DIS 07 16-20 April 2007 Munchen

Spin Structure Function g_1 at small x and arbitrary Q^2: Total Resummation of Leading Logarithms vs DGLAP

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talk based on results obtained in collaboration with M. Greco and S.I. Troyan

Spin-dependent part of $W_{\mu\nu}$ is parameterized by two structure functions:

$$W_{\mu\nu}^{spin} = \frac{m}{pq} i\varepsilon_{\mu\nu\lambda\rho} q_{\lambda} \left[S_{\rho} g_1(x, Q^2) + \left(S_{\rho} - \frac{Sq}{pq} p_{\rho} \right) g_2(x, Q^2) \right]$$

where m, p and S are the hadron mass, momentum and spin; q is the virtual photon momentum ($Q^2 = -q^2 > 0$). Again both functions depend on Q^2 and $x = Q^2/2pq$, $0 < x < 1$. They measure asymmetries

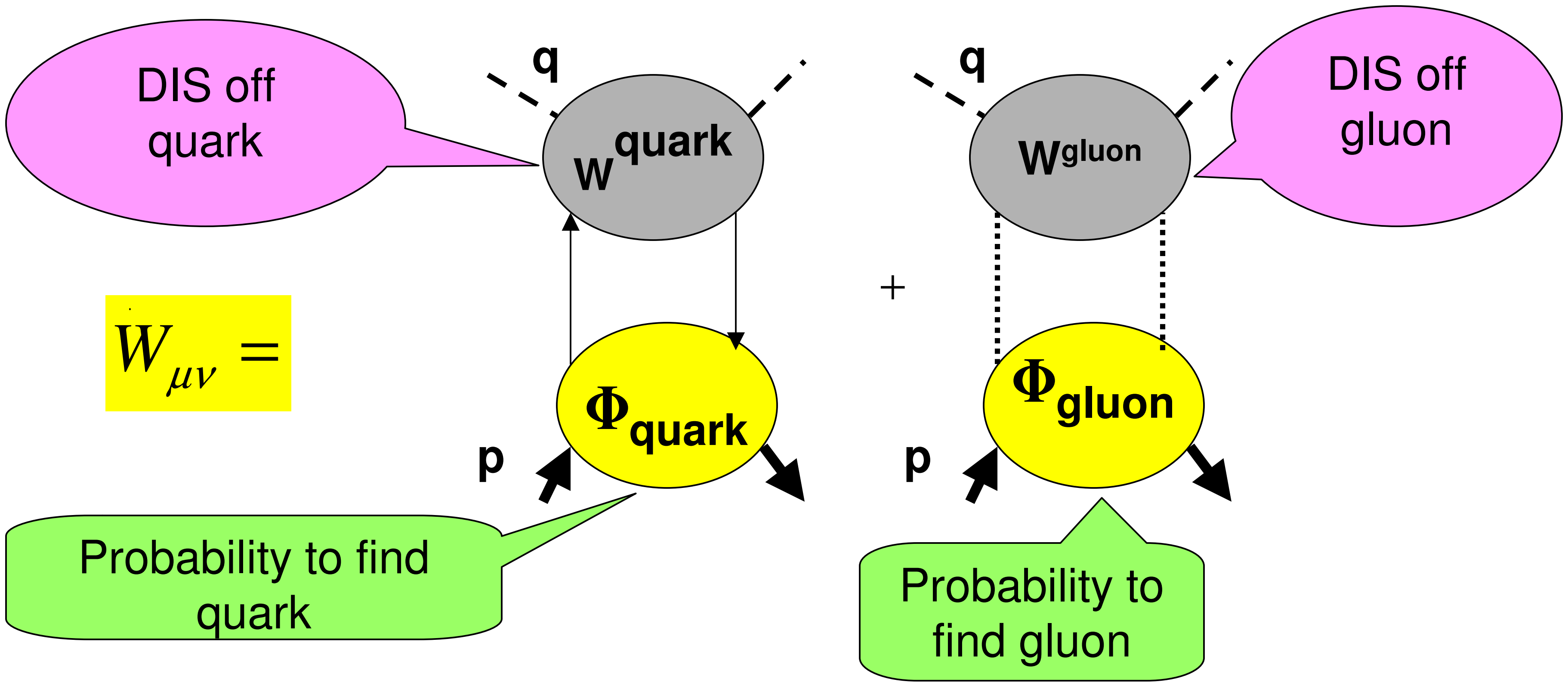
g_1 measures the longitudinal spin flip

$$g_1 \propto \sigma_{L\uparrow\uparrow} - \sigma_{L\uparrow\downarrow}$$

$g_1 + g_2$ measures the transverse spin flip

$$g_1 + g_2 \propto \sigma_{T\uparrow\uparrow} - \sigma_{T\uparrow\downarrow}$$

FACTORISATON: $W_{\mu\nu}$ **is a convolution of the the partonic tensor and probabilities to find a polarized parton (quark or gluon) in the hadron :**



DIS off quark and gluon can be studied with perturbative QCD, with calculating involved Feynman graphs.

Probabilities, Φ_{quark} and Φ_{gluon} involve non-perturbaive QCD. There is no a regular analytic way to calculate them. Usually they are defined from experimental data at large x and small Q^2, they are called the initial quark and gluon densities and are denoted δq and δg .

So, the conventional form of the hadronic tensor is:

$$W_{\mu\nu} = W_{\mu\nu}^{quark} \otimes \delta q + W_{\mu\nu}^{gluon} \otimes \delta g$$

Initial quark density

Initial gluon density

DIS off the quark,

DIS off the gluon

are calculated with Pert QCD

Standard Approach

includes the DGLAP Evolution Equations and the Standard Fits for initial parton densities

DGLAP Evolution Equations

Altarelli-Parisi,Gribov-Lipatov,
Dokshitzer

$$g_1(x,Q^2) = C_q(x/y) \otimes \Delta q(y,Q^2) + C_g(x/y) \otimes \Delta g(y,Q^2)$$

Evolved quark distribution

Evolved gluon distribution

Coefficient function

Coefficient function

DGLAP evolution equations

$$\frac{d\Delta q}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{qg} \otimes \Delta g$$

$$\frac{d\Delta g}{d\ln Q^2} = \frac{\alpha_s}{2\pi} P_{gq} \otimes \Delta q + \frac{\alpha_s}{2\pi} P_{gg} \otimes \Delta g$$

$P_{qq},\ P_{qg},\ P_{gq}, P_{gg}$ are splitting functions

Mellin transformation of the splitting functions
= anomalous dimensions

The Standard Approach includes the DGLAP Evolution Equations and the Standard Fits for initial parton densities. One can say that SA combines Science and Art

SCIENCE

LO splitting functions	Ahmed-Ross, Altarelli-Parisi, Sasaki,
NLO splitting functions	Floratos, Ross, Sachradja, Gonzale- Arroyo, Lopes, Yandurain, Kounnas, Lacaze, Curci, Furmanski, Petronzio, Zijlstra, Mertig, van Neerven, Vogelsang
Coefficient functions $C^{(1)}_k$, $C^{(2)}_k$	Bardeen, Buras, Muta, Duke, Altarelli, Kodaira, Efremov, Anselmino, Leader, Zijlstra, van Neerven

