The curvature of $F_2^p(x, Q^2)$ as a probe of perturbative QCD evolutions in the small-x region *

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- The structure function of the proton F_2 in QCD
- Standard χ^2 analysis
- Curvature of F_2
- Results
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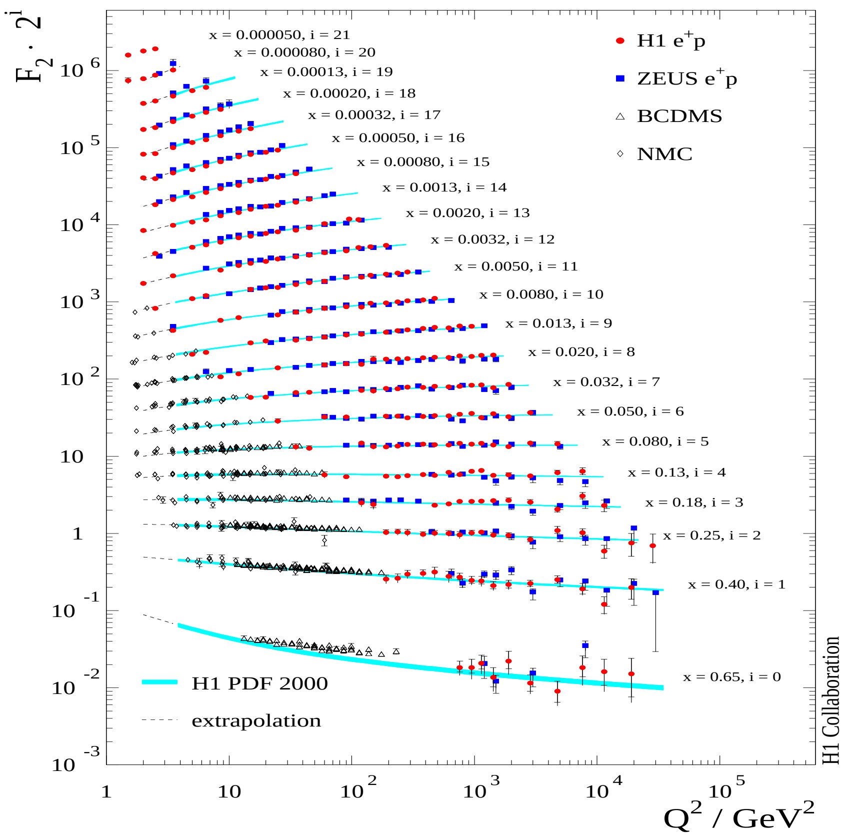
* EPJ C40, 515 (2005) and EPJ C50, 29 (2007); in collab. with M. Glück and E. Reya

The structure function of the proton F_2 in QCD

- At fixed x and $Q^2 \gtrsim$ 1 GeV^2 , the structure function of the proton F_2 appears to depend logarithmically on Q^2



- This behaviour arises from perturbative QCD (pQCD), which dictates the Q^2–evolution of the underlying parton distributions $f(x, Q^2),\ f = q,\ \bar{q},\ g$
- The parton distributions are fixed at a specific input scale $Q^2 = Q_0^2$, mainly by experiment, only their evolution to any $Q^2 > Q_0^2$ being predicted by pQCD

Standard χ^2 analysis

Do NLO and NNLO pQCD Q^2–evolutions agree with recent HERA data on F_2 at $x \lesssim 10^{-3}$?

- In order to answer, the valence $q_v = u_v, d_v$ and sea $w = \bar{q}, g$ distributions are parametrized at the input scale $Q_0^2 = 1.5$ GeV^2 as follows

$$\begin{aligned} x\, q_v(x, Q_0^2) &= N_{q_v} x^{a_{q_v}} (1-x)^{b_{q_v}} (1 + c_{q_v} \sqrt{x} + d_{q_v} x + e_{q_v} x^{1.5}) \\ x\, w(x, Q_0^2) &= N_w x^{a_w} (1-x)^{b_w} (1 + c_w \sqrt{x} + d_w x) \end{aligned}$$

