

Transverse momentum in semi-inclusive DIS

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Essential literature

- *Mulders, Tangerman, NPB 461 (96)*
[no T-odd distribution functions, no Trento conventions]
- *Boer, Mulders, PRD 57 (98)*
[subleading twist not complete, no Trento conventions]
- *Bacchetta, Mulders, Pijlman, PLB 595 (04)*
[no transverse polarization, no Trento conventions]
- *Goeke, Metz, Schlegel, PLB 618 (05)*
[no cross sections]

*A. B., Diehl, Goeke, Metz, Mulders, Schlegel,
JHEP 02 (2007) 093*

Outline

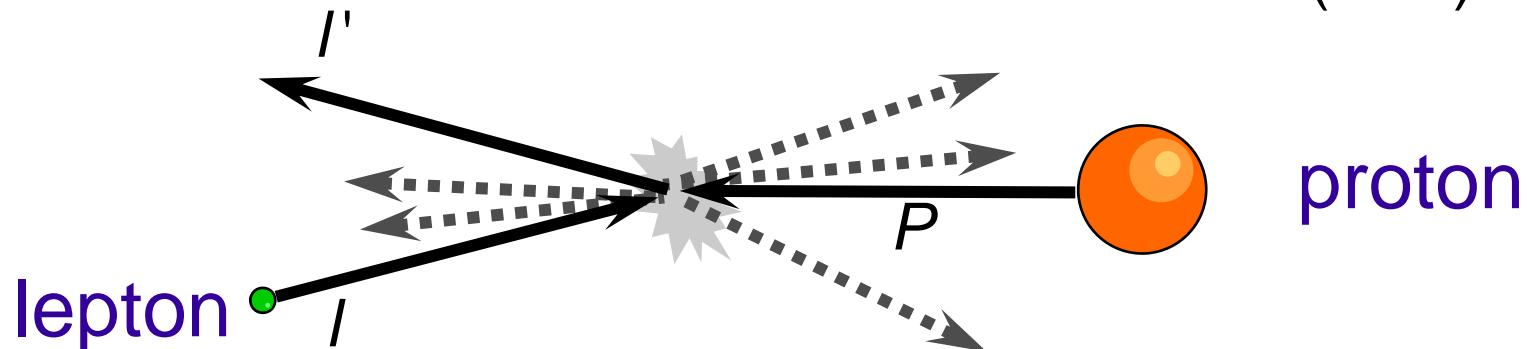
- Inclusive and semi-inclusive DIS
- Cross section in terms of structure functions
- Selected examples of structure functions in terms of transverse-momentum-dependent (TMD) partonic functions
- Conclusions