ART

= the art of composing the fits for initial parton densities

Altarelli-Ball-Forte-Ridolfi, Blumlein- Botcher, Leader- Sidorov-Stamenov, Hirai et al

There are different fits for initial parton densities. For example,

$$\delta q = N x^{-\alpha} [(1 - x)^{\beta} (1 + \gamma x^{\delta})]$$

$$\delta q = N [\ln^{\alpha}(1/x) + \gamma x \ln^{\beta}(1/x)]$$

Altarelli-Ball-Forte-Ridolfi,

Parameters $N, \alpha, \beta, \gamma, \delta$ should be fixed from experiment

This combination of Science and Art works well at large and small x, though strictly speaking, DGLAP is not supposed to work at the small- x region:

For example, for the simplest case: the non-singlet $\mathbf{g_1}$

Initial quark density

$$g_1^{NS}(x,Q^2) = (e_q^2/2) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i}\left(\frac{1}{x}\right)^{\omega} C(\omega)\,\delta q(\omega) \exp\left[\int_{\mu^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\,\gamma(\omega,\alpha_s(k_\perp^2))\right]$$

Coefficient function

Anomalous dimension

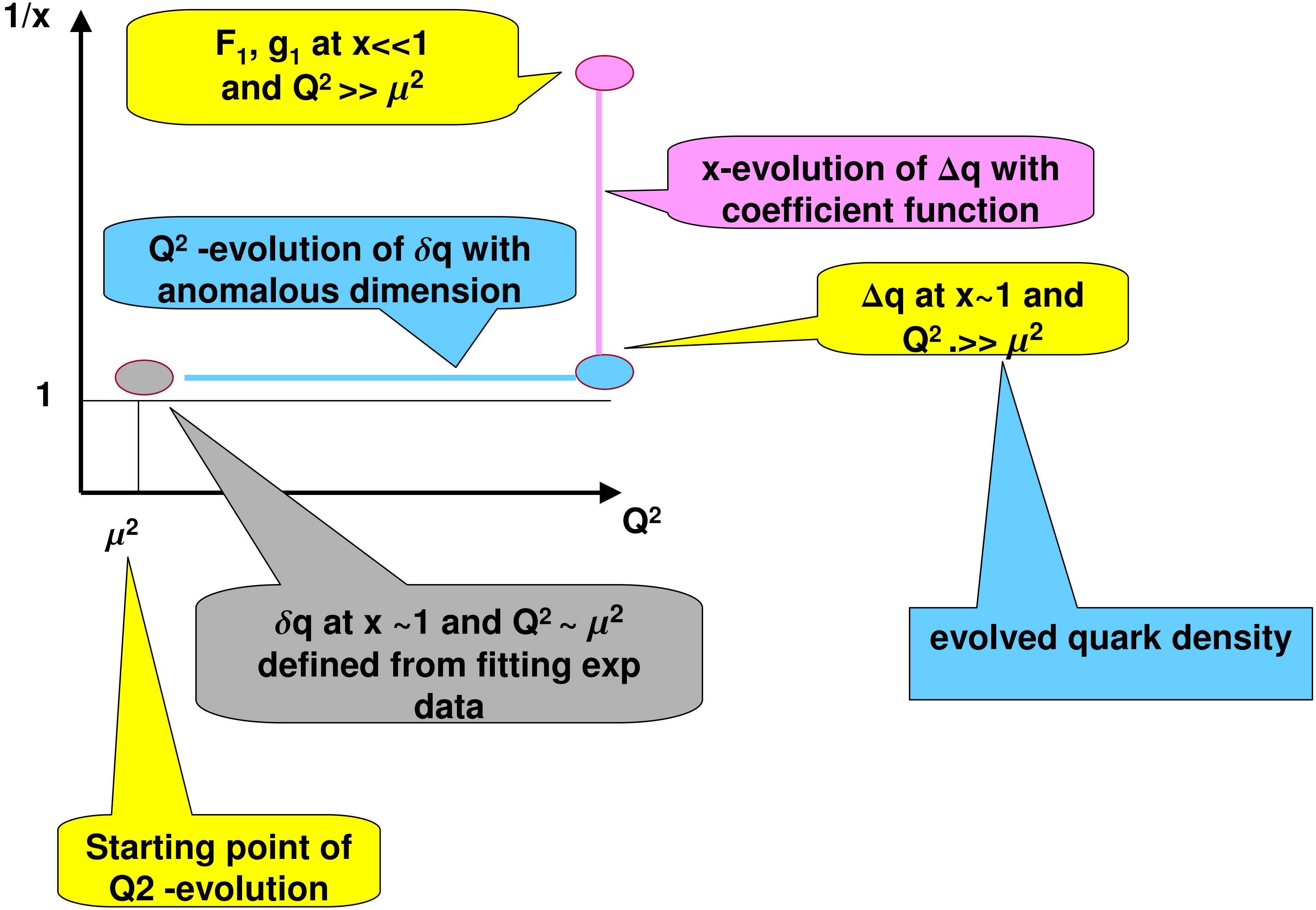
Non-Pert QCD

Pert QCD

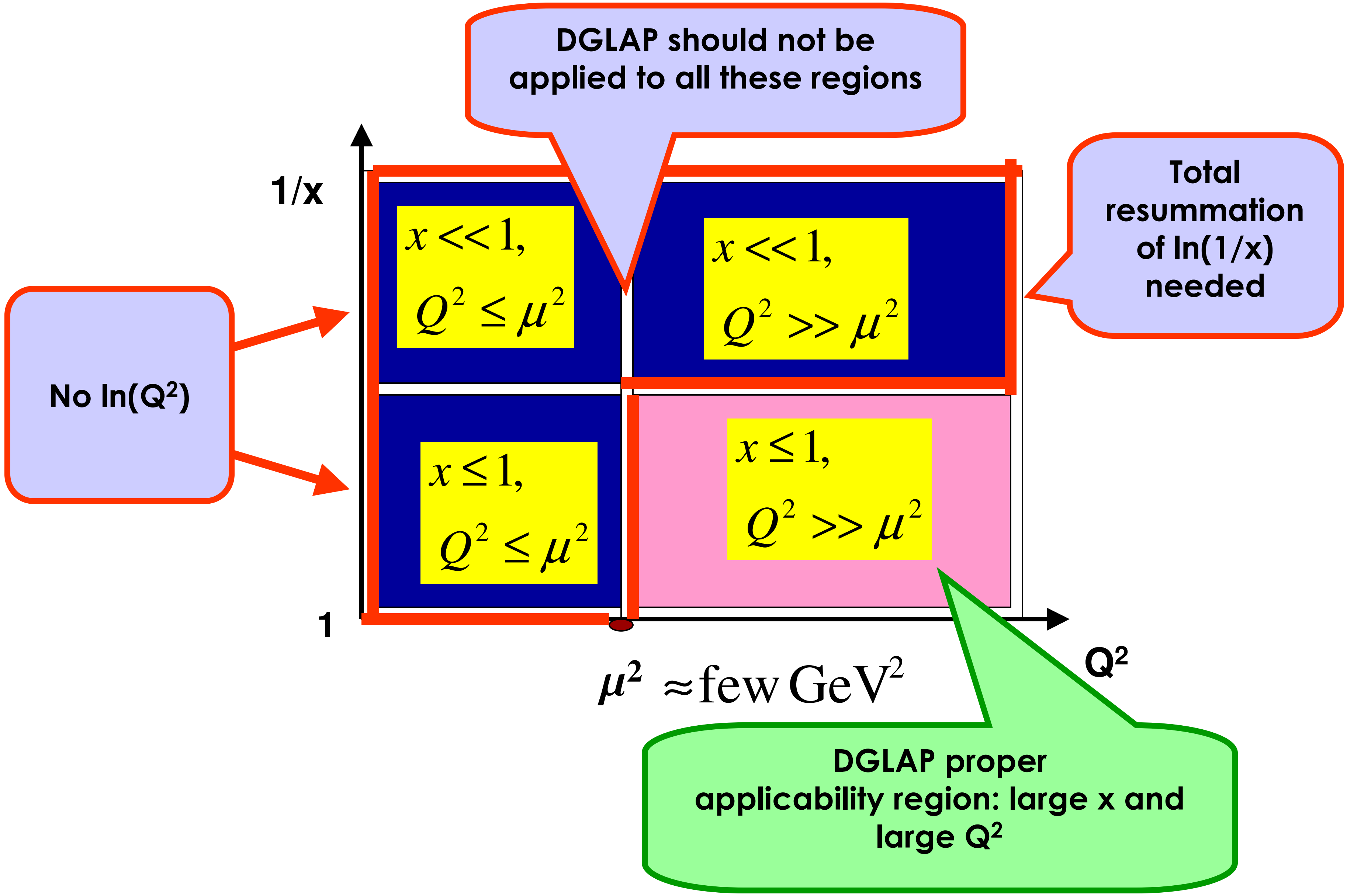
In DGLAP, coefficient functions and anomalous dimensions are known with LO and NLO accuracy, often at integer ω = n

$$C(\omega) = \overbrace{1}^{\text{LO}} + \overbrace{(\alpha_s(Q^2)/2\pi)\, C^{(1)}(\omega)}^{\text{NLO}} + \ldots$$

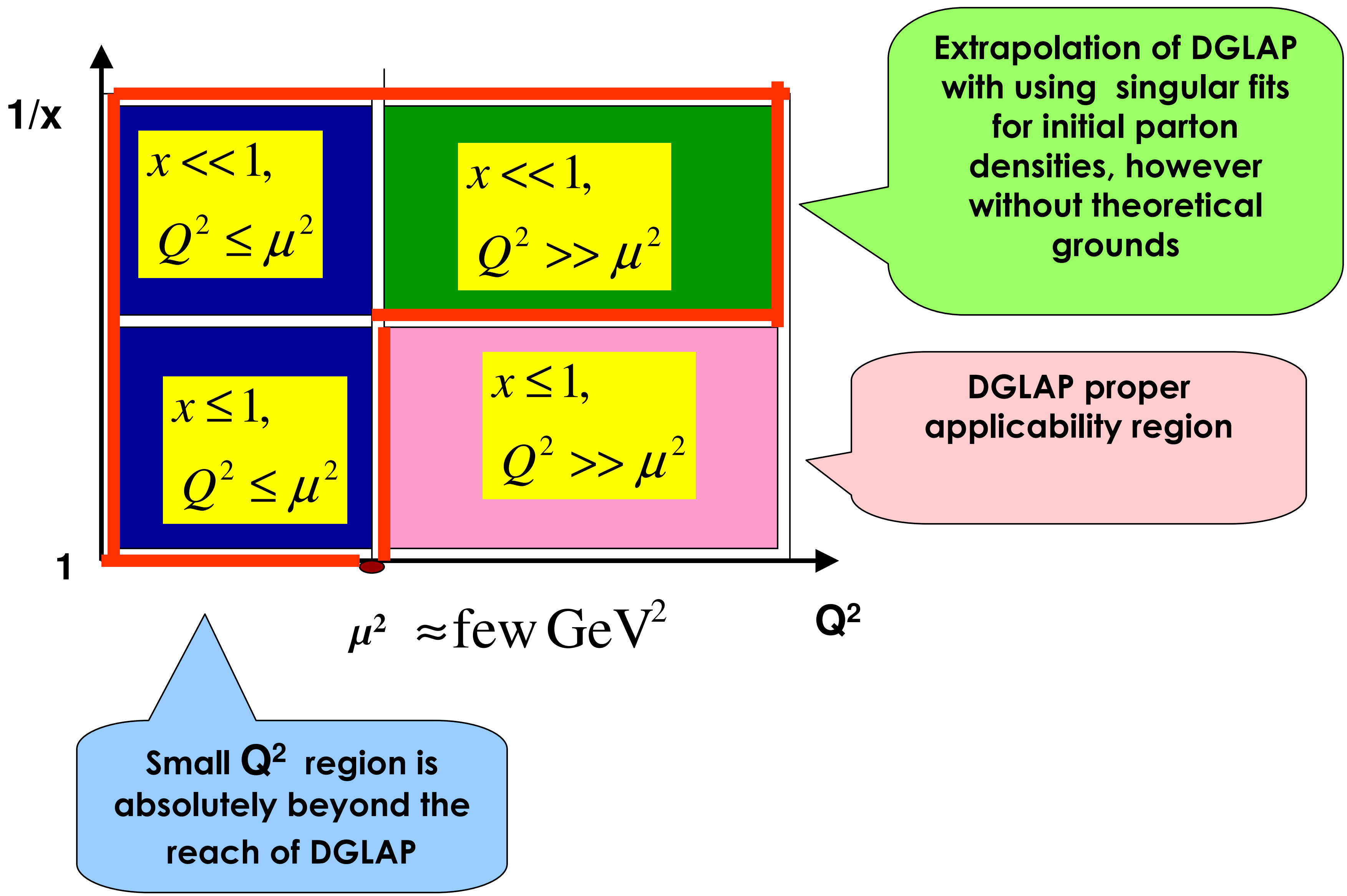
$$\gamma(\omega) = \underbrace{(\alpha_s(Q^2)/4\pi)\gamma^{(0)}(\omega)}_{\text{LO}} + \underbrace{(\alpha_s(Q^2)/2\pi)^2 \gamma^{(1)}(\omega)}_{\text{NLO}} + \ldots$$

