

# Non-forward BK equation and Vector Mesons

Cyrille Marquet, Gregory Soyez (BNL)  
and Robi Peschanski (Saclay), <sup>a</sup>

DIS 2007

Wednesday, 18 April 2007

- Motivation

*QCD and Saturation at non-zero transfer*

- Balitsky-Kovchegov Eq. in full momentum space

*Formalism and Traveling Wave Solutions*

- From Theory to Phenomenology

*Exclusive Vector Meson Production*

- Conclusion and Prospects

---

<sup>a</sup>C.M., G.S., R.P. hep-ph/0702171,  
and for the Theory: hep-ph/0504117, C.M. and G.S., hep-ph/0504080.

# The BK Equation in full momentum space

- The Non-Linear BK Equation:

$$\partial_Y \mathcal{N}(b, r; Y) = \bar{\alpha} \int d^2 z \frac{(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2 (\mathbf{z} - \mathbf{y})^2} [\mathcal{N}(\mathbf{x}, \mathbf{z}) + \mathcal{N}(\mathbf{z}, \mathbf{y}) - \mathcal{N}(\mathbf{x}, \mathbf{y}) - \mathcal{N}(\mathbf{x}, \mathbf{z})\mathcal{N}(\mathbf{z}, \mathbf{y})]$$

- BK Equation  $\Rightarrow$  momentum-space

C.Marquet, G.Soyez, 2005

$$\frac{2\pi}{\bar{\alpha}} \partial_Y \tilde{\mathcal{N}}(\mathbf{k}, \mathbf{q}) = \int \frac{2d^2 k'}{(\mathbf{k} - \mathbf{k}')^2} \left\{ \tilde{\mathcal{N}}(\mathbf{k}', \mathbf{q}) - \frac{1}{2} \left[ \frac{k^2}{k'^2 + (\mathbf{k} - \mathbf{k}')^2} + \frac{(\mathbf{q} - \mathbf{k})^2}{(\mathbf{q} - \mathbf{k}')^2 + (\mathbf{k} - \mathbf{k}')^2} \right] \tilde{\mathcal{N}}(\mathbf{k}, \mathbf{q}) \right\} - \int d^2 k' \tilde{\mathcal{N}}(\mathbf{k}, \mathbf{k}') \tilde{\mathcal{N}}(\mathbf{k} - \mathbf{k}', \mathbf{q} - \mathbf{k}')$$

- Traveling Wave Dictionnary (at zero transfer)

S.Munier, R.P, 2003

$$\text{Time} = t \sim Y$$

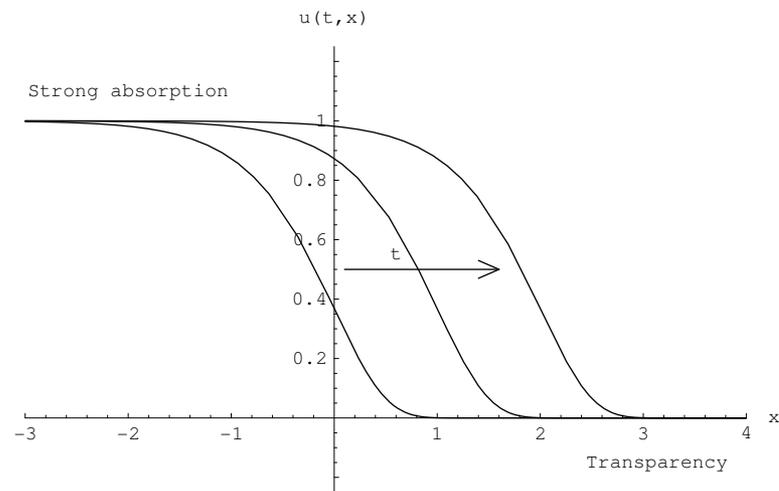
$$\text{Space} = x \sim \text{Log } k^2$$

$$\text{Traveling Wave} = u(t, x) \sim u(t - v_c x) \propto \mathcal{N}_{q^2=0} \left( \frac{k}{Q_s(Y)} \right)$$

$$\text{Saturation Scale} = Q_s(Y)$$

$$Q_s^2(Y) \underset{Y \rightarrow \infty}{\sim} \exp \left[ \bar{\alpha} \frac{\chi(\gamma_c)}{\gamma_c} Y - \frac{3}{2\gamma_c} \log(Y) - \frac{3}{\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha} \chi''(\gamma_c)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y) \right]$$