- Sea breaking effects are not considered: $\bar{q} \equiv \bar{u} = \bar{d}$ and $s = \bar{s} = 0.5\bar{q}$
- The normalizations N_{u_v}, N_{d_v} and N_g are fixed by $(\Sigma(x, Q^2) \equiv \Sigma_{q=u,d,s}(q + \bar{q}))$:

$$\int_0^1 u_v dx = 2, \qquad \int_0^1 d_v dx = 1, \qquad \int_0^1 x(\Sigma + g) dx = 1$$

- All Q^2-evolutions are performed in Mellin n-moment space, the program QCD-PEGAUS has been used for the NNLO evolutions
  A. Vogt, Comput. Phys. Commun. 170, 65 (2005)

- The choice of a factorization scheme in NLO might imply similar effects as the additional NNLO contributions: NLO analysis in both $\overline{\mathrm{MS}}$ and DIS schemes. NNLO only in the $\overline{\mathrm{MS}}$ factorization scheme

Parameter values

- The following data sets from DIS processes have been used:
  small-x and large-x H1 F_2^p data
  fixed target BCDMS data for F_2^p and F_2^n
  proton and deuteron NMC data
- Total of 740 data points; degrees of freedom dof = 720, χ^2 evaluated by adding in quadrature statistical and systematic errors

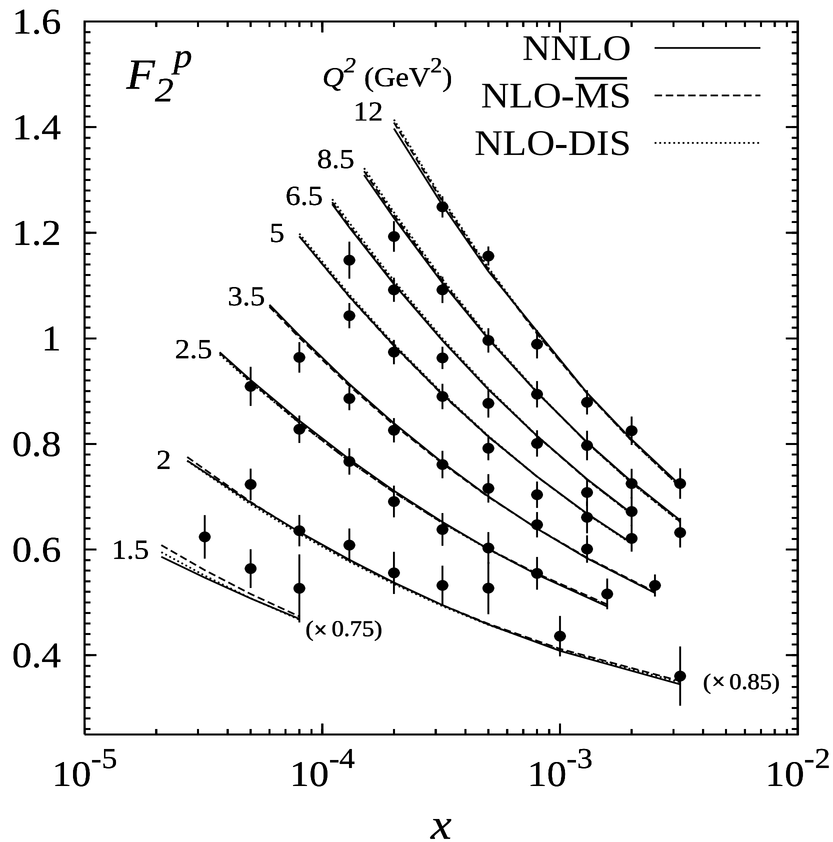
	NNLO($\overline{\mathrm{MS}}$)				NLO($\overline{\mathrm{MS}}$)			
	u_v	d_v	$\bar{q}$	g	u_v	d_v	$\bar{q}$	g
N	0.250	3.620	0.120	2.120	0.430	0.396	0.055	2.379
a	0.252	0.925	-0.149	-0.012	0.286	0.538	-0.218	-0.012
b	3.629	6.711	3.728	6.514	3.550	5.797	3.311	5.639
c	4.764	6.723	0.621	2.092	1.112	22.50	5.309	0.879
d	24.18	-24.24	-1.135	-3.089	15.61	-52.70	-5.905	-1.771
e	9.049	30.11	—	—	4.241	69.76	—	—
χ^2/dof	0.989				0.993			
$\alpha_s(M_Z^2)$	0.112				0.114			

