

Recent developments in heavy flavour production

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- Fixed Order Perturbation Theory (FFNS)
- Massless Variable Flavour Number Scheme (ZM-VFNS)
- Massive Variable Flavour Number Scheme (GM-VFNS)

3 HARD SCATTERING COEFFICIENTS WITH HEAVY-QUARK MASSES

- Massless Limit
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- Fragmentation Functions
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OVERVIEW

Subject of this talk:

- One-particle inclusive production of heavy hadrons $H = D, B, \Lambda_c, \dots$
- General-Mass Variable Flavour Number Scheme (GM-VFNS):
 - ▶ Collinear logarithms of the heavy-quark mass $\ln \mu/m_h$ are subtracted and resummed [1]
 - ▶ Finite non-logarithmic m_h/Q terms are kept in the hard part/taken into account
 - ▶ Scheme guided by the factorization theorem of Collins with heavy quarks [2]

Ongoing effort to compute all relevant processes in the GM-VFNS at NLO:

- $\gamma + \gamma \rightarrow D^{*+} + X$: direct process [3]
- $\gamma + \gamma \rightarrow D^{*+} + X$: single-resolved process [4]
- $\gamma + p \rightarrow D^{*+} + X$ [5,6]
- $p + \bar{p} \rightarrow (D^0, D^{*+}, D^+, D_s^+, \Lambda_c^+) + X$ [1,7]
- $p + \bar{p} \rightarrow B^+ + X$ [8]

[1] Kniehl,G.K.,Schienbein,Spiesberger, PRD71(2005)014018; EPJC41(2005)199

[2] J. Collins, PRD58(1998)094002

[3] G.K.,Spiesberger, EPJC22(2001)289; [4] EPJC28(2003)495; [5] EPJC38(2004)309

[6] Kniehl,G.K.,Schienbein,Spiesberger, to be published,
results compared to H1 data in hep-ex/0608042, submitted to EPJC

[7] Kniehl,G.K.,Schienbein,Spiesberger, PRL96(2006)012001

[8] Kniehl,G.K.,Schienbein,Spiesberger, in preparation

OVERVIEW -CONTINUED-

Input for the computation: Fragmentation Functions (FFs) into heavy hadrons H

- FFs from fits to e^+e^- data from LEP and SLD directly in x -space
- Use new fits with initial scale $\mu_0 = m_h$ (instead of $\mu_0 = 2m_h$)
→ important for gluon fragmentation → consistency with PDFs
- FFs from fits to $e^+e^- \rightarrow B + X$ data from LEP and SLD
- FFs from fits to $e^+e^- \rightarrow D^*, D, D_s, \Lambda_c$ data from LEP

Work in progress:

- new extraction of FFs for production of D mesons in the GM-VFNS (→ include low-energy data)

T. Kneesch, Univ. Hamburg

Goal:

- Test pQCD formalism/universality of FFs in as many processes as possible

MOTIVATION

WHY HEAVY FLAVOUR PRODUCTION?

Heavy Quarks: $h = c, b, t$

... are interesting:

- $m_h \gg \Lambda_{\text{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}) \ll 1$ (asymptotic freedom)
- m_h sets hard scale; acts as long distance cut-off

⇒ PERTURBATION THEORY (PQCD) APPLICABLE!

Heavy quark production processes are:

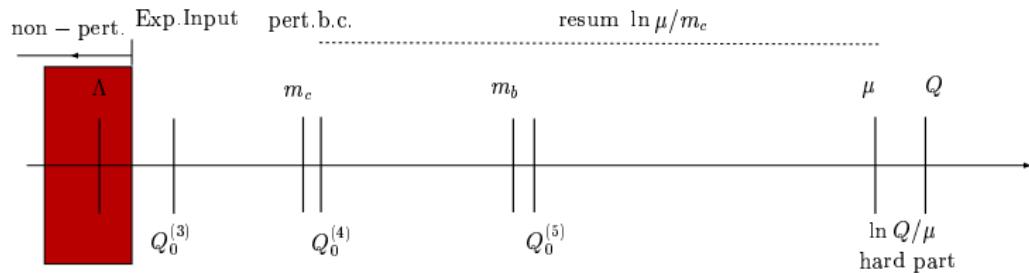
- Fundamental elementary particle processes
- Important background to New Physics searches at the LHC

MOTIVATION

HEAVY QUARKS AND PQCD

Fixed Order Perturbation Theory:

- finite collinear logs $\ln Q/m_h$, $h = b, c$ arise → can be kept in hard part
Of course need exp. Input for u, d, s, g PDFs at scale $Q_0^{(3)}$



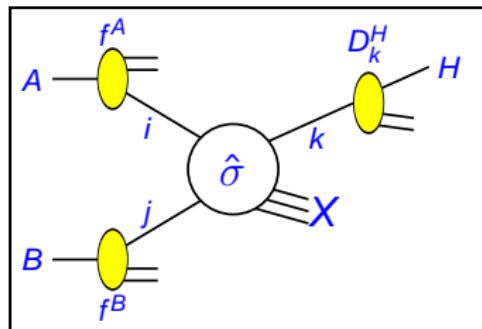
Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: multi-scale problems
For $Q \gg m_h$: write $\ln Q/m_h = \ln Q/\mu + \ln \mu/m_h$, subtract $\ln \mu/m_h$ and resum $\ln \mu/m_h$ ($h=b,c$)
by introducing charm and/or bottom PDF (FF) at $Q_0^{(4)} \simeq m_c$ and/or $Q_0^{(5)} \simeq m_b$ using a perturbative boundary condition

