

Recent developments in small- x QCD evolution

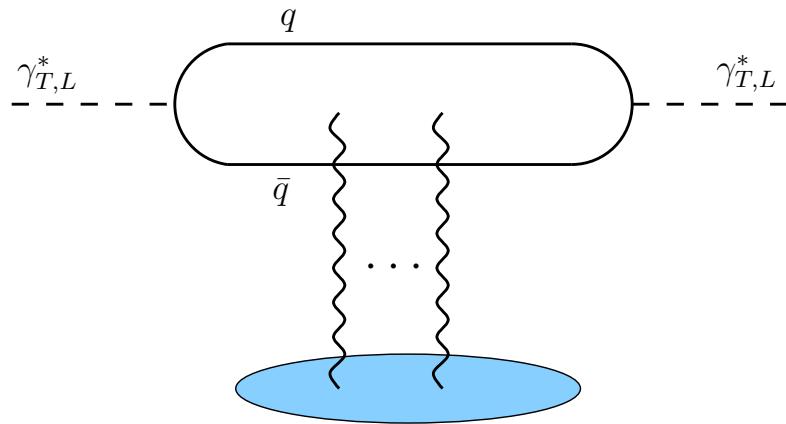
Arif Shoshi

Bielefeld University, Germany

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Deep Inelastic Scattering

- A proton (nucleus) probed with a dipole:



- Total cross section:

$$\sigma^{\gamma_{T,L}^* p}(Q, x_{Bj}) = \int d^2\mathbf{r} \int dz |\psi_{T,L}(z, Q)|^2 2 \int d^2\mathbf{b} T(\mathbf{r}, x_{Bj}, \mathbf{b})$$

- How does $T(\mathbf{r}, Y)$ change if $Y \rightarrow Y + dY$?

- Kinematics:

$$Q^2 = -q^2$$

– photon virtuality

$$s = (p + q)^2$$

– total collision energy

$$x_{Bj} \approx \frac{Q^2}{s} \ll 1$$

– Bjorken variable (small- x_{Bj})

$$Y = \ln(1/x_{Bj}) \sim \ln(s)$$

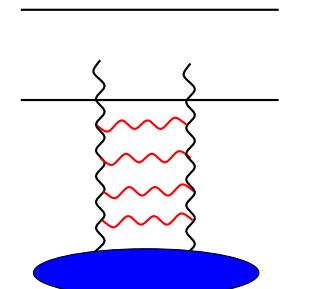
– total rapidity

“Mean Field Equations”

- BFKL equation:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s \langle T \rangle_Y$$

* $\langle T \rangle_Y \sim \exp[c\bar{\alpha}_s Y] \rightarrow$ unitarity violation!

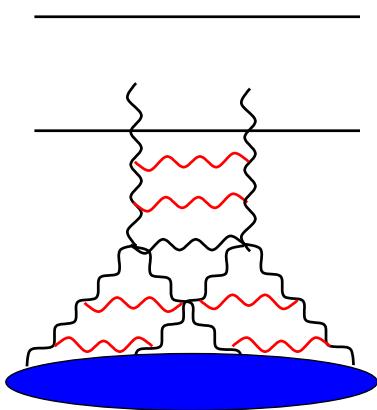


- Kovchegov equation:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s [\langle T \rangle_Y - \langle T \rangle_Y \langle T \rangle_Y]$$

* $\langle T \rangle_Y \langle T \rangle_Y$; non-linear evolution, $\langle T \rangle_Y \leq 1$

* $\langle T \rangle_Y \ll 1$; linear BFKL evolution



- B-JIMWLK equations:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \bar{\alpha}_s [\langle T \rangle_Y - \langle T T \rangle_Y] ,$$

$$\frac{\partial}{\partial Y} \langle T T \rangle_Y \propto \bar{\alpha}_s [\langle T T \rangle_Y - \langle T T T \rangle_Y] , \dots$$

* Fluctuations: $\langle T T \rangle_Y \neq \langle T \rangle_Y \langle T \rangle_Y$?

