Vector Meson production from NLL BFKL

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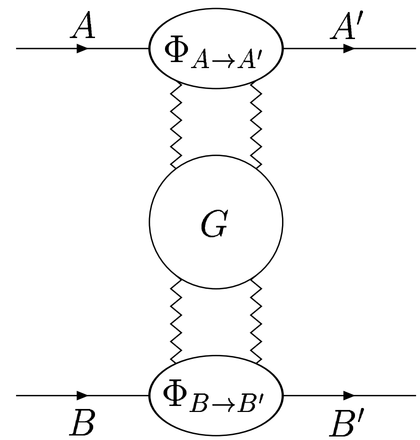
Introduction and motivations

Scattering $A + B \longrightarrow A' + B'$ in the Regge kinematical region $s \to \infty$, t fixed

BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles.



Valid both in
LLA (resummation of all terms $(\alpha_s \ln(s))^n$)
NLA (resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$).

The Green's function is determined through the BFKL equation.
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

The kernel of the BFKL equation is completely known in the NLA for the forward ($t = 0$) case.
[V.S. Fadin, L.N. Lipatov (1998)]
[G. Camici, M. Ciafaloni (1998)]

Impact factors have been calculated in the NLA for

- **colliding partons** **[V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]** **[M. Ciafaloni and G. Rodrigo (2000)]**
- **forward jet production** **[J. Bartels, D. Colferai, G.P. Vacca (2003)]**

Colorless NLA impact factors

- $\gamma^* \to \gamma^*$, **close to completion**
  [J. Bartels, D. Colferai, S. Gieseke, A. Kyrieleis (2002)]
  [V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
  [J. Bartels, A. Kyrieleis (2004)]
- $\gamma^* \to V$, **with** $V = \rho^0$, ω, ϕ, **forward case**
  [D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]

The (forward) $\gamma^*\gamma^* \to VV$ is the first amplitude of physical process completely calculable within perturbative QCD in the NLA.

Theoretical importance:

- **possibility to understand the role and the optimal choice of energy scales in the BFKL approach;**
- **comparison between different approaches (BFKL *vs.* DGLAP, etc.)**

Phenomenological interest:

- **first step toward the application of the BFKL approach to the description of**
- $\gamma^* p \to V p$**, at HERA**
- $\gamma^*\gamma^* \to VV$ **or** $\gamma^*\gamma \to V J/\Psi$**, at high-energy** e^+e^- **and** $e\gamma$ **colliders**

Kinematics and BFKL amplitude

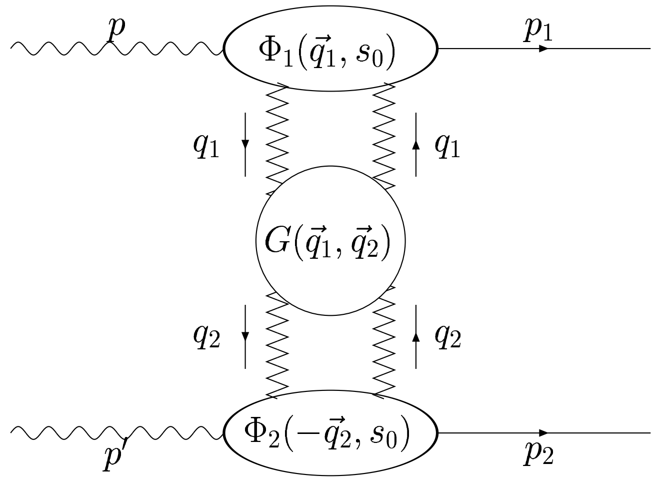
$\gamma^*(p)\gamma^*(p') \to V(p_1)V(p_2)$

$p_1^2 = p_2^2 = 0\ , \quad 2(p_1 p_2) = s$
(p_1 and p_2 Sudakov vectors)

virtual photon momenta:

$p \simeq p_1 - \frac{Q_1^2}{s} p_2\ , \quad p' \simeq p_2 - \frac{Q_2^2}{s} p_1$

$s \gg Q_{1,2}^2 \gg \Lambda_{QCD}^2$



Longitudinally polarized vector mesons are produced by longitudinally polarized photons; other helicity amplitudes power suppressed by $\sim m_\rho / Q_{1,2}$;
[D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]

forward scattering, i.e. zero transverse momenta of the produced mesons.