# Traveling waves



- General Method

- i) solve the linear equation:  $u(t, x, q^2) \sim \int d\gamma e^{\gamma[x - v(\gamma, q^2)t]}$
- ii) find the critical (minimal) velocity  $\min v(\gamma, q^2) = v(\gamma_c, q^2)$
- iii) sharp enough initial conditions  $\gamma_0 > \gamma_c(q^2)$

- Mathematical properties

**Universality:** Independence from initial data,

from nonlinear damping, from “noncritical” features.

**Universality classes** Different equations  $\rightarrow$  same asymptotics.

**Validity:** “Interior” of the wave and late times.

# Traveling Waves at non-zero transfer

C.M, G.S, R.P, 2005

- **Near-Forward region**  $q \ll k_{\text{Target}} \ll k_{\text{Projectile}}$

$$Q_s^2(Y) \underset{Y \rightarrow \infty}{\sim} k_{\text{Target}}^2 \exp \left[ \bar{\alpha} \frac{\chi(\gamma_c)}{\gamma_c} Y - \frac{3}{2\gamma_c} \log(Y) - \frac{3}{\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha} \chi''(\gamma_c)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y) \right]$$

- **Intermediate transfer region**  $k_{\text{Target}} \ll q \ll k_{\text{Projectile}}$

$$Q_s^2(Y) \underset{Y \rightarrow \infty}{\sim} q^2 \exp \left[ \bar{\alpha} \frac{\chi(\gamma_c)}{\gamma_c} Y - \frac{3}{2\gamma_c} \log(Y) - \frac{3}{\gamma_c^2} \sqrt{\frac{2\pi}{\bar{\alpha} \chi''(\gamma_c)}} \frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y) \right]$$

