

# Transversity and Collins function: from $e^+e^- \rightarrow h_1h_2X$ to SIDIS processes

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## Outline

- Azimuthal Asymmetries in  $e^+e^- \rightarrow h_1h_2X$ :  $\mathbf{k}_\perp$  helicity formalism
- Single Spin Asymmetries in  $\ell p \rightarrow \ell' hX$ :  $\mathbf{k}_\perp$  helicity formalism (**Collins effect**)
- Global fit of HERMES, COMPASS, BELLE data
  - $\Rightarrow \Delta_T q(x)$  (transversity)
  - $\Rightarrow \Delta^N D_{\pi/q^\uparrow}(z, p_\perp)$  (Collins function)
- Comparison with other extractions
- Predictions for JLAB and COMPASS
- Conclusions and outlook

$e^+e^- \rightarrow h_1 h_2 X$  cross sections: helicity formalism with  $k_\perp$

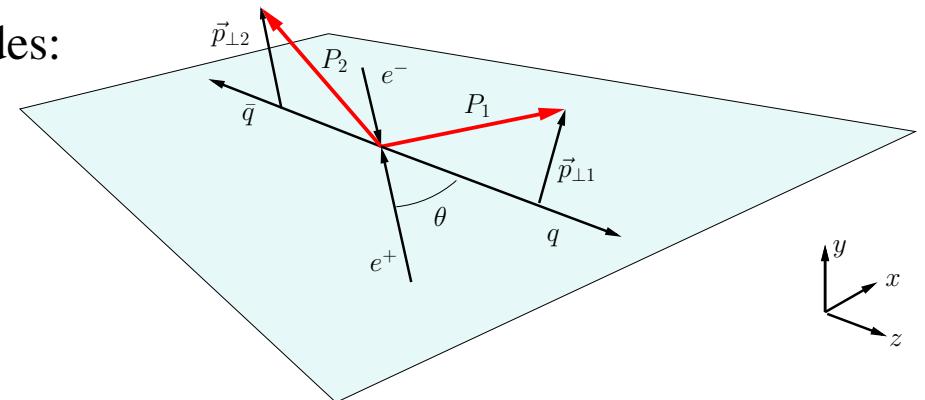
The cross section for unpolarized leptons reads

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{p}_{\perp 1} d^2\mathbf{p}_{\perp 2} d\cos\theta} \simeq \sum_{\{\lambda\}} \hat{M}_{\lambda_q \lambda_{\bar{q}}; \lambda_+ \lambda_-} \hat{M}_{\lambda'_q \lambda'_{\bar{q}}; \lambda_+ \lambda_-}^* D_{\lambda_q \lambda'_q}^{h_1/q}(z_1, \mathbf{p}_{\perp 1}) D_{\lambda_{\bar{q}} \lambda'_{\bar{q}}}^{h_2/\bar{q}}(z_2, \mathbf{p}_{\perp 2})$$

only two non-zero, independent helicity amplitudes:

$$\hat{M}_{+-;+-} = \hat{M}_{-+;-+} = e^2 e_q (1 + \cos\theta)$$

$$\hat{M}_{-+;+-} = \hat{M}_{+-;-+} = e^2 e_q (1 - \cos\theta)$$



$D_{\lambda_q \lambda'_q}^{h/q}(z, \mathbf{p}_{\perp 1})$  are probability densities for the fragmentation of  $q$  into  $h$

$$D_{\pm\pm}^{h/q}(z, \mathbf{p}_{\perp}) = D_{h/q}(z, p_{\perp}) \quad \text{unpolarized frag. function}$$

$$-2i D_{+-}^{h/q}(z, p_{\perp}) = \Delta^N D_{h/q\uparrow}(z, p_{\perp}) \equiv \frac{2p_{\perp}}{zM_h} H_1^{\perp} \quad \text{Collins function}$$

$$\frac{d\sigma^{e^+ e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2 \mathbf{p}_{\perp 1} d^2 \mathbf{p}_{\perp 2} d \cos \theta} \simeq \sum_q e_q^2 \left\{ (1 + \cos^2 \theta) D_{h_1/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \right.$$

$$\left. + \frac{1}{4} \sin^2 \theta \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(\varphi_1 + \varphi_2) \right\}$$