- Rather small quantitative difference between the NLO(DIS) and NLO($\overline{\mathrm{MS}}$)

- H1 data in the very small-x region:

$$1.5\ \mathrm{GeV}^2 \le Q^2 \le 12\ \mathrm{GeV}^2, \qquad 3 \times 10^{-5} \lesssim x \lesssim 3 \times 10^{-3}$$

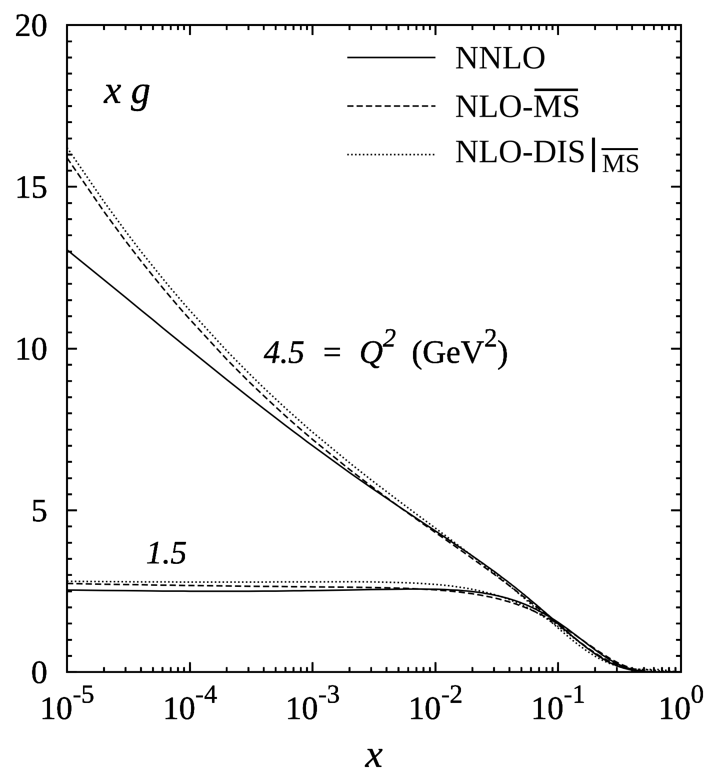
C. Adloff et al., H1 Collab., EPJ C21, 33 (2001)

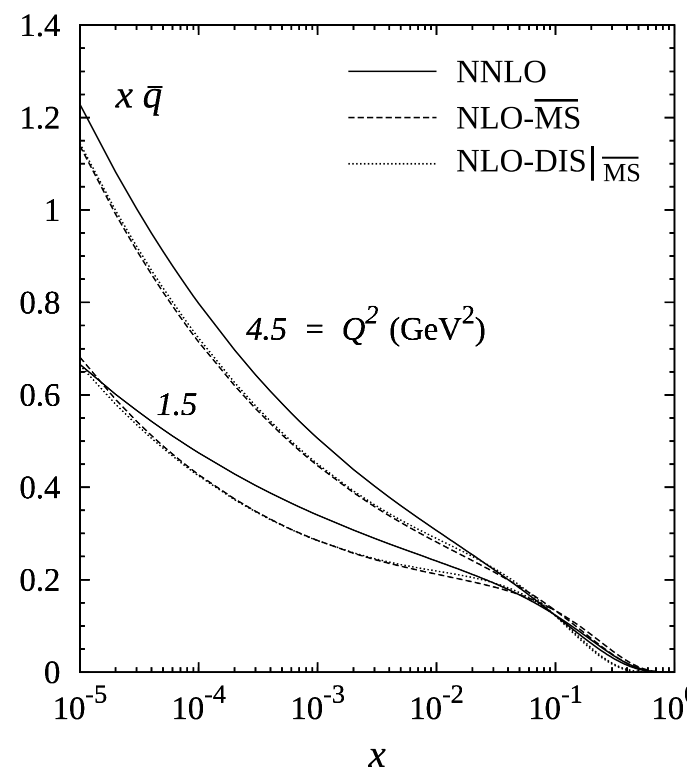


- Perturbatively stable predictions compatible with the data, comparable χ^2' s: agreement between the Q^2–evolutions of $f(x, Q^2)$ and the measured Q^2–dependence of $F_2(x, Q^2)$

