

# Soft gluon resummation and a novel asymptotic formula for double-spin asymmetries in dilepton production at small transverse momentum

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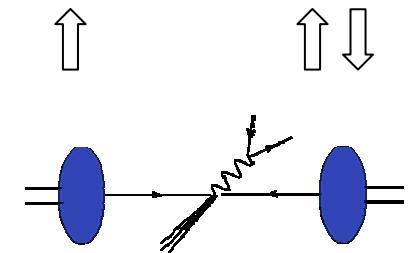
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# Transversely Polarized Drell-Yan process

➤ tDY (RHIC,J-PARC,GSI ...)

$$P^\uparrow + P^\uparrow (\bar{P}^\uparrow) \rightarrow l + \bar{l} + X$$

$$A_{TT} \simeq \frac{\cos(2\phi)}{2} \frac{\sum_i e_i^2 \left[ \delta q_i(x_1) \delta \bar{q}_i(x_2) + \delta \bar{q}_i(x_1) \delta q_i(x_2) \right]}{\sum_i e_i^2 \left[ q_i(x_1) \bar{q}_i(x_2) + \bar{q}_i(x_1) q_i(x_2) \right]}$$



## — Transversity

$$\delta q(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+ \xi^-} \langle PS | \bar{q}(0) \gamma^+ \gamma^i \gamma_5 q(0, \xi^+, 0_\perp) | PS \rangle$$



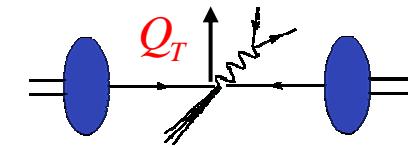
- Chiral-odd : not measured in inclusive DIS
- First global fit (LO): Anselmino et al. ('07)  
SIDIS at HERMES, COMPASS  
+ Collins function from BELLE
- Lattice study by QCDSF
- Future DY data can provide an direct access to  $\delta q$ .

# Asymmetries at small $Q_T$

- $A_{TT}$  in “ $Q_T$ -integrated” cross sections at NLO

$$A_{TT}^{NLO} = \frac{\Delta_T d\sigma^{NLO}}{dQ^2 dy d\phi} \Bigg/ \frac{d\sigma^{NLO}}{dQ^2 dy d\phi}$$

- RHIC :  $A_{TT}$  small (high energy, pp)
- J-PARC : sea pol. at large-x
- GSI : valence pol. at large-x



Martin et al. ('99)  
Barone et al. ('06)  
Shimizu et al. ('05)

- $A_{TT}(Q_T)$  at small  $Q_T$

- Bulk of dileptons produced. Can be larger than  $A_{TT}^{NLO}$ .
- Soft gluon effects are dominant : “universal”  
→ Extraction of  $\delta q(x)$  can be simpler.

- “recoil logs” must be resummed →  $Q_T$  resummation

$$\left. \frac{d\hat{\sigma}}{dQ^2 dQ_T^2} \right|_{Q_T^2 \ll Q^2} \simeq \sum_n \alpha_s^n \left[ c_n \delta(Q_T^2) + \frac{1}{Q_T^2} \sum_{k=0}^{2n-1} d_{nk} \ln^k \left( \frac{Q^2}{Q_T^2} \right) \right]$$

- Spin asymmetries with  $Q_T$  resummation

cf. Boer ('00), Koike,Nagashima,Vogelsang ('06), etc.

# $Q_T$ resummation at NLL

DDT ('78), Parisi, Petronzio ('79),...  
 Kodaira, Trentadue ('82),...  
 Callins, Soper, Sterman ('82), ...