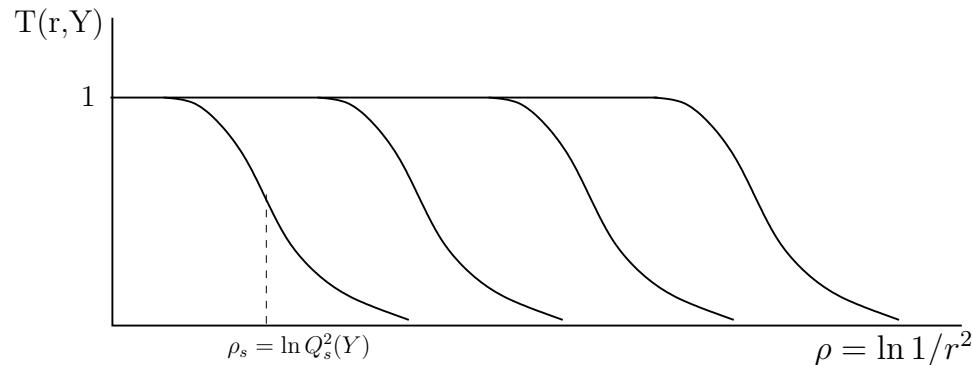
* Mean field approximation: $\langle T T \rangle_Y \approx \langle T \rangle_Y \langle T \rangle_Y \rightarrow$ Kovchegov Eq.

• Numerical result [Rummukainen, Weigert 04]: $\langle T \rangle_Y^{\text{Kovchegov}} \approx \langle T \rangle_Y^{\text{B-JIMWLK}}$

Results from “Mean Field Equations”

[Mueller, Triantafyllopoulos 2002], [Munier, Peschanski 2003]

- Physical picture:



- Geometric scaling:

$$T(r, Y) = C [Q_s^2(Y) r^2]^{1-\lambda_0} \left[\ln\left(\frac{1}{Q_s^2(Y) r^2}\right) + c \right]$$

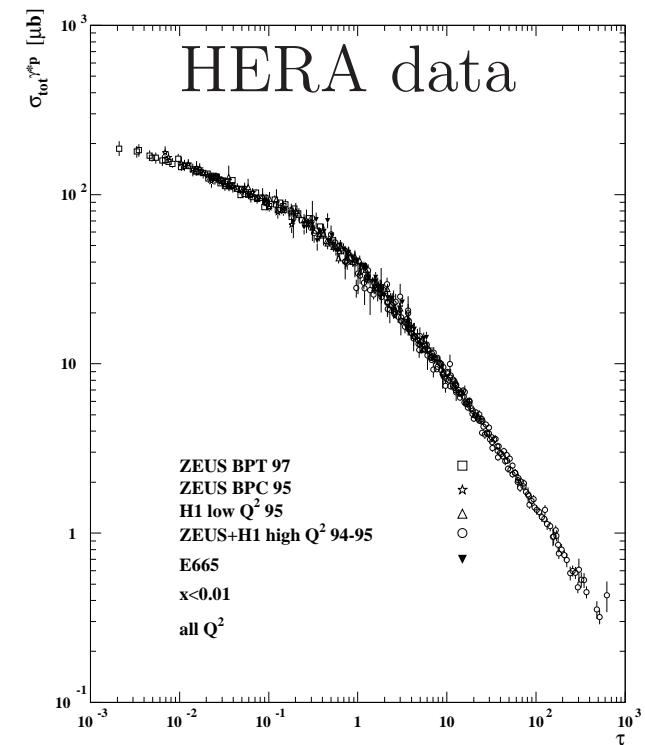
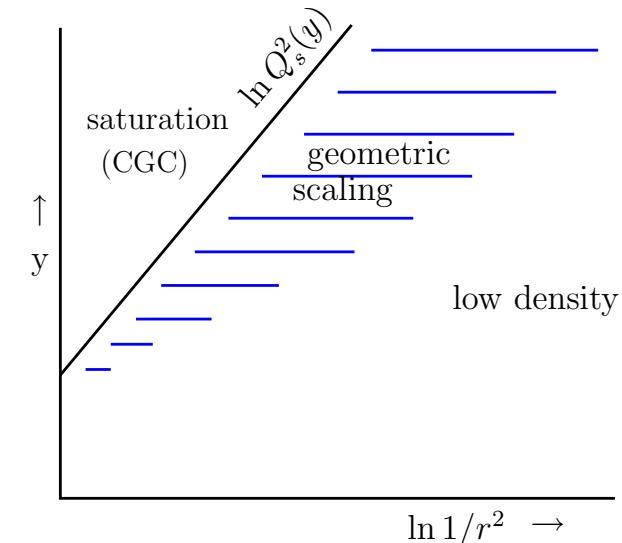
- $\sigma^{\gamma^* p}(Q, x_{Bj}) \Rightarrow \sigma^{\gamma^* p}(Q/Q_s(Y))!$