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2} \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_1(\vec{q}_1, s_0) \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_2(-\vec{q}_2, s_0) \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

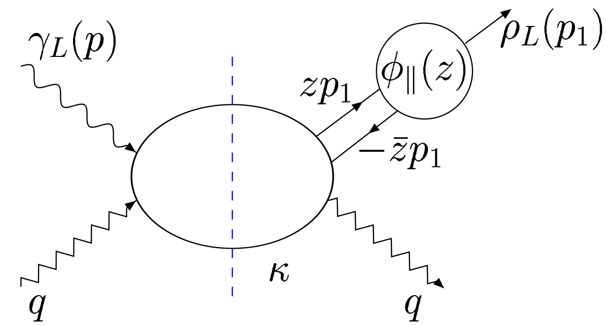
The $\gamma^* \to V$ impact factor

$$\Phi_{1,2}(\vec{q}) = \alpha_s\, D_{1,2} \left[C^{(0)}_{1,2}(\vec{q}^{\,2}) + \bar{\alpha}_s C^{(1)}_{1,2}(\vec{q}^{\,2}) \right]$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}$$

$$e_q \longrightarrow \frac{e}{\sqrt{2}}\,, \quad \frac{e}{3\sqrt{2}}\,, \quad -\frac{e}{3}$$

$$\rho^0\,, \qquad \omega\,, \qquad \phi$$



Leading order (photon virtuality Q^2): $C^{(0)}(\vec{q}^{\,2}) = \int\limits_0^1 dz\, \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + z\bar{z}Q^2} \phi_\parallel(z, \mu_F)$

Next-to-leading order: $C^{(1)}(\vec{q}^{\,2}) = \frac{1}{4N_c} \int\limits_0^1 dz\, \frac{\vec{q}^{\,2}}{\vec{q}^{\,2} + z\bar{z}Q^2} [\tau(z) + \tau(1-z)] \phi_\parallel(z, \mu_F)$

$\tau(z)$ – **see next pages**

$\phi_\parallel(z, \mu_F)$ **is the twist-2 meson distribution amplitude** $\longrightarrow \phi^{as}_\parallel(z) = 6z(1-z)$

$$
\begin{aligned}
\tau(z) = C_F \ln\left(\frac{Q^2}{\mu_F^2}\right) \left[\frac{3}{2} + \frac{(1-2\,z)\,(\alpha-\bar{z}z)}{4\,c\,\bar{z}\,z} \ln\left(\frac{1+2\,c-2\,z}{2\,c-1+2\,z}\right) - \frac{(\alpha+\bar{z}z)}{4\,c\,\bar{z}\,z} \ln\left(\frac{2\,c+1}{2\,c-1}\right)\right. \\
\left. - \frac{(\alpha-\bar{z}z)}{2\,\bar{z}\,z} \ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)\right] - \frac{\beta_0}{2} \ln\left(\frac{Q^2}{\mu_R^2}\right) + n_f \left[-\frac{5}{9} + \frac{\ln\alpha}{3}\right] + C_F \left[-\frac{\ln^2\alpha}{2} - 3\ln\left(\frac{\alpha+\bar{z}z}{z}\right)\right. \\
+ 4\ln^2\left(c - z + \frac{1}{2}\right) - 2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right) + \frac{(\alpha+\bar{z}z)}{2\,\alpha\,\bar{z}\,z}\left(\ln\alpha + 2\,c\ln\left(\frac{2\,c+1}{2\,c-1}\right)\right) \\
- \frac{(1-2\,z)\,(\alpha+\bar{z}z)}{2\,c\,\bar{z}\,z}\ln(1+2\,c-2\,z) - \frac{(\alpha+\bar{z}z)}{2\,\bar{z}\,z}\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right) + \frac{(\alpha+\bar{z}z)}{\bar{z}\,(\alpha+z)}\ln\left(\frac{\alpha+\bar{z}z}{\alpha\,z}\right) \\
+ \frac{(\alpha+\bar{z}z)}{2\,c\,\bar{z}\,z}\left(\frac{\pi^2}{6} + \ln 4\ln\alpha + \frac{1}{2}\ln\left(\frac{1+2\,c}{2\,c-1}\right) + 2\ln c\ln\left(\frac{2\,c-1}{4}\right) - \ln^2(2\,c+1) - \ln^2 c\right. \\
\left. + \frac{1}{4}\ln^2\left(\frac{2\,c+1}{2\,c-1}\right) - 2\mathrm{Li}_2\left(\frac{2\,c-1}{4\,c}\right) - \mathrm{Li}_2\left(\frac{2}{1+2\,c}\right)\right) + \frac{(\alpha+\bar{z}z)}{2\,\bar{z}\,z}\left(\frac{\ln^2\alpha}{2}\right. \\
\left. - 2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right) + \ln^2\left(\frac{z}{\alpha}\right) - \ln^2\left(\frac{\alpha+\bar{z}z}{z}\right) + 2\mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) + 2\mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)\right) \\
- \frac{(1-2\,z)\,(\alpha-\bar{z}z)}{2\,c\,\bar{z}\,z}\left(\ln\left(\frac{z}{4\alpha+1}\right)\ln\left(\frac{2\alpha+(1-2\,c)\,z}{2\alpha+(1+2\,c)\,z}\right) - \ln(4\alpha+1)\ln\left(\frac{2\,c+1}{2\,c-1}\right)\right. \\
\left.\left. + 2\mathrm{Li}_2\left(\frac{1-2\,c-2\,z}{1+2\,c-2\,z}\right) - \mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) + \mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)\right)\right] + N_c\left[\ln\left(s_0/Q^2\right)\ln\left(\frac{(\alpha+\bar{z}z)^2}{z^2\alpha}\right)\right. \\
+ \frac{20}{9} - \frac{\ln\alpha}{3} + \frac{1}{2}\ln^2\left(\frac{\bar{z}}{z}\right) + \frac{1}{2}\ln^2\left(\frac{\alpha+\bar{z}z}{\alpha}\right) - 2\ln\left(\frac{\alpha+z}{z}\right)\ln\left(\frac{\alpha+\bar{z}z}{\alpha\;z}\right) - \ln^2\left(\frac{\alpha+\bar{z}z}{z}\right) + 3\ln z\ln\left(\frac{\alpha+\bar{z}z}{\alpha\,z}\right) \\
+ 2\ln^2 z - \frac{(\alpha+\bar{z}z)}{2\,\alpha\,\bar{z}\,z}\left(z\ln\left(\frac{\alpha}{z}\right) + \sqrt{z\,(4\,\alpha+z)}\ln\left(\frac{z+\sqrt{z\;(4\,\alpha+z)}}{-z+\sqrt{z\;(4\,\alpha+z)}}\right)\right) + \mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right) + \mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)
\end{aligned}
$$

