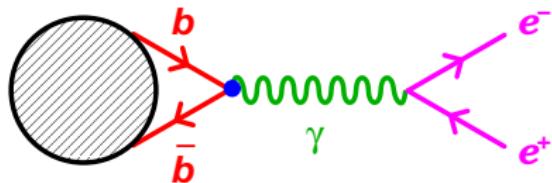


# Heavy quarkonium decay on and off the lattice



Alistair Hart

SUPA: University of Edinburgh

DIS2007, Munich, 18.iv.07



# Outline

Dudek and Edwards

First LQCD estimate of two photon widths

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Two photon decay widths of  $\eta_{c,b}$  and excited states

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Oliveira and Coimbra

Charmonium from classical Yang–Mills

“Conclude with a feature on charm” (6 letters)

# LQCD simulations of heavy quarks

*b*-quarks:

- $\hbar c/m_b \sim 0.04$  fm  $\ll$  simulable lat. spacings
- $v^2/c^2 \sim 0.1$ : use eff. ths. (NRQCD, HQET)

*c*-quarks:

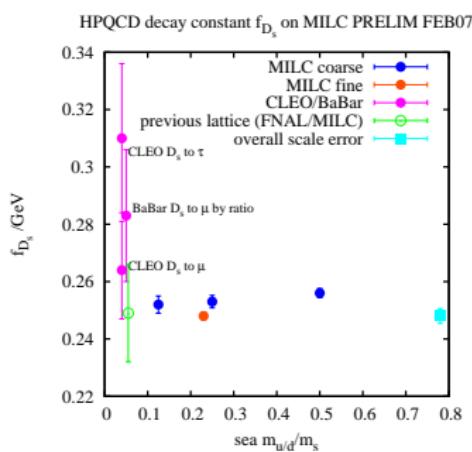
- $v^2/c^2 \sim 0.3$ : effective theories less good
- $\hbar c/m_c \sim 0.1$  fm: relativistic simuls now poss.

## Relativistic sea quarks

- Impr. stag. (ASQTAD): cheap but rooted
- Domain wall: expensive
- Wilson-Clover, Tw. mass: cheaper

## Relativistic valence quarks for charm

- Impr. stag. (HISQ): no rooting ambiguity
- DWF, overlap: good chiral symmetry
- Wilson-Clover: cheaper



## Dudek and Edwards

First LQCD estimate of  $\Gamma_{\gamma\gamma}(\eta_c, \chi_{c0})$

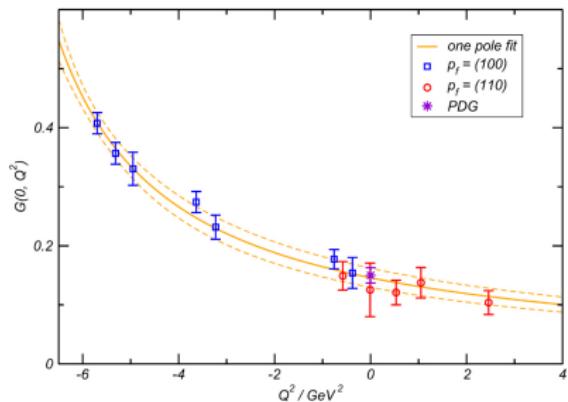
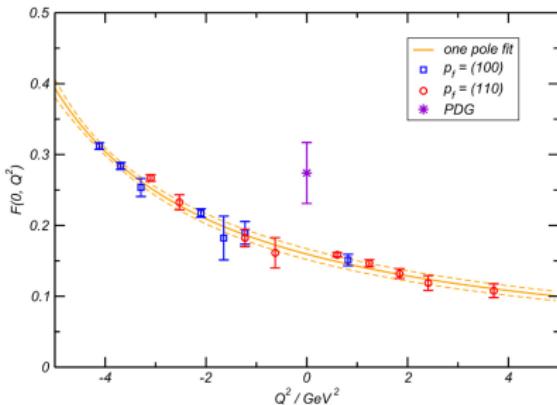
- Phys. Rev. Lett. 97 (2006) 172001 [hep-ph/0607140]

