

$$\langle \rho_L^0(k) | \bar{q}(x) \gamma^\mu q(0) | 0 \rangle = \frac{f_\rho}{\sqrt{2}} k^\mu \int_0^1 dz e^{iz(kx)} \phi(z)$$

with  $\phi(z) = 6z(1-z) \left( 1 + \sum_{n=1}^{\infty} a_{2n} C_{2n}^{3/2} (2z-1) \right)$

## Amplitude of the process in the collinear factorization with DA



The scattering amplitude reads

$$\mathcal{A} = T^{\mu\nu} \epsilon_\mu(q_1) \epsilon_\nu(q_2)$$

where

$$T^{\mu\nu} = \frac{1}{2} g_T^{\mu\nu} (T^{\alpha\beta} g_{T\alpha\beta}) + (p_1^\mu + \frac{Q_1^2}{s} p_2^\mu) (p_2^\nu + \frac{Q_2^2}{s} p_1^\nu) \frac{4}{s^2} (T^{\alpha\beta} p_{2\alpha} p_{1\beta})$$

and

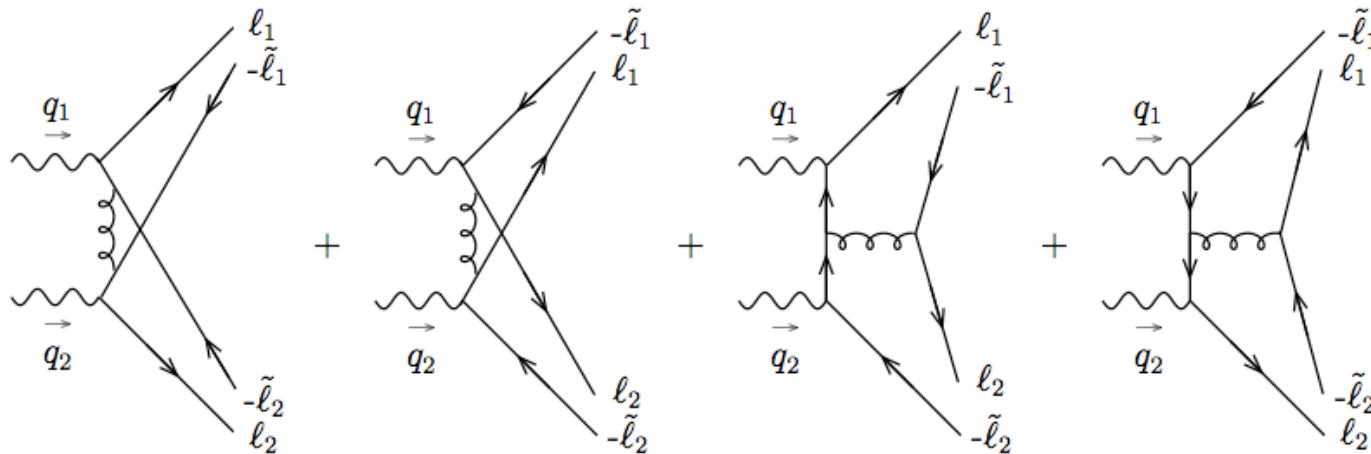
$$g_T^{\mu\nu} = g^{\mu\nu} - (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) / (p_1 \cdot p_2)$$

## longitudinally polarized photons

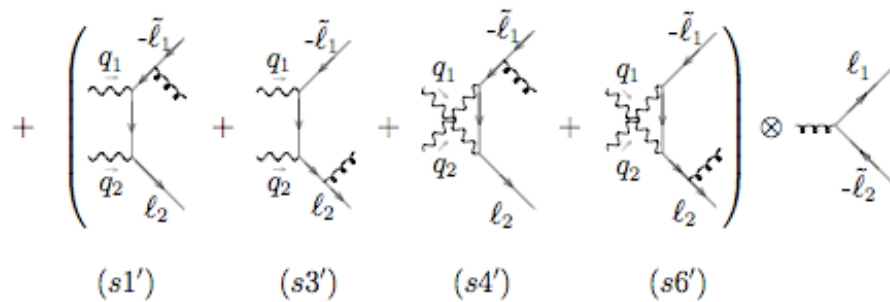
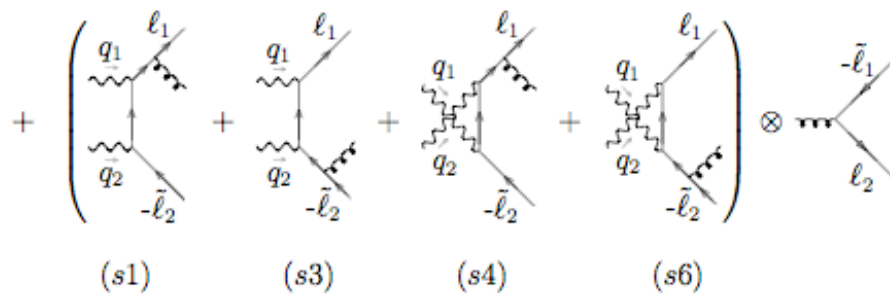
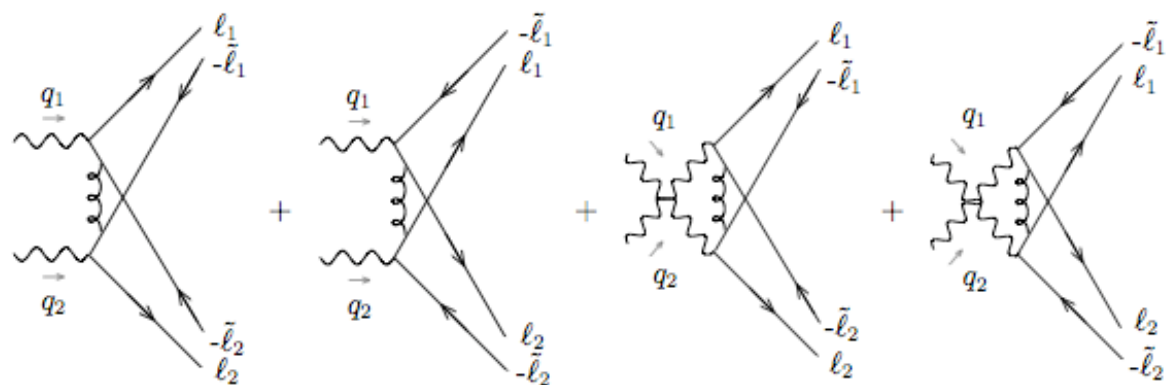
Their polarization vectors read

$$\epsilon_{\parallel}(q_1) = \frac{1}{Q_1}q_1 + \frac{2Q_1}{s}p_2 \quad \text{and} \quad \epsilon_{\parallel}(q_2) = \frac{1}{Q_2}q_2 + \frac{2Q_2}{s}p_1$$

The electromagnetic gauge invariance and the forward kinematics reduce the number of diagrams.