Inclusive DIS

$$\ell(l) + p^\uparrow(P) \rightarrow \ell(l') + X$$

$$-(l - l')^2 = Q^2 = \text{virtuality of photon}$$

$$x = \frac{Q^2}{2P \cdot (l - l')}$$

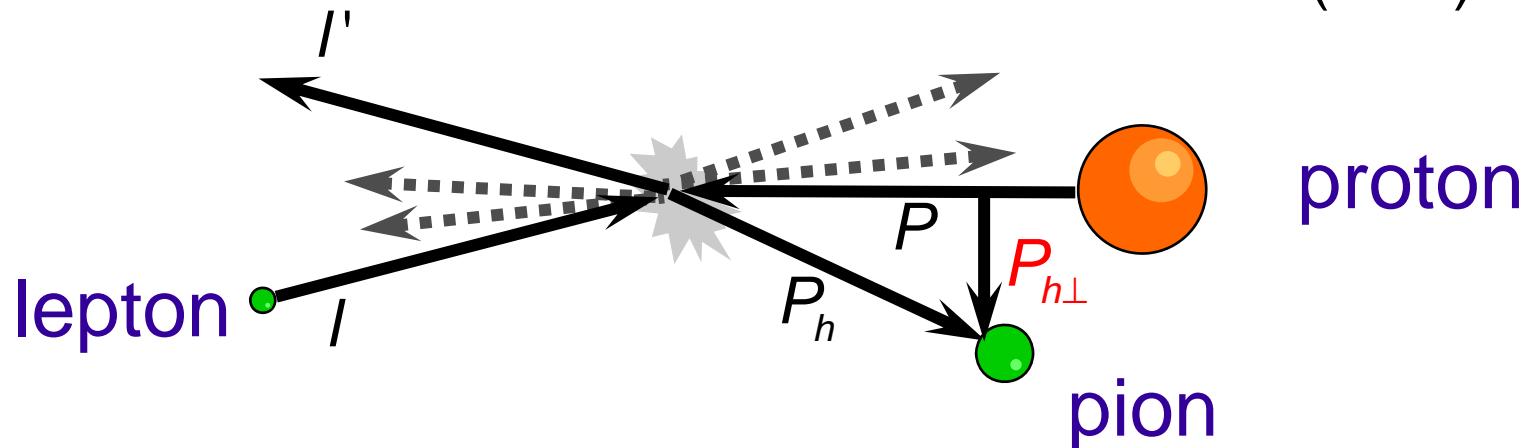


Semi-inclusive DIS (SIDIS)

