

*Obtaining Generalized Parton Distributions  
from hadronic observables and lattice QCD*

Simonetta Liuti

University of Virginia

# Collaborators

Saeed Ahmad (graduate student, physics)

Heli Honkanen (post-doc, physics)

Swadhin Taneja (graduate student, physics)

# Outline

- Motivation
- Zero Skewness: GPDs from form factors and PDFs
- Non-Zero Skewness: usage of lattice results
- Non-Zero Skewness: Present observables
- Conclusions

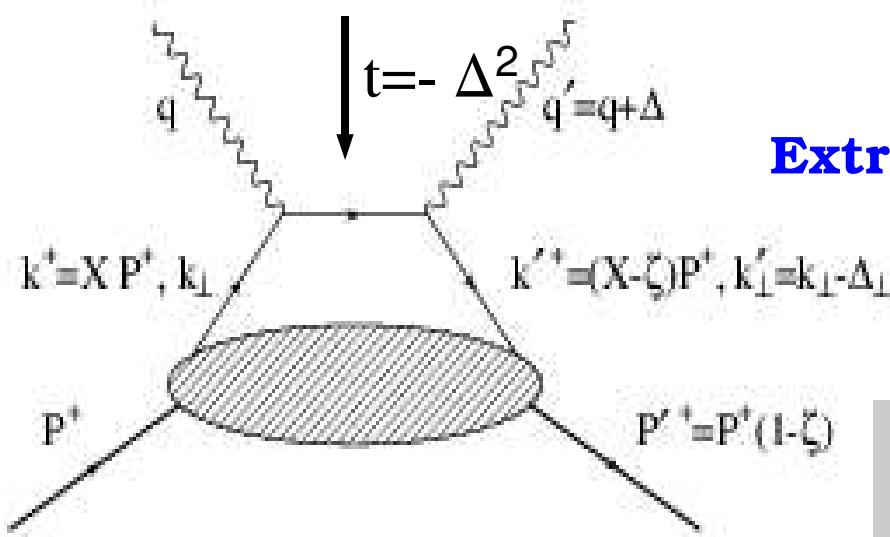
# Motivation

- ➊ DVCS and other types of “exclusive” experiments add a whole *new dimension* to studies of hadronic structure:

One can in principle access the spatial distributions of partonic configurations!

- ➋ DVCS experiments are hard and lengthy: What are the prospects for obtaining spatial configurations from experiment?
- ➌ Need to devise new/alternative strategies

# DVCS and Generalized Parton Distributions

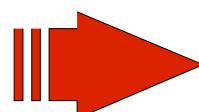


**Extract “Generalized Parton Distributions”**

$$\bar{P}'^+ \int \frac{d\xi^-}{2\pi} e^{ixP'^+\xi^-} \langle P', S' | \psi \left( -\frac{\xi^-}{2} \right) \gamma^+ \psi \left( \frac{\xi^-}{2} \right) | P, S \rangle =$$

$$\bar{u}(P', S') \left[ \gamma^+ H(x, \xi, -\Delta^2) + \frac{i\sigma^{+\nu} q_\nu}{2M} E(x, \xi, -\Delta^2) \right] u(P, S)$$

- GPDs are hybrids of PDFs and FFs: describe simultaneously  $x$  and  $t$ -dependences !
- GPDs give access to spatial d.o.f. of partons !
- GPDs give access to orbital angular momentum of partons!



$$\int dx H_q(x, \zeta, t) + E_q(x, \zeta, t) = 2J_q$$

X. Ji

# Proposed Strategy

- Similarly to the inception of PDFs analyses:

Construct theoretically motivated parametrizations  
at a given *low* initial scale

- Merge data/information from:

→ Form factors constraints  $\zeta=0$

→ PDF constraints  $\zeta=0$

→ Higher GPD moments from lattice calculations  $\zeta \neq 0$

→ DVCS data  $\zeta \neq 0$

- Apply PQCD evolution to connect different sets of data

# Summary of Constraints

## Constraints from Form Factors

$$\int_0^1 dX H^q(X, t) = F_1^q(t) \quad \text{Dirac}$$
$$\int_0^1 dX E^q(X, t) = F_2^q(t), \quad \text{Pauli}$$

$$F_{1(2)}^p(t) = \frac{2}{3}F_{1(2)}^u(t) - \frac{1}{3}F_{1(2)}^d(t) + \frac{1}{3}F_{1(2)}^s(t) \quad \text{Dirac(Pauli) proton}$$

$$F_{1(2)}^n(t) = -\frac{1}{3}F_{1(2)}^u(t) + \frac{2}{3}F_{1(2)}^d(t) + \frac{1}{3}F_{1(2)}^s(t), \quad \text{Dirac(Pauli) neutron}$$

Sachs

$$G_E^{p(n)}(t) = F_1^{p(n)}(t) + \frac{t}{4M^2}F_2^{p(n)}(t)$$
$$G_M^{p(n)}(t) = F_1^{p(n)}(t) + F_2^{p(n)}(t).$$

## Constraints from PDFs

$$q(x) = H_q(x, 0, 0)$$

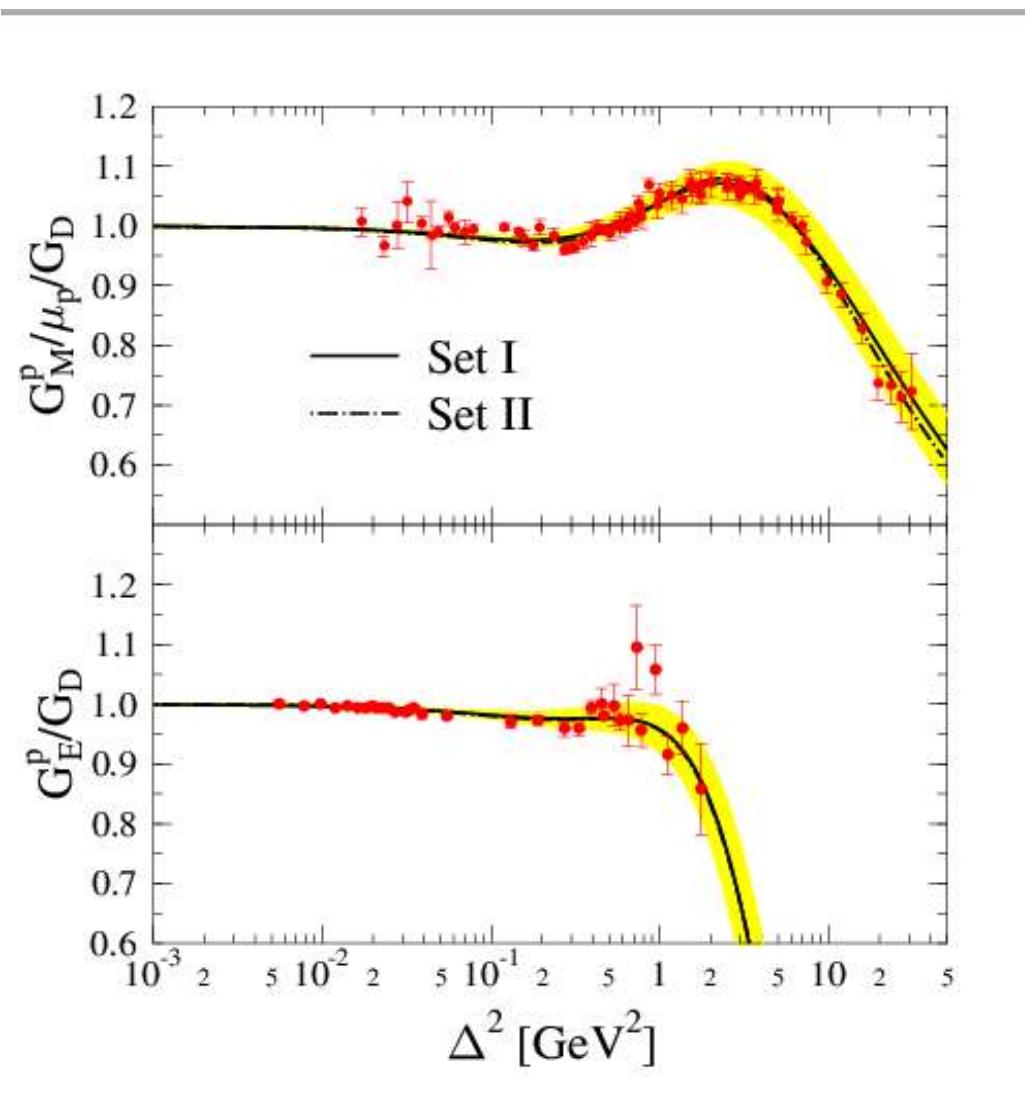
## Constraints from higher moments (n=2,3, ...)