# transversally polarized photons





## Analytic expressions in collinear approximation

Transverse photons

$$T^{\alpha\beta} g_{T\alpha\beta} = -\frac{e^2(Q_u^2+Q_d^2)g^2C_Ff_\rho^2}{4N_c s} \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2)$$

$$\left\{ 2 \left(1 - \frac{Q_2^2}{s}\right) \left(1 - \frac{Q_1^2}{s}\right) \left[ \frac{1}{(z_2 + \bar{z}_2 \frac{Q_1^2}{s})^2 (z_1 + \bar{z}_1 \frac{Q_2^2}{s})^2} + \frac{1}{(\bar{z}_2 + z_2 \frac{Q_1^2}{s})^2 (\bar{z}_1 + z_1 \frac{Q_2^2}{s})^2} \right] + \right. \\ \left. \left( \frac{1}{\bar{z}_2 z_1} - \frac{1}{\bar{z}_1 z_2} \right) \left[ \frac{1}{1 - \frac{Q_2^2}{s}} \left( \frac{1}{\bar{z}_2 + \frac{z_2 Q_1^2}{s}} - \frac{1}{z_2 + \frac{\bar{z}_2 Q_1^2}{s}} \right) - \frac{1}{1 - \frac{Q_1^2}{s}} \left( \frac{1}{\bar{z}_1 + \frac{z_1 Q_2^2}{s}} - \frac{1}{z_1 + \frac{\bar{z}_1 Q_2^2}{s}} \right) \right] \right\}$$

Non-zero values of  $Q_1$  and  $Q_2$

Behaviour of the rho DAs

→ no end-point singularities

## Analytic expressions in collinear approximation

Longitudinal photons

$$T^{\alpha\beta} p_{2\alpha} p_{1\beta} = -\frac{s^2 f_\rho^2 C_F e^2 g^2 (Q_u^2 + Q_d^2)}{8N_c Q_1^2 Q_2^2} \int_0^1 dz_1 dz_2 \phi(z_1) \phi(z_2) \\ \times \left\{ \frac{(1 - \frac{Q_1^2}{s})(1 - \frac{Q_2^2}{s})}{(z_1 + \bar{z}_1 \frac{Q_2^2}{s})(z_2 + \bar{z}_2 \frac{Q_1^2}{s})} + \frac{(1 - \frac{Q_1^2}{s})(1 - \frac{Q_2^2}{s})}{(\bar{z}_1 + z_1 \frac{Q_2^2}{s})(\bar{z}_2 + z_2 \frac{Q_1^2}{s})} + \frac{1}{z_2 \bar{z}_1} + \frac{1}{z_1 \bar{z}_2} \right\}$$

Non-zero values of  $Q_1$  and  $Q_2$

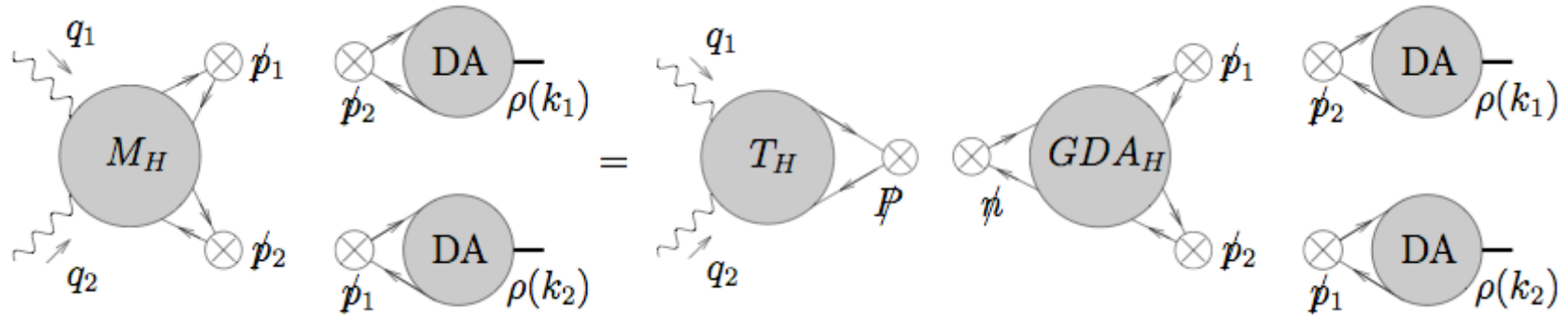
Behaviour of the rho DAs

→ no end-point singularities

# Factorization of the amplitude in terms of a GDA in the generalized Bjorken limit for transverse photons

In the kinematical region where the scattering energy is smaller than the highest photon virtuality

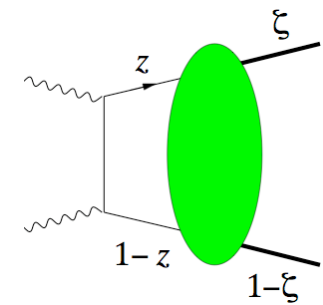
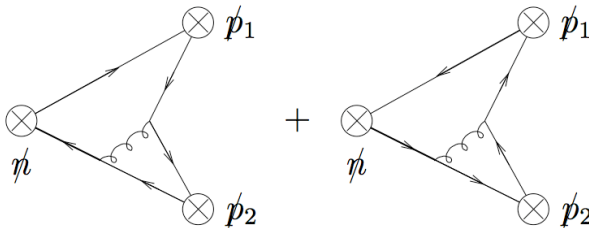
$$\frac{W^2}{Q_1^2} = \frac{s}{Q_1^2} \left(1 - \frac{Q_1^2}{s}\right) \left(1 - \frac{Q_2^2}{s}\right) \approx 1 - \frac{Q_1^2}{s} \ll 1$$



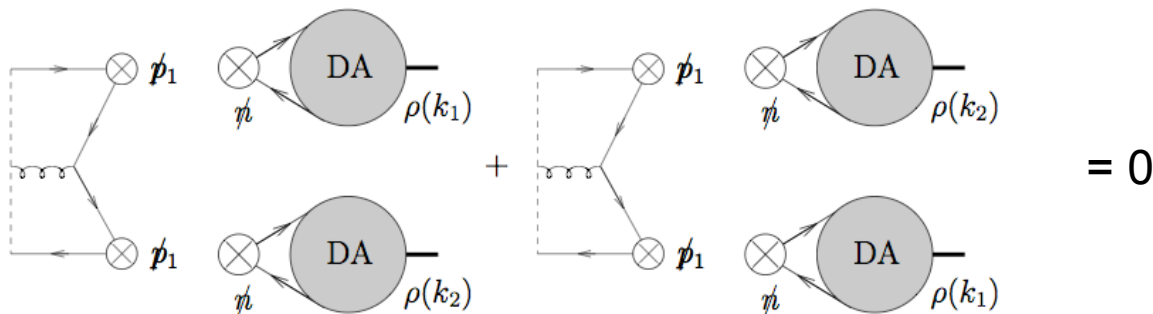
Definition of the leading twist **GDA** calculated in the Born order of the perturbation theory

$$\langle \rho_L^0(k_1) \rho_L^0(k_2) | \bar{q}(-\alpha n/2) \hat{n} \exp\left(ig \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} dy n_\nu A^\nu(y)\right) q(\alpha n/2) | 0 \rangle$$