- **High transfer region**  $k_{\text{Target}} \ll k_{\text{Projectile}} \ll q$

*no traveling wave solution  $\Rightarrow$  no scaling*

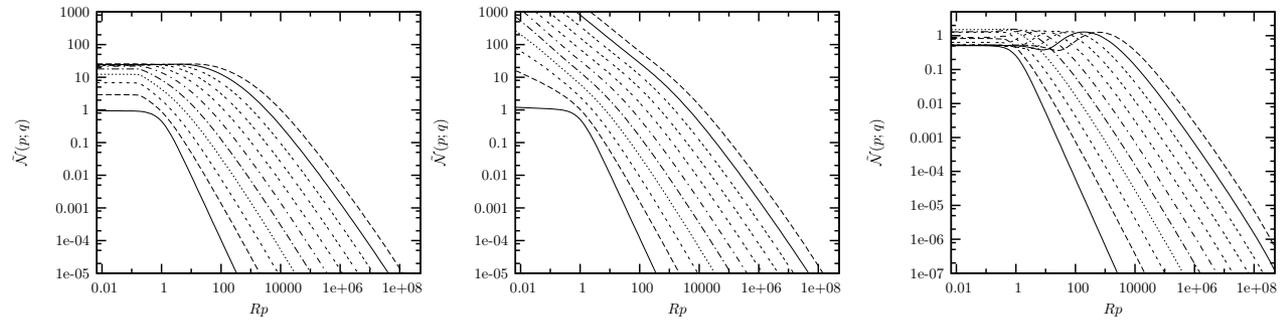
- **Prediction**

*Geometric Scaling at “semi – hard”  $q^2$*

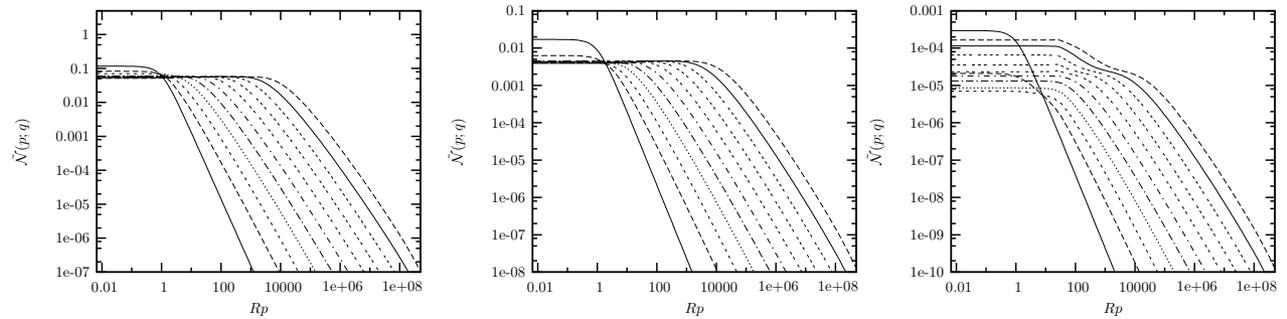
# Numerical Simulations

C.M, G.S, 2005

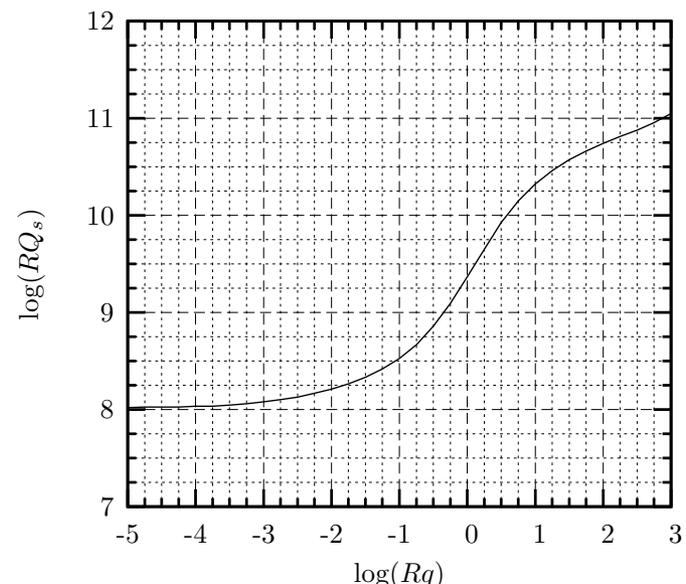
Small  $q$



Large  $q$



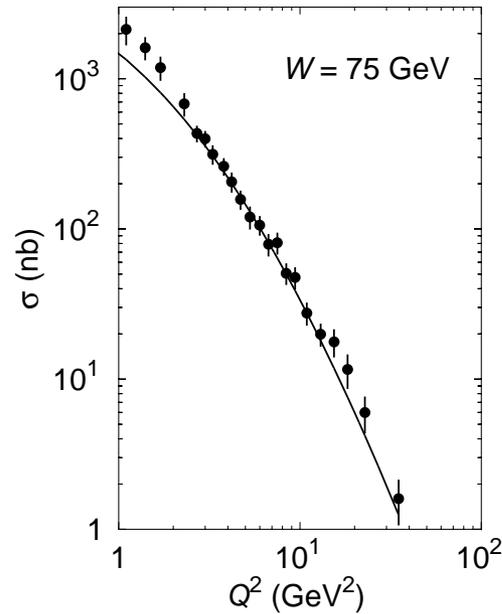