**BELLE analysis** (Seidl talk):

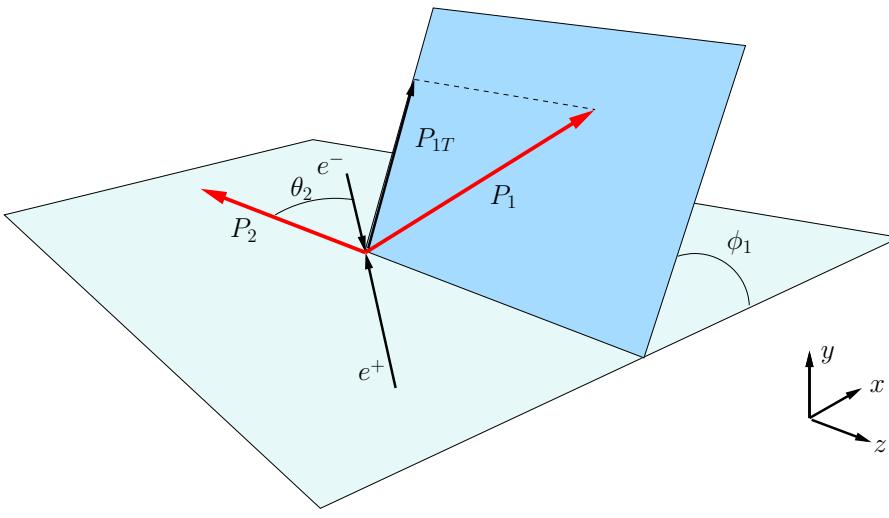
- $(\varphi_1, \varphi_2) \rightarrow (\varphi_1, \varphi_1 + \varphi_2)$ ;
- integrated over  $p_{\perp 1}, p_{\perp 2}, \varphi_1$  and normalized to  $\langle d\sigma \rangle$
- ratio of unlike-sign to like-sign pion pair production,  $A_U$  and  $A_L$
- $(\Delta^N)D_{\text{fav}} \equiv (\Delta^N)D_{\pi^+/u, \bar{d}}$  and  $(\Delta^N)D_{\text{unf}} \equiv (\Delta^N)D_{\pi^+/d, \bar{u}, \text{s}}$

$$R \equiv \frac{A_U}{A_L} \simeq 1 + \cos(\varphi_1 + \varphi_2) \frac{1}{4} \frac{\langle \sin^2 \theta \rangle}{\langle 1 + \cos^2 \theta \rangle} (P_U - P_L)$$

$$= 1 + \cos(\varphi_1 + \varphi_2) A_{12}(z_1, z_2)$$

$$P_U \simeq [5 \Delta^N D_{\text{fav}}(z_1) \Delta^N D_{\text{fav}}(z_2) + 7 \Delta^N D_{\text{unf}}(z_1) \Delta^N D_{\text{unf}}(z_2)] / [\Delta^N D \rightarrow D]$$

$$\Delta^N D(z) = \int d^2 \mathbf{p}_\perp \Delta^N D(z, p_\perp)$$



Complementary analysis in a different reference frame, [Boer et al. (1997)]:  $e^+e^- \rightarrow q\bar{q}$  out of the  $\hat{x}\hat{z}$  plane: helicity scattering amplitudes involve an azimuthal phase  $\varphi_2$ . No reconstruction of the quark direction.

At  $\mathcal{O}(p_\perp/z\sqrt{s})$ :

$$\frac{d\sigma^{e^+e^- \rightarrow h_1 h_2 X}}{dz_1 dz_2 d^2\mathbf{p}_{\perp 1} d^2\mathbf{p}_{\perp 2} d\cos\theta_2} \simeq \sum_q e_q^2 \left\{ (1+\cos^2\theta_2) D_{h_1/q}(z_1, p_{\perp 1}) D_{h_2/\bar{q}}(z_2, p_{\perp 2}) \right. \\ \left. + \frac{1}{4} \sin^2\theta_2 \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1}) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2}) \cos(2\varphi_2 + \phi_q^{h_1}) \right\},$$

$\phi_q^{h_1}$ : azimuthal angle of the hadron  $h_1$  around the direction of the fragmenting quark,  $q$ .

It can be expressed in terms of  $\mathbf{p}_{\perp 2}$  and  $P_{1T}$ .

Integrated over  $\mathbf{p}_{\perp 2}$  and  $P_{1T}$ ; normalized; ratio of unlike- to like-sign pion pairs:

$$R \simeq 1 + \cos(2\phi_1) A_0(z_1, z_2),$$

with

$$A_0(z_1, z_2) = \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} (P_U - P_L),$$

Configuration adopted in previous analysis [Efremov et al. (2005)]

2 ways of extracting the hadron-hadron azimuthal correlation in  $e^+e^-$ .

(helicity TMD approach with full  $\mathbf{k}_\perp$  kinematics under completion)