$$\ell(l) + p^\uparrow(P) \rightarrow \ell(l') + h(P_h) + X$$

$$-(l - l')^2 = Q^2 = \text{virtuality of photon}$$

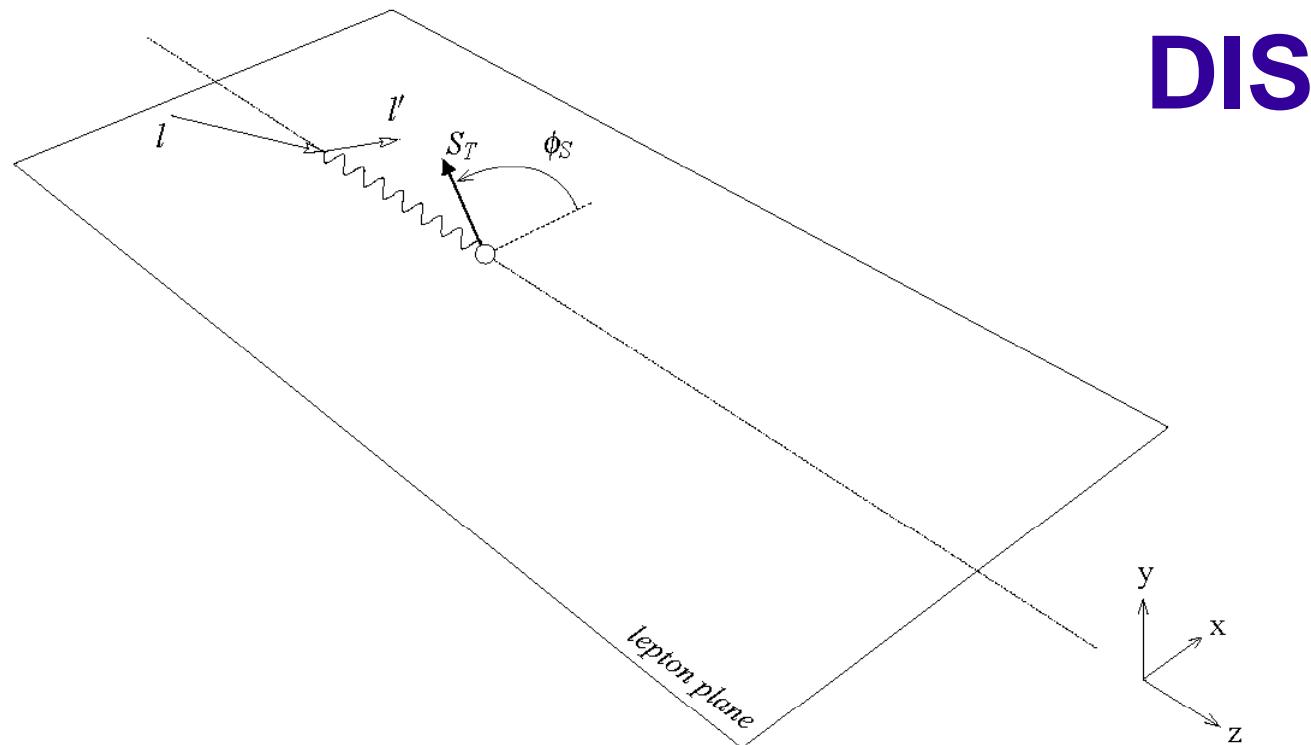
$$x = \frac{Q^2}{2P \cdot (l - l')}$$



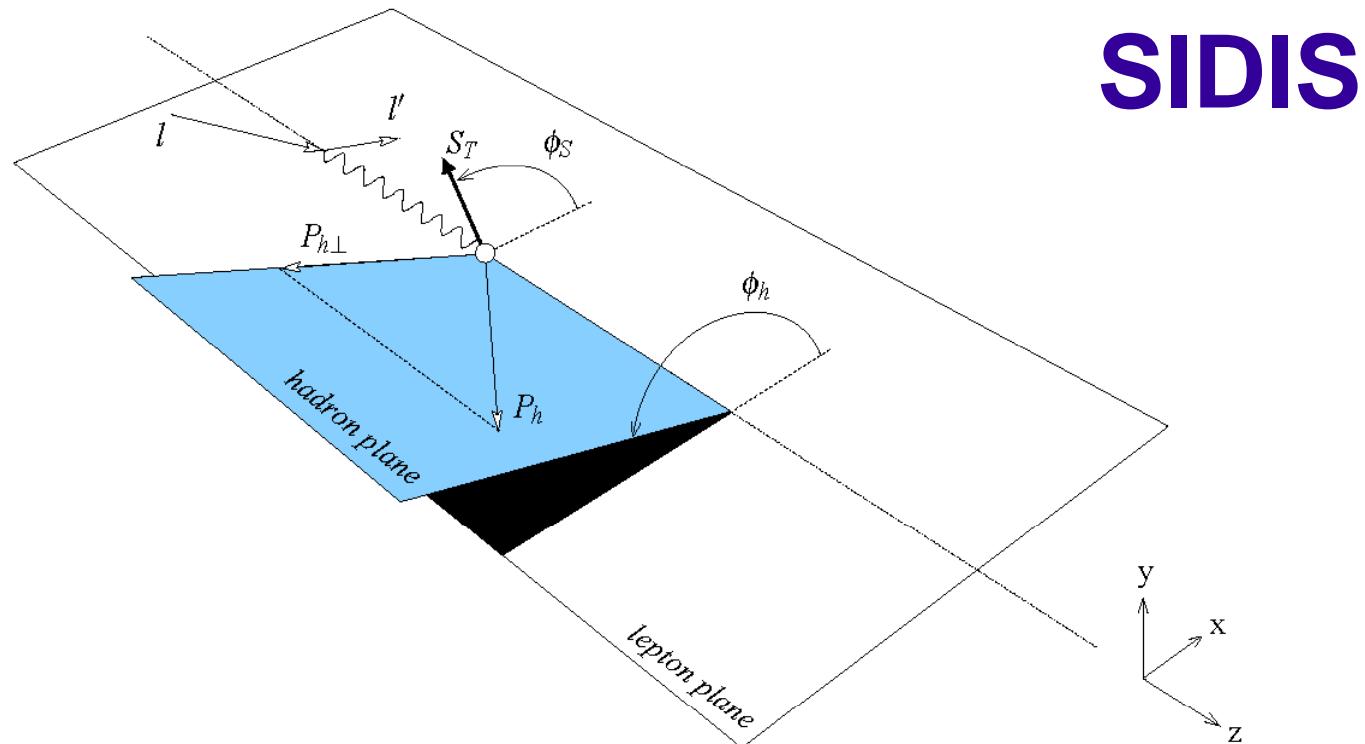
$$z = \frac{P \cdot P_h}{P \cdot (l - l')}$$

$P_{h\perp}$ = transverse momentum of pion

Vectors and angles involved



Vectors and angles involved



DIS cross section

$$\frac{d\sigma}{dx dy d\phi_S} = \frac{2 \alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_T + \varepsilon F_L + S_L \lambda_e \sqrt{1-\varepsilon^2} 2x(g_1 - \gamma^2 g_2) + S_T \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S 2x\gamma(g_1 + g_2) \right\}$$

$F_T(x, Q^2)$

Any other modulation (e.g. $\sin \phi_S$) is
a sign of two-photon exchange

Full SIDIS cross section

$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} + S_L \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] + S_T \left[\sin(\phi_h - \phi_S) (F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)}) \right. \\
 &+ \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} \\
 &+ \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \Big] + S_T \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 &\quad \left. \left. + \sqrt{2\varepsilon(1-\varepsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} + \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
 \end{aligned}$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

*Kotzinian, NPB 441 (95)
Diehl, Sapeta, EPJC 41 (05)*

18 structure functions

10 already some measurements

Regime $P_{h\perp} \ll Q$

- Factorization proof (leading twist)
Ji, Ma, Yuan, PRD 71 (04)
- Subleading twist is included, even though factorization is not guaranteed
- Structure functions can be interpreted as convolutions of TMD parton distribution functions
 - 8 PDFs are leading twist and occur in 8 structure functions
 - 16 PDFs are subleading twist and occur in 8 structure functions
 - 2 structure functions are sub-subleading twist
- For large $P_{h\perp}$ see talks by *Koike, Tanaka*