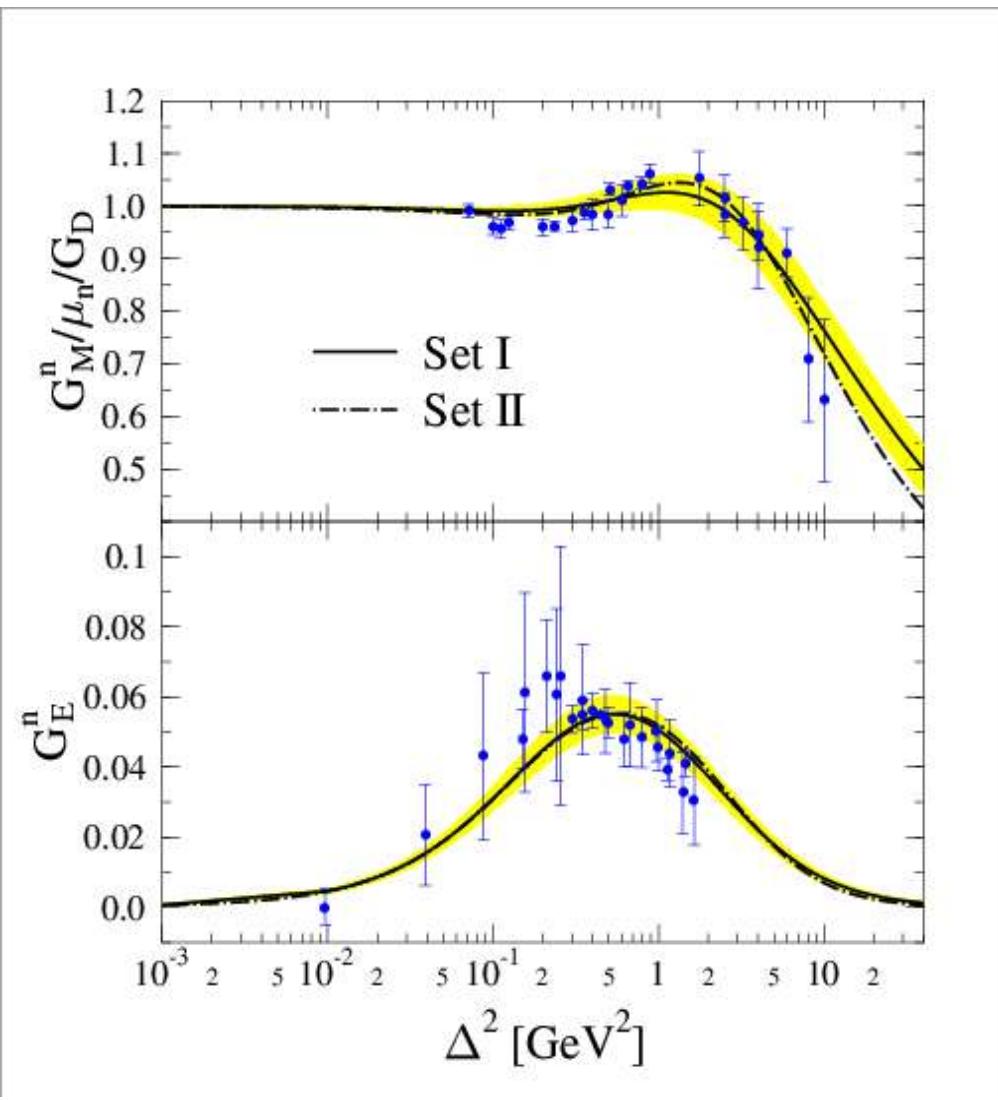
$$H_n^q(\zeta, t) = \int_0^1 dX X^{n-1} H^q(X, \zeta, t)$$
$$E_n^q(\zeta, t) = \int_0^1 dX X^{n-1} E^q(X, \zeta, t),$$