$$\frac{\Delta_T d\sigma}{dQ^2 dy dQ_T^2 d\phi} = N \cos(2\phi) \left[ \underline{\Delta_T X^{NLL}(Q^2, y, Q_T^2)} + \underline{\Delta_T Y(Q^2, y, Q_T^2)} \right]$$

resummed part      finite part      b : impact parameter

$$\Delta_T X^{NLL}(Q^2, y, Q_T^2) = \int db \frac{b}{2} J_0(bQ_T) e^{S(b,Q)} \sum_{ijk} C_{ij} \otimes \delta q_j \left( x_1^0, \frac{b_0^2}{b^2} \right) \cdot C_{ik} \otimes \delta q_k \left( x_2^0, \frac{b_0^2}{b^2} \right)$$

$$x_{1,2}^0 = \frac{Q^2}{S} e^{\pm y}$$

$$b_0 = 2e^{-\gamma_E}$$

Sudakov factor

$$S(b, Q) = - \int_{b_0^2/b^2}^{Q^2} \frac{d\lambda^2}{\lambda^2} \left\{ \ln \left( \frac{Q^2}{\lambda^2} \right) A_q \left( \alpha_s(\lambda^2) \right) + B_q \left( \alpha_s(\lambda^2) \right) \right\}$$

$$A_q = C_F \frac{\alpha_s}{\pi} + \frac{1}{2} C_F \left\{ \left( \frac{67}{18} - \frac{\pi}{6} \right) - \frac{5}{9} N_f \right\} \left( \frac{\alpha_s}{\pi} \right)^2 \quad B_q = -\frac{3}{2} C_F \alpha_s \quad \text{universal}$$

Coeff. function

$$C_{qq}(z, \alpha_s) = \delta(1-z) + \frac{\alpha_s}{4\pi} C_F (\pi^2 - 8) \quad \text{for tDY}$$

Kodaira, Shimizu, Tanaka, HK ('06)

NLL resummation  $\leftrightarrow$

$$\frac{\alpha_s^n}{Q_T^2} \ln^m \left( \frac{Q^2}{Q_T^2} \right) \quad (m = 2n-1, 2n-2, 2n-3)$$

# New prescription for b-integration

➤  **$b_*$  formalism**    Collins,Soper,Sterman ('82)

$$b_* = \frac{b}{\sqrt{1 + b^2/b_{lim}^2}}$$

$$\Delta_T X^{NLL}(Q^2, y, Q_T^2) \sim \int_0^\infty db \frac{b}{2} J_0(bQ_T) e^{S(b_*, Q) - g_{NP} b^2} \sum_i e_i^2 \delta q_i \left( x_1^0, \frac{b_0^2}{b_*^2} \right) \delta \bar{q}_i \left( x_2^0, \frac{b_0^2}{b_*^2} \right)$$

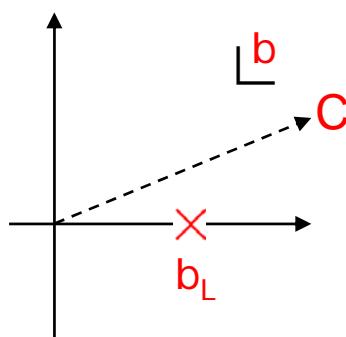


$g_{NP}$  : non-perturbative parameter

➤ Contour deformation method

$$\Delta_T X^{NLL}(Q^2, y, Q_T^2) \sim \int_C db \frac{b}{2} H_0^{(1)}(bQ_T, Q) e^{S(b, Q) - g_{NP} b^2} \sum_i e_i^2 \delta q_i \left( x_1^0, \frac{b_0^2}{b^2} \right) \delta \bar{q}_i \left( x_2^0, \frac{b_0^2}{b^2} \right) + (c.c.)$$

$$\left( J_0(bQ_T) = \frac{1}{2} [H_0^{(1)}(bQ_T) + H_0^{(2)}(bQ_T)] \right)$$



→ resummation at the parton level

$$\text{Landau pole : } b_L = \frac{1}{Q} e^{1/2 \beta_0 \alpha_s(Q)}$$

Laenen, Vogelsang, Sterman ('99)

Kulesza, Vogelsang, Sterman ('01)

Bozzi, Catani, Grazzini, De Florian ('05)