Structure functions $F_{UU,T}$ & $F_{UU,L}$

$$F_{UU,T} = \mathcal{C}[\textcolor{red}{f_1 D_1}]$$

$$= x \sum_a e_a^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z) \textcolor{red}{f_1^a(x, p_T^2)} D_1^a(z, k_T^2)$$

$F_{UU,T}(x, z, P_{h\perp}^2)$ generalizes the familiar $F_T(x)$

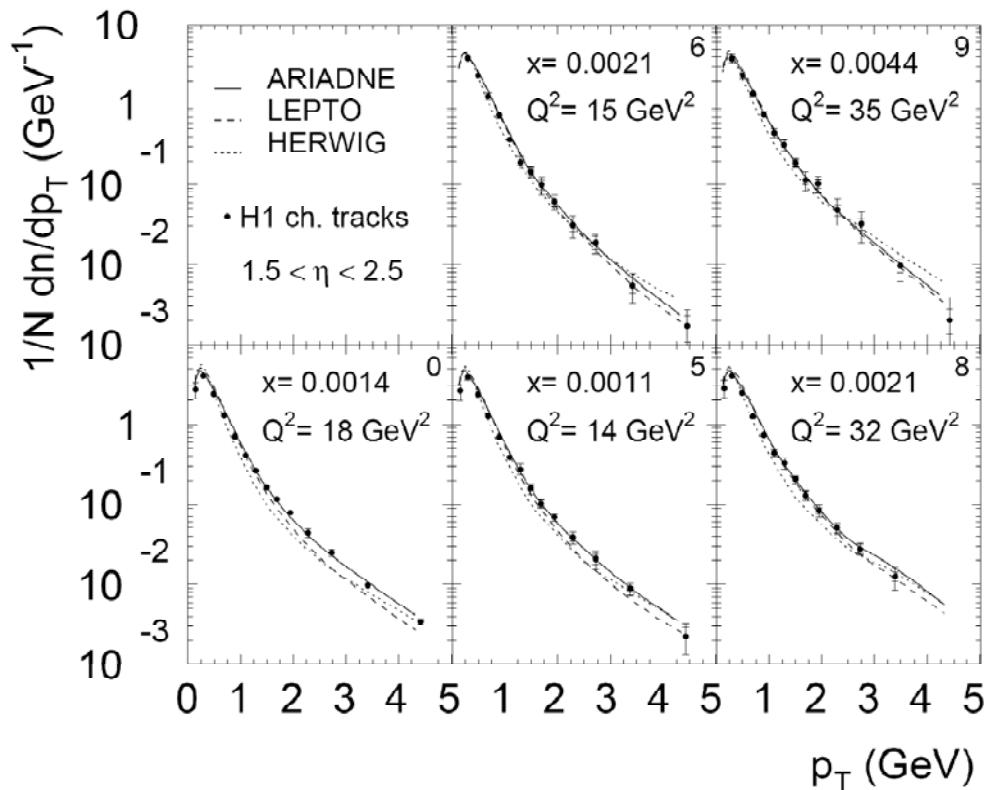
$$F_{UU,L} = \mathcal{O}\left(\frac{1}{Q^2}\right)$$