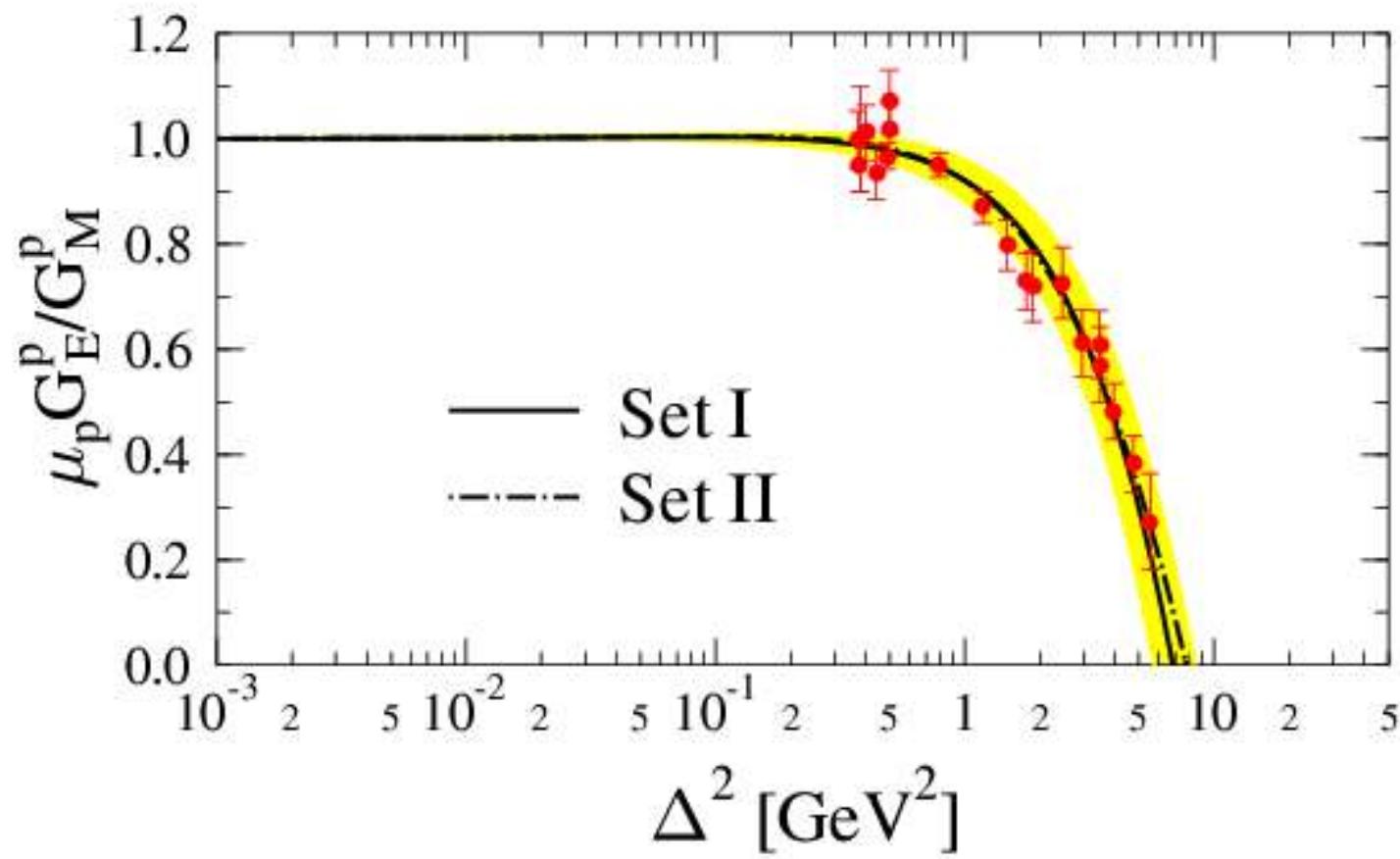
$\zeta = 0$

## *GPDs from available data 1*

### Nucleon Form Factors







## *GPDs from available data 2*

### Parton Distribution Functions

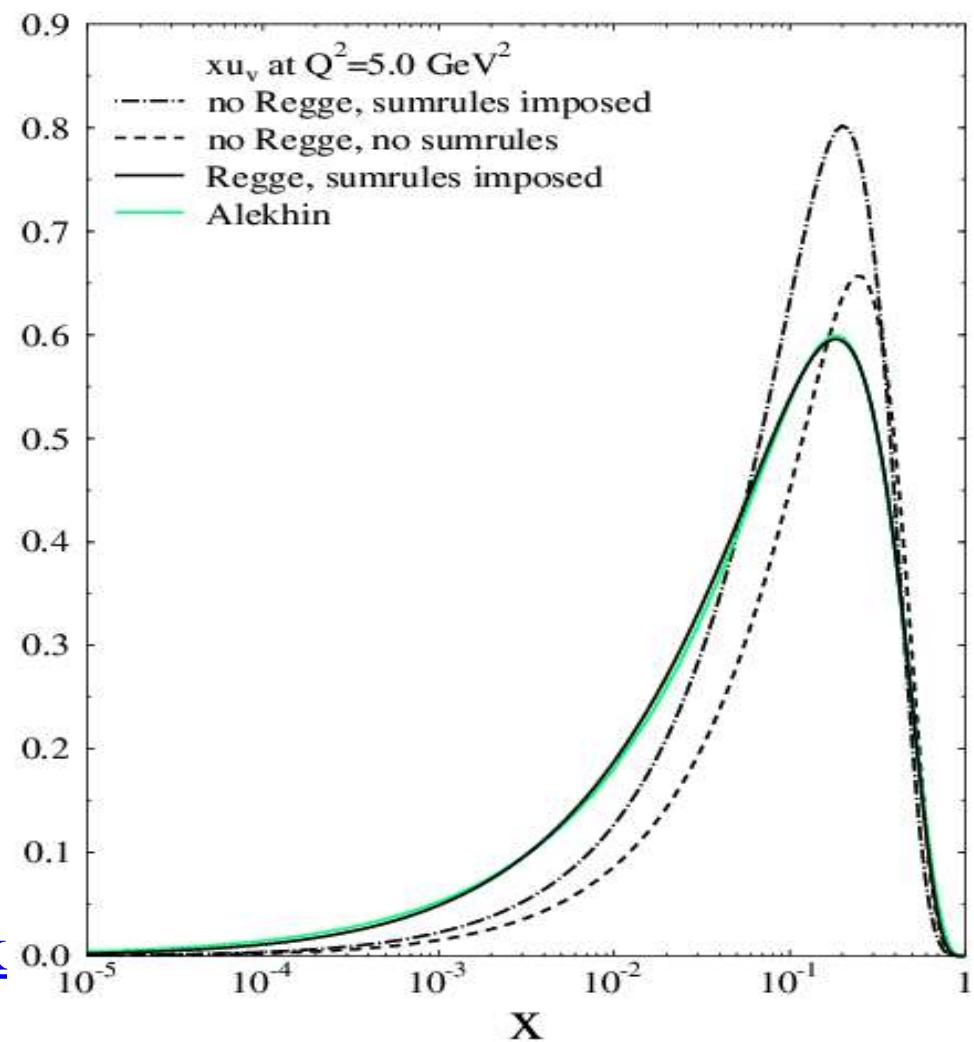
**Notice!** GPD parametric form is given at  $Q^2 = \mu^2$  and evolved to  $Q^2$  of data.