$$+2\mathrm{Li}_2\left(-\frac{z^2}{\alpha+\bar{z}z}\right)-2\mathrm{Li}_2\left(-\frac{z}{\alpha}\right)+3\,\mathrm{Li}_2\left(-\frac{\bar{z}z}{\alpha}\right)\Bigg]+\frac{1}{N_c}\left[\frac{5}{2}+\left(\frac{\alpha+\bar{z}z}{\bar{z}\,z}-\frac{3}{2}\right)\ln\alpha\right.$$

$$+\frac{1}{2}\ln\left(\frac{4\,\alpha}{(2\,c-1)^2}\right)\ln\left(\frac{2\,c+1}{2\,c-1}\right)+\frac{1}{2}\ln^2\left(\frac{1+2\,c-2\,z}{2\,c-1+2\,z}\right)+\frac{c\,(1-2\,z)\,(\alpha+\bar{z}z)}{\bar{z}^2\,z^2}\ln(1+2\,c-2\,z)\Bigg]$$

$$+\frac{(1-2\,z)\,(\alpha+\bar{z}z)}{\bar{z}^2 z}\ln\left(\frac{z}{\alpha+\bar{z}z}\right)+\mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)+\ln z\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)+\frac{(\alpha+\bar{z}z)}{4\,\bar{z}^2 z^2}\left(2\,c\,\ln\left(\frac{2\,c+1}{2\,c-1}\right)\right.$$

$$\left.-\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)\right)+\mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right)-\frac{\left(\alpha^2+\alpha\,z-\bar{z}\,z^3\right)}{2\,\bar{z}\,z^3}\left(2\ln\left(c-z+\frac{1}{2}\right)\ln\left(c+z-\frac{1}{2}\right)\right.$$

$$-2\ln\left(c-\frac{1}{2}\right)\ln\left(c+\frac{1}{2}\right)+\ln\left(\frac{2\,c+1}{2\,c-1}\right)\ln\left(\frac{2\,\alpha+(1+2\,c)\;z}{2\,\alpha+(1-2\,c)\;z}\right)-\ln\left(\frac{\alpha\,\bar{z}^2\,z^2}{(\alpha+\bar{z}z)^2}\right)\ln\left(\frac{\alpha+\bar{z}z}{\alpha}\right)$$

$$+2\ln\left(\frac{\alpha+z}{\alpha}\right)\ln\left(\frac{\alpha\,z}{\alpha+\bar{z}z}\right)-2\mathrm{Li}_2\left(-\frac{z}{\alpha}\right)+4\mathrm{Li}_2\left(\frac{2\,z}{1-2\,c}\right)+4\mathrm{Li}_2\left(\frac{2\,z}{1+2\,c}\right)-2\mathrm{Li}_2\left(-\frac{\bar{z}z}{\alpha}\right)$$

$$\left.\left.+2\mathrm{Li}_2\left(-\frac{z^2}{\alpha+\bar{z}z}\right)\right)\right]$$

$$\alpha=\frac{\vec{q}^{\,2}}{Q^2}\,,\qquad\qquad c=\sqrt{\alpha+1/4}$$

[D.Yu. Ivanov, M.I. Kotsky, A. P. (2004)]

BFKL Green's function

$$\mathcal{I}m_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^2}\int\frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\Phi_1(\vec{q}_1,s_0)\int\frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\Phi_2(-\vec{q}_2,s_0)\int\limits_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1,\vec{q}_2)$$

BFKL equation: $\delta^2(\vec{q}_1-\vec{q}_2) = \omega\, G_\omega(\vec{q}_1,\vec{q}_2) - \int d^2\vec{q}\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_2)$