## SSA in SIDIS: helicity formalism with $k_\perp$

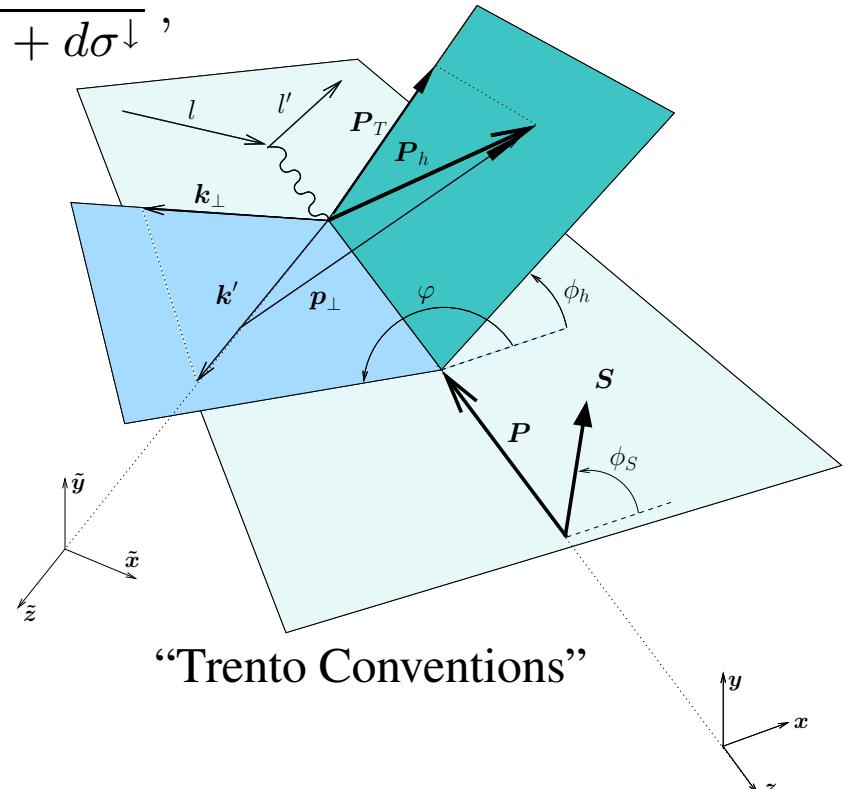
The transverse single spin asymmetry (SSA) is defined as

$$A_{UT} = \frac{d^6\sigma^{\ell p^\uparrow \rightarrow \ell' h X} - d^6\sigma^{\ell p^\downarrow \rightarrow \ell' h X}}{d^6\sigma^{\ell p^\uparrow \rightarrow \ell' h X} + d^6\sigma^{\ell p^\downarrow \rightarrow \ell' h X}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow},$$

$$d^6\sigma^{\ell p^\uparrow \rightarrow \ell h X} \equiv d^6\sigma/dx_B dy dz_h d^2\mathbf{P}_T d\phi_S$$

and the  $\sin(\phi_S + \phi_h)$  weighted asymmetry:

$$A_{UT}^{\sin(\phi_S + \phi_h)} = 2 \frac{\int d\phi_S d\phi_h [d\sigma^\uparrow - d\sigma^\downarrow] \sin(\phi_S + \phi_h)}{\int d\phi_S d\phi_h [d\sigma^\uparrow + d\sigma^\downarrow]}$$



TMD factorization (**no soft factor**) at  $\mathcal{O}(k_\perp/Q)$  (focusing on the numerator)

$$d\sigma^\uparrow - d\sigma^\downarrow \simeq \int d^2 k_\perp \Delta_T q(x, k_\perp) d(\Delta\hat{\sigma}) \Delta^N D_{h/q}^\uparrow(z, p_\perp) \sin(\phi_S + \varphi + \phi_q^h) + \dots$$

$$d(\Delta\hat{\sigma}) \equiv \frac{d\hat{\sigma}^{\ell q \uparrow \rightarrow \ell q \uparrow}}{dy} - \frac{d\hat{\sigma}^{\ell q \uparrow \rightarrow \ell q \downarrow}}{dy} = \frac{4\pi\alpha^2}{sxy^2} (1-y) \text{ (lepton-quark asymmetry)}$$

$\sin(\phi_S + \varphi + \phi_q^h)$  from phase dependences in:

- $\Delta_T q(x, k_\perp)$      $k_\perp = (k_\perp, \varphi)$
- **non-planar**  $\ell q \rightarrow \ell q$  scattering amplitudes
- $\Delta^N D_{h/q}^\uparrow(z, p_\perp)$   $\hat{s} \cdot (\hat{p}_q \times \hat{p}_\perp)$

along the general approach: Anselmino et. al (2006)

$\phi_q^h$ : azimuthal angle of the final hadron  $h$  in the quark helicity frame

$$p_\perp \cos \phi_q^h \simeq P_T \cos(\phi_h - \varphi) - z k_\perp$$

Notice  $\phi_q^h \equiv \phi_h|_{k_\perp=0}$

Simple factorized (Gaussian) parameterizations

$$\Delta_T q(x, k_\perp) = \mathcal{N}_q^T(x) \frac{1}{2} [q(x) + \Delta q(x)] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \mathcal{N}_q^C(z) 2 D_{h/q}(z) \sqrt{2e} \frac{p_\perp}{M} \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle_C}}{\pi \langle p_\perp^2 \rangle} ,$$

with  $\left[ \langle p_\perp^2 \rangle_C = \frac{M^2 \langle p_\perp^2 \rangle}{M^2 + \langle p_\perp^2 \rangle} \right]$

$$\mathcal{N}_q^T(x) \simeq N_q^T x^\alpha (1-x)^\beta \quad q = u, d \quad [4]$$

$$\mathcal{N}_q^C(z) \simeq N_q^C z^\gamma (1-z)^\delta \quad q = u, d, s \quad [h = \pi : \text{ fav } - \text{ unfav}] \quad [4]$$

Soffer bound and positivity bound are automatically fulfilled.

$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2$  and  $\langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2$  from Cahn effect  $\rightarrow$  9 parameters.