**Notice!** We provide a parametrization for GPDs that simultaneously fits the PDFs:

$$H_q(X, t) = R(X, t) + G(X, t)$$

Regge

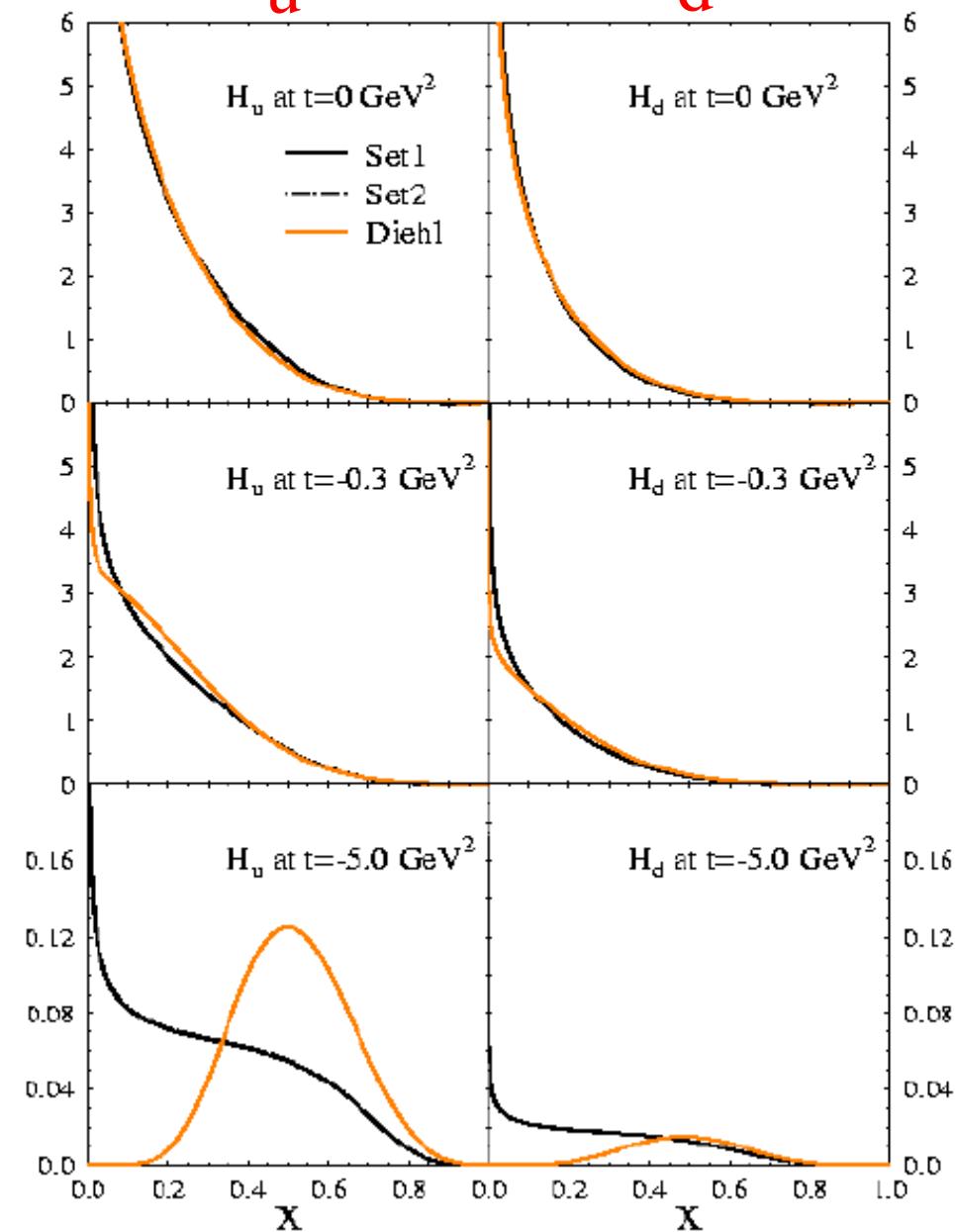
Quark-Diquark



$H_q(x, 0, t)$ 

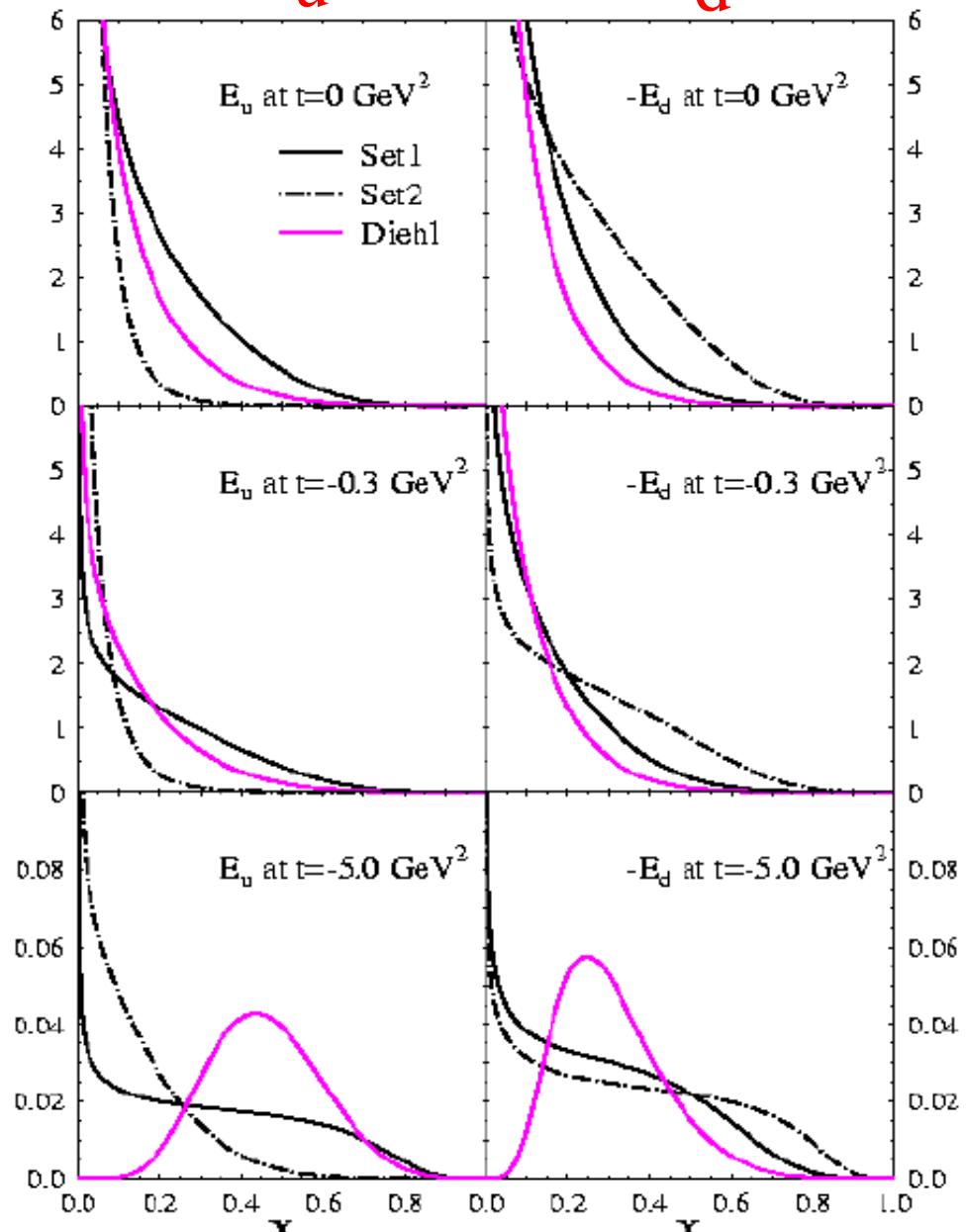
u

d

 $E_q(x, 0, t)$  (spin flip)