Saturation Scale Evolution w.r.t.  $q$



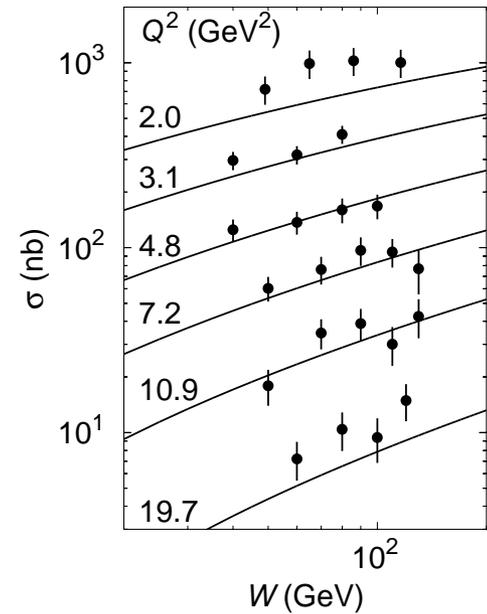


# Exclusive VM production (1)

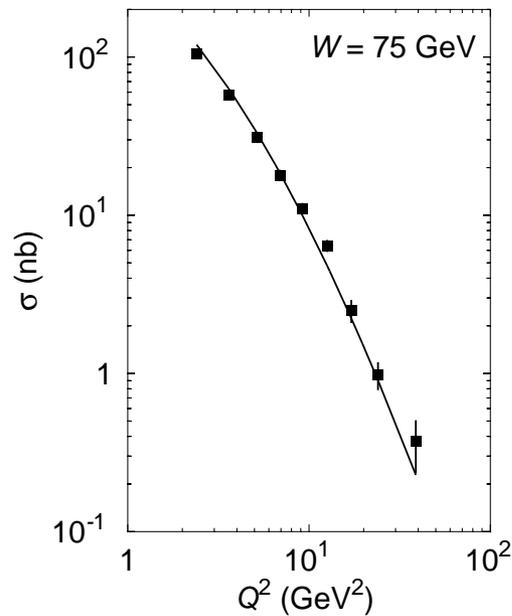
$\rho$  (H1) and  $\phi$  (ZEUS) elastic cross-sections



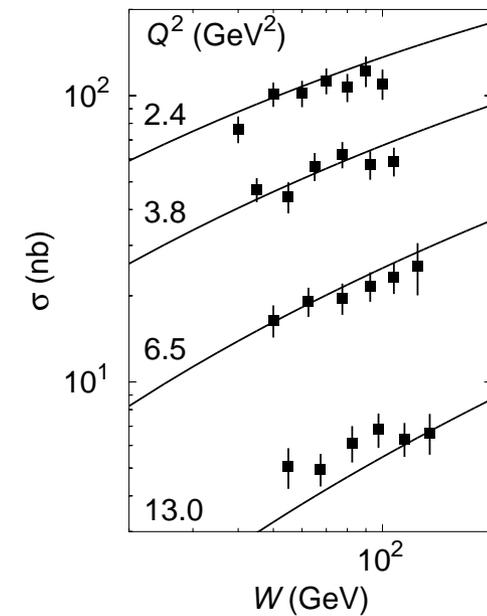
(a)  $\gamma^* p \rightarrow \rho p$  at fixed  $W$



(b)  $\gamma^* p \rightarrow \rho p$  at fixed  $Q^2$



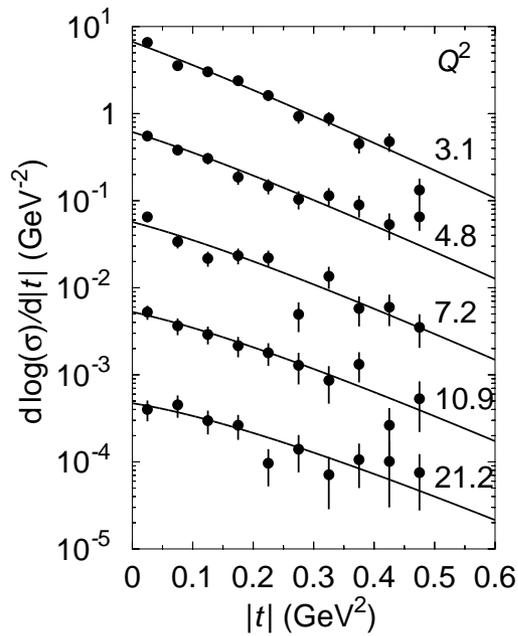
(c)  $\gamma^* p \rightarrow \phi p$  at fixed  $W$