generalizes the familiar $F_L(x)$

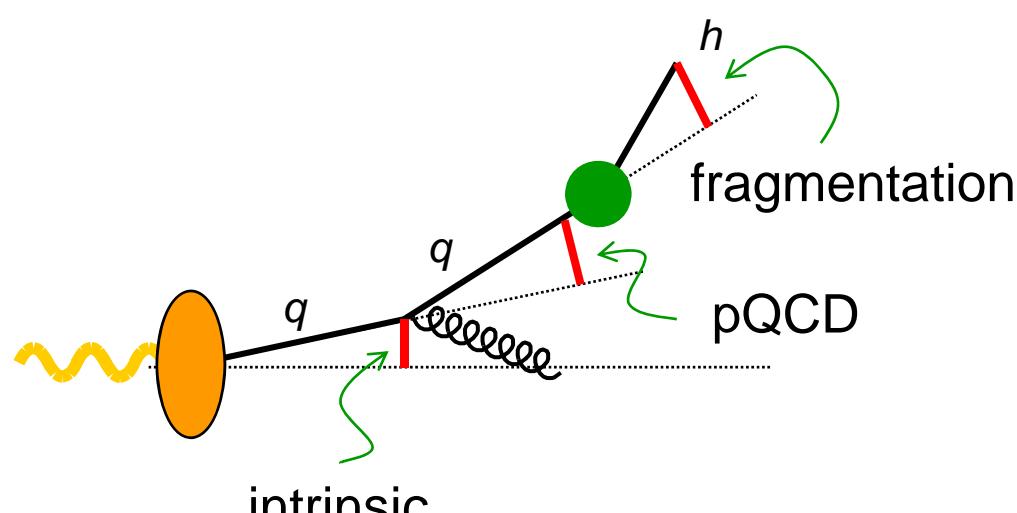
Structure function $F_{UU,T}$

$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

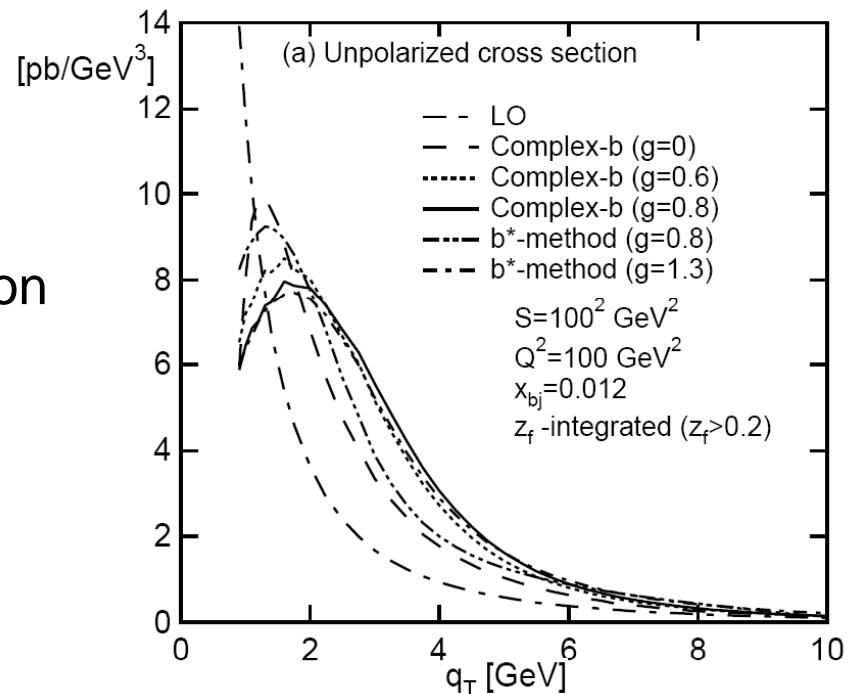
Access intrinsic transverse momentum



Structure function $F_{UU,T}$



Koike, Nagashima, Vogelsang, NPB744 (06)



See talk by C.P. Yuan (Structure functions)

Unpolarized beam and target

$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

$$F_{UU,L} = \mathcal{O}\left(\frac{1}{Q^2}\right)$$

$$F_{UU}^{\cos \phi_h} = \frac{M}{Q} \mathcal{C} \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f_1^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$

ZEUS, hep-ex/0608053

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C} \left[-\frac{2 \hat{h} \cdot \mathbf{k}_T \hat{h} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_1^\perp H_1^\perp \right]$$

ZEUS, hep-ex/0608053

Boer-Mulders

Collins

See talk by Gamberg

Longitudinally pol. beam

$$F_{LU}^{\sin \phi_h} = \frac{M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(xe H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(xg^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

CLAS, PRD 69 (04)

$$F_{UL}^{\sin \phi_h} = \frac{M}{Q} \mathcal{C} \left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_h} \left(xh_L H_1^\perp + \frac{M_h}{M} g_{1L} \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(xf_L^\perp D_1 - \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{H}}{z} \right) \right]$$

HERMES, PLB 622 (05)