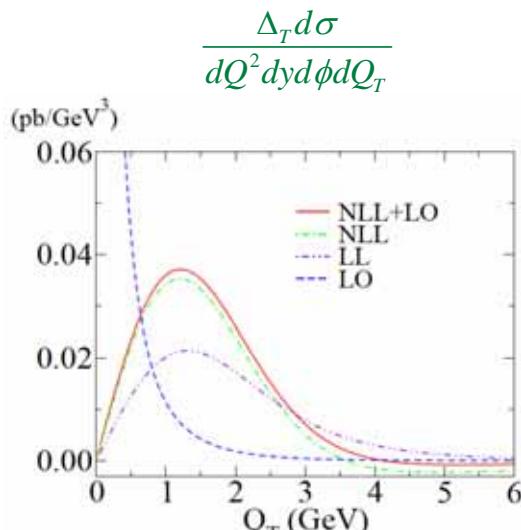
# $Q_T$ distributions in tDY

Kodaira, Shimizu, Tanaka, HK ('06)

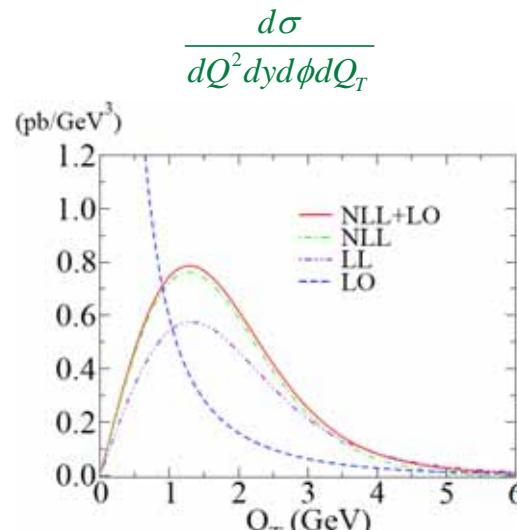
- Numerical study with a model saturating Soffer bound

$$\delta q(x, \mu_0^2) = \frac{1}{2} [q(x, \mu_0^2) + \Delta q(x, \mu_0^2)] \text{ at } \mu_0^2 = 0.4 \text{ GeV}^2 + \text{NLO evolution}$$

(GRV98+GRSV00)



pol.



unpol.

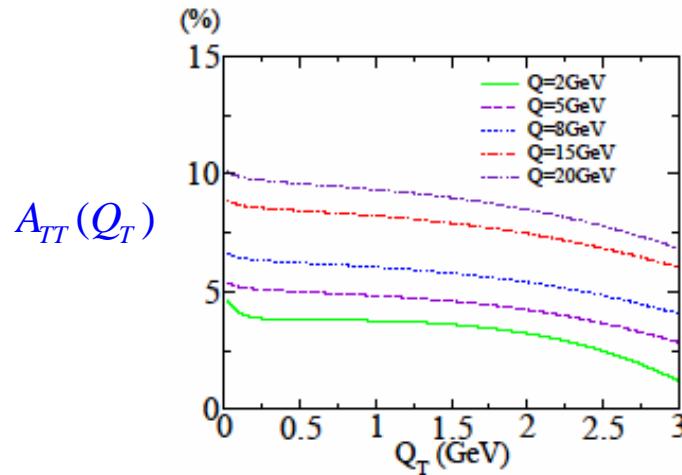
pp collision @ RHIC    $\sqrt{s} = 200 \text{ GeV}$ ,  $Q = 5 \text{ GeV}$ ,  $y=2$ ,  $\phi=0$  with  $g_{\text{NP}}=0.5 \text{ GeV}^2$

Koike et al. ('96)  
Kumano et al. ('96)  
Vogelsang ('97)

NLL+LO:  $X^{\text{NLL}}+Y$   
NLL:  $X^{\text{NLL}}$   
LL:  $X^{\text{LL}}$   
LO:  $X+Y$