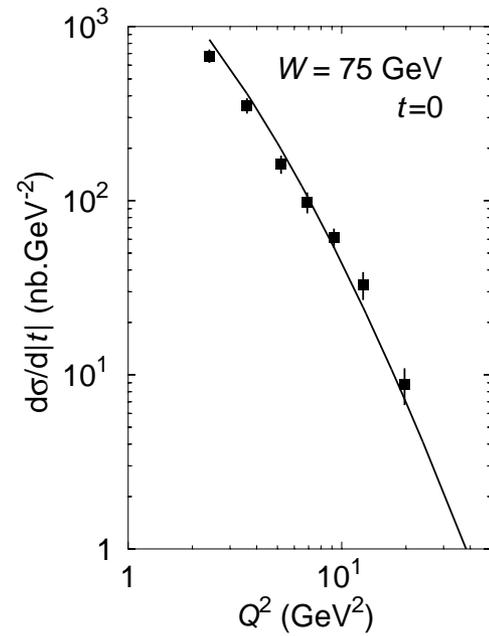
(d)  $\gamma^* p \rightarrow \phi p$  at fixed  $Q^2$

# Exclusive VM production (2)

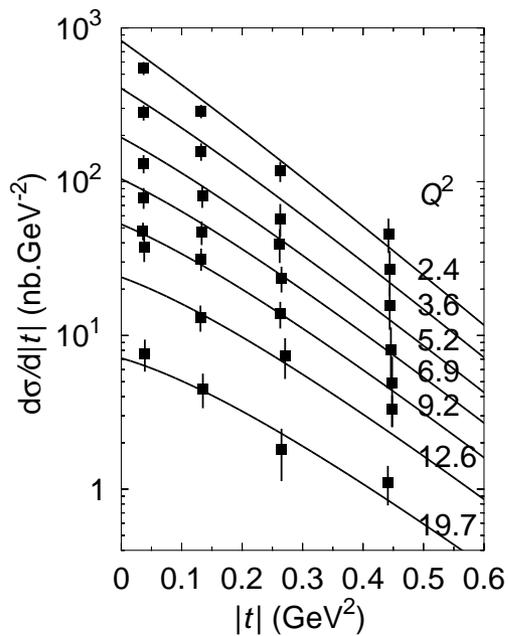
$\rho$  (H1) and  $\phi$  (ZEUS) differential cross-sections