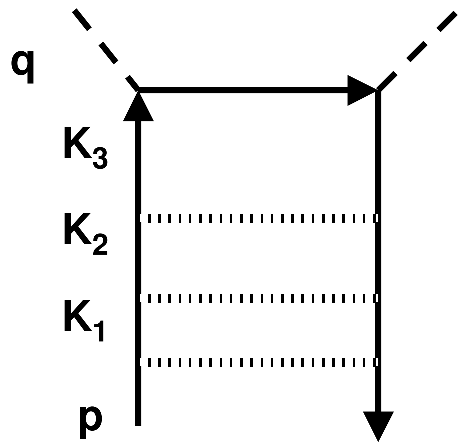

1/x
F_1, g_1 at x<<1 and $Q^2 >> \mu^2$
x-evolution of Δq with coefficient function
Q^2 -evolution of δq with anomalous dimension
Δq at x~1 and Q^2 .>> μ^2
1
Q^2
μ^2
δq at x ~1 and $Q^2 \sim \mu^2$ defined from fitting exp data
evolved quark density
Starting point of Q2 -evolution

From theoretical grounds:



In practice:


1/x
1
$x << 1,$
$Q^2 \le \mu^2$
$x << 1,$
$Q^2 >> \mu^2$
$x \le 1,$
$Q^2 \le \mu^2$
$x \le 1,$
$Q^2 >> \mu^2$
$\mu^2 \approx \text{few GeV}^2$
Q^2
Extrapolation of DGLAP with using singular fits for initial parton densities, however without theoretical grounds
DGLAP proper applicability region
Small Q^2 region is absolutely beyond the reach of DGLAP



DGLAP –ordering:

$$\mu^2 < \mathrm{k}_{1\perp}^2 < \mathrm{k}_{2\perp}^2 < \mathrm{k}_{3\perp}^2 < Q^2$$

good approximation for large **x** when logs of **x** can be neglected. At **x << 1** the ordering has to be lifted. It makes possible to account for leading logs of x

DL contributions

SL contributions

$$(\alpha_s \ln^2(1/x))^k,$$

$$(\alpha_s \ln (1/x)\ln(Q^2/\mu^2))^k \qquad (\alpha_s \ln(1/x))^k,$$

$$k = 1,2..\infty$$

NB: Lifting DGLAP –ordering $\longrightarrow$ **infrared divergences in gluon ladders and non-ladder quark and gluon graphs**

NEXT IMPORTANT STEP:

What is appropriate parameterization of α_s at small x ?

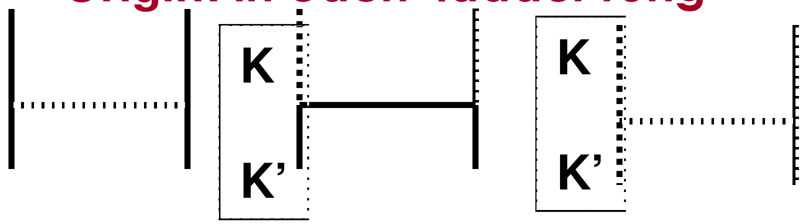
Standard parameterization $\alpha_s = \alpha_s(Q^2)$

DGLAP-parameterization

Arguments in favor of the Q^2- parameterization:

Amati-Bassetto-Ciafaloni-Marchesini - Veneziano; Dokshitzer-Shirkov

Origin: in each ladder rung



$$\alpha_s = \alpha_s(k_\perp^2)$$

DGLAP-parameterization

Ermolaev-Greco-Troyan

However, such a parameterization is good for large x only. At small **x** :

$$\alpha_s = \alpha_s((k-k')^2) \approx \alpha_s((k_\perp^2 + k_\perp^{'2})/x) \approx \alpha_s(k_\perp^2)$$

time-like argument

Participates in the Mellin transform

When DGLAP-ordering is used and x ~1

DGLAP -parametrization

$$\alpha_s = \alpha_s(Q^2)$$

space-like argument,

no Mellin transform

Example: quark ladder in the Born approximation No Q^2 at all

$$M_B = \alpha_s(s)\frac{s}{s-\mu^2+i\varepsilon} \to \frac{A(\omega)}{\omega}$$

The coupling participates in the Mellin transform

$$\alpha_s(s) \to A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty d\rho \frac{\exp(-\omega\rho)}{(\rho+\eta)^2+\pi^2}\right]$$

where

$$\eta = \ln(\mu^2 / \Lambda^2_{QCD})$$

instead of DGLAP-parameterization

$$\alpha_s = \alpha_s(Q^2)$$

It is valid when

$$\mu^2 > \Lambda^2_{QCD}$$

This restriction guarantees the applicability of Pert QCD

Expression for the non-singlet g_1 at large Q^2: $Q^2 >> 1$ GeV^2

Initial quark density

$$g_1^{NS} = \frac{e_q^2}{2}\int_{-i\infty}^{i\infty}\frac{d\omega}{2\pi i}\left(\frac{1}{\mathrm{x}}\right)^{\omega}\left(\frac{\omega}{\omega - \mathrm{H}(\omega)}\right)\delta \mathrm{q}(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H(\omega)}$$

Coefficient function

Anomalous dimension

New coefficient function and anomalous dimension sum up leading logarithms to all orders in α_s

Compare our non-singlet anomalous dimension to the LO DGLAP one:

expand C and H into series in $1/\omega$

$$H = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] + \ldots$$

coincide, save the treatment of α_s

$$\gamma_{NS}^{\text{LO DGLAP}} = \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n(n+1)} + \frac{3}{2} - S_2(n)\right] \approx \frac{\alpha_s(Q^2)C_F}{2\pi}\left[\frac{1}{n} + \frac{1}{2} + O(n)\right]$$

when n < 1

where

$$S_k(n) = \sum_{j=1}^{n}\frac{1}{j^k}$$

small/large x ⟷ **small/large n**

Compare our coefficient function and the NLO DGLAP one

$$C = \frac{\omega}{\omega - H(\omega)} = 1 + \frac{A(\omega) C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right] + \ldots.$$

coincide, save the treatment of α_s

LO

NLO

$$C_{NS}^{\mathrm{DGLAP}} = 1 + \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + \frac{1}{2n+1} - \frac{9}{2} + \left(\frac{3}{2} - \frac{1}{n(1+n)}\right) S_1(n) + S_1^2(n) - S_2(n)\right]$$

when n < 1

$$\approx \frac{\alpha_s(Q^2) C_F}{2\pi}\left[\frac{1}{n^2} + \frac{1}{2n} + O(n)\right]$$

Expression for the singlet g_1 at large Q^2:

$$g_1^S = \frac{<e_q^2>}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega}$$

$$\left[\left(C_q^{(+)} \delta q + C_q^{(+)} \delta g \right) \left(\frac{Q^2}{\mu^2} \right)^{\Omega^{(+)}} + \left(C_q^{(-)} \delta q + C_q^{(-)} \delta g \right) \left(\frac{Q^2}{\mu^2} \right)^{\Omega^{(-)}} \right]$$