Transverse momentum notation: $\hat{\vec{q}}\,|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1-\vec{q}_2) \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})$$

$$\hat{1} = (\omega-\hat{K})\hat{G}_\omega \qquad \longrightarrow \qquad \hat{G}_\omega = (\omega-\hat{K})^{-1}$$

$$\hat{K} = \bar{\alpha}_s\hat{K}^0 + \bar{\alpha}_s^2\hat{K}^1\,, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

With NLA accuracy

$$\hat{G}_\omega = (\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + (\omega-\bar{\alpha}_s\hat{K}^0)^{-1}\left(\bar{\alpha}_s^2\hat{K}^1\right)(\omega-\bar{\alpha}_s\hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2\hat{K}^1\right)^2\right]$$

Basis of eigenfunctions of the LLA kernel: $\{|\nu\rangle\}$

$$\boxed{\hat{K}^0|\nu\rangle = \chi(\nu)|\nu\rangle} \qquad \chi(\nu) = 2\psi(1) - \psi\left(\tfrac{1}{2}+i\nu\right) - \psi\left(\tfrac{1}{2}-i\nu\right)$$

$$\langle\vec{q}|\nu\rangle = \frac{1}{\pi\sqrt{2}}\left(\vec{q}^{\,2}\right)^{i\nu-\frac{1}{2}} \qquad \langle\nu'|\nu\rangle = \int \frac{d^2\vec{q}}{2\pi^2}\left(\vec{q}^{\,2}\right)^{i\nu-i\nu'-1} = \delta(\nu-\nu')$$

Action of the full NLA kernel on the LLA eigenfunctions:

$$\boxed{\begin{aligned}\hat{K}|\nu\rangle &= \bar{\alpha}_s(\mu_R)\chi(\nu)|\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(\nu) + \frac{\beta_0}{4N_c}\chi(\nu)\ln(\mu_R^2)\right)|\nu\rangle \\ &+ \bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(\nu)\left(i\frac{\partial}{\partial\nu}\right)|\nu\rangle\end{aligned}}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\,N_c}\left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu)$$

$$\bar{\chi}(\nu) = -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)} + \frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) + 4\phi(\nu)\right]$$

$$\phi(\nu) = 2\int_0^1 dx\,\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right]$$

$$\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)$$

$$\times\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\left(\bar{\chi}(\nu)\right.\right.$$

$$\left.\left.+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}+i\frac{d\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{d\nu}+2\ln(\mu_R^2)\right]\right)\right]$$

$|\nu\rangle$ **representation for impact factors:**

$$\frac{C_1^{(0)}(\vec{q}^{\,2})}{\vec{q}^{\,2}} = \int_{-\infty}^{+\infty} d\,\nu'\, c_1(\nu')\langle\nu'|\vec{q}\rangle \qquad \frac{C_2^{(0)}(\vec{q}^{\,2})}{\vec{q}^{\,2}} = \int_{-\infty}^{+\infty} d\,\nu\, c_2(\nu)\,\langle\vec{q}|\nu\rangle$$

$$c_1(\nu) = \int d^2\vec{q}\; C_1^{(0)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{i\nu-\frac{3}{2}}}{\pi\sqrt{2}} \qquad c_2(\nu) = \int d^2\vec{q}\; C_2^{(0)}(\vec{q}^{2})\frac{\left(\vec{q}^{2}\right)^{-i\nu-\frac{3}{2}}}{\pi\sqrt{2}}$$

(analogous definitions for $c_1^{(1)}(\nu)$ and $c_2^{(1)}(\nu)$)

Leading-order: $c_{1,2}(\nu) = \frac{\left(Q_{1,2}^2\right)^{\pm i\nu - \frac{1}{2}}}{\sqrt{2}} \frac{\Gamma^2[\frac{3}{2} \pm i\nu]}{\Gamma[3 \pm 2i\nu]} \frac{6\pi}{\cosh(\pi\nu)}$

Next-to-leading-order: $c_{1,2}^{(1)}(\nu)$ **numerical calculation**

The amplitude contains also terms beyond the NLA.
If only the "allowed" terms in the NLA are kept, $(\alpha_s \ln(s))^n)$ **and** $\alpha_s(\alpha_s \ln(s))^n$**,**
the dependence on s_0 **and** μ_R **disappears.**

Uncertainties of NLA BFKL amplitude related to:

- **the particular representation for of the amplitude**
- **the choice of the scales** s_0 **and** μ_R

Series representation for the amplitude

$$\frac{Q_1 Q_2}{D_1 D_2}\frac{\mathcal{I}m_s \mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\Bigg[b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n \, b_n \left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0,\mu_R)\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\Bigg]$$

- **LLA**

$$\frac{b_n}{Q_1 Q_2} = \int\limits_{-\infty}^{+\infty} d\nu \, c_1(\nu) c_2(\nu) \frac{\chi^n(\nu)}{n!}$$

- **NLA**

  $d_n(s_0,\mu_R)$ – ν **integral, containing the NLA impact factors** $c^{(1)}_{1,2}$ **and NLA BFKL kernel eigenvalues**