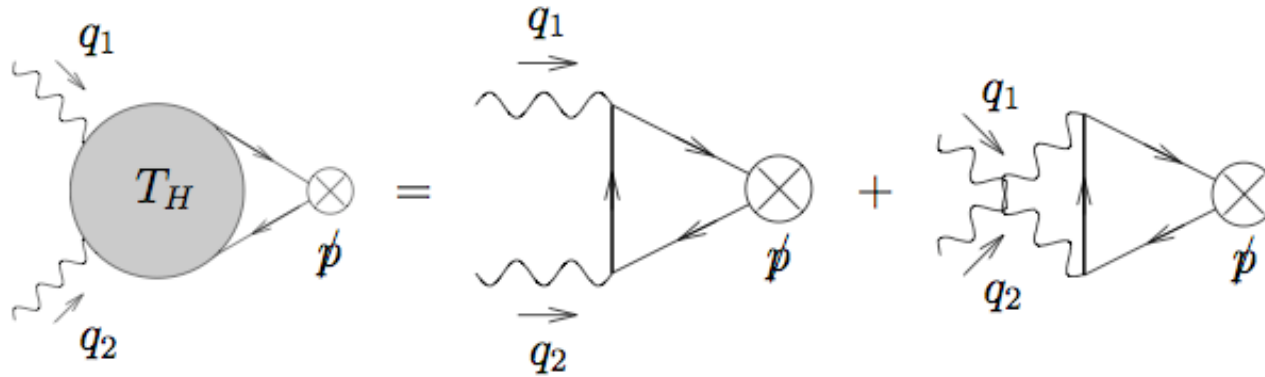
$$= \int_0^1 dz e^{-i(2z-1)\alpha(nP)/2} \Phi^{\rho_L \rho_L}(z, \zeta, W^2)$$



→ The **QCD Wilson line** vanishes



Expansion of the **Hard Part** at Born order



For a one flavored quarks , it equals

$$T_H(z) = -4 e^2 N_c Q_q^2 \left( \frac{1}{\bar{z} + z \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z} \frac{Q_2^2}{s}} \right)$$

## Summary of the factorization with GDA for transverse virtual photons

factorization of  $T^{\alpha\beta} g_{T\alpha\beta}$  into the **Hard Part** and the **GDA**

$$T^{\alpha\beta} g_{T\alpha\beta} = \frac{e^2}{2} (Q_u^2 + Q_d^2) \int_0^1 dz \left( \frac{1}{\bar{z} + z \frac{Q_2^2}{s}} - \frac{1}{z + \bar{z} \frac{Q_2^2}{s}} \right) \Phi^{\rho_L \rho_L}(z, \zeta \approx 1, W^2)$$

$$\text{with } \Phi^{\rho_L \rho_L}(z, \zeta \approx 1, W^2) = -\frac{f_\rho^2 g^2 C_F}{2 N_c W^2} \int_0^1 dz_2 \phi(z) \phi(z_2) \left[ \frac{1}{z \bar{z}_2} - \frac{1}{\bar{z} z_2} \right]$$

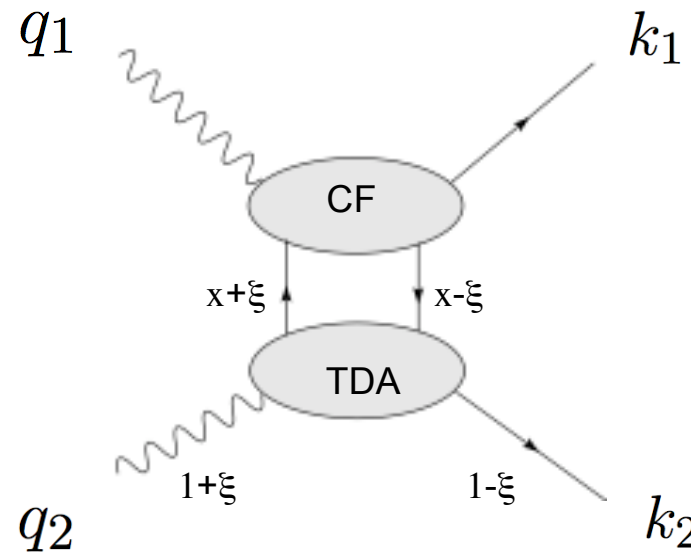
Limiting case of the original equation by [D.Müller et al \(1994\)](#)

Extension of the studies of  $\gamma^* \gamma \rightarrow \pi\pi$  by [M.Diehl et al \(2000\)](#)  
for virtual photons at  $t = t_{min}$

# Factorization of the amplitude in terms of a TDA for longitudinal virtual photons

- Kinematical regime  $Q_1^2 \gg Q_2^2$  ,  $t = t_{min}$   
 $\longrightarrow$  second factorization

- Amplitude = convolution of Hard Part and **TDA**

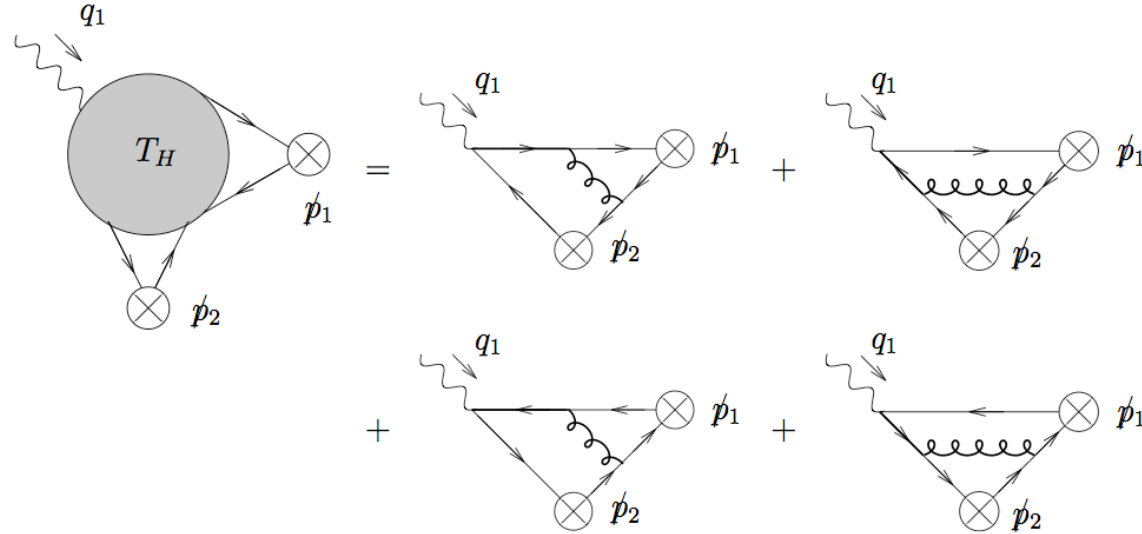


**TDA** kinematics = **GPD** kinematics

**TDA**  $\longrightarrow \int \frac{dz^-}{2\pi} e^{-ixP^+z^-} \langle \rho(p_2) | \bar{q}(-z^-/2) \gamma^+ q(z^-/2) | \gamma(q_2) \rangle$