- Saturation momentum:

$$Q_s(x_{Bj}) = Q_0^2 (1/x_{Bj})^\lambda,$$

Experiment: $\lambda \approx 0.3$

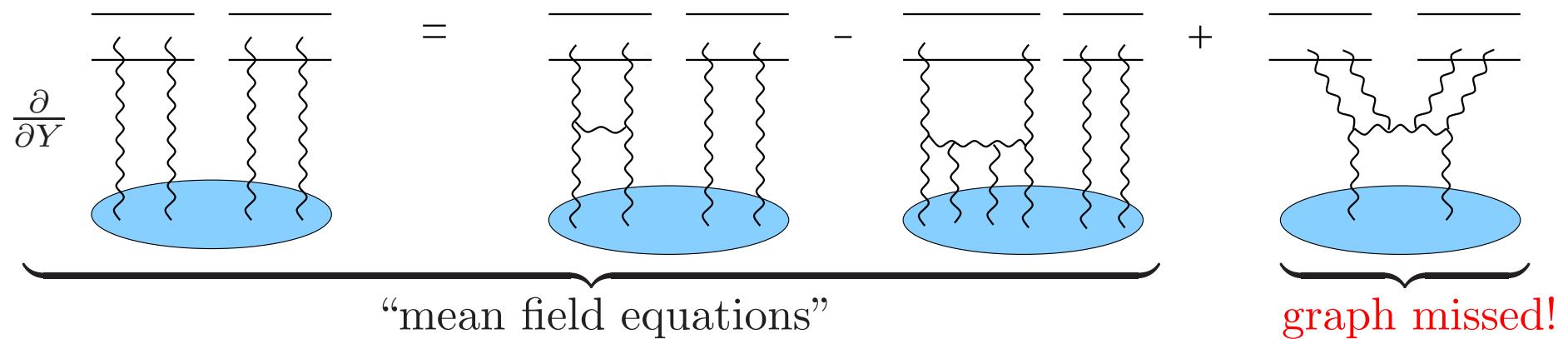
Theory [Triantafyllopoulos 02]: λ close to 0.3



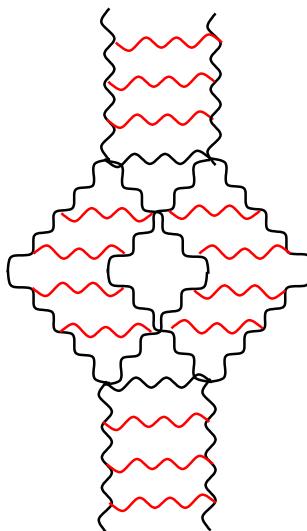
Shortcomings of “mean field equations”: Pomeron loops

[Iancu, Triantafyllopoulos 2005]

Two dipoles scattering off a target:



- Pomeron loops missed!