Exponentiated form of the amplitude

$$\frac{\mathcal{I}m_s\left(\mathcal{A}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2}\int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)+\bar{\alpha}_s^2(\mu_R)\left(\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}\right]\right)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)$$

$$\times\left[1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu)\left(i\frac{d\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{d\nu}+2\ln(\mu_R^2)\right)\right]$$

Numerical analysis

$Q_1 = Q_2 \equiv Q$ **"pure" BFKL regime**

LLA: b_n coefficients (Q-independent)

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$
$$b_5 = 10.5698 \quad b_6 = 4.54792 \quad b_7 = 1.69128 \quad b_8 = 0.554475$$

NLA: $d_n(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, $n_f = 5$)

$$d_1 = -3.71087 \quad d_2 = -11.3057 \quad d_3 = -23.3879 \quad d_4 = -39.1123$$
$$d_5 = -59.207 \quad d_6 = -83.0365 \quad d_7 = -111.151 \quad d_8 = -143.06$$

NLA: $d_n^{\rm imp}(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, impact factor contribution)

$$d_1^{\rm imp} = -3.71087 \quad d_2^{\rm imp} = -8.4361 \quad d_3^{\rm imp} = -13.1984 \quad d_4^{\rm imp} = -18.0971$$
$$d_5^{\rm imp} = -23.0235 \quad d_6^{\rm imp} = -27.9877 \quad d_7^{\rm imp} = -32.9676 \quad d_8^{\rm imp} = -37.9618$$

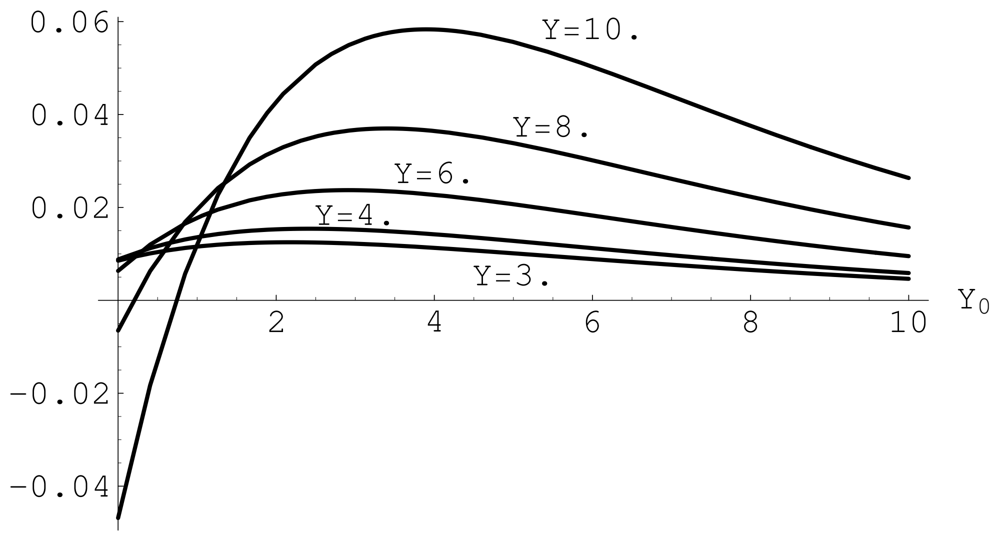
- **Large NLA corrections!**
- **Optimization of perturbative expansion needed!**

- **Principle of minimal sensitivity (PMS) [P.M. Stevenson (1981)]:** **require the minimal sensitivity to the change of both s_0 and μ_R.**

- **Strategy: for each fixed s calculate the amplitude for varying s_0 at fixed μ_R and viceversa, up to finding the optimal values for which the amplitude is least sensitive to variations of them.**

- **In practice, there are wide regions in s_0 and μ_R where the amplitude is very weekly dependent on s_0 and μ_R.**