THEORETICAL BASIS: FACTORIZATION FORMULAE FOR $A + B \rightarrow H + X$

$$d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\hat{\sigma}(ij \rightarrow kX) \otimes D_k^H(z)$$

Sum over all possible subprocesses $i + j \rightarrow k + X$



- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), [\frac{m_h}{p_T}])$: hard scattering cross sections, **perturbatively** computable, free of long-distance physics $\rightarrow m_h$ can be kept
- PDFs $f_i^A(x_1, \mu_F), f_j^B(x_2, \mu_F)$: **universal, non-perturbative** input
 - ▶ Direct Photon: $i, j = \gamma, f_\gamma^Y(x, \mu_F) = \delta(1 - x)$
 - ▶ Resolved Photon/Proton: $i, j = g, q, \dots$ [$q = u, d, s$]
- Fragmentation functions $D_k^H(z, [\mu'_F])$: **universal, non-perturbative** input $k = h, \dots$
- Error:
 - ▶ light hadrons: $\mathcal{O}((\Lambda/p_T)^p)$ with p_T hard scale, Λ hadronic scale, $p = 1, 2$
 - ▶ heavy hadrons: $\mathcal{O}((m_h/p_T)^p)$ if m_h neglected in $d\hat{\sigma}$

Details (which subprocesses, PDFs, FFs; mass terms) depend on the **Heavy Flavour Scheme**

Two basic approaches:

- Fixed Order Perturbation Theory (FFNS)
- Parton Model (ZM-VFNS)

Interpolating scheme combining the good features:

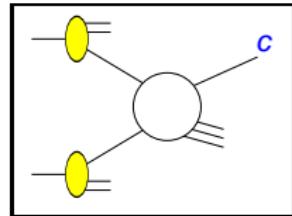
- Parton Model with quark masses (GM-VFNS, ACOT)

Glossary:

- ZM: Zero Mass
- GM: General Mass
- VFNS: Variable Flavour Number Scheme
- FFNS: Fixed Flavour Number Scheme

FIXED FLAVOUR NUMBER SCHEME (FFNS)

- $m_c \neq 0$, $n_f = 3$ fixed
- Partons: g, u, d, s
[NO charm parton; Charm (only) in final state]
- collinear logarithms $\ln \frac{s}{m_c^2}$ finite
 - No factorization; no conceptual necessity for FFs
 - fixed order perturbation theory; **no resummation**
- Usually c treated in on-shell scheme ($\overline{\text{MS}}_m$)

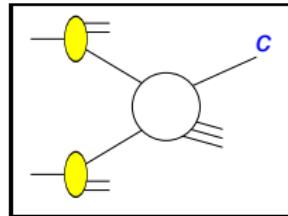


Pro and Contra:

- + $(\frac{m_c}{p_T})^n$ terms included; correct threshold suppression
 - ⇒ valid for $0 \leq p_T^2 \lesssim m_c^2$ ⇒ σ_{tot} calculable
- fixed order logarithms $\ln \frac{p_T^2}{m_c^2}$ large for $p_T^2 \gg m_c^2$;
resummation of these large logarithms necessary
 - ⇒ breaks down for $p_T^2 \gg m_c^2$
- non-perturbative function $D_c^H(z)$, describing the hadronisation $c \rightarrow H$ needed to match data;
 - not based on factorization theorem (no AP evolution)
 - universal?

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CONVENTIONAL PARTON MODEL (ZM-VFNS)

- $m_c = 0 \rightarrow$ 'Zero Mass'
- Number of active partons depends on scale $\mu_F \rightarrow$ 'VFNS'
 - ▶ $\mu_F < m_c$: $n_f = 3$, Partons: g, u, d, s
 - ▶ $m_c \leq \mu_F < m_b$: $n_f = 4$, Partons: g, u, d, s, c
 - ▶ $m_b \leq \mu_F$: $n_f = 5$, Partons: g, u, d, s, c, b
- Matching conditions at transition scale $Q_0 = m_c$ (similar at m_b): $n_f = 3 \rightarrow n_f = 4$
$$\left. \begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array} \right\} @ Q_0 = m_c \quad \boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0} \text{ pert. b.c.}$$
- Collinear divergences related to c lines factorized into non-perturbative PDFs and FFs

Pro and Contra:

- + large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^*}(x, \mu^2)$ to LL and NLL accuracy
⇒ $\boxed{\text{good for large } \mu^2 \simeq p_T^2 \gg m_c^2}$
- + Universality of PDFs and FFs guaranteed by factorization theorem → predictive power, global data analysis
- $(\frac{m_c}{p_T})^n$ terms neglected in the hard part
⇒ $\boxed{p_T^2 \lesssim m_c^2} \Rightarrow \text{No } \sigma_{\text{tot}}$

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 - ▶ $m_b \leq \mu_F$: $n_f = 5$, Partons: g, u, d, s, c, b
- Matching conditions at transition scale $Q_0 = m_c$ (similar at m_b): $n_f = 3 \rightarrow n_f = 4$

$$\left. \begin{array}{l} \alpha_s^{(3)} \rightarrow \alpha_s^{(4)} = \alpha_s^{(3)} + \mathcal{O}(\alpha_s^3) \\ f_i^{(3)} \rightarrow f_i^{(4)} = f_i^{(3)} + \mathcal{O}(\alpha_s^2) \end{array} \right\} @ Q_0 = m_c \quad \boxed{f_c^{(4)}(x, Q_0^2 = m_c^2) = 0} \text{ pert. b.c.}$$