Large Q^2 means

$$Q^2 > \mu^2; \ \mu \approx 5 \text{ GeV}$$

$$\Omega^{(+)} > \Omega^{(-)}$$

Small –x asymptotics of g_1: when $x \to 0$, the saddle-point method leads to

$$g_1^{NS} \sim \frac{e_q^2}{2}(1/x)^{\Delta_{NS}}\left(Q^2/\mu^2\right)^{\Delta_{NS}/2}\delta q$$

Nonsinglet intercept $\Delta_{NS} = 0.42$

At large x, g_1^{NS} and g_1^{S} are positive

$\delta q > 0$ → $g_1^{NS} > 0$ **In the whole range of x at any Q^2**

Asymptotics of the singlet g_1 are more involved

$$g_1^S \sim \frac{\langle e_q^2 \rangle}{2} S(\Delta_S) \left(1/x\right)^{\Delta_S} \left(Q^2/\mu^2\right)^{\Delta_S/2}$$

With intercept $\Delta_S = 0.86$

and $S(\Delta_S) = -\delta q - 0.064\ \delta g$

Interplay between the quark and gluon densities can lead to different sign of g_1 singlet at $x \ll 1$

Values of the intercepts perfectly agree with results of several groups who fitted experimental data.

non-singlet intercept	Soffer-Teryaev, Kataev-Sidorov-Parente, Kotikov-Lipatov-Parente-Peshekhonov-Krivokhijine-Zotov,
singlet intercept	Kochelev-Lipka-Vento-Novak-Vinnikov

Anatomy of the singlet intercept

A. Graphs with gluons only: $\Delta_S = 1.1$ → violates unitarity → similar to LO BFKL

B. All graphs $\Delta_S = 0.86$ → No violation of unitarity

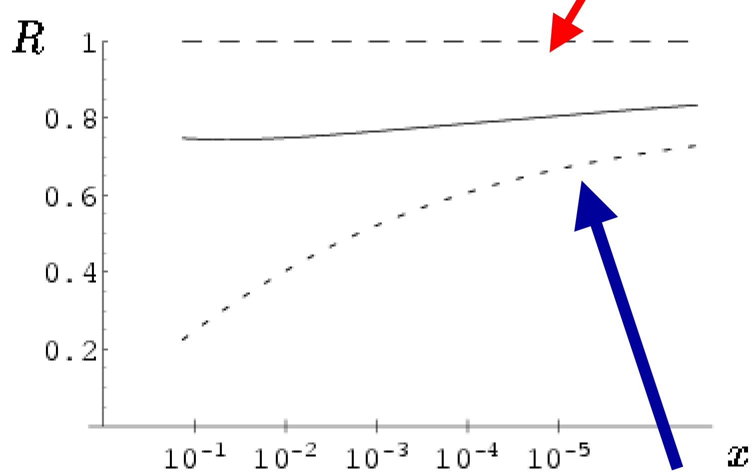
However, using the asymptotics is not reliable at available x:

$$g_1^{AS} = \Pi(\Delta_{NS})\frac{e_q^2}{2}(1/x)^{\Delta_{NS}}\left(Q^2/\mu^2\right)^{\Delta_{NS}/2}$$

Let us compare g_1^{NS} to its small-x asymptotics:

$R^{AS} = g_1^{AS}/g_1$ without Π_{NS}

Q² = 20 GeV² at the plots, though no big difference at other Q²



g_1^{AS}/g_1, including Π_{NS}

Conclusion: using asymptotics is reliable for $x<10^{-5}$

Now:
Compare our results with DGLAP without using asymptotic formulae

Comparison depends on the assumed shape of initial parton densities.

The simplest option: use the bare quark input

$$\delta q(x) = \delta(\mathrm{x}) \quad \Longleftrightarrow \quad \delta q(\omega) = 1$$

in x- space

in Mellin space

Numerical comparison shows that the impact of the total resummation of logs of x becomes quite sizable at x = 0.05 approx.

Hence, DGLAP cannot work well at x < 0.05.
However, in practice DGLAP works at x < 0.05

Puzzle

Solution to the puzzle

In order to understand the reason for success of DGLAP at small **x**, let us consider in more detail

standard fits for initial parton densities.

$$\delta q(x) = N\, x^{-\alpha} [(1+\gamma x^{\delta})(1-x)^{\beta}]$$

Altarelli-Ball-Forte-Ridolfi

normalization

singular factor

regular factors

parameters $\alpha \approx 0.58,\ \beta \approx 2.7, \gamma \approx 34.3,\ \delta \approx 0.75$

are fixed from fitting experimental data at large **x**

In the Mellin space this fit is

$$\delta q(\omega) = N[(\omega-\alpha)^{-1} + \sum_{k=1}^{\infty} c_k((\omega+k-\alpha)^{-1} + \gamma(\omega+k+1-\alpha)^{-1})]$$

Leading pole

α=0.58 >0

Non-leading poles

-k +α<0

So, actually the small-x DGLAP asymptotics of g_1 is

$$g_1^{DGLAP} \sim (1/\mathrm{x})^{\alpha}(\ln Q^2)^{\gamma(\alpha)}$$

Regge behavior

Instead of the well-known DGLAP asymptotics

$$g_1^{DGLAP} \sim \exp[\ln(1/\mathrm{x})\ln\ln(\mathrm{Q}^2/\Lambda^2_{\mathrm{QCD}})]^{1/2}$$

Comparison it to our asymptotics: both asymptotics are of the Regge type

$$g_1^{DGLAP} \sim (1/x)^{\alpha} (\ln Q^2)^{\gamma(\alpha)}$$

Phenomenological intercept

$$g_1 \sim (1/x)^{\Delta} \left(Q^2/\mu^2\right)^{\Delta/2}$$

calculations

CONCLUSION: the singular factors in the DGLAP fits mimic the total resummation of ln(1/x) .

MISCONCEPTION: the total resummation is not relevant at available x

ACTUALLY: the resummation has always been accounted for through the standard fits, however without realizing it

MISCONCEPTION: fits for δq are singular but defined and large x, then convoluting them with coefficient functions weakens the singularity

$$C(x,y) \otimes \delta q(y) = \Delta q(x)$$

ACTUALLY: The both distributions are singular equally

Structure of DGLAP fit once again:

$$\delta q(x) = N\, \mathrm{x}^{-\alpha}[(1+\gamma \mathrm{x}^{\delta})(1-x)^{\beta}]$$