“Pomeron Loop Equations”

[Mueller,Shoshi,Wong 2005], [Iancu,Triantafyllopoulos 2005], [Kovner, Lublinsky 2005]

- Hierarchy of equations:

$$\frac{\partial}{\partial Y} \langle T \rangle_Y \propto \alpha_s [\langle T \rangle_Y - \langle T T \rangle_Y]$$
$$\frac{\partial}{\partial Y} \langle T T \rangle_Y \propto \alpha_s [\langle T T \rangle_Y - \langle T T T \rangle_Y + \alpha_s^2 \langle T \rangle_Y]$$

- Langevin-type version:

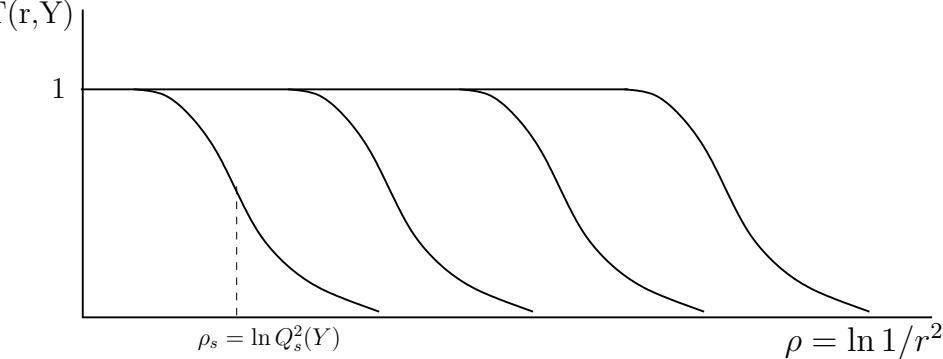
$$\frac{\partial}{\partial Y} T_Y \propto \alpha_s \left[T_Y - T_Y T_Y + \sqrt{\alpha_s T} \nu \right]$$

⇒ Gluon number fluctuations from event to event!

Gluon number fluctuations \Rightarrow Diffusive Scaling

[Mueller,Shoshi 2004], [Iancu,Mueller,Munier 2004]