Resulting gluon and sea distributions

- The gluon distributions at $Q^2 = 4.5$ GeV^2 conform to the rising shape as $x \to 0$ (tamed at NNLO) obtained in most available analyses published so far





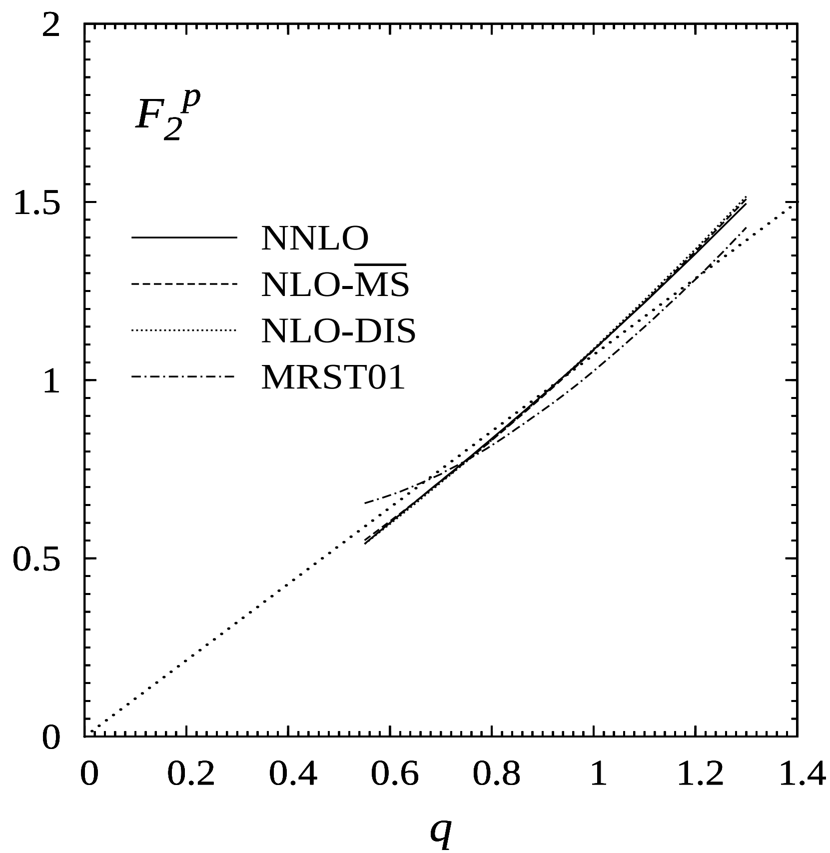
- NLO-DIS$|_{\overline{\text{MS}}}$: NLO(DIS) results transformed to the $\overline{\text{MS}}$ factorization scheme
- It is possible to conceive a valence–like gluon at some very–low Q^2 scale, but even in this extreme case the gluon ends up as non valence–like at $Q^2 > 1$ GeV^2

Curvature of F_2

- At $x = 10^{-4}$ most measurements lie along a straight (dotted) line, if plotted versus