# Proof of the factorization in terms of a TDA

Hard Part



$$T_H(z_1, x) = -i f_\rho g^2 e Q_q \frac{C_F \phi(z_1)}{2 N_c Q_1^2} \epsilon^\mu(q_1) \left( 2\xi n_{2\mu} + \frac{1}{1+\xi} n_{1\mu} \right) \\ \times \left[ \frac{1}{z_1(x + \xi - i\epsilon)} + \frac{1}{\bar{z}_1(x - \xi + i\epsilon)} \right]$$



# Proof of the factorization in terms of a TDA

definition of the **TDA**

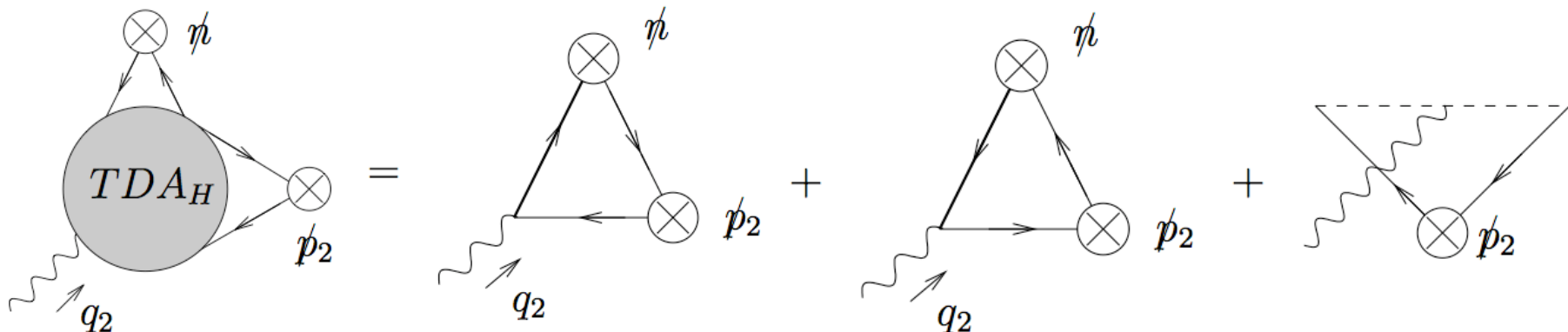
$$\int \frac{dz^-}{2\pi} e^{ix(P \cdot z)} \langle \rho_L^q(k_2) | \bar{q}(-z/2) \hat{n} e^{-ieQ_q \int_{z/2}^{-z/2} dy_\mu A^\mu(y)} q(z/2) | \gamma^*(q_2) \rangle$$

$$= \frac{e Q_q f_\rho}{P^+} \frac{2}{Q_2^2} \epsilon_\nu(q_2) \left( (1 + \xi) n_2^\nu + \frac{Q_2^2}{s(1 + \xi)} n_1^\nu \right) T(x, \xi, t_{min})$$

with  $T(x, \xi, t_{min}) \equiv$

$$N_c \left[ \Theta(1 \geq x \geq \xi) \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1+x}{1-\xi}\right) \right]$$

QED Wilson line



# Summary of the factorization with TDA for longitudinal virtual photons

Factorized form of the amplitude involving a **TDA**

$$T^{\alpha\beta} p_{2\alpha} p_{1\beta} = -i f_\rho^2 e^2 (Q_u^2 + Q_d^2) g^2 \frac{C_F}{8N_c} \int_{-1}^1 dx \int_0^1 dz_1 \left[ \frac{1}{\bar{z}_1(x-\xi)} + \frac{1}{z_1(x+\xi)} \right] \phi(z_1) T(x, \xi, t_{min})$$

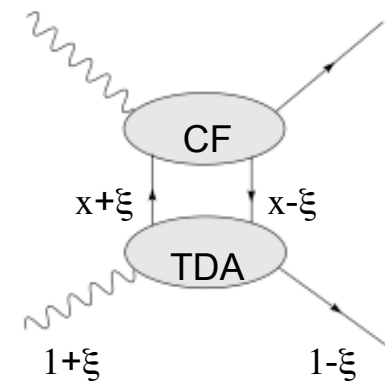
with  $T(x, \xi, t_{min}) \equiv$

$$N_c \left[ \Theta(1 \geq x \geq \xi) \phi\left(\frac{x-\xi}{1-\xi}\right) - \Theta(-\xi \geq x \geq -1) \phi\left(\frac{1+x}{1-\xi}\right) \right]$$

Only the **DGLAP** part of the TDA contributes because we use the  $\rho$ -mesons DAs.

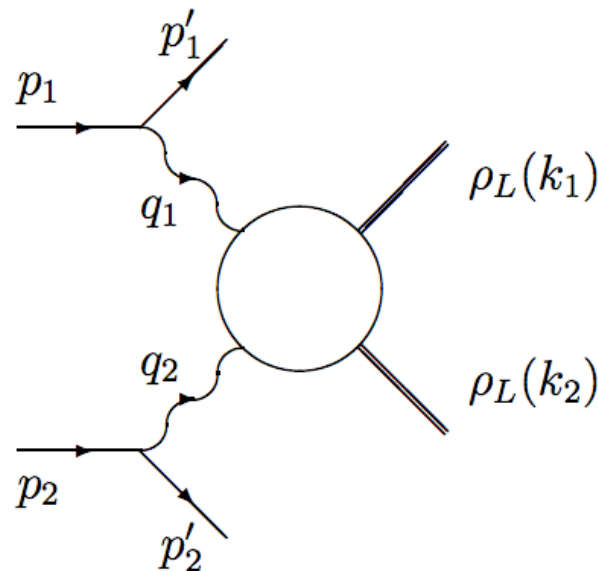
$$\begin{aligned} -\xi &\geq x \geq -1 \\ 1 &\geq x \geq \xi \end{aligned}$$

We get the same kind of factorization for an opposite ordering of the photons virtualities.



## Second part: Diffractive mesons production with leptons tagging for studying the BFKL Pomeron

- We go back to the process  $e^+e^- \rightarrow e^+e^- \rho_L \rho_L$



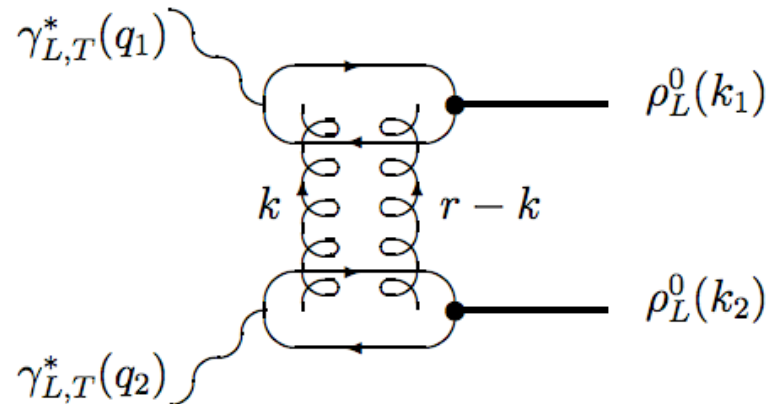
In the Regge limit, we expect to 'observe' an exchange of a BFKL Pomeron in the t-channel.