u

d

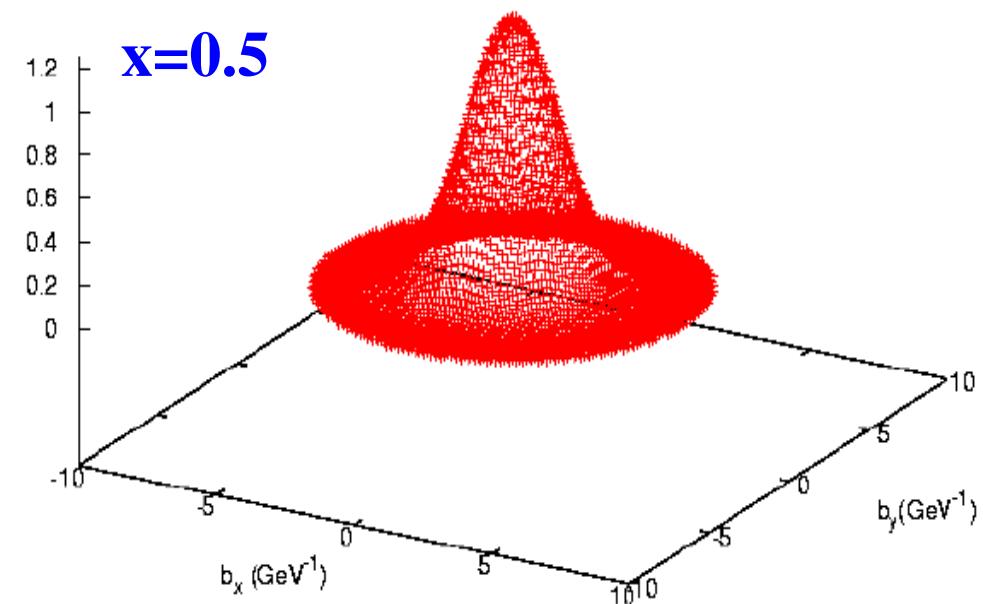
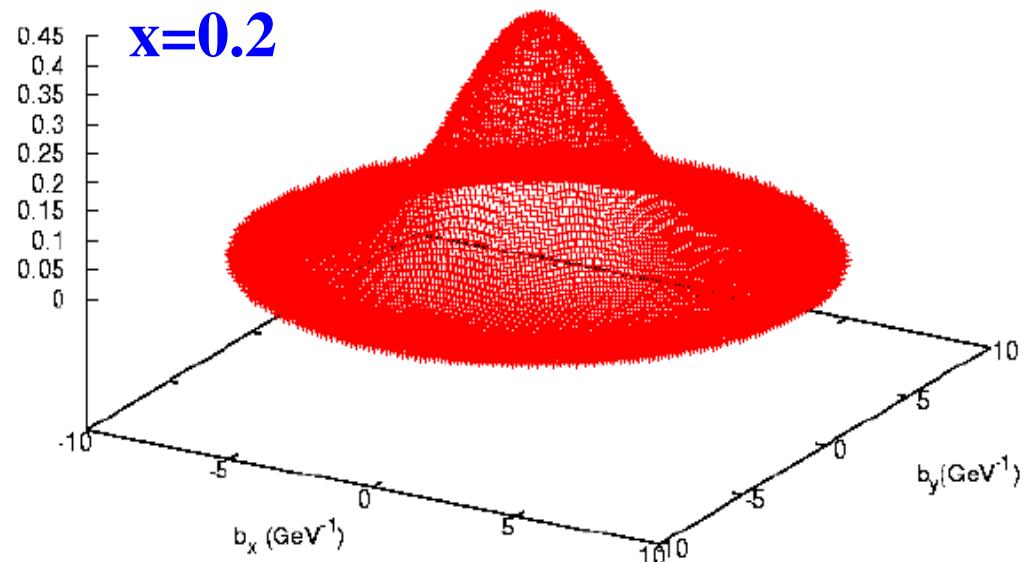


# DVCS and Generalized Parton Distributions: IPPDFs

$$\zeta = 0$$

$$q(x, \mathbf{b}) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i \mathbf{b} \cdot \Delta} H_q(x, 0, -\Delta^2)$$

$$\langle \mathbf{b}^2(x) \rangle = \mathcal{N}_b \int d^2 \mathbf{b} q(x, \mathbf{b}) \mathbf{b}^2$$



H<sub>q</sub> (x,t) from Ahmad, Honkanen, S.L., Taneja (2006)

$\zeta \neq 0$

Use information from Lattice QCD:

(1) chiral extrapolate dipole masses

### Dipole fit

$$G(Q^2) = \frac{G(0)}{(1 + Q^2/\Lambda^2)^2}$$

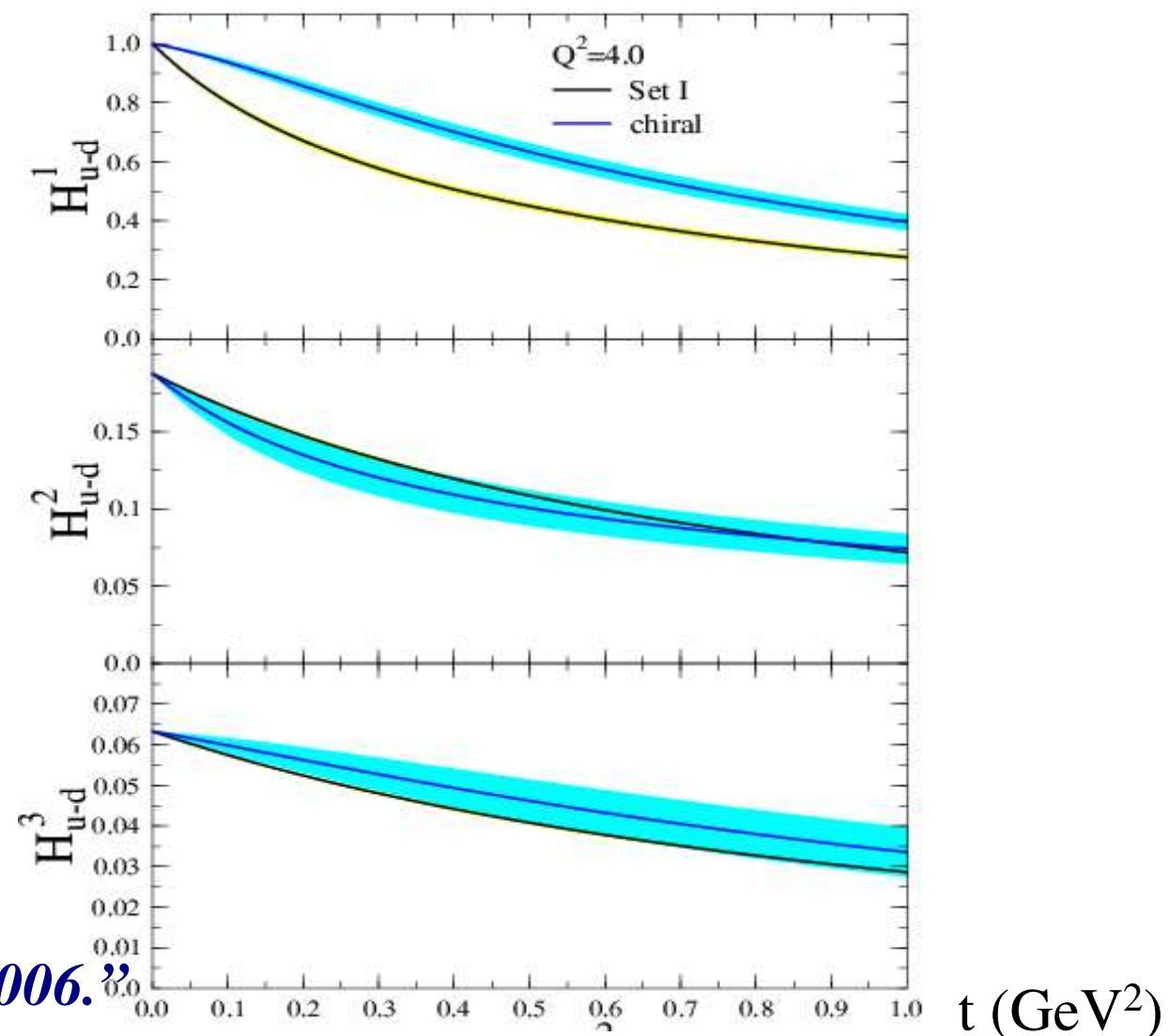
Lattice results from:

M. Gockeler et al. (2006);

J. Zanotti: hep-ph/0501209;

and “44<sup>th</sup> Winter School in

Schladming, Austria, March 2006.”