$$q = \log_{10}\left(1 + \frac{Q^2}{0.5\,\mathrm{GeV}^2}\right)$$

D. Haidt, EPJ C35, 519 (2004)

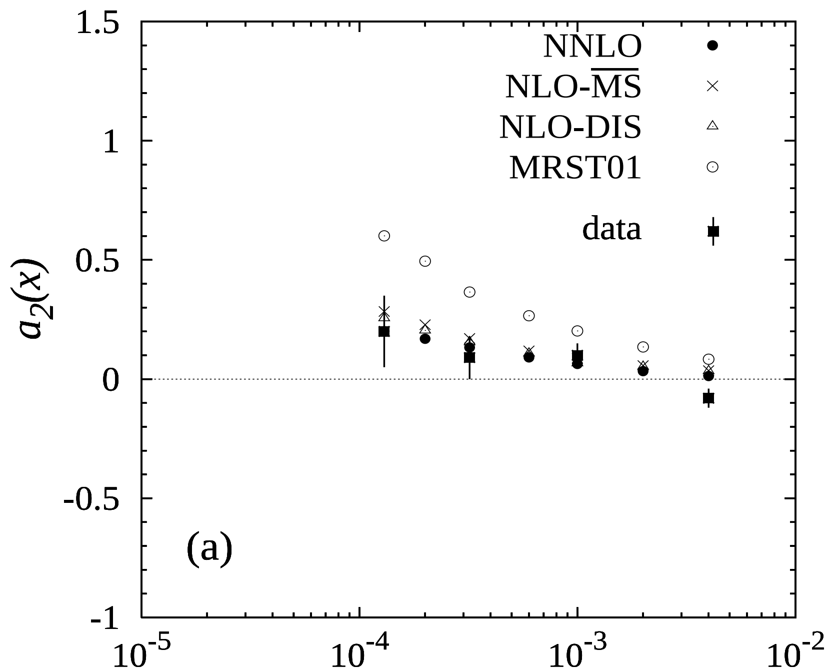


- MRST01 fit (NLO-$\overline{\mathrm{MS}}$): sizable curvature for F_2, incompatible with the data, mainly caused by the valence–like input gluon distribution at $Q_0^2 = 1$ GeV^2

Calculation of the curvature

- The curvature $a_2(x) = \frac{1}{2}\,\partial_q^2 F_2(x,Q^2)$ is evaluated by fitting the predictions for $F_2(x,Q^2)$ at fixed values of x to a (kinematically) given interval of q, as

$$F_2(x,Q^2) = a_0(x) + a_1(x)q + a_2(x)q^2$$

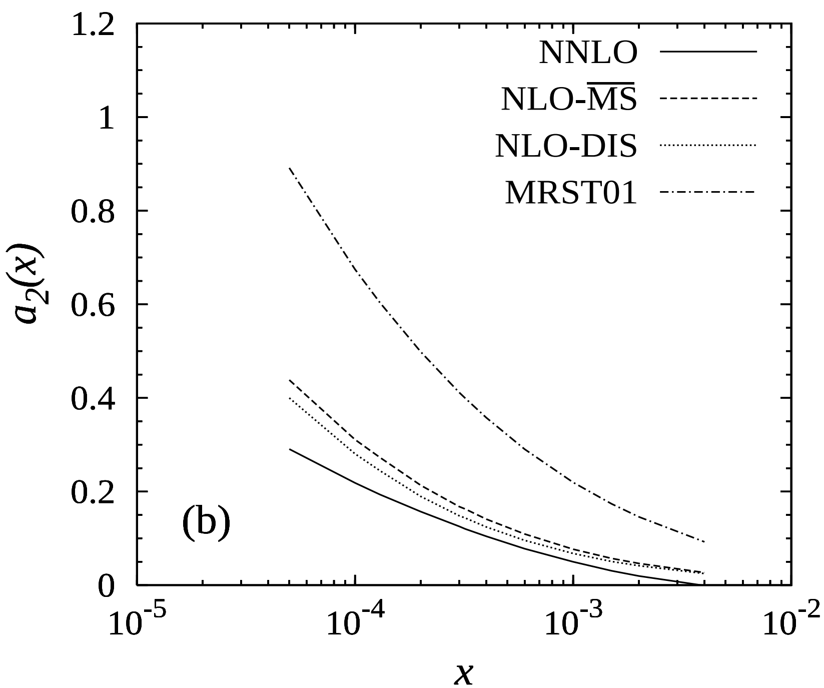


$$0.7 \le q \le 1.4 \quad \text{for} \quad 2\times10^{-4} < x < 10^{-2}$$
$$0.7 \le q \le 1.2 \quad \text{for} \quad 5\times10^{-5} < x \le 2\times10^{-4}$$

- (a): The average value of q decreases with decreasing x due to the kinematically more restricted Q^2 range accessible experimentally
- Our fits agree with the experimental curvatures, as calculated by Haidt using H1 data

Results

- (b): For comparison $a_2(x)$ is also shown for an x–independent fixed q–interval