We compute the scattering amplitude in a complete analytical way at the Born order .

This process has already been studied until NLO only in the forward case.

D.Ivanov, A.Papa

## Study of the process



Selection of events in which two vectors  $\rho$  mesons with longitudinal polarization are produced in the final state with a big gap in rapidity.

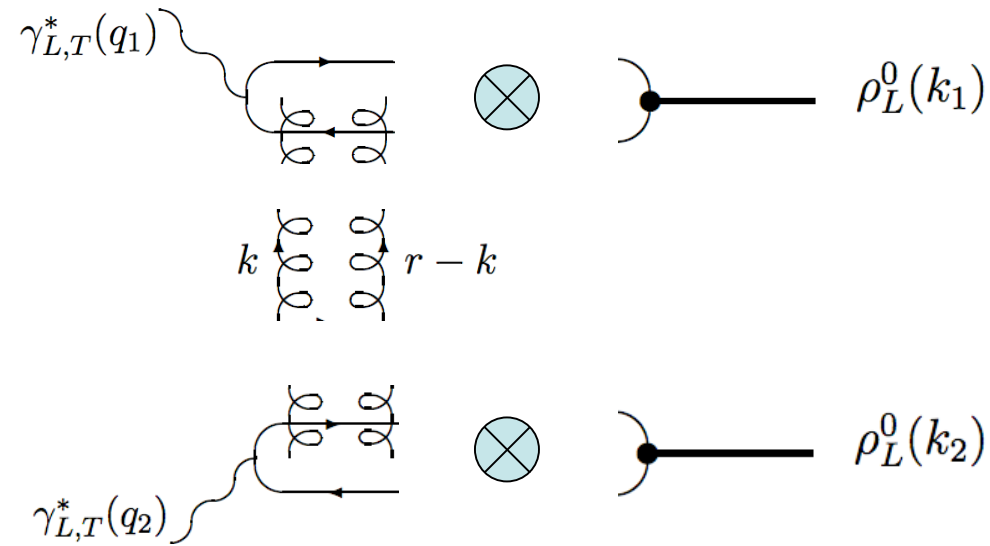
Double tagging of final leptons  $\rightarrow$  photons polarization

Highly virtual photons  $Q_1^2, Q_2^2 \gg \Lambda_{QCD}^2 \rightarrow$  perturbative computation

$Q_1^2 \sim Q_2^2 \rightarrow$  neglect DGLAP partonic evolution

$\rightarrow$  In the Regge limit  $s \gg -t, Q_1^2, Q_2^2$ , the process is dominated by BFKL evolution.

## Amplitude of the process at the Born order



Integration over the internal moments :

- ♦ Sudakov basis  $\underline{k} = \alpha q'_1 + \beta q'_2 + k_\perp$   $q_1'^2 = q_2'^2 = 0$
- ♦ In the BFKL dynamics the longitudinal momenta of the gluons are strongly ordered.  
 ➔ **kT-factorization** in transverse momentum cf.  $\int d^4k = \int d\alpha d\beta d\vec{k}^2$
- ♦ **Collinear approximation** ➔ we neglect transverse relative momentum of quark inside the mesons.

## Factorization of the amplitude

$$\mathcal{J}^{\gamma_{L,T}^*(q_1) \rightarrow \rho_L^0(k_1)} \longrightarrow \gamma_{L,T}^*(q_1) \left\{ \begin{array}{c} \text{quark-antiquark loop} \\ \text{with two gluon exchanges} \end{array} \right\} \otimes \text{meson vertex} \longrightarrow \rho_L^0(k_1)$$

$k$        $r - k$

$$\mathcal{J}^{\gamma_{L,T}^*(q_2) \rightarrow \rho_L^0(k_2)} \longrightarrow \gamma_{L,T}^*(q_2) \left\{ \begin{array}{c} \text{quark-antiquark loop} \\ \text{with two gluon exchanges} \end{array} \right\} \otimes \text{meson vertex} \longrightarrow \rho_L^0(k_2)$$

$$\mathcal{M} = i s \int \frac{d^2 \underline{k}}{(2\pi)^4 \underline{k}^2 (\underline{r} - \underline{k})^2} \mathcal{J}^{\gamma_{L,T}^*(q_1) \rightarrow \rho_L^0(k_1)}(\underline{k}, \underline{r} - \underline{k}) \mathcal{J}^{\gamma_{L,T}^*(q_2) \rightarrow \rho_L^0(k_2)}(-\underline{k}, -\underline{r} + \underline{k})$$

Every impact factor  $\mathcal{J}^{\gamma_{L,T}^*(q_1) \rightarrow \rho_L^0(k_1)}$  is written as a convolution of the DA of the meson with the coefficient function corresponding to the quark-antiquark opened pair production from one polarized photon with two gluons exchanged in the t channel.

We use the matrix element of the non local correlator of quarks fields on the light cone to define the DA of the meson as :

$$\langle \rho(k_2) | \bar{q}(-\frac{z}{2}) \gamma^\mu q(\frac{z}{2}) | 0 \rangle = f_\rho k_2^\mu \int_0^1 du e^{i(1-2u)(k_2 \frac{z}{2})} \phi(u)$$

- In the case of **longitudinally** polarized photons, they read :

$$\mathcal{J}_{L}^{\gamma^*(q_i) \rightarrow \rho_L(k_i)}(\underline{k}, \underline{r} - \underline{k})$$

$$= 8\pi^2 \alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} Q_i f_\rho \alpha(k_i) \int_0^1 dz_i z_i \bar{z}_i \phi(z_i) P_P(z_i, \underline{k}, \underline{r}, \mu_i)$$

with  $P_P(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{1}{z_i^2 \underline{r}^2 + \mu_i^2} + \frac{1}{\bar{z}_i^2 \underline{r}^2 + \mu_i^2} - \frac{1}{(z_i \underline{r} - \underline{k})^2 + \mu_i^2} - \frac{1}{(\bar{z}_i \underline{r} - \underline{k})^2 + \mu_i^2}$

where  $\mu_i^2 = Q_i^2 z_i \bar{z}_i + m^2$

- For **transversely** polarized photons, one obtains :

$$\mathcal{J}_{T}^{\gamma^*(q_i) \rightarrow \rho_L(k_i)}(\underline{k}, \underline{r} - \underline{k})$$

$$= 4\pi^2 \alpha_s \frac{e}{\sqrt{2}} \frac{\delta^{ab}}{2N_c} f_\rho \alpha(k_i) \int_0^1 dz_i (z_i - \bar{z}_i) \phi(z_i) \underline{\epsilon} \cdot \underline{Q}(z_i, \underline{k}, \underline{r}, \mu_i)$$

with  $\underline{Q}(z_i, \underline{k}, \underline{r}, \mu_i) = \frac{z_i \underline{r}}{z_i^2 \underline{r}^2 + \mu_i^2} - \frac{\bar{z}_i \underline{r}}{\bar{z}_i^2 \underline{r}^2 + \mu_i^2} + \frac{\underline{k} - z_i \underline{r}}{(z_i \underline{r} - \underline{k})^2 + \mu_i^2} - \frac{\underline{k} - \bar{z}_i \underline{r}}{(\bar{z}_i \underline{r} - \underline{k})^2 + \mu_i^2}$

Both Impact factor vanish when  $\underline{k} \rightarrow 0$  or  $\underline{r} - \underline{k} \rightarrow 0$  due to QCD gauge invariance

To compute the scattering amplitude  $M_{\lambda_1 \lambda_2}$  we have to perform analytically the 2D integration over the transverse momentum.