By performing the  $k_\perp$  integration analytically:

$$\sin(\phi_S + \varphi + \phi_q^h) \rightarrow \sin(\phi_S + \phi_h) \text{ and } \int d\phi_S d\phi_h \text{ gives}$$

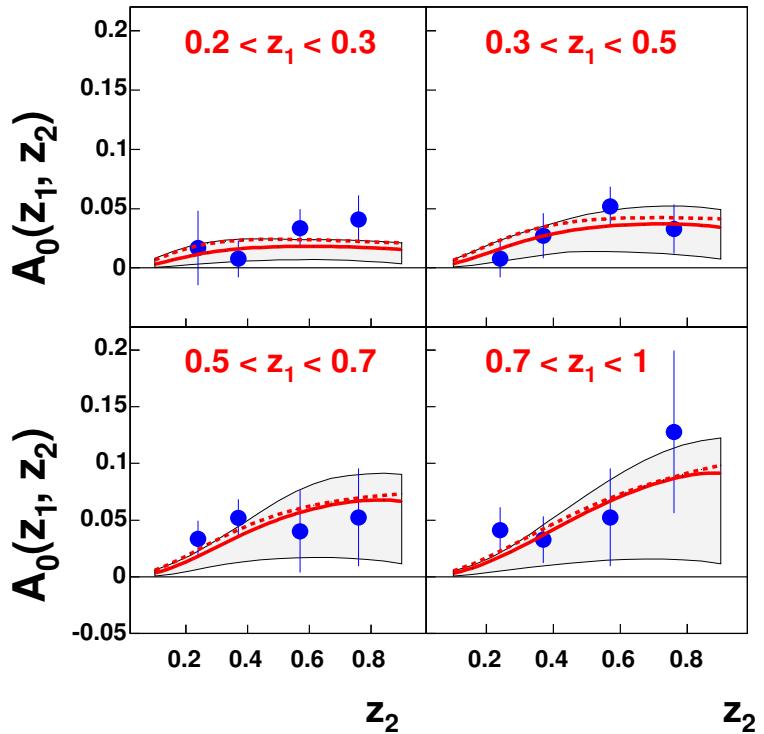
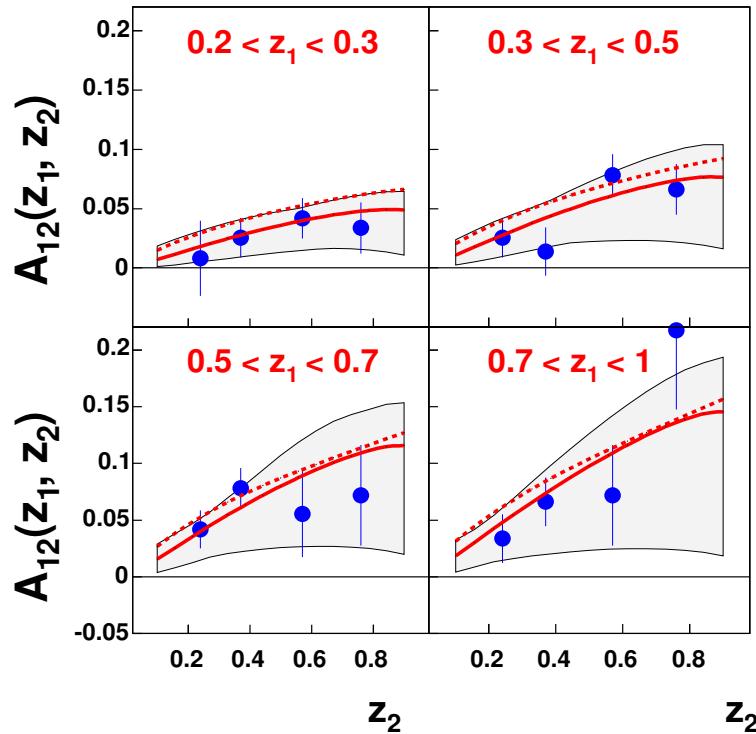
$$A_{UT}^{\sin(\phi_S + \phi_h)} = S_{\text{gaus}} \frac{P_T}{M} \frac{2(1-y)}{1+(1-y)^2} \frac{\sum_q e_q^2 \mathcal{N}_q^T(x) [q(x) + \Delta q(x)] \mathcal{N}_q^C(z) D_{h/q}(z)}{\sum_q e_q^2 q(x) D_{h/q}(z)}$$

where

$$S_{\text{gaus}} = \sqrt{2e} \frac{\langle p_\perp^2 \rangle_C^2}{\langle P_T^2 \rangle_C^2} \frac{\langle P_T^2 \rangle}{\langle p_\perp^2 \rangle} \exp \left[ -P_T^2 \frac{\langle P_T^2 \rangle - \langle P_T^2 \rangle_C}{\langle P_T^2 \rangle \langle P_T^2 \rangle_C} \right]$$

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle, \quad \langle P_T^2 \rangle_C = \langle p_\perp^2 \rangle_C + z^2 \langle k_\perp^2 \rangle.$$