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MASSIVE VFNS (GM-VFNS)

- VFNS with $m_c \neq 0$
- Partons: g, u, d, s, c (\exists charm parton: $f_c \neq 0$)
- collinear $\ln \frac{\mu^2}{m_c^2}$ terms:
subtracted from hard part (avoid double counting!) and
resummed by AP evolution equations ($\rightarrow f_c \neq 0$)
- $D_c^{D^*}(z, \mu_F'^2)$ evolved

Pro and Contra:

- technically more involved:
 - ▶ calculation with $m_c \neq 0$
 - ▶ subtraction of collinear parts \leftrightarrow 'IR-safe' hard parts
 - Mass factorization with massive regularization
 - ▶ kinematics: factorization with massive partons \rightarrow 'ACOT- χ ' in DIS
 - + large collinear logarithms $\ln \frac{\mu^2}{m_c^2}$ resummed in evolved $f_c(x, \mu^2)$ and $D_c^{D^*}(x, \mu^2)$
 - + $(\frac{m_c}{p_T})^n$ included
- \Rightarrow good for all p_T : $0 \leq p_T^2 \lesssim m_c^2$ and $p_T^2 \gg m_c^2$

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Two ways to derive them:

- Massless limit of fixed order calculation
- Mass factorization with massive regularization

OUR THEORETICAL BASIS FOR $p\bar{p} \rightarrow D^* X$ AND $p\bar{p} \rightarrow BX$

Factorization Formula:

[1]

$$d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^p(x_1) f_j^{\bar{p}}(x_2) \times \\ d\hat{\sigma}(ij \rightarrow kX) D_k^{D^*}(z) + \mathcal{O}(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)$$

Q: hard scale, $p = 1, 2$

replace D^* by B in the case of B production

- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections free of long-distance physics $\rightarrow m_h$ kept
- PDFs $f_i^p(x_1, \mu_F), f_j^{\bar{p}}(x_2, \mu_F)$: $i, j = g, q, c$ [$q = u, d, s$]
- FFs $D_k^{D^*}(z, \mu'_F)$: $k = g, q, c$

\Rightarrow need short distance coefficients including heavy quark masses

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

LIST OF SUBPROCESSES: GM-VFNS

Only light lines

- 1 $gg \rightarrow qX$
- 2 $gg \rightarrow gX$
- 3 $qg \rightarrow gX$
- 4 $qg \rightarrow qX$
- 5 $q\bar{q} \rightarrow gX$
- 6 $q\bar{q} \rightarrow qX$
- 7 $qg \rightarrow \bar{q}X$
- 8 $qg \rightarrow \bar{q}'X$
- 9 $qg \rightarrow q'X$
- 10 $qq \rightarrow gX$
- 11 $qq \rightarrow qX$
- 12 $q\bar{q} \rightarrow q'X$
- 13 $q\bar{q}' \rightarrow gX$
- 14 $q\bar{q}' \rightarrow qX$
- 15 $qq' \rightarrow gX$
- 16 $qq' \rightarrow qX$

Heavy quark initiated ($m_Q = 0$)

- 1 -
- 2 -
- 3 $Qg \rightarrow gX$
- 4 $Qg \rightarrow QX$
- 5 $Q\bar{Q} \rightarrow gX$
- 6 $Q\bar{Q} \rightarrow QX$
- 7 $Qg \rightarrow \bar{Q}X$
- 8 $Qg \rightarrow \bar{q}X$
- 9 $Qg \rightarrow qX$
- 10 $QQ \rightarrow gX$
- 11 $QQ \rightarrow QX$
- 12 $Q\bar{Q} \rightarrow qX$
- 13 $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- 14 $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- 15 $Qq \rightarrow gX, qQ \rightarrow gX$
- 16 $Qq \rightarrow QX, qQ \rightarrow qX$

Mass effects: $m_Q \neq 0$

- 1 $gg \rightarrow QX$
- 2 -
- 3 -
- 4 -
- 5 -
- 6 -
- 7 -
- 8 $qg \rightarrow \bar{Q}X$
- 9 $qg \rightarrow QX$
- 10 -
- 11 -
- 12 $q\bar{q} \rightarrow QX$
- 13 -
- 14 -
- 15 -
- 16 -

⊕ charge conjugated processes

ADOPTED PROCEDURE

- Calculate $m \rightarrow 0$ limit of massive 3-FFNS calculation (at partonic level) [1]

Only keep m as regulator in $\ln \frac{m^2}{s}$

Partonic subprocesses in 3-FFNS:(4-FFNS)

- Leading Order (LO):

1. $gg \rightarrow c\bar{c}$ resp. $gg \rightarrow b\bar{b}$
2. $q\bar{q} \rightarrow c\bar{c}$ ($q = u, d, s$) resp. $q\bar{q} \rightarrow b\bar{b}$ ($q = u, d, s, c$)

- Next-To-Leading Order (NLO):

1. $gg \rightarrow c\bar{c}g$ resp. $gg \rightarrow b\bar{b}g$
2. $q\bar{q} \rightarrow c\bar{c}g$ resp. $q\bar{q} \rightarrow b\bar{b}g$
3. $gq \rightarrow c\bar{c}q$ resp. $gq \rightarrow b\bar{b}q$

New@NLO

Limiting procedure non-trivial:

- Map from PS-slicing to Subtraction method
- care needed to recover $\delta(1-w)$, $(\frac{1}{1-w})_+$, $(\frac{\ln(1-w)}{1-w})_+$

Checks:

- Compare Abelian parts with results in [2]
- Numerical tests

[1] Bojak, Stratmann, PRD67(2003)034010; FORTRAN code provided by I. Bojak

[2] G. K. Spiesberger, EPJC22(2001)289; EPJC28(2003)495

ADOPTED PROCEDURE –CONTINUED–

- Compare $m \rightarrow 0$ limit of massive calculation with massless $\overline{\text{MS}}$ calculation [1]

$$\lim_{m \rightarrow 0} d\sigma(m) = d\hat{\sigma}(\overline{\text{MS}}) + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{SUB}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\sigma(m) - d\hat{\sigma}(\overline{\text{MS}})$$

- Subtract $d\sigma_{\text{SUB}}$ from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\sigma(m) - d\sigma_{\text{SUB}}$$

→ $d\hat{\sigma}(m)$ short distance coefficient including m

→ allows to use PDFs and FFs with $\overline{\text{MS}}$ factorization ⊕ massive short dist. cross sections

- Treat contributions with charm in the initial state with $m_c = 0$;
~~ scheme choice of practical importance; tiny effect in DIS [2]

[1] Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105

[2] Kretzer, Schienbein, PRD58(1998)094035

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SUBTRACTION TERMS VIA $\overline{\text{MS}}$ MASS FACTORIZATION: $a(k_1)b(k_2) \rightarrow Q(p_1)X$ [1]

Sketch of kinematics:

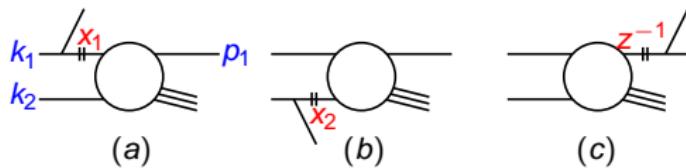


Fig. (a): $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_1 f_{a \rightarrow i}^{(1)}(x_1, \mu_F^2) d\hat{\sigma}^{(0)}(ib \rightarrow QX)[x_1 k_1, k_2, p_1]$

$$\equiv f_{a \rightarrow i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(ib \rightarrow QX)$$

Fig. (b): $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_2 f_{b \rightarrow j}^{(1)}(x_2, \mu_F^2) d\hat{\sigma}^{(0)}(aj \rightarrow QX)[k_1, x_2 k_2, p_1]$

$$\equiv f_{b \rightarrow j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \rightarrow QX)$$

Fig. (c): $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dz d\hat{\sigma}^{(0)}(ab \rightarrow kX)[k_1, k_2, z^{-1} p_1] d_{k \rightarrow Q}^{(1)}(z, \mu_F'^2)$

$$\equiv d\hat{\sigma}^{(0)}(ab \rightarrow kX) \otimes d_{k \rightarrow Q}^{(1)}(z)$$

1. initial state:

$$\begin{aligned} f_{g \rightarrow Q}^{(1)}(x, \mu^2) &= \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(x) \ln \frac{\mu^2}{m^2} \\ f_{Q \rightarrow Q}^{(1)}(x, \mu^2) &= \frac{\alpha_s(\mu)}{2\pi} C_F \left[\frac{1+z^2}{1-z} \left(\ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+ \quad [2] \\ f_{g \rightarrow g}^{(1)}(x, \mu^2) &= -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x) \end{aligned}$$

2. final state:

$$\begin{aligned} d_{g \rightarrow Q}^{(1)}(z, \mu^2) &= \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(z) \ln \frac{\mu^2}{m^2} \\ d_{Q \rightarrow Q}^{(1)}(z, \mu^2) &= C_F \frac{\alpha_s(\mu)}{2\pi} \left[\frac{1+z^2}{1-z} \left(\ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+ \quad [1, 2, 3] \end{aligned}$$

- Other distributions are zero to this order in α_s
- Analogous for photon splitting: $g \rightarrow \gamma$, $\alpha_s \rightarrow \alpha$, color factors

[1] Mele, Nason, NPB361(1991)626; Ma, NPB506(1997)329 [$\gamma^* \rightarrow c\bar{c}g$][2] Kretzer, Schienbein, PRD58(1998)094035; D59(1999)054004 [$c\gamma^* \rightarrow cg$][3] Melnikov, Mitov, PRD70(2004)034027; Mitov, PRD71(2005)054021 [$\mathcal{O}(\alpha_s^2)$]

Existing calculations: (based on DGLAP evolution)

- ZM-VFNS: Binnewies et al.; Cacciari et al.
- FFNS: Frixione et al.
- GM-VFNS:
 - ▶ direct part: G. K., Spiesberger
 - ▶ resolved part: Kniehl, G. K., Schienbein, Spiesberger (from hadroproduction)

Status:

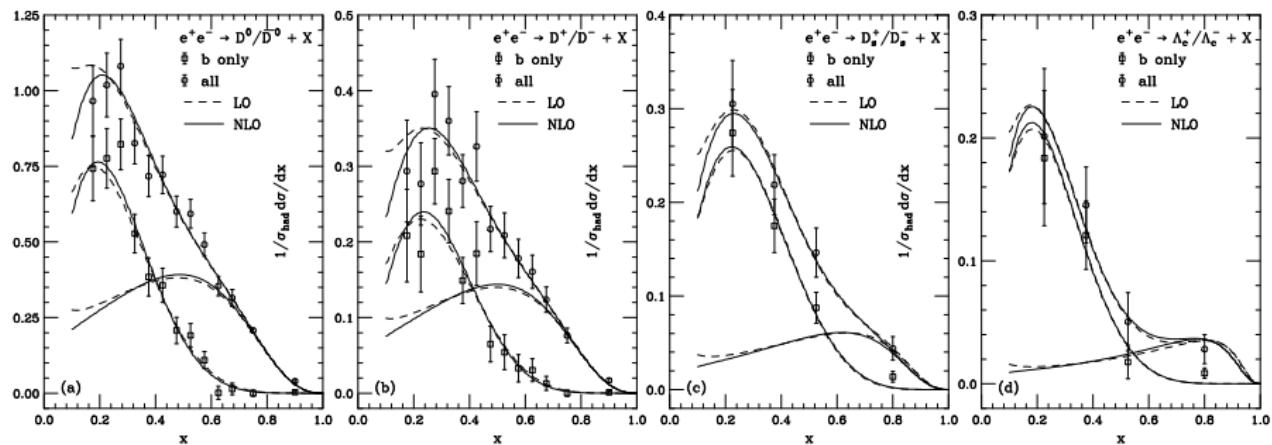
- For comparison of GM-VFNS with preliminary ZEUS data (p_T, y, W, z distributions) see G. K., Spiesberger, EPJC38(2004)309
- Including resolved part in GM-VFNS will help to improve results at small p_T
- New FFs with initial scale $\mu_0 = m_h$ used, results compared with recent H1 data in H1 publication: A. Aktas et al., DESY 06-110, hep-ex/0608042, will be shown below

NUMERICAL RESULTS

Input parameters:

- $\alpha_s(M_Z) = 0.1181$
- $m_c = 1.5 \text{ GeV}$, $m_b = 4.5 \text{ GeV}$
- PDFs: CTEQ6.1M (NLO)
- FFs: NLO FFs from fits to LEP1-OPAL data (D -mesons);
from fits to LEP1-ALEPH and OPAL, SLC-SLD data (B -mesons)
- **initial scale for evolution:** $\mu_0 = m_c$ (D -mesons) resp. $\mu_0 = m_b$ (B -mesons)
- Default scale choice: $\mu_R = \mu_F = \mu'_F = m_T$ where $m_T = \sqrt{p_T^2 + m^2}$

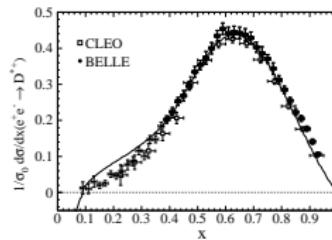
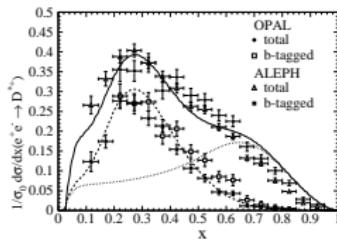
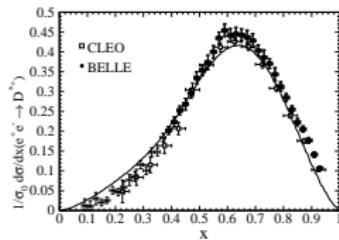
- $(1/\sigma_{\text{tot}})d\sigma/dx$, ZM-VFNS
- $\mu_R, \mu_F' = \sqrt{S}$
- Fit to LEP1 data from OPAL [2]



[1] Kniehl, G. K., PRD71(2005)094013

[2] Alexander et al., OPAL Collaboration, ZPC72(1996)1

- $(1/\sigma_{\text{tot}})d\sigma/dx$, GM-VFNS, $\mu_R, \mu_F' = \sqrt{S}$
- Corrected for photon radiation in the initial state
- Fit to data from BELLE and CLEO [2], combined fit, compared to ALEPH and OPAL data from LEP1 [3]
- Bowler parametrisation [4]: $D_c^{D^*}(x, \mu^2) = N(1-x)^\alpha x^{-(1+\gamma^2)} \exp(-\gamma^2/x)$ at $\mu = m_c$



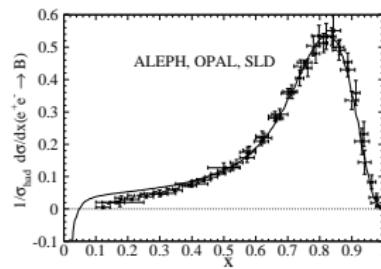
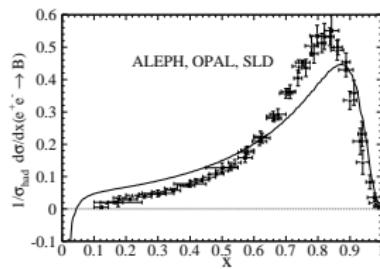
[1] Kneesch, Kniehl, G. K., to be published

[2] Seuster et al., BELLE Collaboration, PRD73 (2006) 032002; M. Artuso et al., CLEO Collaboration, PRD70 (2004) 112001

[3] K. Ackerstaff et al., OPAL Collaboration, EPJC1 (1998) 439; R. Barate et al., ALEPH Collaboration, EPJC16 (2000) 597

[4] M. G. Bowler, Z.Phys. C11 (1981)

- $(1/\sigma_{\text{tot}}) d\sigma/dx$, ZM-VFNS
- $\mu_R, \mu_F' = \sqrt{S}$
- Fit to LEP1 data from ALEPH [2], OPAL [3] and SLC data from SLD [4]
- Full points: ALEPH, squares: OPAL, triangles: SLD
- Left figure: Peterson ansatz, right figure: power ansatz
- Starting scale $\mu = m_b$ in both
- Peterson can not, power can describe the data