Analytical computation of the 2D integrals involved is performed after the use of conformal transformations in the transverse momentum space.

This reduces the number of propagators .

For example , we have to compute this kind of integrals :

$$J_{3m} = \int \frac{d^2 \underline{k}}{\underline{k}^2 (\underline{k} - \underline{r})^2} \left( \frac{1}{(\underline{k} - \underline{r}a)^2 + m^2} - \frac{1}{\underline{r}^2 + m^2} + (a \leftrightarrow \bar{a}) \right)$$

Inversion on the integration variable and vector parameter

$$\underline{k} \rightarrow \frac{\underline{K}}{\underline{K}^2}, \quad \underline{r} \rightarrow \frac{\underline{R}}{\underline{R}^2}, \quad m \rightarrow \frac{1}{M}$$

Then we perform the shift of variable  $\underline{K} = \underline{R} + \underline{k}'$

And an other inversion

Finally we obtain

$$J_{3m} = \frac{1}{r^2} \int \frac{d^2 \underline{k}}{\underline{k}^2} \left[ \frac{(\underline{r} + \underline{k})^2}{(r^2 a^2 + m^2) \left( \left( \underline{k} - \underline{r} \frac{r^2 a \underline{a} - m^2}{r^2 \underline{a}^2 + m^2} \right)^2 + \frac{m^2 r^4}{(r^2 \underline{a}^2 + m^2)^2} \right)} - \frac{1}{a^2 r^2 + m^2} + (a \leftrightarrow \bar{a}) \right]$$

It is now possible to compute this integral by using standard technique



## Non-forward cross-sections at ILC for

$$e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$$

We use the same Sudakov basis

$$s_{e^+e^-} \sim s/(y_1 y_2)$$

$$\begin{aligned} & \frac{d\sigma(e^+e^- \rightarrow e^+e^- \rho_L \rho_L)}{dy_1 dy_2 dQ_1^2 dQ_2^2} \\ &= \frac{1}{y_1 y_2 Q_1^2 Q_2^2} \left( \frac{\alpha}{\pi} \right)^2 [l_1(y_1) l_2(y_2) \sigma(\gamma_L^* \gamma_L^* \rightarrow \rho_L \rho_L) + t_1(y_1) l_2(y_2) \sigma(\gamma_T^* \gamma_L^* \rightarrow \rho_L \rho_L) \\ &+ l_1(y_1) t_2(y_2) \sigma(\gamma_L^* \gamma_T^* \rightarrow \rho_L \rho_L) + t_1(y_1) t_2(y_2) \sigma(\gamma_T^* \gamma_T^* \rightarrow \rho_L \rho_L)] . \end{aligned}$$

with the usual photons flux factors

$$t_i = \frac{1 + (1 - y_i)^2}{2}, \quad l_i = 1 - y_i$$

Weizsacker-Wiliams

Kinematical constraints coming from experimental features of the ILC collider are used to perform the phase-space integration.

Photons momentum fractions

$$y_i = \frac{E - E'_i \cos^2(\theta_i/2)}{E}$$

and virtualities

$$Q_i^2 = 4EE'_i \sin^2(\theta_i/2)$$

In the cms frame, kinematical constraints coming from the minimal detection angle around the beampipe and from the conditions on the energies of the scattered leptons and the Regge limit.

$$y_{i\max} = 1 - \frac{E_{\min}}{E}$$

$$y_{1\min} = \max\left(f(Q_1), 1 - \frac{E_{\max}}{E}\right) \quad \text{with} \quad f(Q_i) = 1 - \frac{Q_i^2}{s \tan^2(\theta_{\min}/2)}$$

$$y_{2\min} = \max\left(f(Q_2), 1 - \frac{E_{\max}}{E}, \frac{c Q_1 Q_2}{s y_1}\right)$$