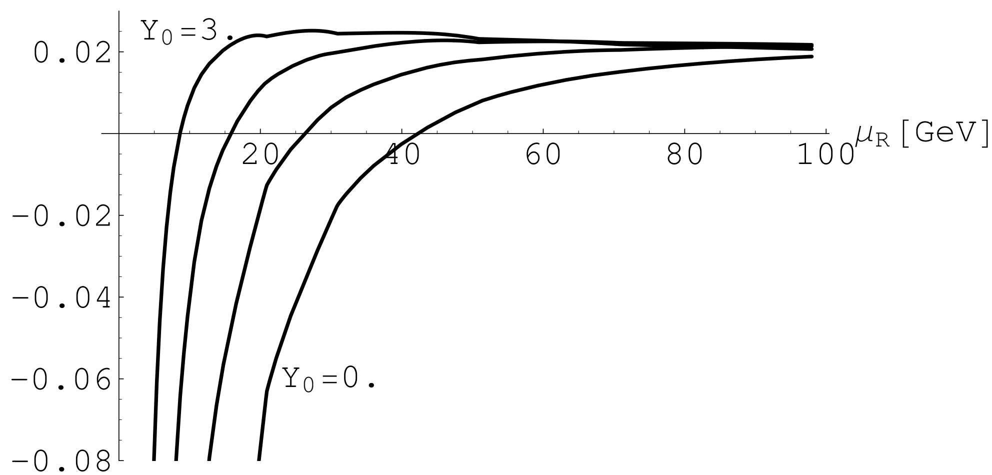
Example: Q^2=24 GeV^2, $n_f = 5$



$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ *vs* Y_0

Y**=10, 8, 6, 4, 3**

$\mu_R = 10\,Q$



$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ *vs* μ_R

Y_0**=3, 2, 1, 0**

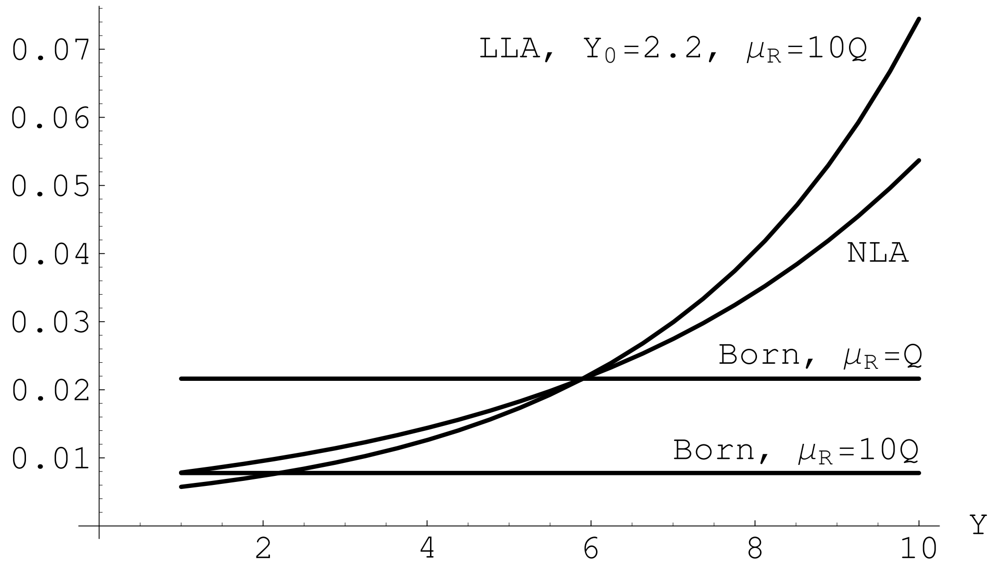
Y**=6**

$$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\left[b_0 + \sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left((Y-Y_0)^n + d_n(s_0,\mu_R)\,(Y-Y_0)^{n-1}\right)\right]$$

$$Y \equiv ln\left(\frac{s}{Q^2}\right)\,, \quad Y_0 \equiv ln\left(\frac{s_0}{Q^2}\right)$$

$$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}\ vs\ Y$$

Q^2**=24 GeV**^{2}**,** $n_f = 5$



Lessons

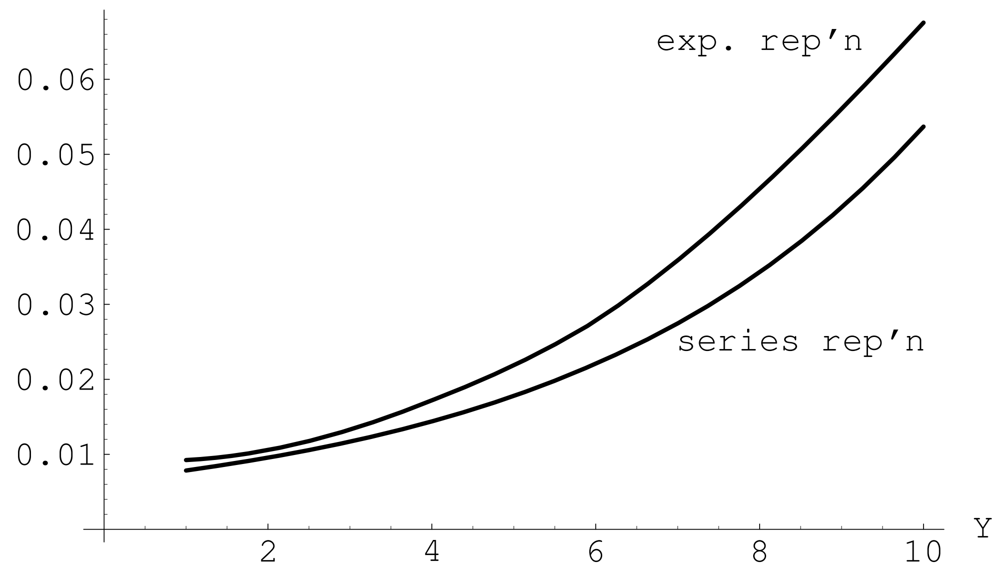
The Born approximation does not give necessarily the estimate from below.