Use method of Ji and Jung (2001):

- accurate perturbative expansion of photon-quark coupling
- express photon as superposition of QCD eigenstates

Lattice calculation:

- quenched:  $a \simeq 0.05$  fm, no sea quark effects
- valence quarks: relativistic, non-chiral



$\Gamma_{\gamma\gamma}/\text{keV}$	D & E	(stat)	(disc)	(quench)	PDG
$\eta_c$	2.65	(26)	(80)	(53)	7.14 (2.49)
$\chi_{c0}$	2.41	(58)	(72)	(48)	2.87 (0.40)

Quenched: no sea quark contributions

- Incorrect running of  $\alpha_s \Rightarrow$  depleted wavefn at origin

One lattice spacing:  $a = 0.05 \text{ fm}$

- Disc. error: c. 15% from  $\mathcal{O}(am_c)$  effects

# Dudek, Edwards and Richards

Calculations of decay constants, EM transition rates

- Phys. Rev. D 73 (2006) 074507 [hep-ph/0601137]

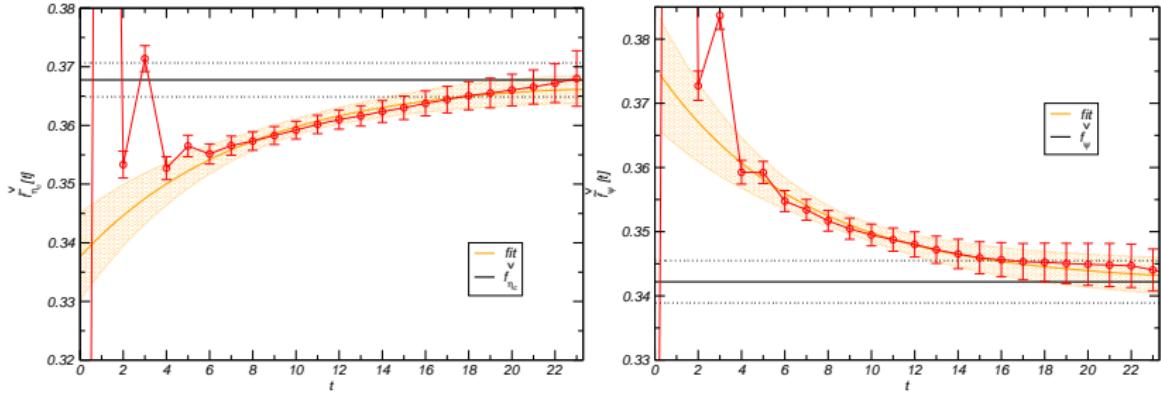
Simulation details:

- Quenched: no sea quark contributions
  - Error: claimed small  $\Rightarrow$  corrected masses, phase space.
- DWF fermions (relativistic):
  - Exact chiral symmetry (almost) — 5% violation of  $Z_V = Z_A$
- One lattice spacing:  $a_s = 0.10$  fm,  $a_t = 0.03$  fm
  - Disc. error: claimed small as DWF  $\mathcal{O}(a^2)$  improved

Lattice charm mass  $\hat{m}_c = a_t m_c$  not properly tuned

- ground state masses 5% too low
- leads to 20% ambiguity in radiative widths ( $\Gamma \sim 1/M^2$ )
  - phenomenological attempt to correct these