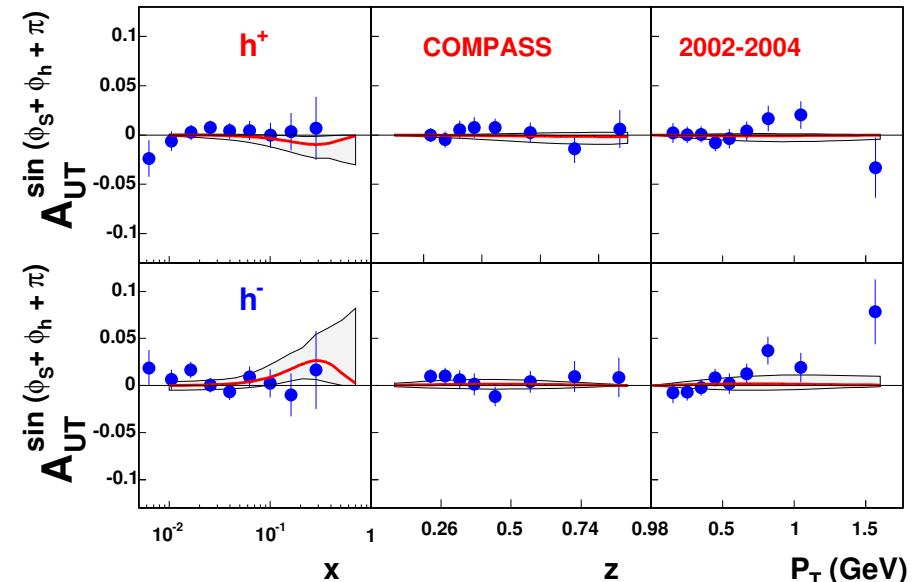
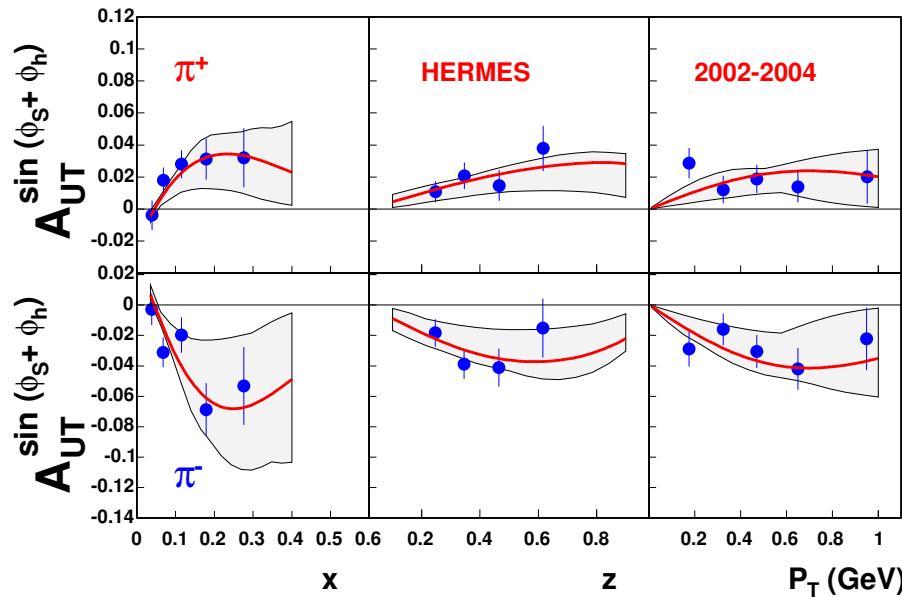
## *Global Fit to Data*



Best fit to data on two different azimuthal correlations in  $e^+e^- \rightarrow h_1 h_2 X$  processes, as measured by BELLE Collaboration [Seidl et al. (2006)].

**Solid (dashed) lines from fitting the  $A_{12}(A_0)$  asymmetry.**

The shaded area corresponds to the theoretical uncertainty on the parameters.