The optimal values for μ_R are "unnaturally" larger than Q (new scale or nature of the BFKL series?).

PMS method - Exponentiated representation

$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ *vs* Y

Q^2=**24 GeV**^{2}, $n_f = 5$



- **Agreement with the PMS method applied to the series representation.**
- **The optimal value for μ_R is slightly smaller than in previous case**

FAC method

- **Fast apparent convergence (FAC) [G. Grunberg (1980)]: require that the NLO corrections identically vanish.**

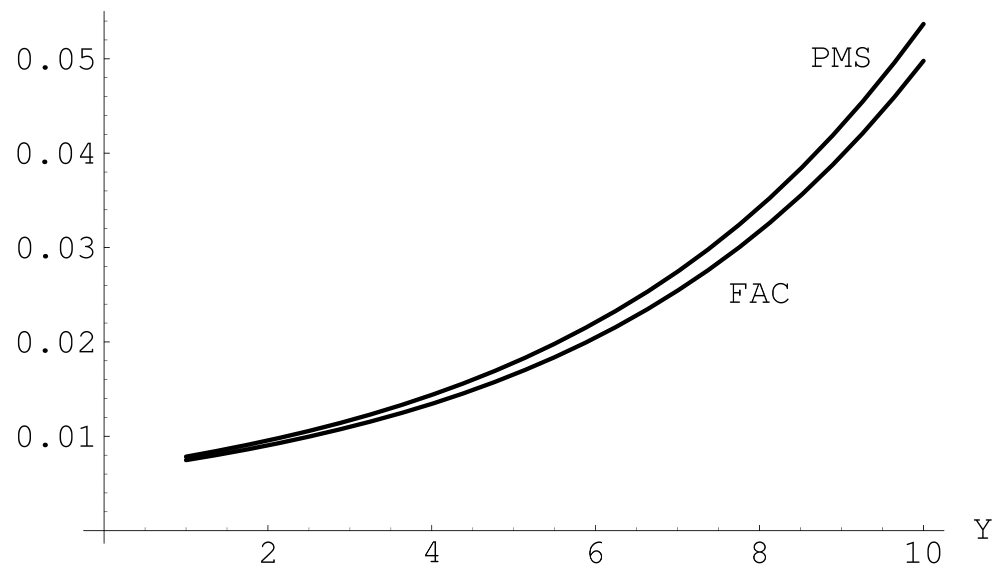
- **Strategy: for each fixed Y determine "the line" of Y_0 and μ_R for which the NLO correction vanish; then, the optimal values of Y_0 and μ_R along this line are chosen according to "minimal sensitivity".**

FAC method - Series representation

$$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\left[b_0 + \sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n\, b_n\left((Y-Y_0)^n + d_n(s_0,\mu_R)\,(Y-Y_0)^{n-1}\right)\right]$$

$\frac{Q^2}{D_1 D_2}\frac{\mathcal{I}m_s\mathcal{A}}{s}$ *vs* Y

Q^2**=24 GeV**^{2}**,** $n_f = 5$



- **Despite the very different strategy, FAC and PMS give quite consistent results.**

FAC method - Exponentiated representation

$$\frac{\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2} \int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(\nu)+\bar\alpha_s^2(\mu_R)\left(\bar\chi(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu)+\frac{10}{3}\right]\right)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)$$

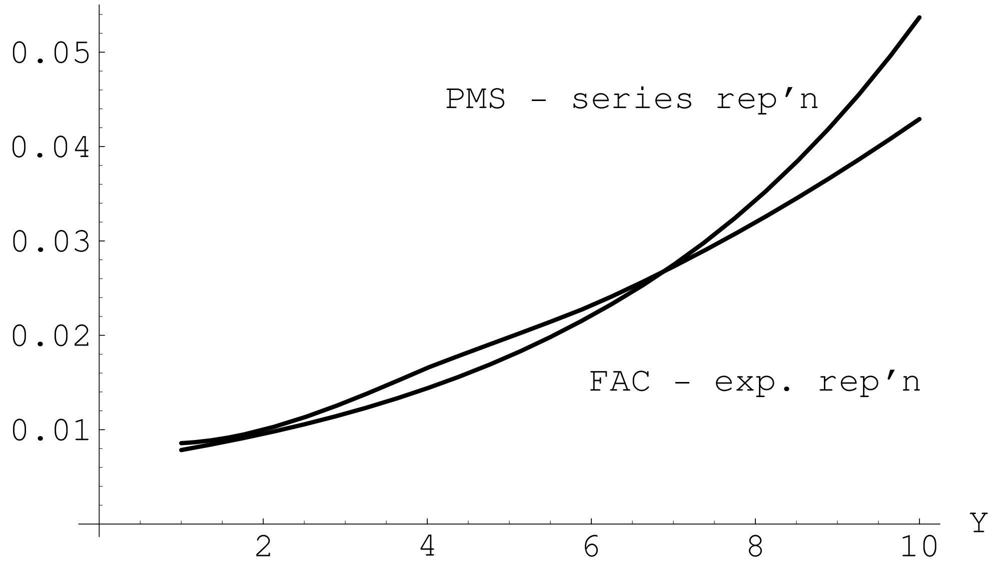
$$\times \left[1+\bar\alpha_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)+\bar\alpha_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu)\left(i\frac{d\ln(\frac{c_1(\nu)}{c_2(\nu)})}{d\nu}+2\ln(\mu_R^2)\right)\right]$$