# Decay constants

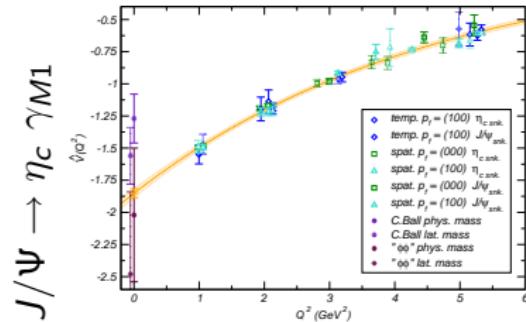
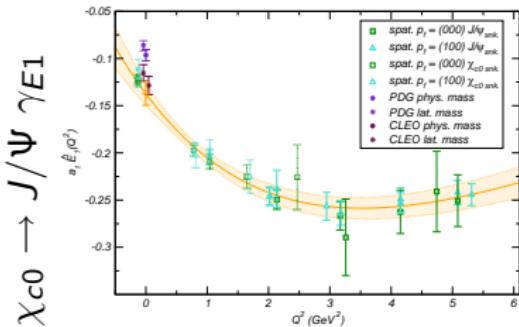


D, E & R <sub>(stat)</sub> ( $Z_V \neq Z_A$ )		expt/MeV
$f_{J/\psi}$	399 (4)	411 (7)
$f_{\eta_c}$	429 (4) (25)	335 (75)
$f_{\psi'}$	143 (81)	279 (8)
$f_{\eta'_c}$	56 (21) (3)	

Heavy Quark Spin Symmetry: supported by  $f_{J/\psi} \simeq f_{\eta_c}$

- NR quark model: differ by  $\mathcal{O}(\frac{v}{c})$ -suppressed spin terms
- Clearly not applicable to radially excited states:  $f_{\eta'_c} \simeq \frac{1}{3} f_{\psi'}$

# EM transition rates



$\Gamma/\text{keV}$	$\chi_{c0} \rightarrow J/\psi \gamma_{E1}$	$\chi_{c1} \rightarrow J/\psi \gamma_{E1}$	$h_c \rightarrow \eta_c \gamma_{E1}$	$J/\psi \rightarrow \eta_c \gamma_{M1}$
lat. mass	288 (60)	600 (178)	663 (132)	1.61 (7)
phys. mass	232 (41)	487 (122)	601 (55)	2.57 (11)
PDG	115 (14)	303 (44)		1.14 (33)
CLEO	204 (31)	364 (31)		2.9 (1.5)

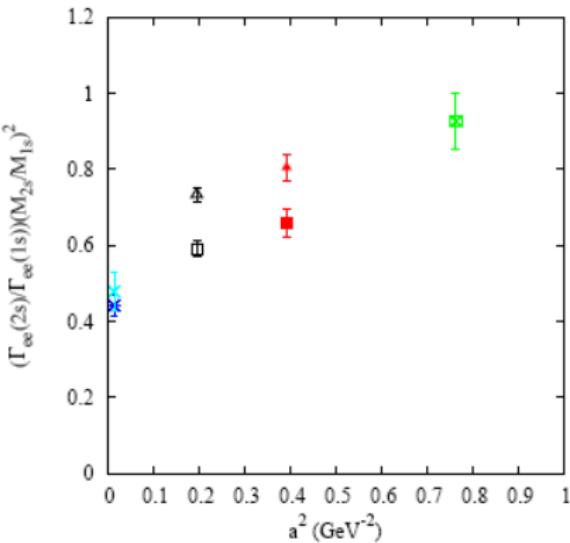
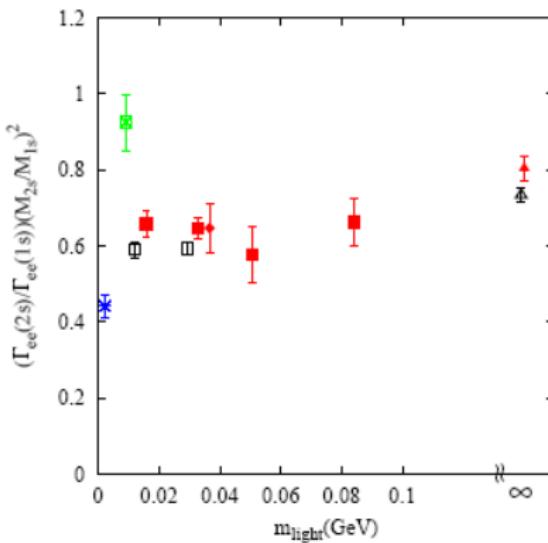
Also predicts