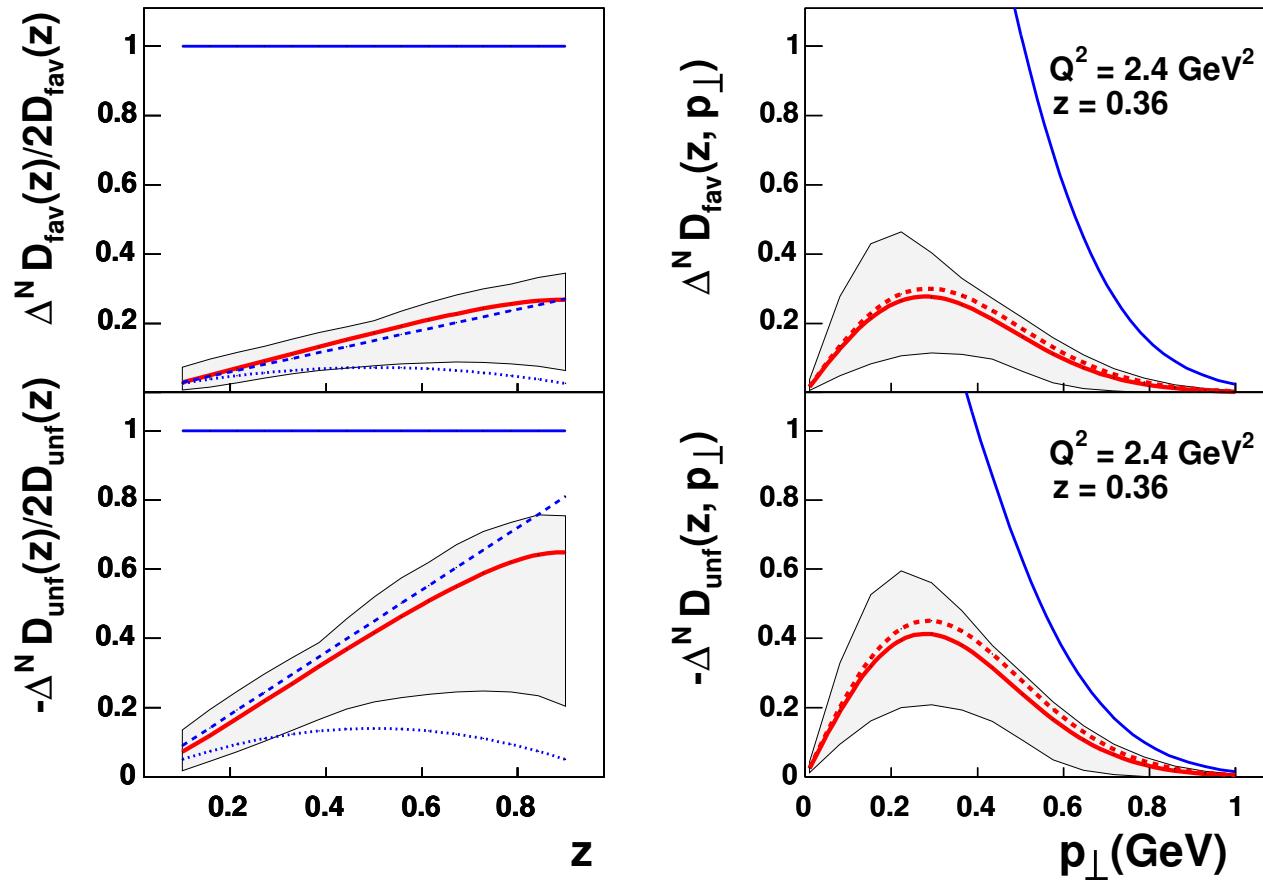


Left: Fit of HERMES data [Airapetian et al. (2004)] on  $A_{UT}^{\sin(\phi_S + \phi_h)}$  for  $\pi^\pm$  production.

Right: Fit of data from COMPASS experiment operating on a deuterium target [Ageev et al. (2007)] for production of positively and negatively charged hadrons.

The shaded area corresponds to the theoretical uncertainty on the parameters.