Can be dropped when ln(x) are resummed

x-dependence is weak at x<<1 and can be dropped

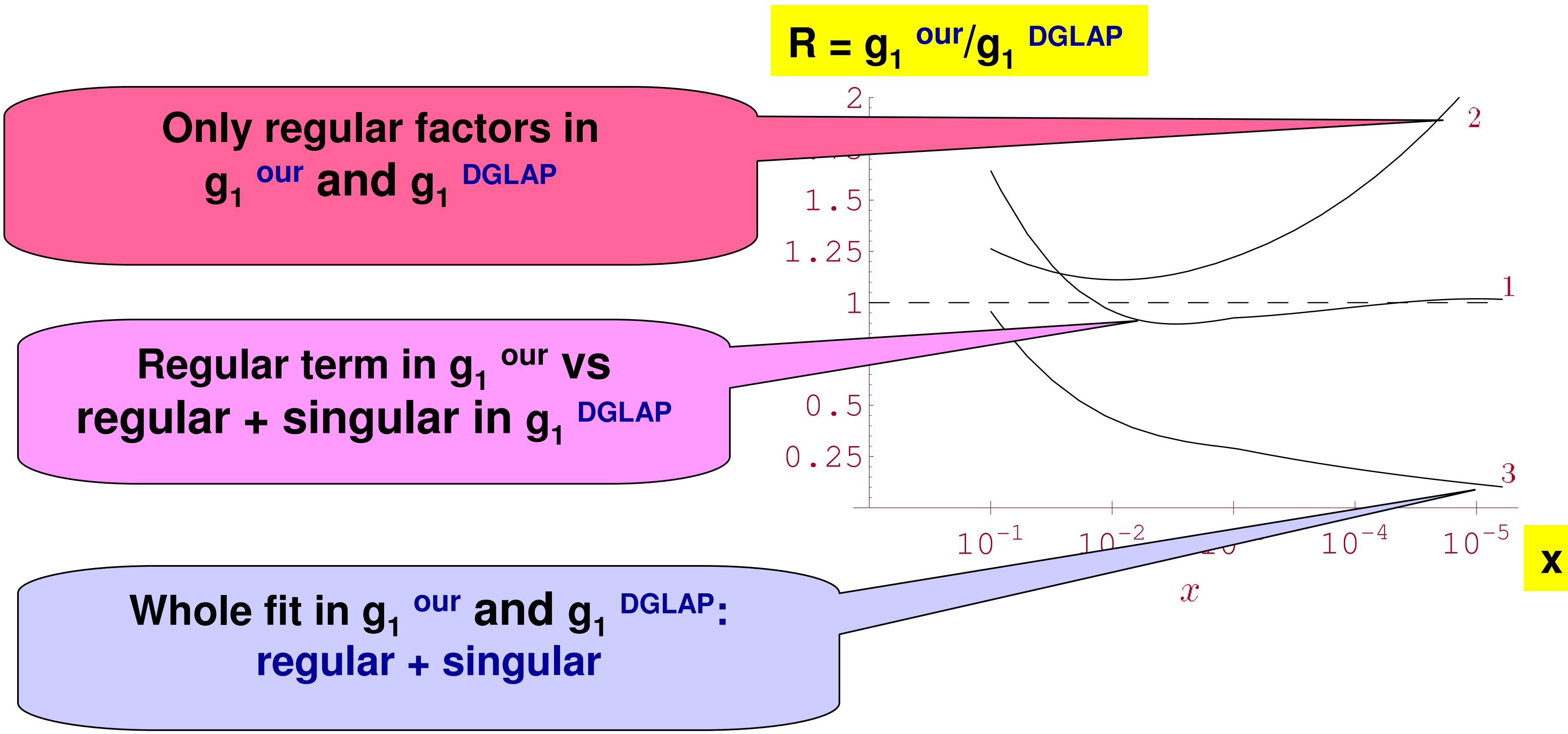
Therefore at x << 1

$$\delta q(x) \approx \mathrm{N}(1+\mathrm{ax})$$

MISCONCEPTON: fits are complicated because they mimic unknown phenomena from Non-Pert QCD

ACTUALLY: they mostly mimic Pert QCD; not much of Non-Pert QCD is at small x

Numerical comparison of DGLAP with our approach at small but finite x, using the same DGLAP fit for initial quark density.



Comparison between DGLAP and our approach at large x

DGLAP	our approach
Good at large x because includes exact two-loop calculations but bad at small x as lacks the total resummaion of ln(x)	Good at small x , includes the total resummaion of ln(x) but bad at large x because neglects some contributions essential in this region

WAY OUT – synthesis of our approach and DGLAP

1. Expand our formulae for coefficient functions and anomalous dimensions into series in the QCD coupling
2. Replace the first- and second- loop terms of the expansion by corresponding DGLAP –expressions

New formulae are equally good at large and small x,
singular fits are not exploited

Our expressions

$$H(\omega) = (1/2)[\omega - (\omega^2 - B(\omega))]^{1/2} \quad C(\omega) = \omega/(\omega - H(\omega))$$

anomalous dimension

coefficient function

First tems of their expansions into the perturbation series

$$H_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega} + \frac{1}{2}\right] \qquad C_1 = \frac{A(\omega)C_F}{2\pi}\left[\frac{1}{\omega^2} + \frac{1}{2\omega}\right]$$

New, "synthetic" formulae:

$$h = H - H_1 + H_{LO\ DGLAP} \quad c = C - C_1 + C_{LO\ DGLAP}$$

New, "synthetic" formulae accumulate all advantages of the both approaches and should equally be good at large and small **x.**

New fits should not involve singular factors

Taken from wwwcompass.cern.ch

COMPASS is a high-energy physics experiment at the Super Proton Synchrotron (SPS) at CERN in Geneva, Switzerland. The purpose of this experiment is the study of hadron structure and hadron spectroscopy with high intensity muon and hadron beams.
On February 1997 the experiment was approved conditionally by CERN and the final Memorandum of Understanding was signed in September 1998. The spectrometer was installed in 1999 - 2000 and was commissioned during a technical run in 2001. Data taking started in summer 2002 and continued until fall 2004. After one year shutdown in 2005, COMPASS will resume data taking in 2006.
Nearly 240 physicists from 11 countries and 28 institutions work in COMPASS

COMPASS operates with small Q^2 ($Q^2 < 3$ GeV^2) and small $x \sim 10^{-3}$
DGLAP cannot be applied here: no logs of Q^2 in this region

To generalize our results to the region of small Q^2 , it is enough to make the shift in our previous formmulae:

$$Q^2 \to Q^2 + \mu^2$$

Infrared cut-off

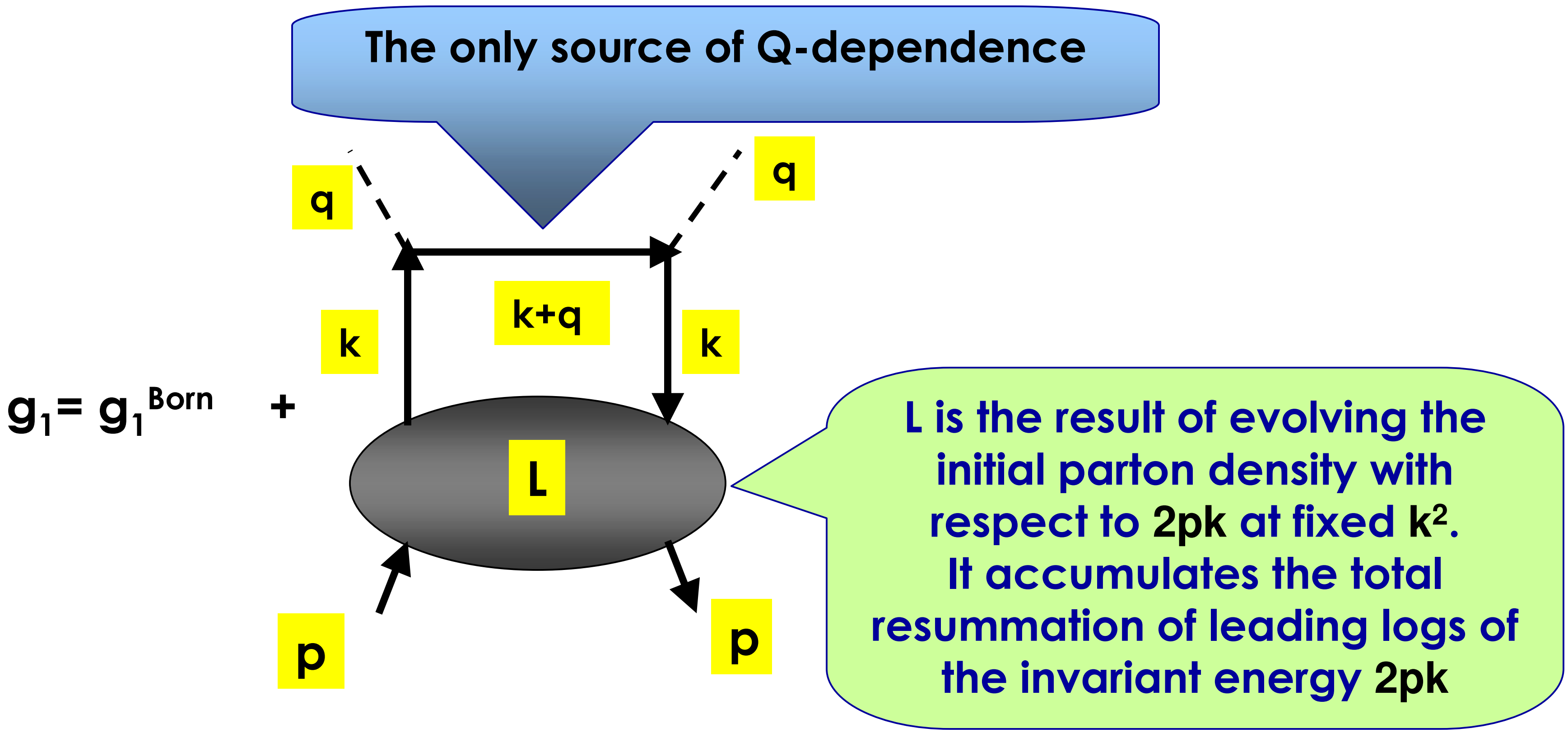
$$Q^2 \to Q^2 + \mu^2 \quad \Rightarrow \quad x \to \bar{x} = (Q^2 + \mu^2)/2pq = x + z$$

Similar to the Nachtmann variable

With the shift, our results describe g_1 at arbitrary Q^2

Proof of the shift

Obviously, g_1 obeys the Bete-Salpeter equation:



$$g_1 = g_1^{Born} + \int \frac{d^4k\, k_\perp^2}{(k^2 - m_q^2)^2} \delta(k^2 + 2qk - Q^2 - m_q^2) L(2pk, k^2)$$

In order to regulate IR singularities in L we introduce the IR cut-off μ Into all diverging (vertical gluon) propagators. We choose $\mu > m_q$ then drop m_q and insert μ into all vertical propagators.

Sudakov variables:

$$k = \alpha q + (\beta + x\alpha) p + k_\perp \approx \alpha q + \beta p + k_\perp$$

integrated out, using δ-function

The leading contribution comes from the region $w > k_\perp^2 > Q^2, w\alpha > k_\perp^2$ therefore

$$g_1 = g_1^{Born} + \int_{Q^2}^{w} \frac{dk_\perp^2}{k_\perp^2 + \mu^2} L(w, k_\perp^2 + \mu^2) = g_1^{Born} + \int_{Q^2+\mu^2}^{w+\mu^2} \frac{dt}{t} L(w, t)$$

Proves the shift

It leads to new expressions: non-singlet g_1 at small Q^2

$$z = \frac{\mu^2}{2pq} >> x = \frac{Q^2}{2pq}$$

$$g_1^{NS} = \frac{e_q^2}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z + x} \right)^{\omega} \left(\frac{\omega}{\omega - H(\omega)} \right) \delta q(\omega) \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{H(\omega)}$$

weak x -dependence

Anomalous dimension

weak Q^2 -dependence

Coefficient function

Initial quark density

Singlet g_1 at small Q^2

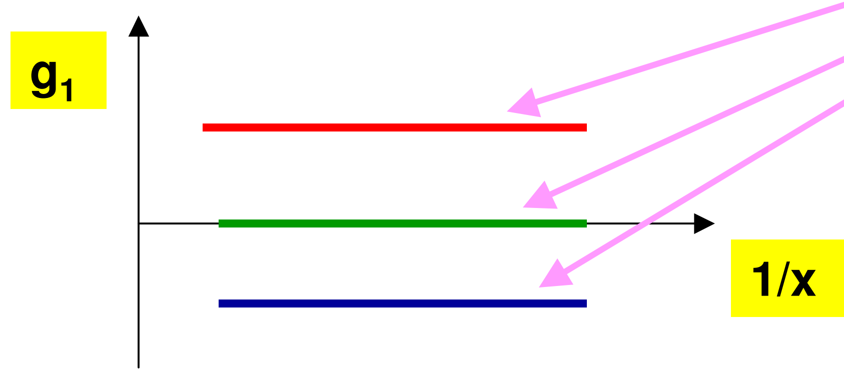
$$z = \frac{\mu^2}{2pq}, \qquad x = \frac{Q^2}{2pq}$$

$$g_1^S = \frac{<e_q^2>}{2} \int \frac{d\omega}{2\pi i} \left(\frac{1}{z+x} \right)^{\omega} \left[C_q \delta q + C_g \delta g \right]$$

$$C_g = C_g^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_g^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

$$C_q = C_q^{(+)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(+)}} + C_q^{(-)} \left(\frac{\mu^2 + Q^2}{\mu^2} \right)^{\Omega^{(-)}}$$

when $Q^2 \ll \mu^2$ both **x**- and **Q^2**- dependences are flat, even for **x<<1**.