$$\frac{\mathcal{I}m_s\left(\mathcal{A}_{LLA}\right)}{D_1 D_2} = \frac{s}{(2\pi)^2} \int\limits_{-\infty}^{+\infty} d\nu \left(\frac{s}{s_0}\right)^{\bar\alpha_s(\mu_R)\chi(\nu)} \alpha_s^2(\mu_R) c_1(\nu) c_2(\nu)$$

$$\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right) = \mathcal{I}m_s\left(\mathcal{A}_{LLA}\right) + \left[\mathcal{I}m_s\left(\mathcal{A}_{NLA}\right) - \mathcal{I}m_s\left(\mathcal{A}_{NLA}\right)\right]$$

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ *vs* Y

Q^2**=24 GeV**^{2}**,** $n_f = 5$



- **Again, good agreement between FAC and PMS.**

BLM method

- [Brodsky, Lepage, Mackenzie (1983)] optimization method:
  perform a finite renormalization to a physical scheme and then choose the renormalization scale in order to remove the β_0- dependent part.

Strategy (applied to the amplitude in the series representation only):

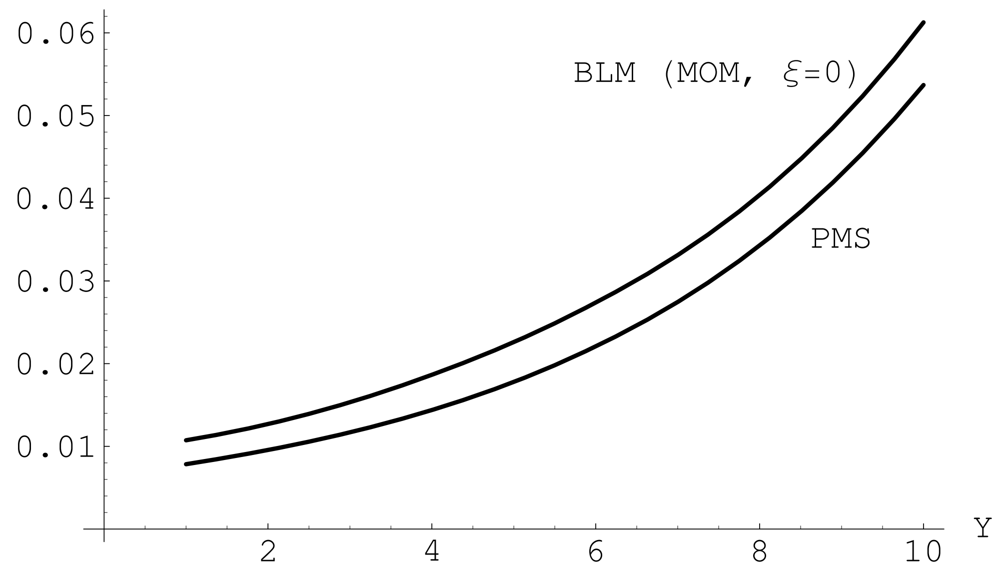
- finite renormalization to the MOM-scheme $(\zeta = 0)$

$$\alpha_S \to \alpha_S \left[1 + T_{MOM}(\zeta = 0)\frac{\alpha_S}{\pi} \right]$$

- Y_0 and μ_R chosen to make in the resulting amplitude the term $\sim \beta_0$ vanish
- optimal values for Y_0 and μ_R determined according to "minimal sensitivity"

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ vs Y

Q^2**=24 GeV**^{2}**,** $n_f = 5$



- **Drawback: for each given** Y**,** Y_0 **"wants" to be as large as Y,** $s_0 \sim s$**!**

Summary

- Closed analytical expression found for the $\gamma^*\gamma^* \to VV$ forward amplitude in the Regge limit with next-to-leading energy log accuracy

- For equal photons' virtualities, i.e. in the BFKL regime
  - NLA corrections are large and of opposite sign with respect to LLA contribution.
  - the PMS optimization method allows to get stable results for the amplitude. But, the optimal values of the scales s_0, μ_R turn out to be much larger than the kinematical scale of the problem. This could be a manifestation of the nature of BFKL series.
  - our predictions for the amplitude are stable under change of both amplitude representation (exponentiated vs series) and of optimization method (FAC, BLM).
- For strongly ordered photon virtualities, $Q_1 \gg Q_2$. Extra $\log(Q_1)$, originating from both the BFKL kernel and the IFs, canceled. The structure of the amplitue is compartible with leading-twist collinear factorization.