- $\chi_{c1} \rightarrow J/\Psi \gamma$ :  $\Gamma(M2)/\Gamma(E1) = -0.199 (121)$
- $\Gamma(h_c \rightarrow \eta_c \gamma) = 663 (132)$  (lat. mass),  $601 (55)$  (phys. mass).

Leptonic width ratio of  $\Upsilon(nS)$  very accurately known:

$$\frac{(M_\Upsilon^2 \Gamma_{ee})(2S)}{(M_\Upsilon^2 \Gamma_{ee})(1S)} = \begin{cases} 0.457 (6) & (\text{CLEO}) \\ 0.48 (5) & (\text{Gray et al., 2005}) \end{cases}$$

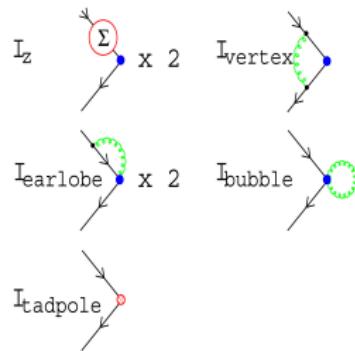
Need to improve annihilation current ME to  $\mathcal{O}(v^2)$ :



Key: **expt.**; ☒/■/□:  $\mathbf{a} \downarrow$ ;  $\mathbf{a = 0}$ ; (◆:  $N_f = 2$ ; △ ▲: qu.)

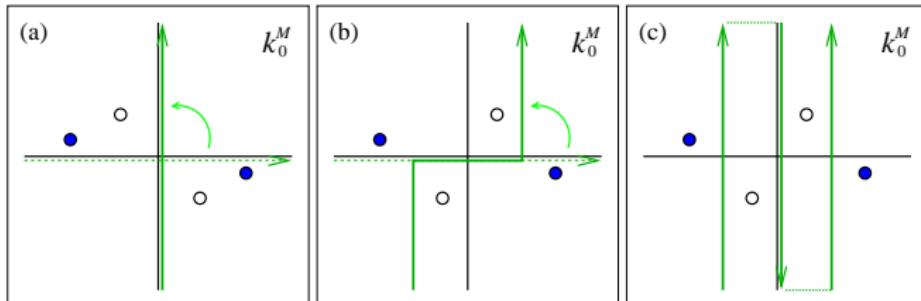
Pert. match LNRQCD MEs to  $\mathcal{O}(\alpha_s, v^2)$ :

- $\langle 0 | \mathbf{J}_{\text{QCD}} | Q \bar{Q} \rangle = \sum_{i=1}^2 a_i \langle 0 | \mathbf{J}_{\text{NRQCD}}^i | Q \bar{Q} \rangle$
- $J_{\text{NRQCD}}^i \propto \left( \frac{\mathbf{D}^2}{m_b^2} \right)^i \sim v^{2i}$ .
- $\mathcal{O}(\alpha_s)$ :  $x = x^{(0)} + \alpha_s x^{(1)}$  for MEs,  $a_i$ .



Features:

- No crossing symm. for Heavy-Heavy  $\Rightarrow$  careful Wick rotation
- Improved NRQCD action: automated Feynman rules
- Application to LNRQCD MEs in progress...



# Lansberg and Pham

Two photon decay widths of  $\eta_{c,b}$  and excited states.