Location of the line is determined by the **z**-dependence

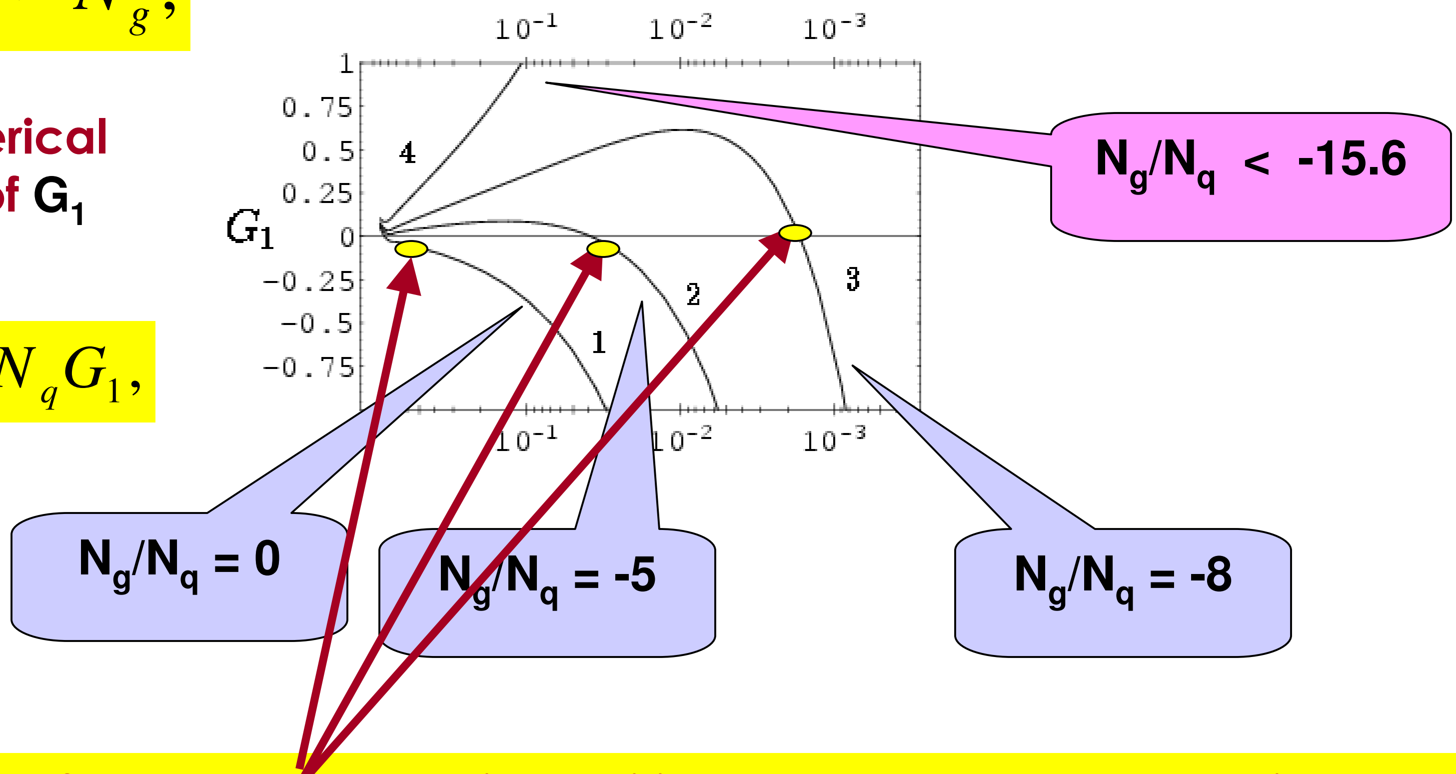
$$g_1(z) = \left(\frac{e_q^2}{2}\right) \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \left(\frac{1}{z}\right)^{\omega} \left[C_q(\omega)\delta q + C_g(\omega)\delta g\right]$$

Approximating

$$\delta q \approx N_q, \delta g \approx N_g,$$

perform numerical calculations of G_1

$$g_1 = (e_q^2/2) N_q G_1,$$



Position of the turning point is sensitive to N_g/N_q, so the experimental detection of it will allow to estimate Ng/Nq

Power Corrections to non-singlet g_1

Leading twist contribution

mass scale: $Q^2 > M^2$

$$g_1(x,Q^2) = g_1^{LT}(x,Q^2)\left[1 + \sum_k C_k \left(\frac{M^2}{Q^2}\right)^k\right]$$

Power corrections

PC are supposed to come from higher twists.
No satisfactory theory
is known for the higher twists

Standard way of obtaining PC from experimental data at small x:

Leader-Stamenov- Sidorov

Compare experimental data to predictions of the Standard Approach and assign the discrepancy to the impact of PC

$$g_1^{LT} = g_1^{DGLAP}$$

Counter-argument:

DGLAP is unreliable at small x, so confronting experiment to it is not productive

Instead:

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{\mathrm{w}}{\mu^2 + Q^2}\right)^{\omega} \mathrm{C}(\omega)\,\delta\mathrm{q}(\omega)\left(\frac{\mu^2 + Q^2}{\mu^2}\right)^{H(\omega)}$$

where w = 2pq and Q^2 can be large or small, μ = 1 GeV

As μ =1 GeV, at $Q^2 > 1$ GeV^2 expansion into series is

$$g_1^{NS} =$$

$$\frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{w}{Q^2}\right)^{\omega} C(\omega)\,\delta q(\omega)\left(\frac{\mu^2}{Q^2}\right)^{H(\omega)}$$

$$\left[1+\sum_{k=1} T_k(\omega)\left(\frac{\mu^2}{Q^2}\right)^{k}\right]$$

Power corrections

Leading contribution For g_1^{NS}

When $Q^2 < 1$ GeV^2, PC are different:

$$g_1^{NS} = \frac{e_q^2}{2}\int \frac{d\omega}{2\pi i}\left(\frac{w}{\mu^2}\right)^{\omega} C(\omega)\,\delta q(\omega)$$

$$\left[1 + \sum_{k=1} T_k(\omega)\left(\frac{Q^2}{\mu^2}\right)^{k}\right]$$

Power corrections

Leading contribution for g_1^{NS} does not depend on Q^2

These power corrections have perturbative origin and should be accounted in the first place. Only after that one can estimate a genuine impact of higher twist contributions

CONCLUSION

DGLAP is theoretically based for describing DIS at large x and large Q^2

Extrapolating DGLAP into the small-x region involves singular fits for the initial parton densities. Discrepance between DGLAP predictions and experiment is often interpreted as the Power Corrections.

The most natural way to describe g_1 in the small-x region is the total resummation of leading logs of x .

The DGLAP fits for initial parton densities are believed to mimic Non-Pert QCD contributions.
Actually, the singular factors in the fits mimic the total resummation of logs of x, ensuring the steep rise of g_1 at small x and lead to the Regge asymptotics with the phenomenological intercepts.
They should be dropped when the resummation is taken into account, which simplifies the fits.

So in a sense, the resummation has always been used in DGLAP at small x, though inexplicitly, through the fits, and without been aware of it.

Combing the resummation with DGLAP provides the expressions for g_1 good at large and small x and does not involve singular fits.

Expressions for g1 at small x and small Q^2 can be obtained from our results for g1 at large Q2 by the shift Q^2 by $Q^2 + \mu^2$. We predict that g_1 does not depend on x at small Q^2 even at x<<1. Singlet g_1 can be positive, negative or zero in this region, depending on the ratio between the quark and gluon initial densities, g_1

Extrapolating DGLAP into the small-x region leads to incorrect estimates for the role of Higher Twists: a good deal of the Power Corrections is actually of the perturbative origin