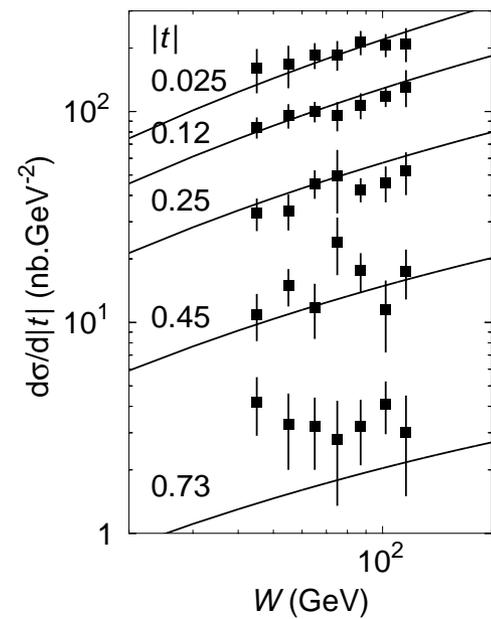
(e)  $\rho$  meson at  $W = 75$



(f)  $\phi$  meson at  $t = 0$



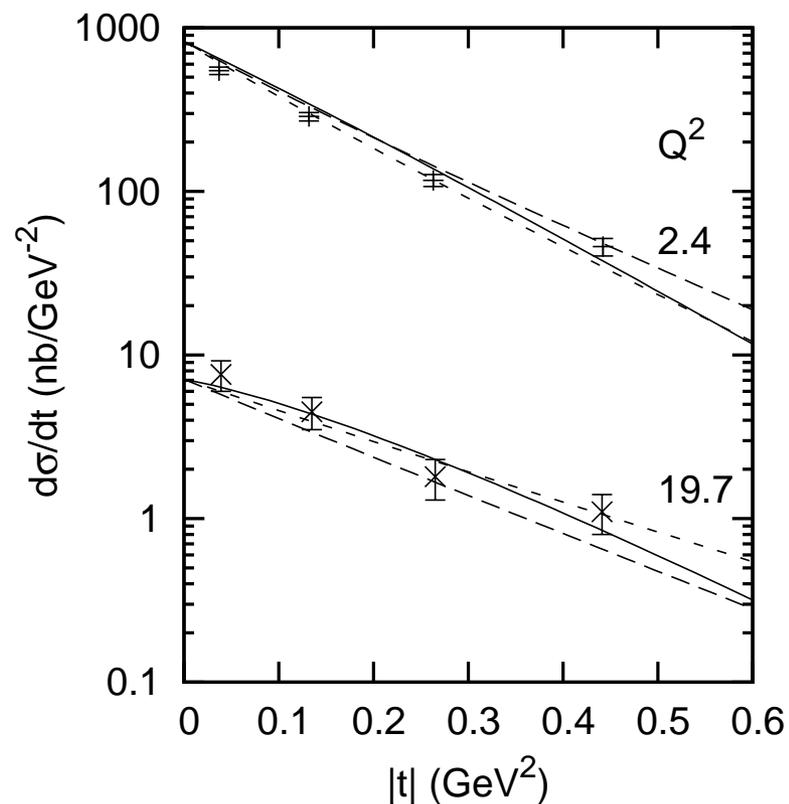
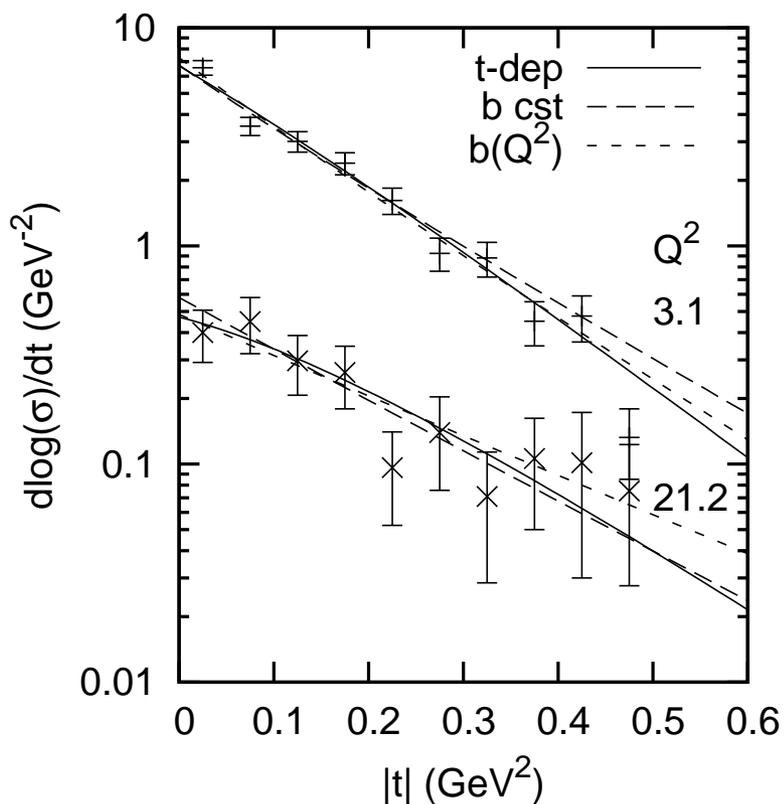
(g)  $\phi$  meson at  $W = 75$



(h)  $\phi$  meson at  $Q^2 = 5$

# Exclusive VM production (3)

## Comparisons



Left:  $\rho$ -meson production at  $Q^2=3.1$  and  $21.2$  GeV. Right:  $\phi$  meson at  $Q^2=2.4$  and  $19.7$  GeV.

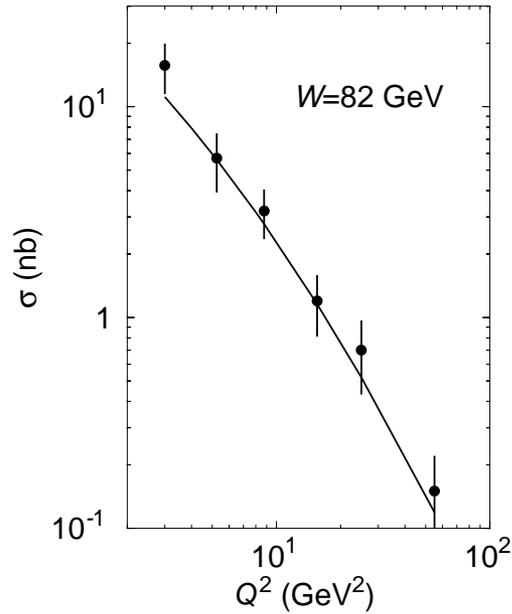
**Continuous:**  $t$ -dependent saturation; **Fat-dashed**  $t$ -independent saturation, fixed slope;