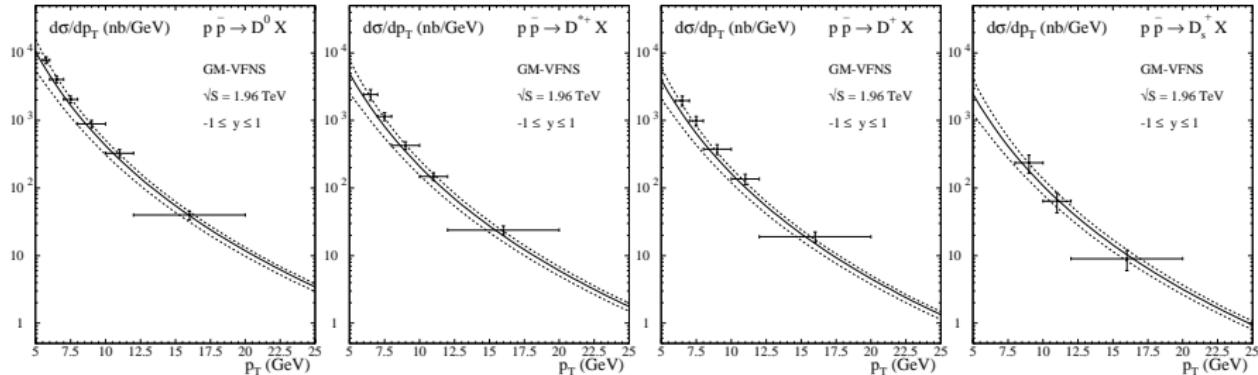
[1] Kniehl, G. K., Schienbein, Spiesberger, to be published

[2] Heister et al., ALEPH Collaboration, Phys. Lett. B512 (2001) 30

[3] Abbiendi et al., OPAL Collaboration, EPJ C29 (2003) 463

[4] Abe et al., SLD Collaboration, Phys. Rev. D65 (2002) 092006, D66 (2002) 079905(E)

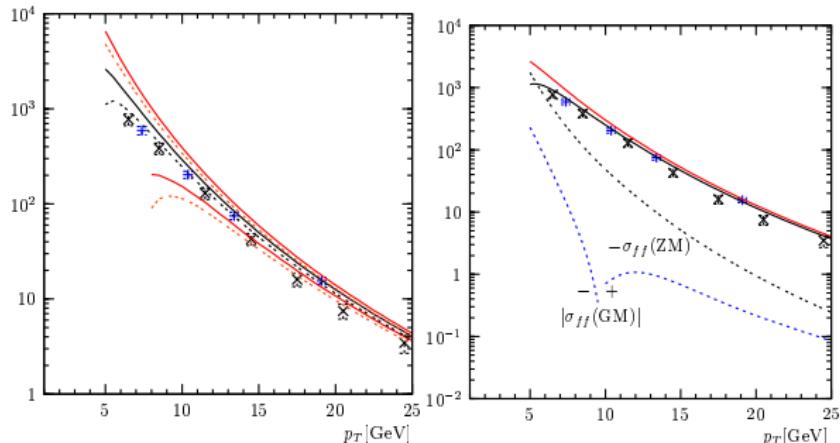
- $d\sigma/dp_T$ (nb/GeV), $|y| \leq 1$, GM-VFNS
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T$, $\xi \in [1/2, 2]$



- Data described by QCD within errors
- New PDFs with intrinsic charm content by Pumplin et al., [3], could be tested with these data

[1] Kniehl, G.K, Schienbein, Spiesberger, PRL96(2006)012001; [2] Acosta et al, PRL91(2003)241804 [3] Pumplin, Lai, Tung, MSY-HEP-070101, hep-ph/0701220

- $d\sigma/dp_T$ (nb/GeV), $|y| \leq 1$, GM-VFNS, four massless flavours, one massive
- Fragmentation function fitted to $e^+ e^-$ data
 $D_b^{B^\pm}(x) = Nx^\alpha(1-x)^\beta$ at $\mu = m_b = 4.5$ GeV
- $\mu = \xi m_T$, $\xi_R = 1$, central : $\xi_I = \xi_F = 1$, lower : $\xi_I = \xi_F = 0.5$, upper : $\xi_I = \xi_F = 2$

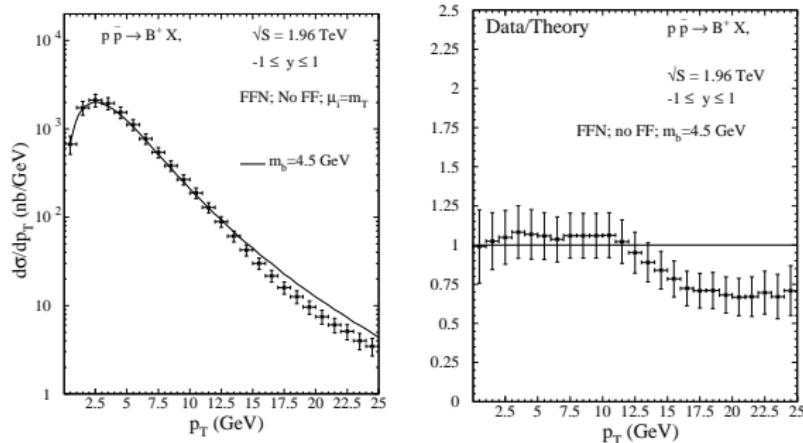


- Data described well by GM-VFNS in range of applicability: $p_T \gtrsim 10$ GeV, no agreement for small p_T
- GM-VFN (full lines) approaches ZM-VFN (dashed lines) at large p_T