# Double-spin Asymmetries at small $Q_T$

Kodaira, Tanaka, HK  
hep-ph/0703079

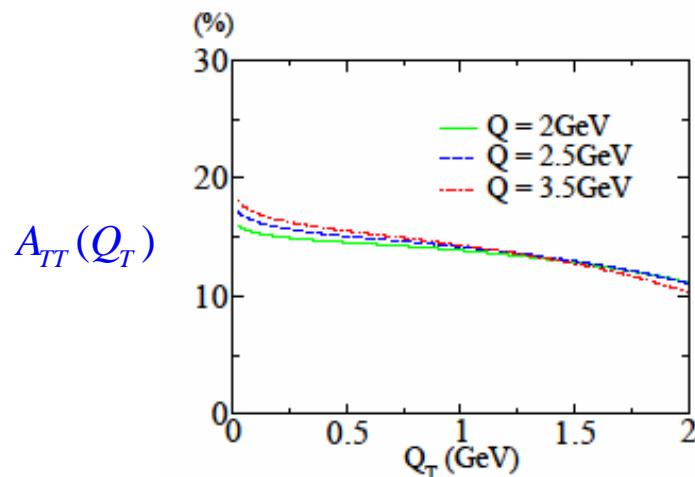


pp collision @ RHIC

$\sqrt{s} = 200 \text{ GeV}, Q=2-20 \text{ GeV}, y=2, \phi=0$

- Larger  $A_{TT}(Q_T)$  for larger  $Q$ .

—  $\frac{\delta \bar{q}(x)}{\bar{q}(x)}$  : suppressed at small  $x$ , due to the steep rise of unpol. sea distribution



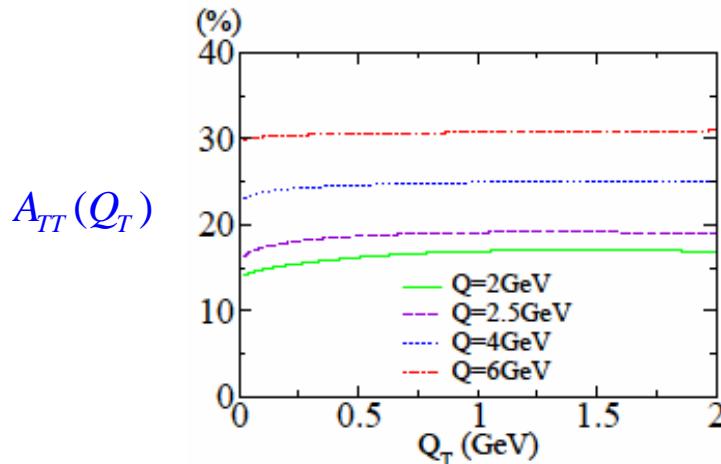
pp collision @ J-PARC

$\sqrt{s} = 10 \text{ GeV}, Q = 2-3.5 \text{ GeV}, y=0, \phi=0$

- $A_{TT}(Q_T) \sim 15\%$  at the peak region.
- pdfs at  $x \geq 0.2$  are probed.

$$\frac{\delta q(x)}{q(x)} \quad \nearrow \quad \frac{\delta \bar{q}(x)}{\bar{q}(x)} \quad \searrow \quad \text{as } x \nearrow$$

# Double-spin Asymmetries at small $Q_T$



ppbar collision @GSI

$\sqrt{s} = 10 \text{ GeV}$ ,  $Q = 2\text{-}6 \text{ GeV}$ ,  $y=0$ ,  $\phi=0$

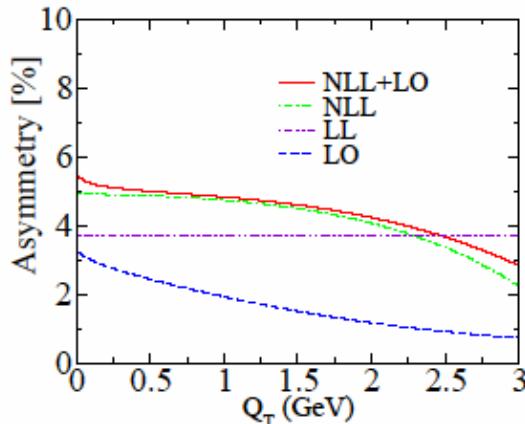
- Larger than other 2 cases.
  - valence polarisation at large-x

## ♠ $A_{TT}(Q_T)$ at small $Q_T$

- Flat at small  $Q_T \leftrightarrow$  Soft gluon corrections are universal.
- $y$  &  $g_{NP}$  dependence small.
- 15-20% larger than  $A_{TT}^{NLO}$  for RHIC & J-PARC, similar for GSI .  
cf. Our results are consistent with  $A_{TT}^{NLO}$  .
- $\delta q$  of which  $(x, \mu)$  determines it ?