Foreseen cms energy :  $\sqrt{s} = 2E = 500 \text{ GeV}$

# Experimental features of the ILC collider

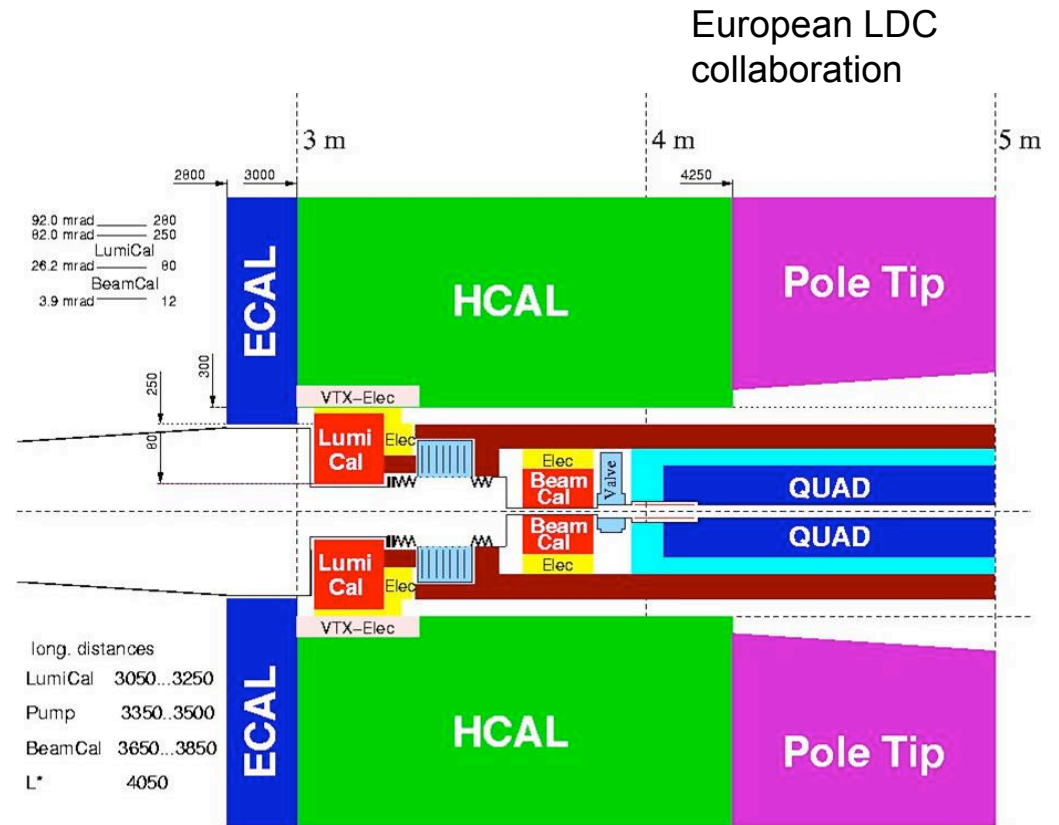
$$\sqrt{s} = 2E = 500 \text{ GeV}$$

LDC detector :  
emc BeamCal around  
the beampipe at 3.65 m from the vertex

F.Richard, R.Poeschl

$$\longrightarrow E_{min} = 100 \text{ GeV}$$

$$\longrightarrow \theta_{min} = 4 \text{ mrad}$$



ECAL, HCAL : hadron calorimeters

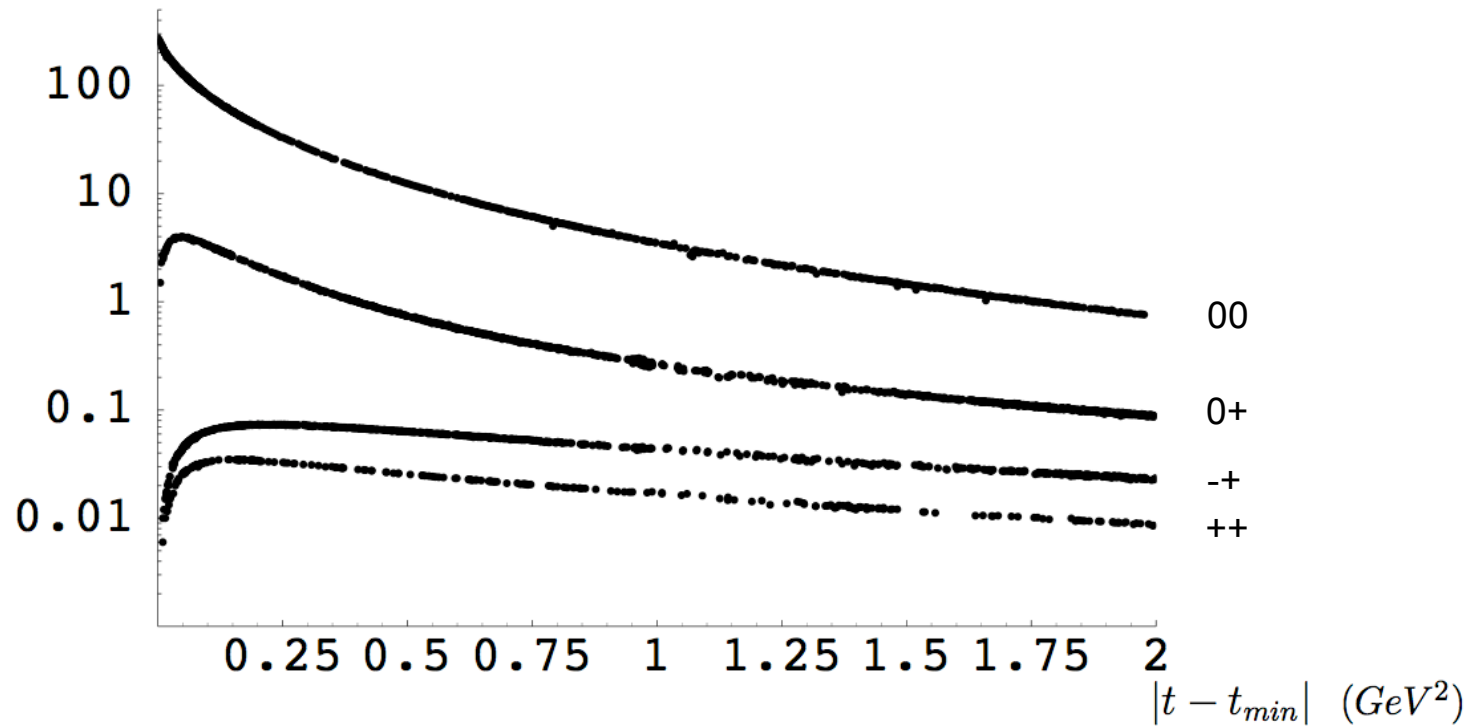
LumiCal, BeamCal : electromagnetic calorimeters

$$\frac{d\sigma^{e^+e^- \rightarrow e^+e^-} \rho_L \rho_L}{dt} = \int_{Q_{1min}^2}^{Q_{1max}^2} dQ_1^2 \int_{Q_{2min}^2}^{Q_{2max}^2} dQ_2^2 \int_{\epsilon}^{y_{max}} dy_1 \int_{\frac{Q_1 Q_2}{sy_1}}^{y_{max}} dy_2 \frac{d\sigma^{e^+e^- \rightarrow e^+e^-} \rho_L \rho_L}{dt dy_1 dy_2 dQ_1^2 dQ_2^2}$$

# Results for non-forward cross-sections at ILC for

$$e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$$

$$\frac{d\sigma^{e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0}}{dt} (fb/GeV^2)$$