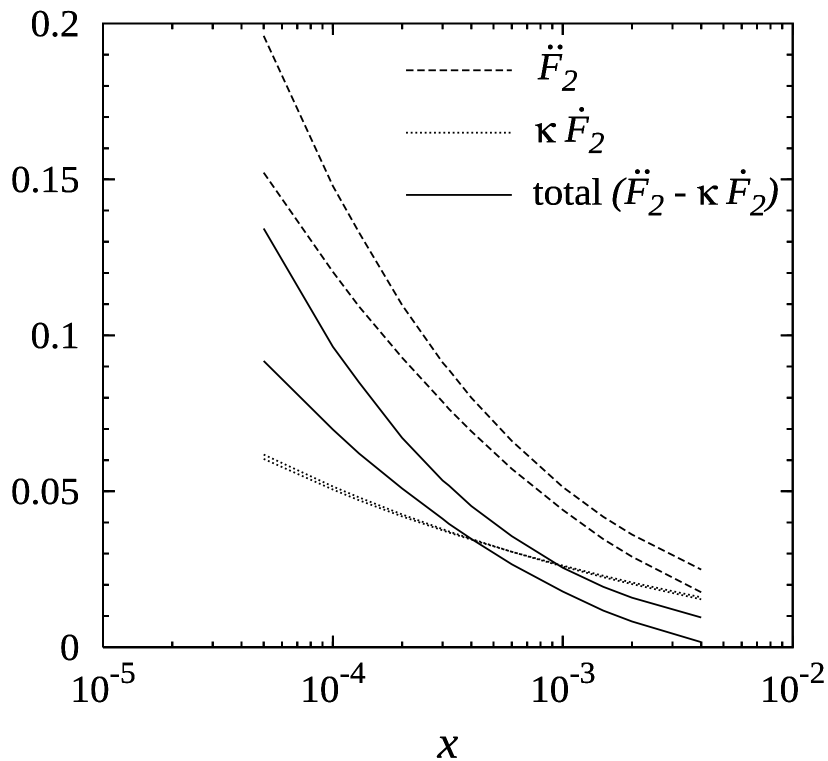
$$0.6 \le q \le 1.4$$
$$(1.5\ \ \mathrm{GeV}^2 \le Q^2 \le 12\ \ \mathrm{GeV}^2)$$

- No sensitive dependence on the factorization scheme ($\overline{\mathrm{MS}}$ or DIS)
- Perturbative stable evolutions result in a positive curvature $a_2(x)$, which increases as x decreases
- Future analyses of present precision measurements in this very small x–region should provide a sensitive test of the range of validity of pQCD evolutions

$\ddot{F}_2^p \equiv \partial^2 F_2^p / \partial(\ln Q^2)^2$

- $\ddot{F}_2^p = \mathcal{O}(\alpha_s^2)$ is directly related to the evolution equations and to experiment
- $\dot{F}_2^p \equiv \partial F_2^p / \partial \ln Q^2 = \mathcal{O}(\alpha_s)$ and $\kappa = 0.5$ GeV$^2/(Q^2 + 0.5$ GeV^2)

$$\partial_q^2 F_2^p = \left(\frac{Q^2 + 0.5\,\mathrm{GeV}^2}{Q^2} \ln 10 \right)^2 \left[-\kappa\, \dot{F}_2^p + \ddot{F}_2^p \right]$$



$\kappa = 0.1$

($Q^2 = 4.5$ GeV^2 or $q = 1$)

At smallest values of x,

upper curves: NLO($\overline{\mathrm{MS}}$)

lower curves: NNLO

- $\ddot{F}_2^p$ dominates over $\kappa \dot{F}_2^p$: $\partial_q^2 F_2^p$ represents a clean test of the curvature of F_2^p

Summary and conclusions

- A dedicated test of the pQCD NLO and NNLO parton evolutions in the small–x region has been performed

- The Q^2-dependence of $F_2(x, Q^2)$ is compatible with recent high-statistics measurements in that region

- The results are perturbatively stable and rather insensitive to the factorization scheme used ($\overline{\mathrm{MS}}$ or DIS)

- A characteristic feature of perturbative QCD is a positive curvature $a_2(x)$, which increases as x decreases

- Present data are indicative for such a behaviour, but they are statistically insignificant for $x < 10^{-4}$.
  The H1 Collab. has found a good agreement between the perturbative NLO evolution and the slope of F_2, $a_1(x)$, i.e. the first derivative $\partial_{Q^2} F_2$

- Future analyses of present precision measurements should provide a sensitive test of the range of validity of pQCD and further information concerning the detailed shapes of the gluon and the sea distributions at very small x