**Thin-dashed:**  $Q^2$ -dependent slope.

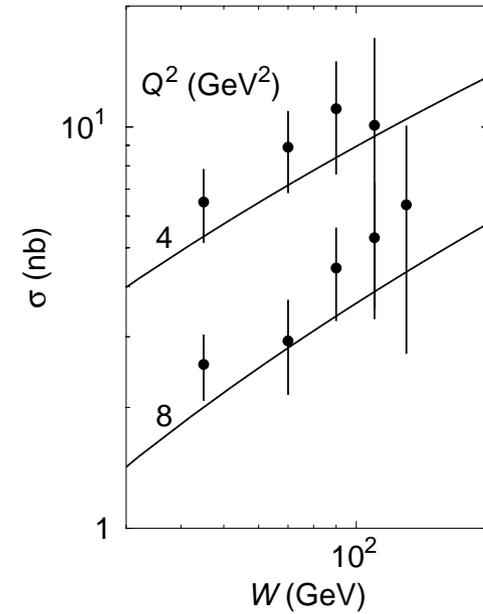
VM	Quantity	$N_{pts}$	$t$ -dependent $Q_s$			$t$ -independent $Q_s$	
			$\chi^2$	$\chi^2/N_{pts}$	LCG	$B=cst.$	$B = B(Q^2 + M_V^2)$
$\rho$	$\sigma_{el}$	47	54.297	1.156	1.688	1.732	1.968
	$\frac{d\sigma}{dt}$	50	69.097	1.382	1.620	1.489	1.405
$\phi$	$\sigma_{el}$	34	44.932	1.322	0.830	2.247	1.036
	$\frac{d\sigma}{dt}$	70	76.307	1.076	0.945	0.931	0.748
Total		201	243.632	1.212	1.267	1.480	1.245