# *Collins function and Transversity*

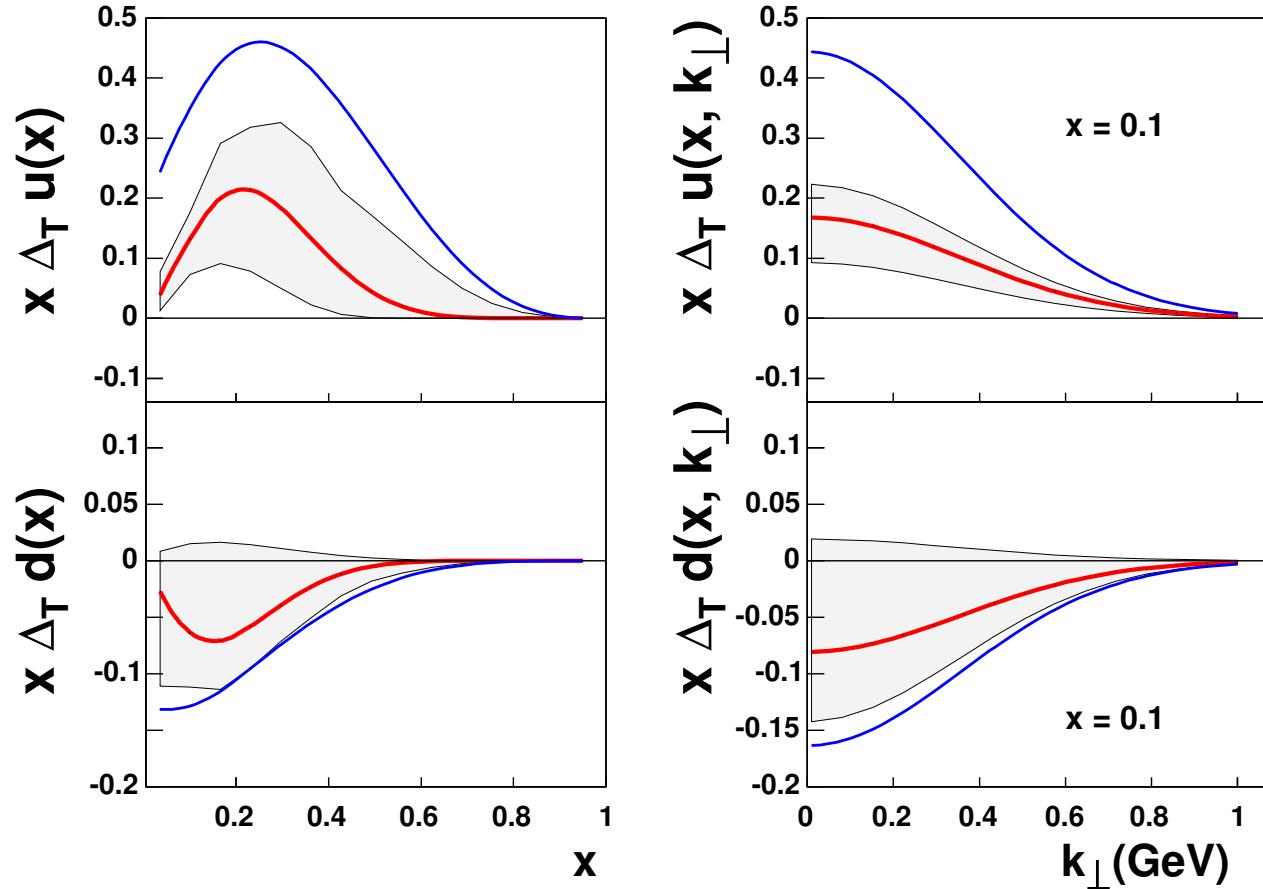


Favored and unfavored Collins ff's from the global fit.

Left:  $z$ -dependence of the  $p_\perp$  integrated Collins functions normalized to their bounds;

dashed line: Efremov et al.(2006); dotted line: Vogelsang-Yuan(2005).

Right:  $p_\perp$ -dependence of Collins ff; solid (dashed) lines from fitting the  $A_{12}(A_0)$  asymmetry.



Transversity distribution functions for  $u$  and  $d$  quarks from our fit.

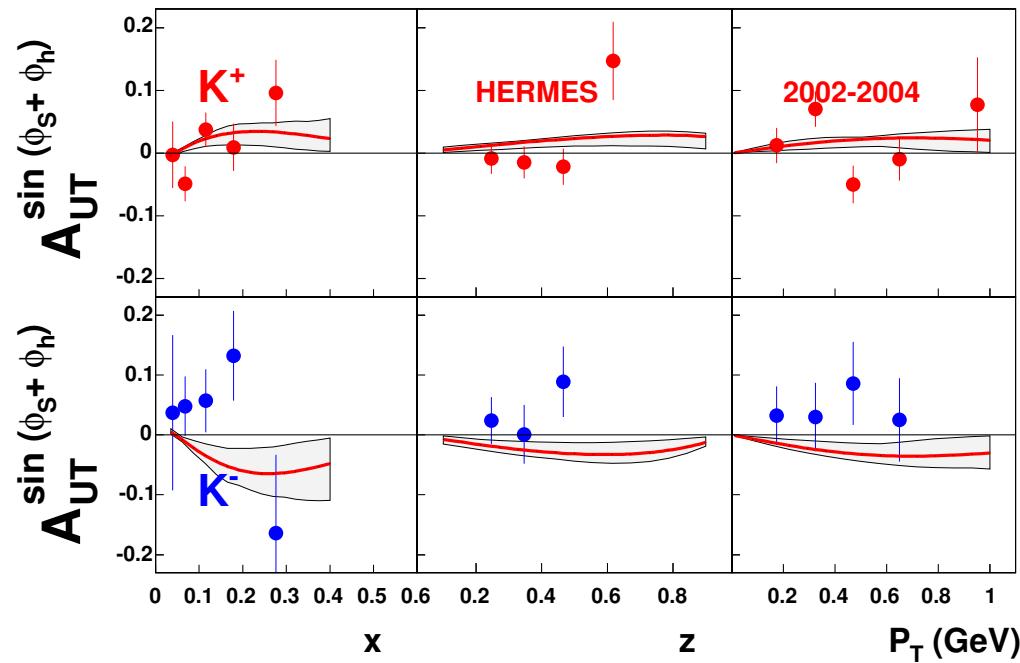
Left:  $x \Delta_T u(x)$  and  $x \Delta_T d(x)$  vs.  $x$  at  $Q^2 = 2.4 \text{ GeV}^2$ .

Right:  $k_\perp$  dependence at fixed  $x = 0.1$  (not fitted but same as unpol. pdf).

**Bold blue line:** Soffer bound.

Notice:

- HERMES data alone tightly constrain  $\Delta_T q$  of  $u$  quarks, the addition of COMPASS data constrain the  $d$  quarks. Although very small, the inclusion of COMPASS SSA data contributes to the extraction of  $\Delta_T q$ .
- Fixing  $\Delta_T q = \Delta q$  or  $\Delta_T q = q_+^+$  lead to a slightly worse description of Belle data.