# What determines $A_{TT}(Q_T)$ at small $Q_T$ ?

- Ratio of each component



pp collision at  $\sqrt{s} = 200$  GeV,  $Q=5$  GeV,  $y=2$ ,  $\phi=0$

$$A_{TT}^{(N)LL}(Q_T) \equiv \frac{\Delta_T X^{(N)LL}}{X^{(N)LL}}, \quad A_{TT}^{LO}(Q_T) \equiv \frac{\Delta_T X + \Delta_T Y}{X + Y}$$

- In the peak region, the asymmetries are dominated by the ratio of the NLL terms.

$$\rightarrow A_{TT}(Q_T) \approx A_{TT}^{NLL}(Q_T)$$

- $A_{TT}^{LL}(Q_T)$  is constant  $\Leftrightarrow$  determined only by  $\frac{\delta q(x, Q^2)}{q(x, Q^2)}$ .

- Consistent with  $A_{TT}^{NLO}$ .

- $A_{TT}^{NLL}(Q_T) \approx A_{TT}^{NLL}(0)$   $\rightarrow$  Saddle-point evaluation

# Saddle point evaluation at NLL

Kodaira, Tanaka, HK,  
hep-ph/0703079

## ➤ Resummation formula at NLL

$$\Delta_T X^{NLL}(Q^2, Q_T^2 = 0) = \int_{C_N} dN_1 x_1^{0-N_1} \int_{C_N} dN_2 x_0^{2-N_2} \\ \times \Delta I_{N_1, N_2}^{NLL}(Q^2, Q_T^2 = 0) \sum_i \delta q_i^{N_1}(Q^2) \cdot \delta \bar{q}_i^{N_2}(Q^2)$$

$$\Delta I_{N_1, N_2}^{NLL}(Q^2, Q_T^2 = 0) = \frac{b_0^2}{4Q^2 \beta_0 \alpha_s(Q^2)} \int_{-\infty}^{\lambda_c} d\lambda e^{-\zeta(\lambda) + h^{(1)}(\lambda) + R_{N_1}(\lambda) + R_{N_2}(\lambda)} \\ \lambda = \beta_0 \alpha_s \ln \left( \frac{Q^2 b^2}{b_0^2} \right)$$

$$\zeta(\lambda) = -\frac{\lambda}{\beta_0 \alpha_s(Q^2)} - \frac{h^{(0)}(\lambda)}{\alpha_s} + \frac{g_{NP} b_0^2}{Q^2} e^{\frac{\lambda}{\beta_0 \alpha_s(Q^2)}} \quad \text{LL terms}$$

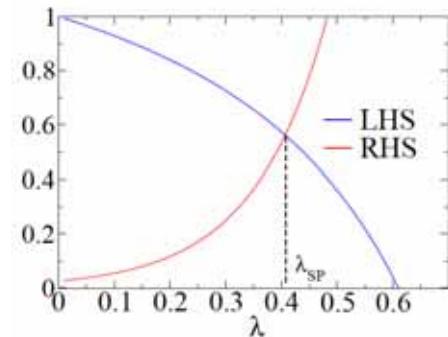
$$h^{(1)}(\lambda) \quad R_N(\lambda) = \frac{\Delta_T P^{(0)}_{qq,N}}{2\pi\beta_0} \ln(1 - \lambda) \quad \text{NLL terms}$$

'  
↑  
pdf evolution

# Saddle point evaluation at NLL

## ➤ Saddle point equation

$$1 - \frac{A_q^{(1)}}{2\pi\beta_0} \frac{\lambda_{SP}}{1 - \lambda_{SP}} = \frac{g_{NP} b_0^2}{Q^2} e^{\beta_0 \alpha_s(Q^2)}$$



The saddle point depends on  $g_{NP}$  &  $Q$  only weakly.