# Exclusive VM production (4)

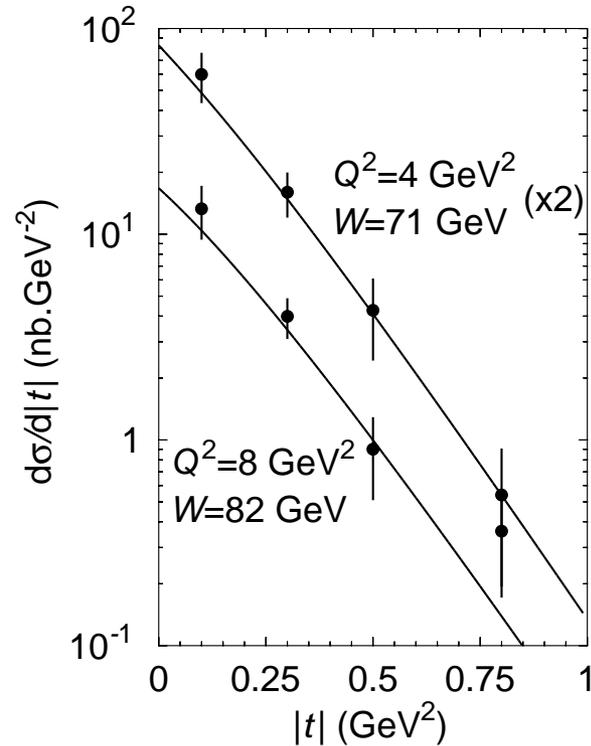
Predictions for the DVCS measurements



(i)  $\gamma^* p \rightarrow \gamma p$  at fixed  $W$



(j)  $\gamma^* p \rightarrow \gamma p$  at fixed  $Q^2$



(k) Differential cross-section

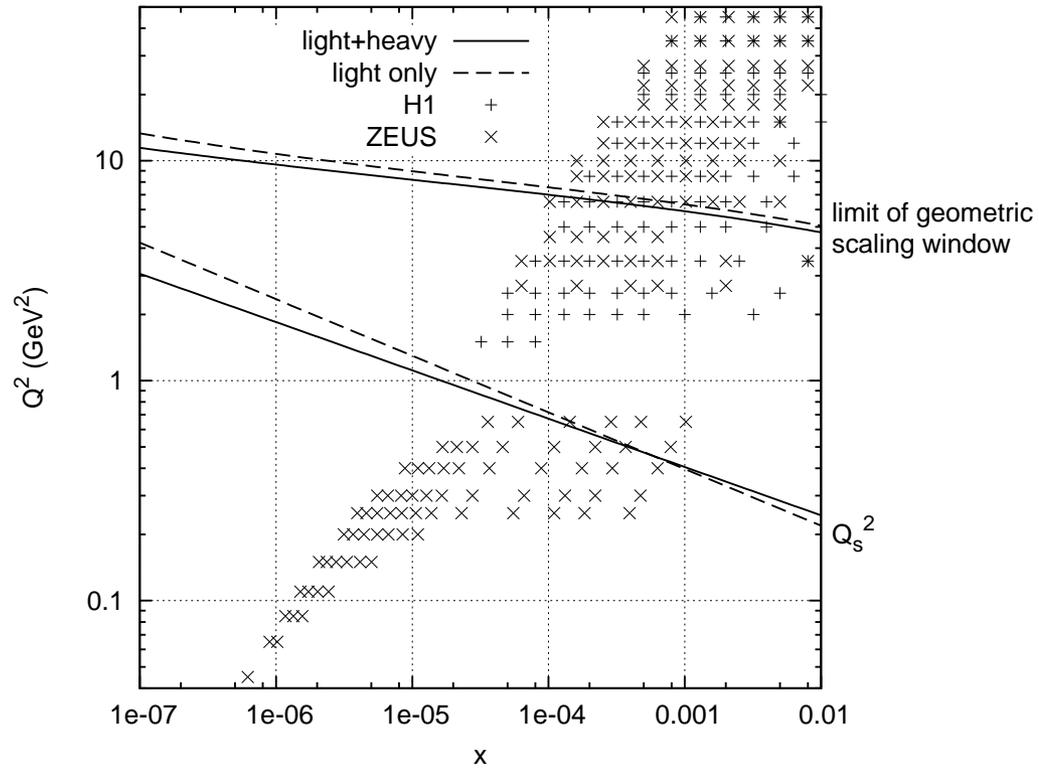
# Conclusions

- **Saturation at non-zero transfer:**  
The BK equation of QCD predicts saturation at “semi-hard” transfer  
Solves the “impact parameter puzzle”
- **Characterisation of the universality class:**  
The same as forward BK with  $k_{\text{Target}}^2 \rightarrow q^2$   
(harder) initial saturation scale
- **Phenomenology of Vector mesons**  
 $q^2$ -QCD Saturation model is OK  
But not required
- **Prospects**  
Phenomenology: Addition of charm, LHC studies?  
Theory: Saturation beyond BK? Including NP QCD?

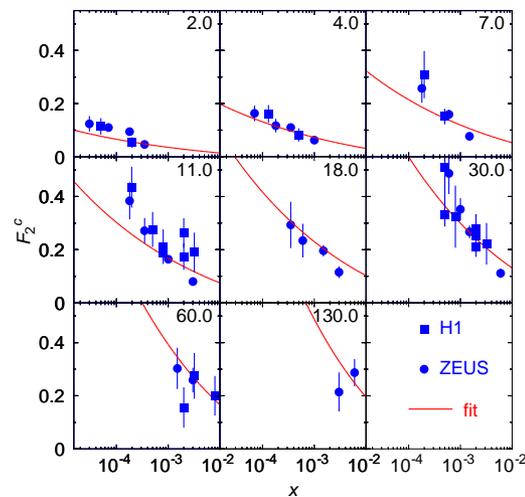
# Note added in proof(s): Including Charm

G.Soyez, to appear

## Geometric Scaling Window



## $F_2^c$ Charm



## $F_2^b$ Beauty