[1] Acosta et al, CDF Collaboration, PRD 71 (2005) 034016 (black points);
 Abulencia et al., CDF Collaboration, hep-ex/0612015, (blue points)

COMPARISON WITH CDF II DATA FOR $p\bar{p} \rightarrow B^+ X \rightarrow J/\psi X'$ [1]

- $d\sigma/dp_T$ (nb/GeV), $|y| \leq 1$, FFN scheme, four massless flavours, one massive
- No fragmentation function: $D_b^{B^\pm}(x) = N\delta(1-x)$, N determined by $BR(b \rightarrow B) = 0.397$
- Theory well known since 1988



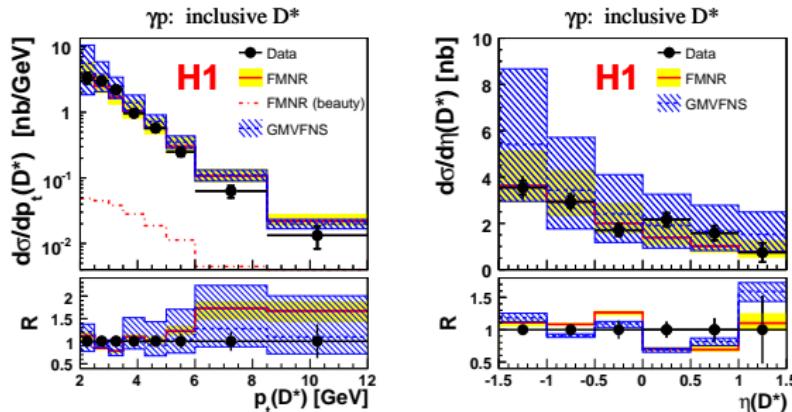
- Data described well by FFNS theory in range of applicability: $p_T \lesssim 10 \text{ GeV}$, disagreement for large p_T

[1] Acosta et al, CDF Collaboration, PRD 71 (2005) 034016

[2] Nason et al., NPB 303 (1988); B 327 (1989) 49; B 335 (1989) 260(E); Beenacker et al., PRD 40 (1989) 54; NPB 351 (1991) 507

COMPARISON WITH H1 DATA FOR $\gamma p \rightarrow D^{*\pm} + X$ [1]

- $d\sigma/dp_T$ (nb/GeV), $|y| \leq 1.5$, FFN scheme and GM-VFN scheme, both three massless flavours, one massive
- Fragmentation function for GM-VFNS: $D_c^{D^*\pm}(x) = Nx^\alpha(1-x)^\beta$ $b \rightarrow D^*$ and Peterson $c \rightarrow D^*$
- Theory FFN [2] GM-VFN [3]



- Data described by FFNS and GM-VFNS theory, GM-VFNS large scale variation for small p_T , $p_T \sim 2$ GeV

[1] Aktas et al, H1 Collaboration, DESY 06-110, hep-ex/0608042

[2] Frixione, Nason, Ridolfi, NPB 454 (1995) 3

[3] Kniehl, G. K., Schienbein, Spiesberger, to be published

- In this talk:
Discussion of one-particle inclusive production of heavy quarks in the GM-VFNS
- Available at NLO in the GM-VFNS:
 - ▶ $\gamma\gamma \rightarrow HX$, $\gamma p \rightarrow HX$, $p\bar{p} \rightarrow HX$
- Work in progress:
 - ▶ $e p \rightarrow HX$
- General expectation:
 - ▶ Improvement at $p_T \gg m_h$ due to updated FFs (and PDFs)
 - ▶ Mass effects: Improve agreement with (HERA and TEVATRON) data for $p_T \gtrsim m_h$
 - ▶ Reduced factorization scale dependence at $p_T \gg m_h$
- No time to discuss other recent work
 - ▶ Work by Bierenbaum, Blümlein and Klein on two-loop operator matrix elements in DIS heavy flavour production: DESY 07-026 and DESY 06-118

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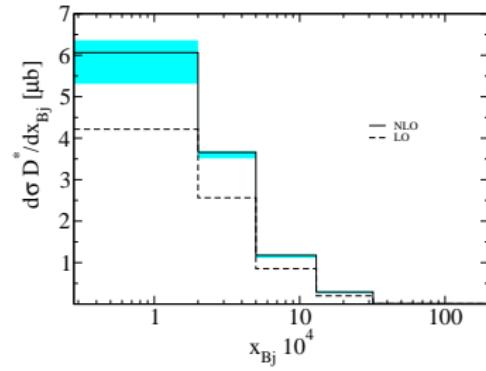
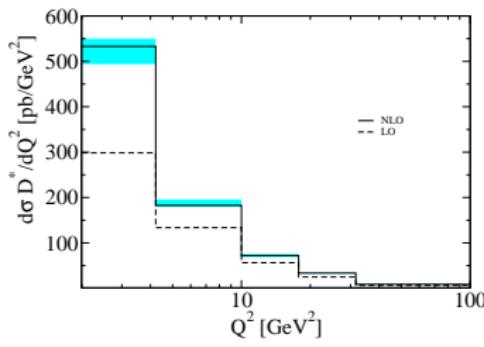
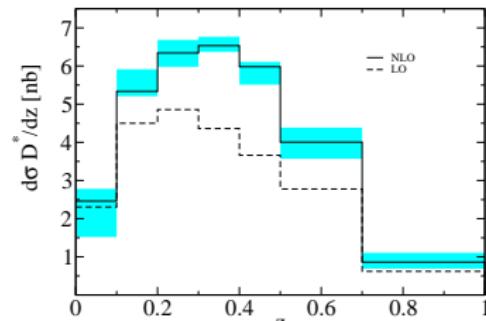
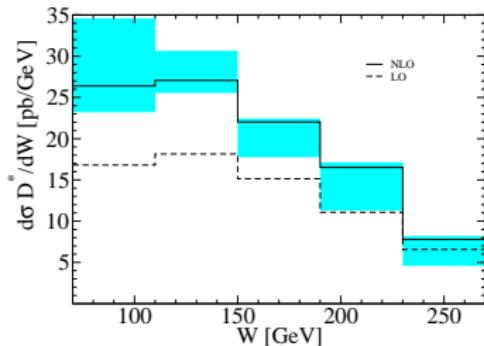
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Backup Slides