# ➤ Result

$$I_{N_1, N_2}(Q^2, 0) = \left( \frac{b_0^2}{4Q^2\beta_0\alpha_s(Q^2)} \sqrt{\frac{2\pi}{\zeta''(\lambda_{SP})}} e^{-\zeta(\lambda_{SP}) + h^{(1)}(\lambda_{SP})} \right) e^{R_{N_1}(\lambda_{SP}) + R_{N_2}(\lambda_{SP})}$$



$$-\delta q(x, Q^2) \rightarrow \delta q\left(x, \frac{b_0^2}{b_{sp}^2}\right)$$

# Asymptotic formula

In the peak region,

$$A_{TT}^{NLL}(0) \approx \frac{\cos(2\phi)}{2} \frac{\sum_i e_i^2 \left[ \delta q_i \left( x_1^0, \frac{b_0^2}{b_{SP}^2} \right) \times \delta \bar{q}_i \left( x_0^2, \frac{b_0^2}{b_{SP}^2} \right) + (x_1^0 \leftrightarrow x_2^0) \right]}{\sum_i e_i^2 \left[ q_i \left( x_1^0, \frac{b_0^2}{b_{SP}^2} \right) \times \bar{q}_i \left( x_2^0, \frac{b_0^2}{b_{SP}^2} \right) + (x_1^0 \leftrightarrow x_2^0) \right]}$$

- $b_0/b_{SP} \approx 1\text{GeV}$  in the present kinematics.
  - $b_0^2/b_{SP}^2 < Q^2 \rightarrow A_{TT}(Q_T) > A_{TT}^{NLO}$
- Depends only on pdfs at a fixed  $(x, \mu)$ 
  - useful for extracting pdf from experimental data.
- **Caveat:** In the NLL accuracy, the evolution from  $Q$  to  $b_0/b$  is given by LO DGLAP kernel.
  - NLO kernel accuracy at RHIC but small at large  $x$  (J-PARC,GSI :  $x \geq 0.2$ ).

# Asymptotic formula vs. Numerical results

## ➤ Comparison of $A_{TT}^{NLL}(0)$

- (1) SP-I : asymptotic formula (NLO pdfs + LO DGLAP for  $Q \rightarrow b_0/b_{SP}$ )
- (2) SP-II : asymptotic formula (NLO pdfs at  $b_0/b_{SP}$ )
- (3) NB : numerical b-integration

$Q$	$\sqrt{S} = 200 \text{ GeV}, y = 2$					$\sqrt{S} = 10 \text{ GeV}, y = 0$		
	2GeV	5GeV	8GeV	15GeV	20GeV	2GeV	2.5GeV	3.5GeV
SP-I	4.3%	5.4%	6.6%	8.7%	9.8%	14.1%	14.5%	14.8%
SP-II	7.3%	8.7%	9.8%	11.8%	12.7%	14.7%	14.8%	14.2%
NB	3.8%	4.9%	6.1%	8.2%	9.4%	13.4%	14.0%	14.9%

pp collision

$Q$	$\sqrt{S} = 14.5 \text{ GeV}, y = 0$			
	2GeV	2.5GeV	4GeV	6GeV
SP-I	17.1%	20.2%	25.6%	30.9%
SP-II	19.0%	21.0%	26.0%	31.0%
NB	17.4%	19.6%	25.3%	30.8%

ppbar collision

- “SP-I” reproduces  $A_{TT}(Q_T)$  in a good accuracy for all cases.
- For J-PARC & GSI , “SP-II” also works well.  
(The difference between LO & NLO kernel is small at large-x.)

# Summary

- We calculated double spin asymmetries at small  $Q_T$  for tDY.  
with soft gluon resummation.
  - complete result at “NLL + LO” level.
- $A_{TT}(Q_T)$  estimated with a model for the transversity  $\delta q(x)$ 
  - flat in small  $Q_T$  region.
  - $y$  dependence is small.
  - can be large at J-PARC (large- $x$  sea quark)  
GSI (large- $x$ , valence pdf )
  - $A_{TT}(Q_T) \approx A_{TT}^{NLL}(Q_T) \approx A_{TT}^{NLL}(0)$
- Based on the saddle-point evaluation at NLL, we derived a simple asymptotic formula for  $A_{TT}^{NLL}(0)$ .
  - PDFs at the scale  $b_0/b_{SP} \sim Q_T$  plays a dominant role.
    - Can be useful to extract  $\delta q$  from experimental data.
    - The formula is general and applicable to other asymmetries such as  $A_{LL}(Q_T)$  and so on.