## Isovector moments

n=1

$$H_1^{u-d} \equiv \int dX (H^u - H^d) = \frac{\tau G_M^V + G_E^V}{1 + \tau}$$
$$E_1^{u-d} \equiv \int dX (E^u - E^d) = \frac{G_M^V - G_E^V}{1 + \tau}.$$

“any” n

$$H_n^{u-d} \equiv \int dX X^{n-1} (H^u - H^d) = \frac{\tau (H_M^V)_n + (H_E^V)_n}{1 + \tau}$$
$$E_n^{u-d} \equiv \int dX X^{n-1} (E^u - E^d) = \frac{(E_M^V)_n - (E_E^V)_n}{1 + \tau},$$

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi} + \chi_2 \ln\left(\frac{m_\pi}{\mu}\right).$$

$$\chi_1 = \frac{g_A^2 m_N}{8\pi f_\pi^2 \kappa_v},$$

$$\chi_2 = -\frac{5g_A^2 + 1}{8\pi^2 f_\pi^2},$$

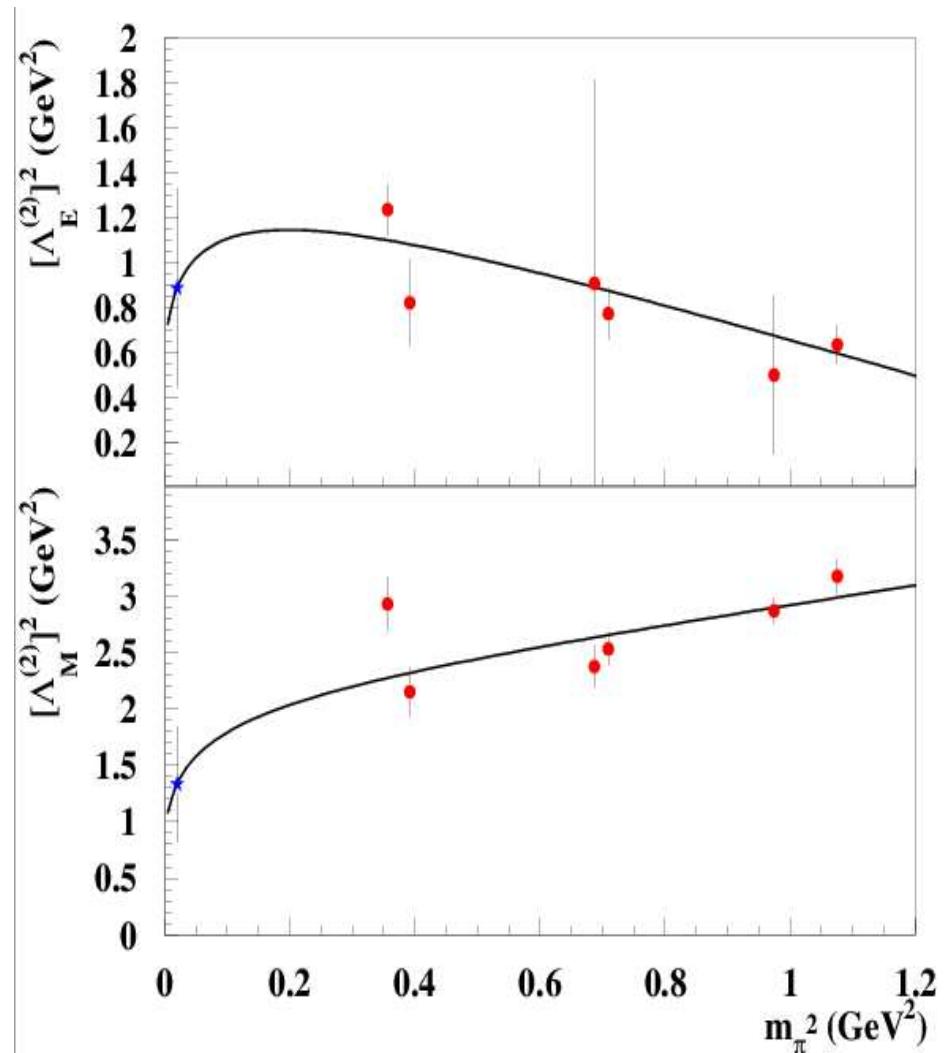
V. Bernard et al. (1995)

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi} \frac{2}{\pi} \arctan(\mu/m_\pi) + \frac{\chi_2}{2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)$$

$$(\Lambda_M^v)^2 = \frac{12}{\langle r^2 \rangle_M^v}.$$

Ashley, Leinweber, Thomas, Young (2003)