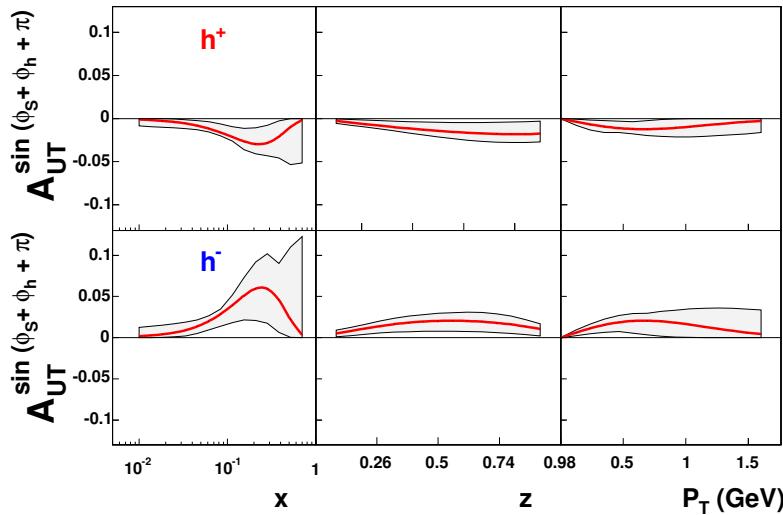
$A_{UT}^{\sin(\phi_S + \phi_h)}$  for  $K^\pm$  production: comparison with HERMES data [Airapetian et al. (2004)].

Data not included in our fit.

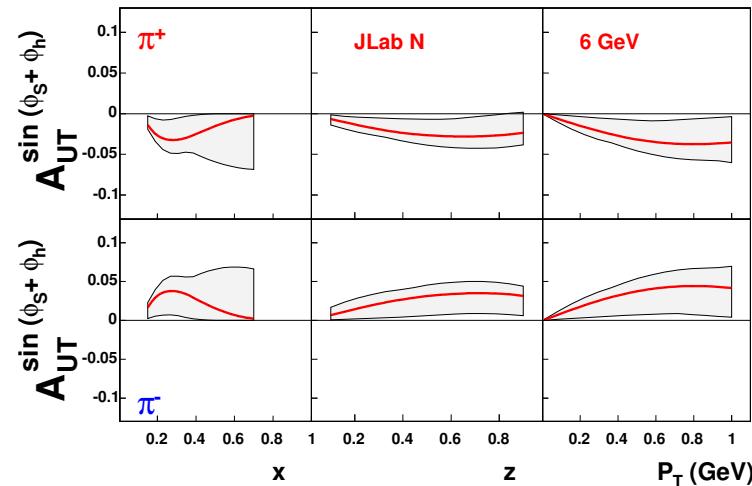
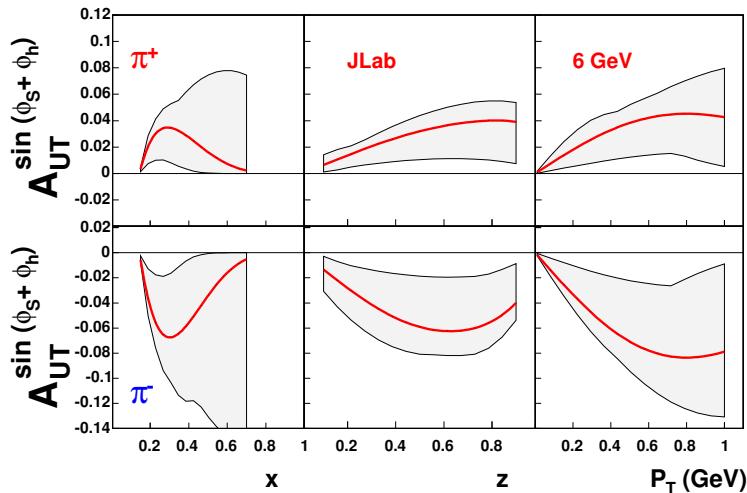
Same  $\mathcal{N}_q^C(z)$ ,  $M$  as for pions with the appropriate unpolarized fragmentation functions.

Fair agreement with data on  $K^+$  production, (dominated by  $u$  quarks);

discrepancies for the  $K^-$  asymmetry (role of  $s$  quarks): sign from unfavored Collins ff.



Predictions for  $A_{UT}^{\sin(\phi_S + \phi_h + \pi)}$  for COMPASS experiment operating with a transversely polarized hydrogen target.



Predictions for  $A_{UT}^{\sin(\phi_S + \phi_h)}$  for JLab operating on polarized hydrogen (proton, left plot) and He<sup>3</sup> (neutron, right plot) targets at a beam energy of 6 GeV: large  $x$  region !!.

## Conclusions and outlook

- Global analysis of exp. data on spin azimuthal asymmetries  $\Rightarrow$  extraction of transversity distributions of  $u$  and  $d$  quarks  
Collins fragmentation functions (favored and unfavored)
- Collins ff in agreement with other analysis: unf  $\simeq$  – fav.
- $|\Delta_T d(x)| < |\Delta_T u(x)|$ , opposite in sign, and smaller than their Soffer bound
- Data on  $K$  production will help in disentangling the role of sea quarks
- Predictions for incoming measurements from COMPASS and JLab experiments.  
Important tests of our complete understanding of the partonic properties which are at the origin of SSA
- Expected data from Belle to study the  $p_\perp$  dependence of the Collins functions
- The combination of data from SIDIS and  $e^+e^- \rightarrow h_1 h_2 X$  processes opens the way to a new phenomenological approach to the study of the nucleon structure and of fundamental QCD properties.