PREDICTIONS FOR $ep \rightarrow D^{*+}X$ IN THE ZM-VFNS (PROVIDED BY MARKOS MANIATIS)



[◀ go back](#)

COMPARISON WITH FONLL

FONLL = FO + (RS - FOM0)G(m, p_T) with

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

GM-VFNS = FO + (RS - FOM0)G(m, p_T) with

$$\tilde{G}(m, p_T) = 1$$

FO: Fixed Order; FOM0: Massless limit of FO; RS \equiv ZM-VFNS: Resummed

- Both approaches interpolate between FO and ZM-VFNS
 - ▶ FONLL: obvious;
 - ▶ GM-VFNS: matching with FO at quark level (see Olness,Scalise,Tung,PRD59(1998)014506)
- Factor $\tilde{G}(m, p_T)$ follows from calculation; $\tilde{G}(m, p_T) = 1 \leftrightarrow$ S-ACOT scheme
- Different point-of-view: GM-VFNS finally needs PDFs and FFs in this scheme.
- Numerical comparisons interesting and should be done!

COMPARISON OF FONLL THEORY WITH DATA FOR $p\bar{p} \rightarrow B^\pm X$ [1]

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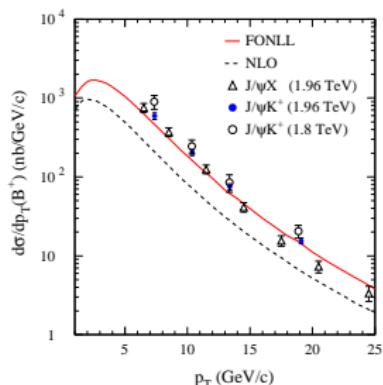
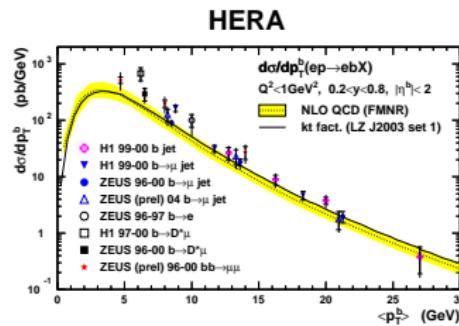


FIG. 10: Measurements of the B^+ differential cross section ($|p_T^{B^+}| \leq 4$) at the Tevatron are compared to the NLO and FONLL theoretical predictions (see text). The result of this experiment (\bullet) is shown together with those of (\triangle) Ref. [30] and (\square) Ref. [9]; the result of Ref. [9] has been increased by 10% to account for the expected increase of the cross section from $\sqrt{s} = 1.8$ to 1.96 TeV [1].



COMPARISON OF FO THEORY WITH DATA FOR

$\gamma p \rightarrow bX$ [1]

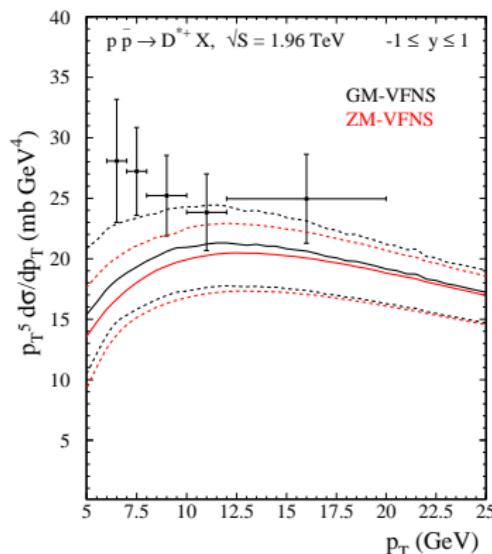


[1] Brugnera, Contribution to the proceedings of HQL06, Munich, hep-ex/0701027

MASS EFFECTS: GM-VFNS vs. ZM-VFNS

$p\bar{p} \rightarrow D^{*+} X$

- Results with old FFs with initial scale $\mu_0 = 2m_c$
- Uncertainty band: independent variation of $\mu_R, \mu_F, \mu'_F = \xi m_T$, $\xi \in [1/2, 2]$



- In this example still $p_T > 3m_c$
- Mass effects bigger for small μ_R (large $\alpha_s(\mu_R)$)

STRONG COUPLING CONSTANT

- PDG'04: $\alpha_s(M_Z) = 0.1187 \pm 0.0020$
- CTEQ6M PDFs: $\alpha_s(M_Z) = 0.118$; MRST03 $\alpha_s(M_Z) = 0.1165$;
- $d\sigma(p\bar{p} \rightarrow DX) \propto \alpha_s^2(1 + \alpha_s K)$