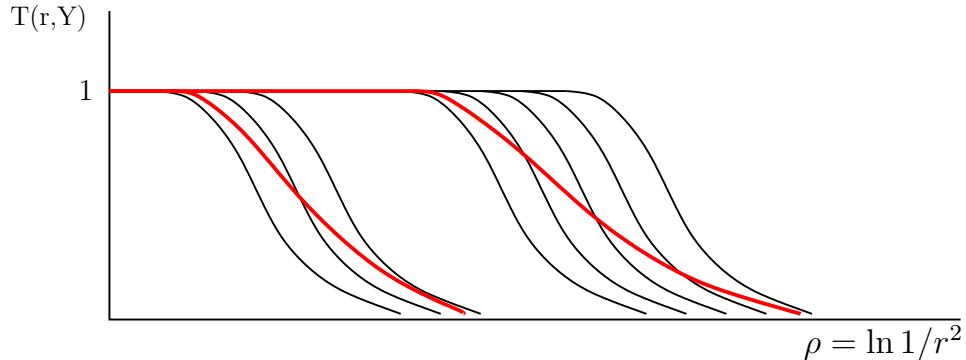
- Single events



Geometric scaling

$$T(r, Y) = T(r^2 Q_s^2(Y))$$

- Average over events



Diffusive scaling

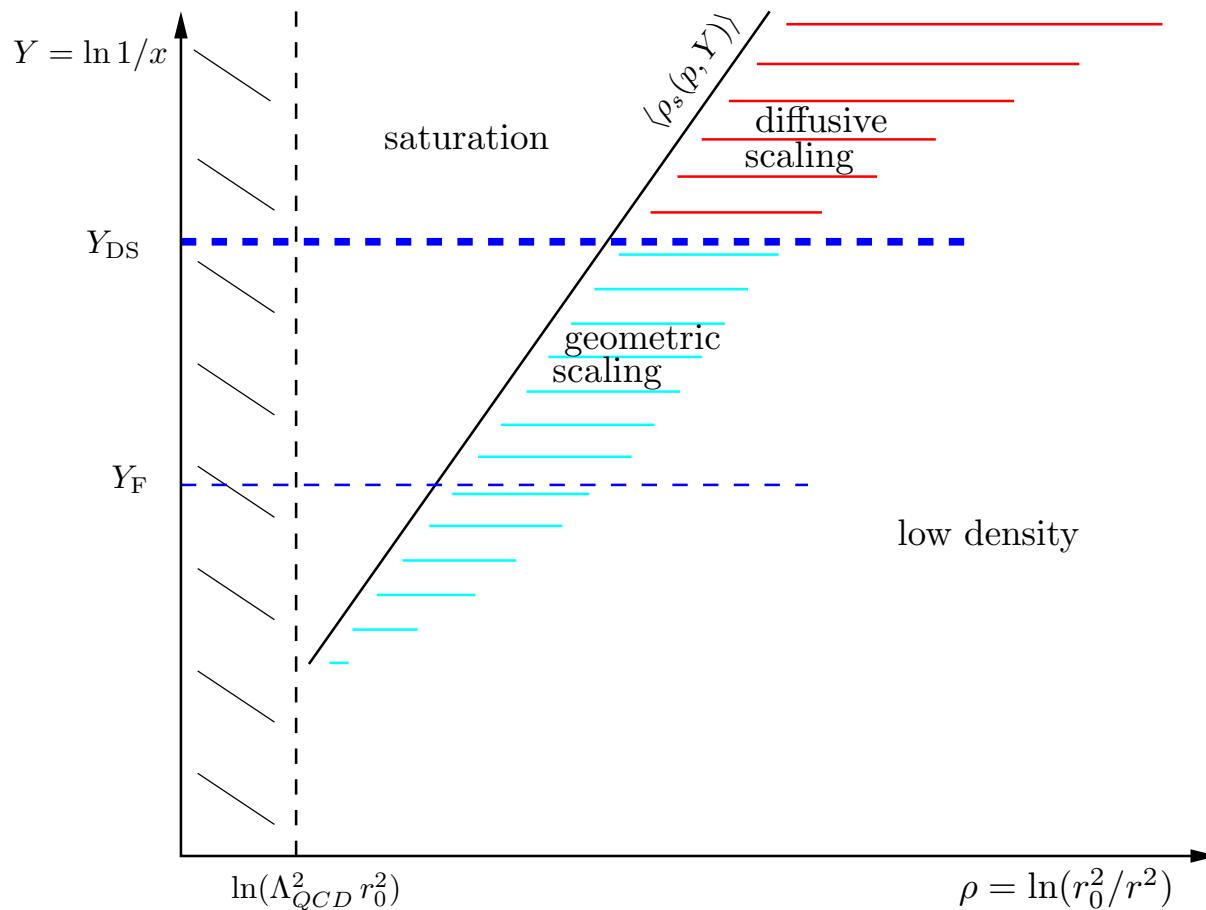
$$\langle T(r, Y) \rangle = T\left(\frac{\ln(\bar{Q}_s^2(Y) r^2)}{\sqrt{DY}}\right)$$

$$\text{for } Y \gg \frac{1}{\alpha_s} \ln^3(1/\alpha_s^2)$$

Statistical physics \Leftrightarrow QCD: $\langle T(\rho - \rho_s) \rangle = \int d\rho_s P(\rho_s) T(\rho - \rho_s)$

- saturation momentum: $\bar{Q}_s^2(Y) = Q_0^2 \left(\frac{1}{x_{Bj}}\right)^\lambda$, $\lambda = \lambda - \frac{C}{\ln^2(1/\alpha_s^2)}$

“Phase diagram”



- Parametrically: $Y_{DS} \sim \frac{1}{\alpha_s} \ln^3 \frac{1}{\alpha_s^2}$

Phenomenological consequences at large energies

- Pomeron loops/fluctuations do strongly change the
 - nuclear modification factor R_{pA} [[Kozlov, Shoshi, Xiao 2006](#)]
 - forward gluon production cross section [[Iancu, Marquet, Soyez 2006](#)]
 - total photon-proton cross section, diffractive cross section, etc.

Conclusion

- Pomeron loop equations
- Large energies:
 - geometric scaling → diffusive scaling
 - energy dependence of saturation momentum
 - phenomenological consequences