## Isovector dipole masses (n=2)

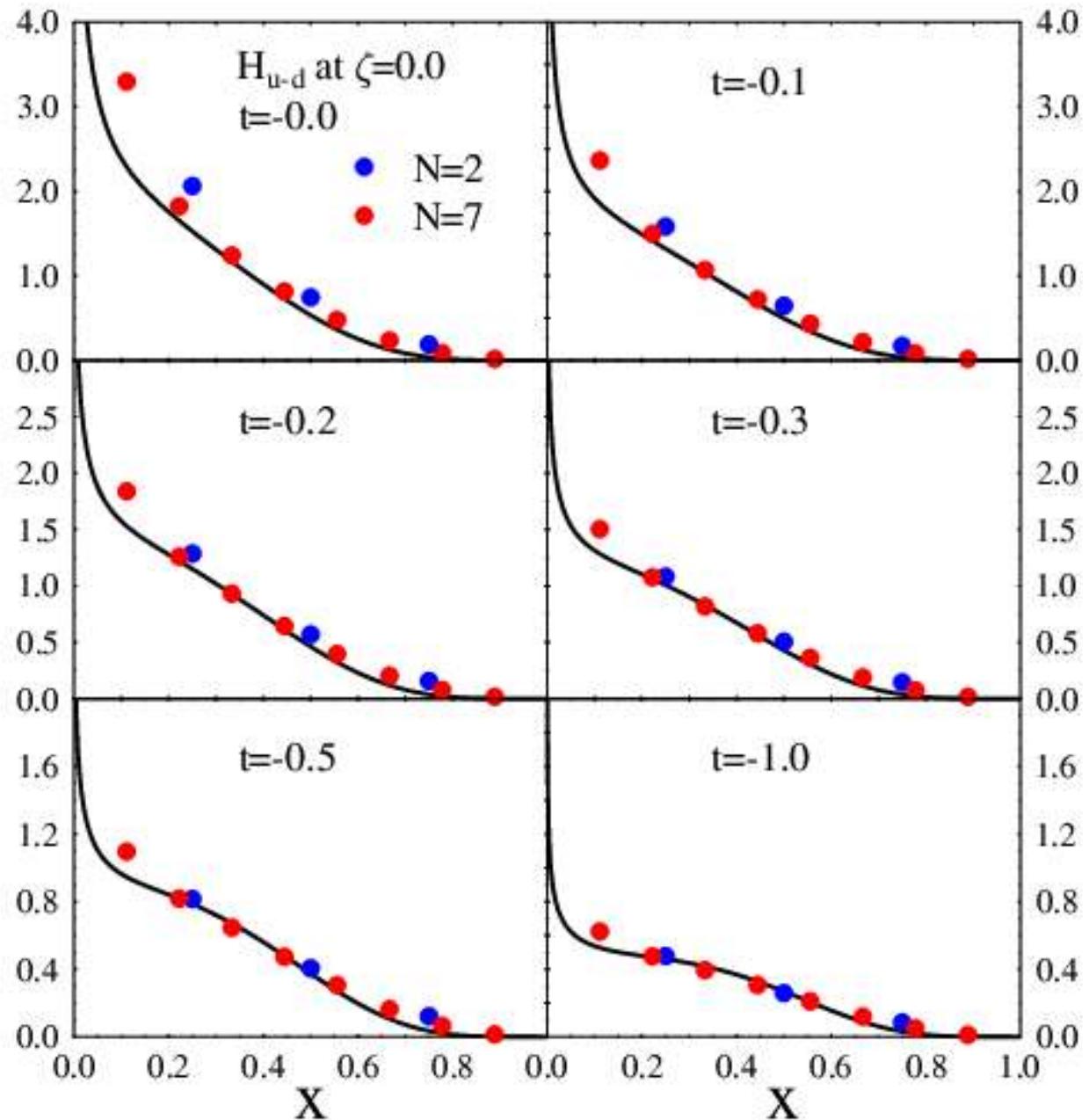


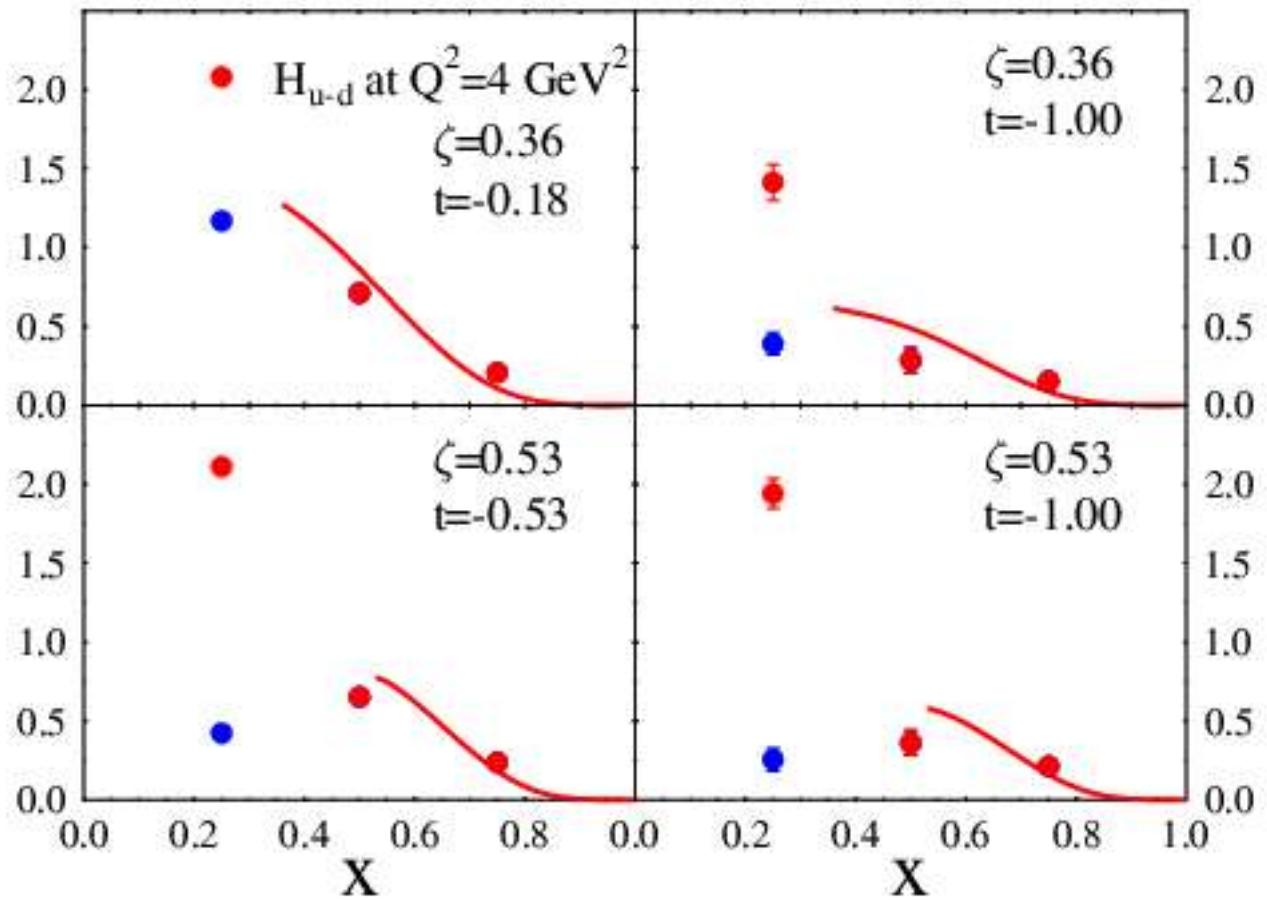
$$[(\Lambda_{M(E)}^V)^2]_n = \frac{12(1 + \alpha_n^{M(E)} m_\pi^2)}{\beta_n^{M(E)} + \gamma_n \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)}$$