- PRD 74 (2006) 034001 [hep-ph/0603113]
- PRD 75 (2007) 017501 [hep-ph/0609268]

Use effective Lagrangian:

- static approximation for  $c, b$  (+ B.E. in props),
- satisfies Heavy Quark Spin Symmetry:
  - 1S: Use  $f_{\eta_c} = f_{J/\Psi}$ , relates  $\Gamma_{\gamma\gamma}(\eta_c)$  to  $\Gamma_{\ell\bar{\ell}}(J/\Psi)$ ,
  - 2S: Assume  $f_{\eta'_c} = f_{\Psi'}$   $\Rightarrow \Gamma_{\gamma\gamma}(\eta'_c) = \Gamma_{\gamma\gamma}(\eta_c) \left( \frac{f_{\Psi'}}{f_{J/\Psi}} \right)^2 + \text{"B.E."}$ .

$f_\eta$	L&P	$\Gamma_{\gamma\gamma}^{\text{expt}}/\text{keV}$	$f_\eta$	L&P
$\eta_c$	7.5 – 10	$7.4 \pm 0.9 \pm 2.1$ (PDG)	$\eta_b$	0.560
$\eta'_c$	3.5 – 4.5	$1.3 \pm 0.6$ (CLEO)	$\eta'_b$	0.269
			$\eta''_b$	0.208

$f_{\eta'_c}$  three times too large:

- Dudek et al.:  $f_{\eta'_c} \simeq \frac{1}{3} f_{\Psi'}$
- Also need to worry about non-static effects.

# Gao, Zhang and Chao

Series of three papers on radiative decays of quarkonium using NRQCD

1.  $\Upsilon \rightarrow \gamma q\bar{q}$  and colour octet mechanism
  - Commun. Theor. Phys. 46 (2006) 1017 [hep-ph/0606170]
2. Radiative decays  $J/\Psi, \eta_{cJ}$  to light mesons
  - Chin. Phys. Lett. 23 (2006) 2376 [hep-ph/0607278]
3. Radiative decays Bottomonium to charmonium and light mesons
  - hep-ph/0701009

## Colour octet mechanism

Calculate 1-lp PT octet vs. singlet contributions to  $BR(\Upsilon \rightarrow \gamma q\bar{q})$ :

$$BR = 0.0002 \times \langle \Upsilon | \mathcal{O}_1(^3S_1) | \Upsilon \rangle + 0.061 \times \langle \Upsilon | \mathcal{O}_8(^3S_1) | \Upsilon \rangle \\ + 0.084 \times \langle \Upsilon | \mathcal{O}_8(^1S_0) | \Upsilon \rangle + 0.043 \times \langle \Upsilon | \mathcal{O}_8(^3P_0) | \Upsilon \rangle$$

- Min. jet energy cut: 1 GeV; min. angle between jets  $\theta > 37^\circ$ .
  - Experimentally: necessary for jet identification
  - Theoretically: desensitised to soft, collinear log. divergences

Colour octet matrix elements from Braaten et al. (2001) predict:

- $BR(\text{singlet}) = 8.2 \times 10^{-4}$
- $BR(\text{octet}) = 4 - 9 \times 10^{-3}$  — “really big!”

Large uncertainties. In particular, use of velocity scaling for MEs.  
RG scaling would make them smaller.

# Radiative decay of charmonium into light mesons

$J/\Psi$ , light mesons both assumed colour-singlet non.-rel. wavefns

- $f_2(1270)$ ,  $f_1(1285)$ : assume  $(u\bar{u} + d\bar{d})/\sqrt{2}$  — no  $s\bar{s}$  mixing
- $f_0(980)$ : pure p-wave dominated  $s\bar{s}$
- Justification not clear

1-lp perturbation theory: also unjustified

Wavefns at origin from potential model calculations

$J/\psi \rightarrow \gamma X$ :	$f_0(980)$	$f_1(1285)$	$f_2(1270)$	$f'_1(1420)$	$f'_2(1525)$
$BR_{th} \times 10^4$	1.6	7.0	8.7	1.8	2.0
$BR_{ex} \times 10^4$		$6.1 \pm 0.8$	$13.8 \pm 1.4$	$7.9 \pm 1.3$	$4.5^{+0.7}_{-0.4}$

- $f_0(1710)$ : too small for  $s\bar{s}$ ,  $\frac{u\bar{u}+d\bar{d}}{\sqrt{2}} \Rightarrow$  “glueball mixing”

Also predictions for  $BR(J/\Psi \rightarrow \eta(\eta')\gamma)$ ,  $BR(\chi_{cJ} \rightarrow \rho(\omega, \phi)\gamma)$ .