$$\sigma^{LL} = 32.4 fb$$

$$\sigma^{LT} = 1.5 fb$$

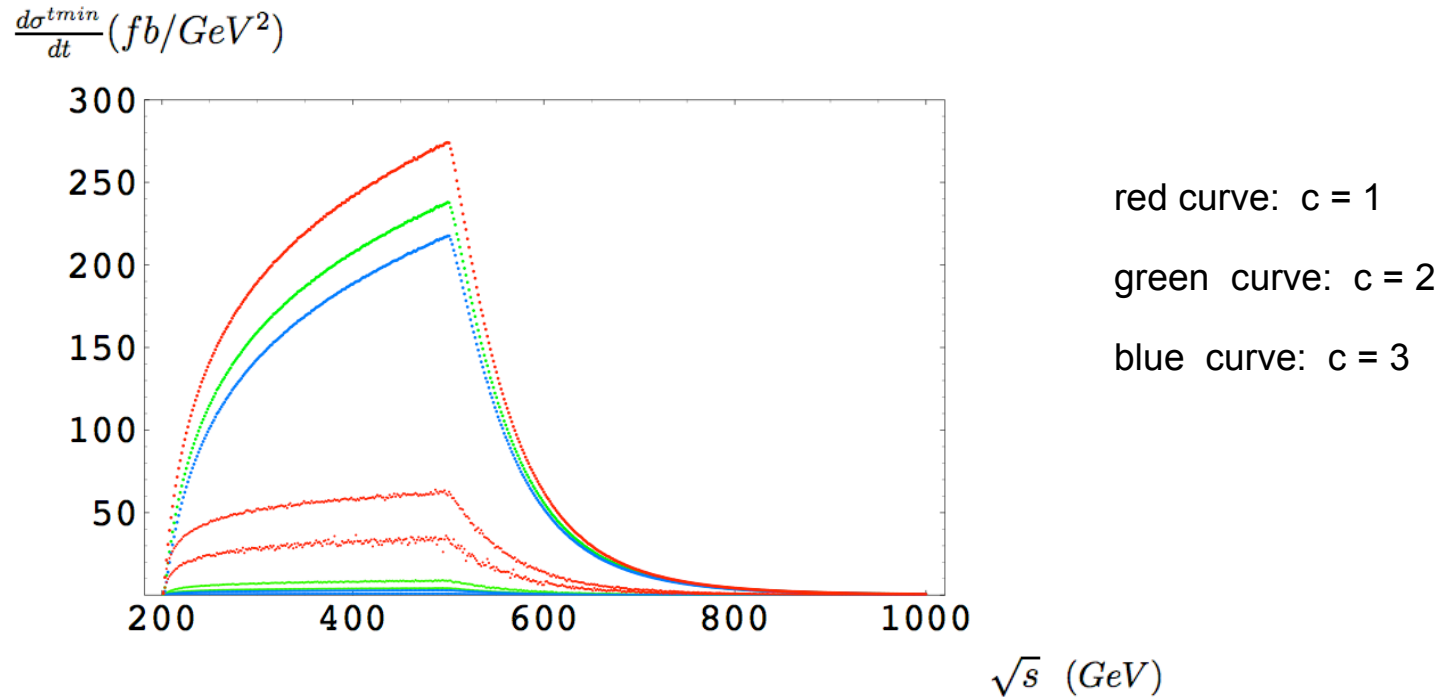
$$\sigma^{TT} = 0.2 fb$$

$$\sigma^{Total} = 34.1 fb$$

with	$\alpha_s(\sqrt{Q_1 Q_2})$ running at three loops
	$\sqrt{s} = 500 GeV$
	$c = 1$

→  $4.26 \cdot 10^3$  events per year with foreseen luminosity

## Effects of parameters and quark exchange contribution to the non-forward cross-sections for $e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$ at $t_{\min}$



For the total cross-section:

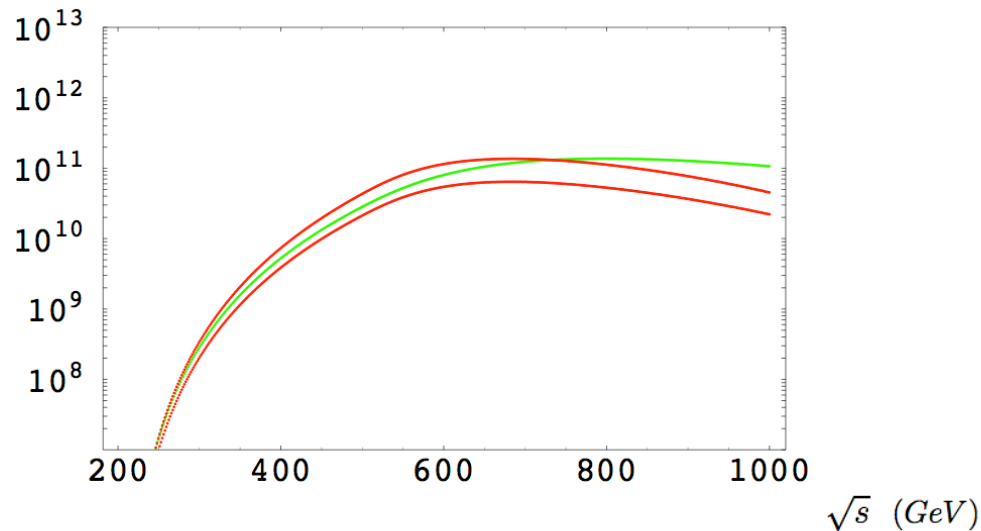
$$\alpha_s(\sqrt{Q_1 Q_2}) \text{ running at three loops} \longrightarrow \sigma^{Total} = 34.1 \text{ fb}$$

$$\alpha_s(\sqrt{Q_1 Q_2}) \text{ running at one loop} \longrightarrow \sigma^{Total} = 35.6 \text{ fb}$$

$$\alpha_s(\sqrt{Q_1 Q_2}) \text{ running at three loops and } c = 2 \longrightarrow \sigma^{Total} = 29.6 \text{ fb}$$

# Effects of parameters on the non-forward cross-sections for $e^+e^- \rightarrow e^+e^- \rho_L^0 \rho_L^0$ with LO BFKL evolution at $t_{\min}$

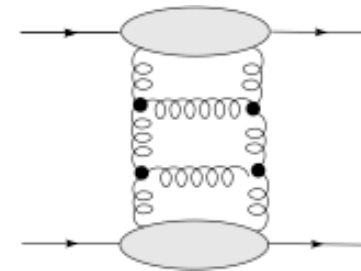
$$\frac{d\sigma^{t_{\min}}}{dt} (fb/GeV^2)$$



upper red curve:  $\alpha_s(\sqrt{Q_1 Q_2})$  running at one loop

lower red curve:  $\alpha_s(\sqrt{Q_1 Q_2})$  running at three loops

green curve: fixed value of  $\alpha_s = 0.46$



Forward case BFKL amplitude in the saddle-point approximation:

$$A(s, t = t_{\min}, Q_1, Q_2) \sim i s \frac{\alpha_s^2 \alpha_{em} f_\rho^2}{Q_1^2 Q_2^2} \frac{e^{4 \ln 2 \bar{\alpha}_s Y}}{\sqrt{14 \bar{\alpha}_s \zeta(3) Y}} \exp \left( -\frac{\ln^2 R}{14 \bar{\alpha}_s \zeta(3) Y} \right)$$

with:  $Y = \ln \left( \frac{c' s y_1 y_2}{Q_1 Q_2} \right)$   
 $\bar{\alpha}_s = \frac{N_c}{\pi} \alpha_s(\sqrt{Q_1 Q_2})$  and  $R = \frac{Q_1}{Q_2}$

# Conclusions

We gave a precise estimation of the two gluon t-channel exchange in the exclusive reaction  $\gamma^* \gamma^* \rightarrow \rho_L^0 \rho_L^0$ , which dominated at HE and corresponds to the BFKL background; since the impact factor are completely known in a perturbative way, not only the behaviour with energy but the complete amplitude can be analytically computed.  $\rightarrow$  **Clean test** of the BFKL resummation scheme at ILC.

We demonstrated the **measurability** of this process at ILC and predicted the number of events.

Born order evaluation  $\longrightarrow$  NLO BFKL evolution for any t.

At lower energy, quark exchange processes appear at lower order in  $\alpha_s$  :

- The perturbative analysis of  $\gamma^* \gamma^* \rightarrow \rho_L^0 \rho_L^0$  at the Born order leads to **two different types** of **factorization**.
- Not only **kinematics** but **polarization** states also dictate the type of the factorization.
- Usually they are applied for two different kinematics but the arbitrariness in choosing the photons virtualities shows that there may exist an intersection region.
- Generalization for transverse mesons, non-forward kinematics and charged mesons pair.