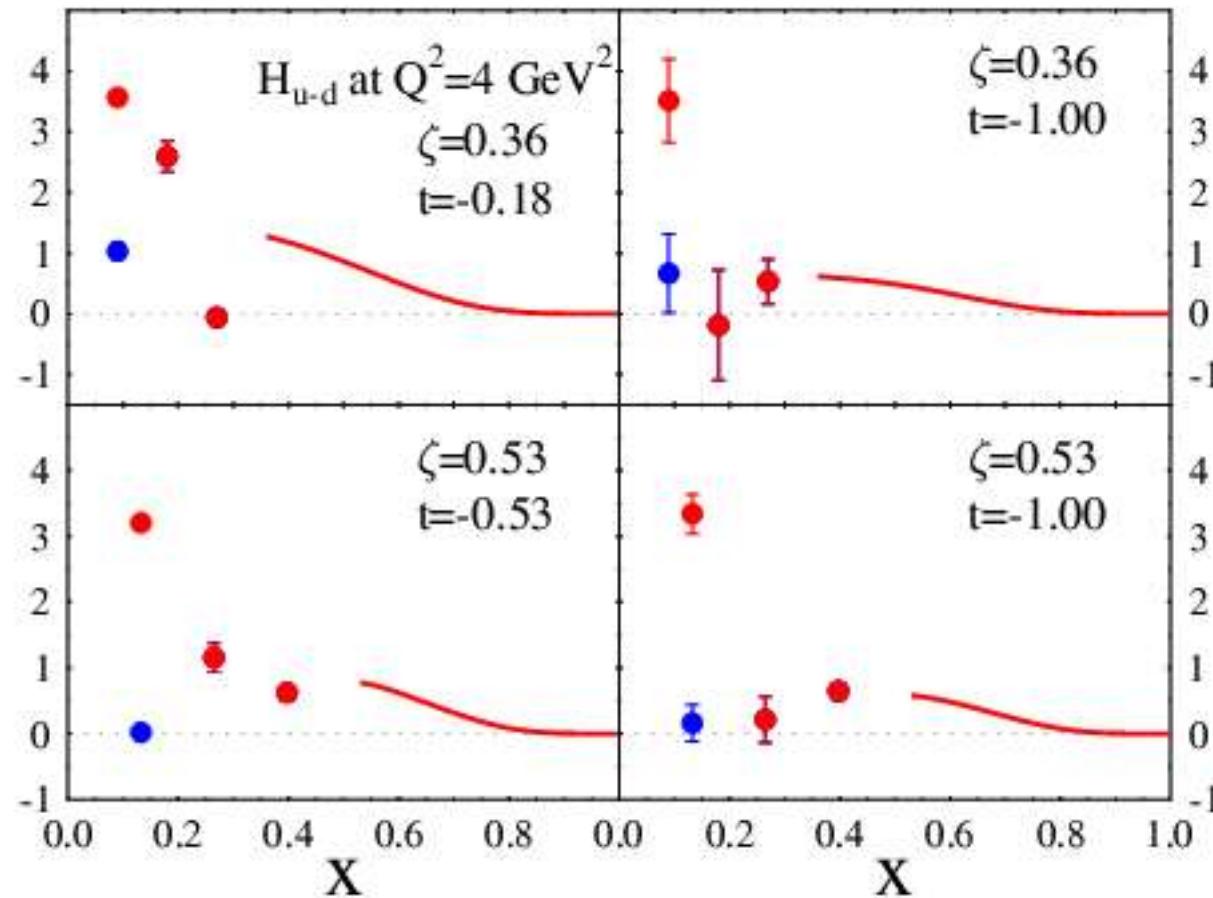
Extension to higher moments

**(2) reconstruct GPD from its moments: Bernstein polynomials**



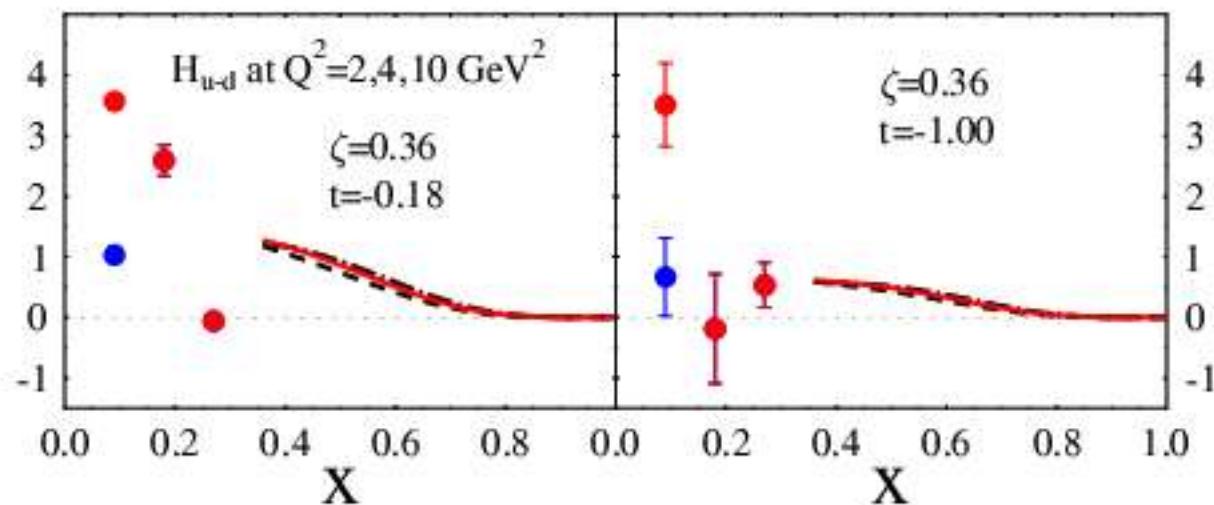


- Reconstruct function in  $x=[0,1]$
- Kinematically Extend parameterization to  $x=[\zeta,1]$
- Kinematic extension fits the moments well!

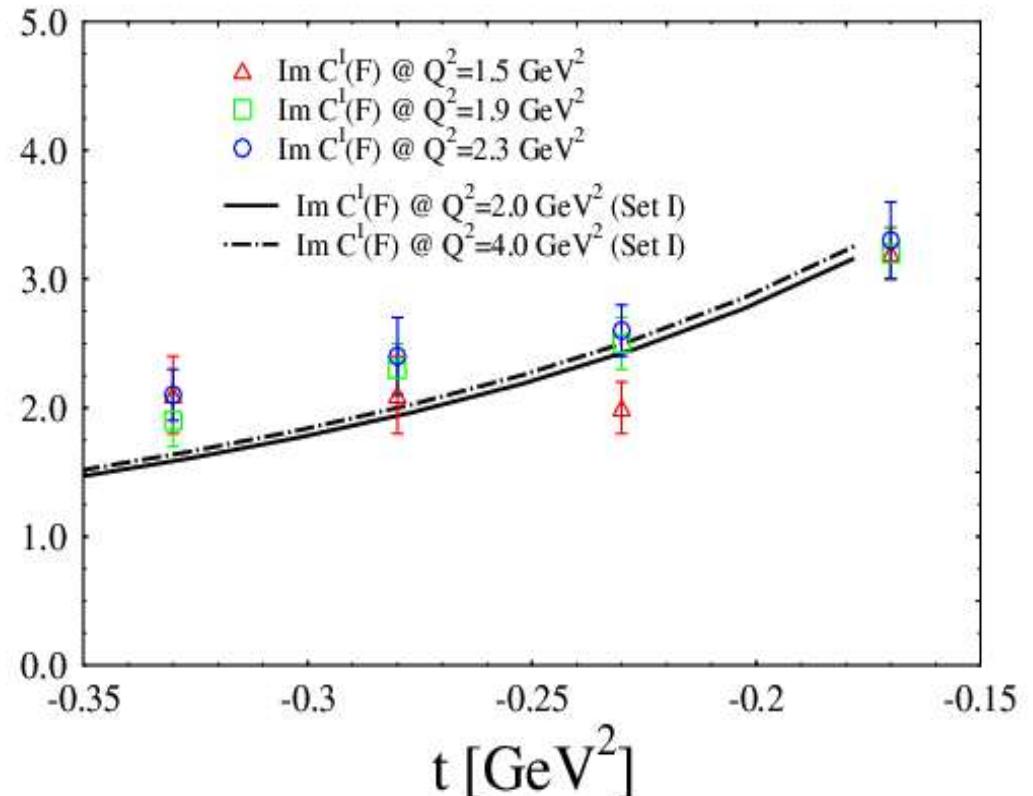
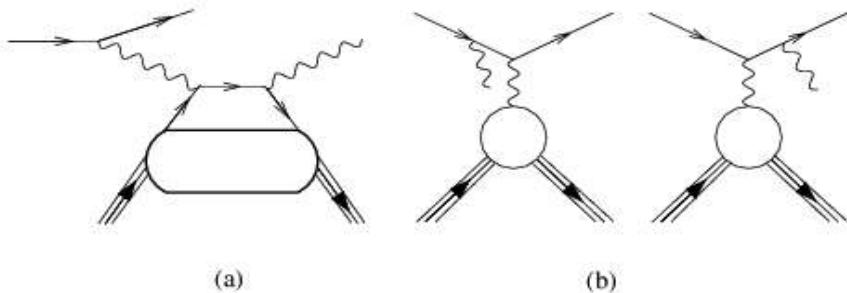


- Form “subtracted” moments = [ (total) - ( $x=[\zeta, 1]$  region)]
- Reconstruct function in  $x=[0, \zeta]$  !
- First model independent extraction of GPDs!!!

# Effect of PQCD evolution in Jlab data range



# Comparison with Jlab Hall A data



- Observable given by Interference Term between DVCS (a) and BH(b):

$$d\sigma^\rightarrow - d\sigma^\leftarrow \propto \sin\phi \left[ F_1(\Delta^2) \mathcal{H} + \frac{x}{2-x} (F_1 + F_2) \tilde{\mathcal{H}} + \frac{\Delta^2}{M^2} F_2(\Delta^2) \mathcal{E} \right]$$

$$\mathcal{H} = \sum_q e_q^2 (H(\xi, \xi, \Delta^2) - H(-\xi, \xi, \Delta^2))$$

# Conclusions

- We presented a method to extract GPDs from presently available experimental data on inclusive experiments, and using constraints from lattice QCD.
- Our analysis is a first attempt to obtain a model independent view of the behavior of GPDs
- Higher “n” lattice moments along with a validation of the chiral extrapolation methods used so far, are crucial for future extractions of GPDs from experiment