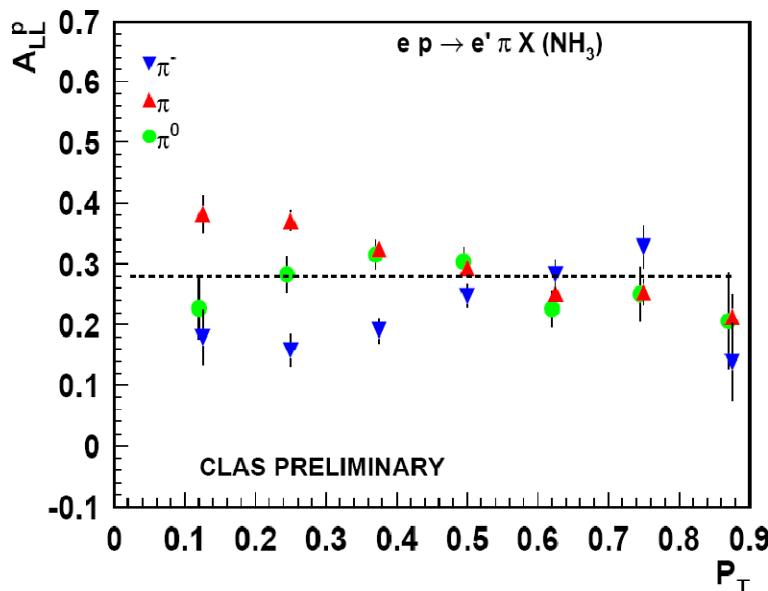
$$F_{UL}^{\sin 2\phi_h} = \mathcal{C} \left[-\frac{2 \hat{\mathbf{h}} \cdot \mathbf{k}_T \hat{\mathbf{h}} \cdot \mathbf{p}_T - \mathbf{k}_T \cdot \mathbf{p}_T}{MM_h} h_{1L}^\perp H_1^\perp \right]$$

HERMES, PRL 84 (00), PLB 562 (03)

Longitudinally pol. beam & target

$$F_{LL} = \mathcal{C}[g_{1L} D_1]$$

generalizes the familiar $g_1(x)$



- Nontrivial interplay between spin and transverse momentum
- Orbital angular momentum involved
- Differences between flavors

H. Avakian, QCD-N'06, Frascati

$$F_{LL}^{\cos \phi_h} = \frac{M}{Q} \mathcal{C} \left[\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x e_L H_1^\perp - \frac{M_h}{M} g_{1L} \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x g_L^\perp D_1 + \frac{M_h}{M} h_{1L}^\perp \frac{\tilde{E}}{z} \right) \right]$$

Transversely pol. target

Sivers

$$F_{UT,T}^{\sin(\phi_h - \phi_S)} = \mathcal{C} \left[-\frac{\mathbf{p}_T \cdot \hat{\mathbf{h}}}{M} f_{1T}^\perp D_1 \right]$$

$$F_{UT,L}^{\sin(\phi_h - \phi_S)} = \mathcal{O}\left(\frac{1}{Q^2}\right)$$

$$F_{UT}^{\sin(\phi_h + \phi_S)} = \mathcal{C} \left[-\frac{\mathbf{k}_T \cdot \hat{\mathbf{h}}}{M_h} h_1 H_1^\perp \right]$$

Transversity

HERMES, PRL 94 (05)

COMPASS, NPB 765 (07)

See talks by Diefenthaler,
Bressan, D'Alesio

Transversely pol. target (cont.)

$$F_{LT}^{\cos(\phi_h - \phi_S)} = C \left[\frac{\mathbf{p}_T \cdot \hat{\mathbf{h}}}{M} g_{1T} D_1 \right]$$

- Similar to Sivers function measurement
- Sensitive to orbital angular momentum

$$F_{UT}^{\sin \phi_S} = \frac{M}{Q} C \left\{ \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) - \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2 M M_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\}$$

+ 4 more

See talk by Kotzinian

Transversely pol. target (cont.)

$$F_{UT}^{\sin \phi_S} = \frac{M}{Q} C \left\{ \left(x f_T D_1 - \frac{M_h}{M} h_1 \frac{\tilde{H}}{z} \right) - \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2 M M_h} \left[\left(x h_T H_1^\perp + \frac{M_h}{M} g_{1T} \frac{\tilde{G}^\perp}{z} \right) - \left(x h_T^\perp H_1^\perp - \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{D}^\perp}{z} \right) \right] \right\}$$

Integrated over $P_{h\perp}$

$$F_{UT}^{\sin \phi_S} = -x \sum_q e_q^2 \frac{M_h}{Q} h_1^q(x) \frac{\tilde{H}^q(z)}{z}$$

In inclusive DIS

$$F_{UT}^{\sin \phi_S} = 0$$

Conclusions

- In one-particle-inclusive DIS there are 18 structure functions to be measured
- Some information about 10 of them is already available
- The structure functions can be interpreted in the low-transverse-momentum regime in terms of transverse-momentum-dependent partonic functions
- Constant progress in experiments, phenomenology and theory
- A lot remains to be done