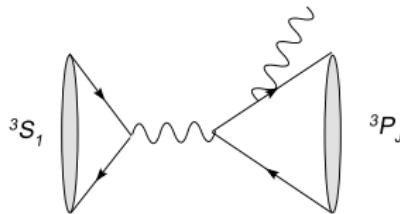
# Radiative decays of bottomonia

Use similar techniques, give predictions for

- $\Upsilon \rightarrow \chi_{cJ}\gamma, \Upsilon \rightarrow f_J\gamma$
- $\chi_{bJ} \rightarrow J/\Psi\gamma, \chi_{bJ} \rightarrow \rho(\omega, \phi)\gamma$
- $\eta_b \rightarrow J/\Psi\gamma$

Include QED contributions

- in many cases significant/dominant e.g.
  - $\chi_{bJ} \rightarrow \rho(\omega, \phi)\gamma$
  - $\Upsilon \rightarrow \chi_{c1}\gamma$



Classical solution → potential model

- Consider only zero energy solns
  - Find classical confining potential
    - Consider only lowest multipole  $\ell = 0$  (sph. symm.)
    - Fit parms from qu. lat. singlet potl.  $0.2 \text{ fm} \leq r \leq 1.0 \text{ fm}$
- Potential model calculation: solve Schr. Eqn.
  - Obtain masses, wavefn. & derivs at origin

Systematic errors — difficult to quantify

- classical,  $\ell = 0$ , static approximations
- quenched lattice parameters.  ${}^3S_1 - {}^1S_0$  HFS/MeV:

PDG	O & C	qu. LQCD	dyn. LQCD
(Follana, hep-lat/0610092)			
117	22	38 – 73	111 (5)

Spectrum of states: within 3% (potential model robustness)

state	identification	$\Delta M/\text{MeV}$
$X(3872)$	$2^3P_1$	+67
$X, Y, Z(3940)$	$2^3P_0, 2^1P_1, 1^3F_2$	-111, +16, +3
$Y(4260)$	$3^3S_1$	-96

Leptonic widths:  $1S$  sets scale

- $2S$ : good. Very robust in potential models
- $3S, 3D$ : 2.4, 2.3 compared to expt. 0.83, 0.86
  - O & C worst of all potential models
- $J^{PC} = 1^{--}$  results improved by mixing

Two photon widths: ( $\Gamma_{\gamma\gamma}(\eta_c)$  as input)

- $\Gamma_{\gamma\gamma}(\chi_{c0}) = 4.06 \text{ keV}$  vs. PDG: 2.87 (40) keV

Gluonic widths: ( $\Gamma_{gg}(\eta_c)$  as input)

- Very poor

# Summary

Exciting time for lattice simulations:

- Relativistic charmonium coming on line
- New techniques for photon widths, radiative transitions
- $\Upsilon$  leptonic width test for LNRQCD
- Moving NRQCD for improved form factors at small  $Q^2$

Models still very relevant:

- provide insight into *mechanisms* e.g. colour octet

Together with perturbative calculations offer:

- Increased predictive power
- Mutual constraints, parameters and mechanism suggestions

We must all talk to each other!

Conclude with a feature on charm (6)<sup>1</sup>

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<sup>1</sup>The Daily Telegraph, 16.iv.07, 12 across.

Conclude with a feature on charm (6)<sup>1</sup>

Endear

(and that is what I shall do)

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<sup>1</sup>The Daily Telegraph, 16.iv.07, 12 across.