

## The small $x$ gluon from exclusive $J/\psi$ production

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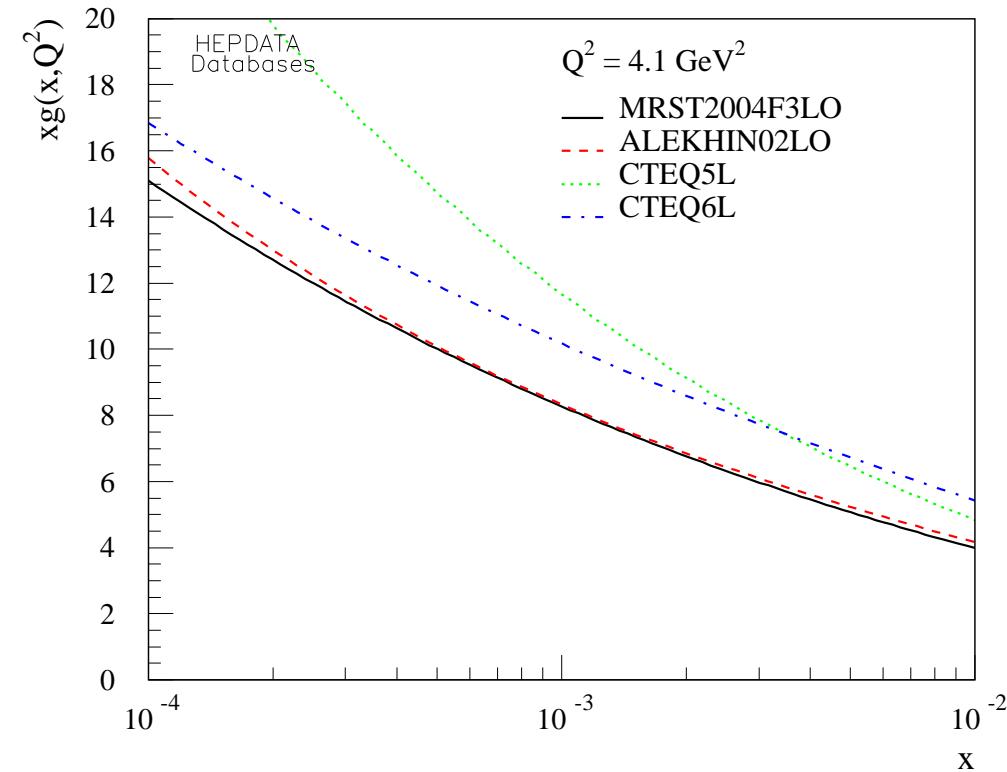
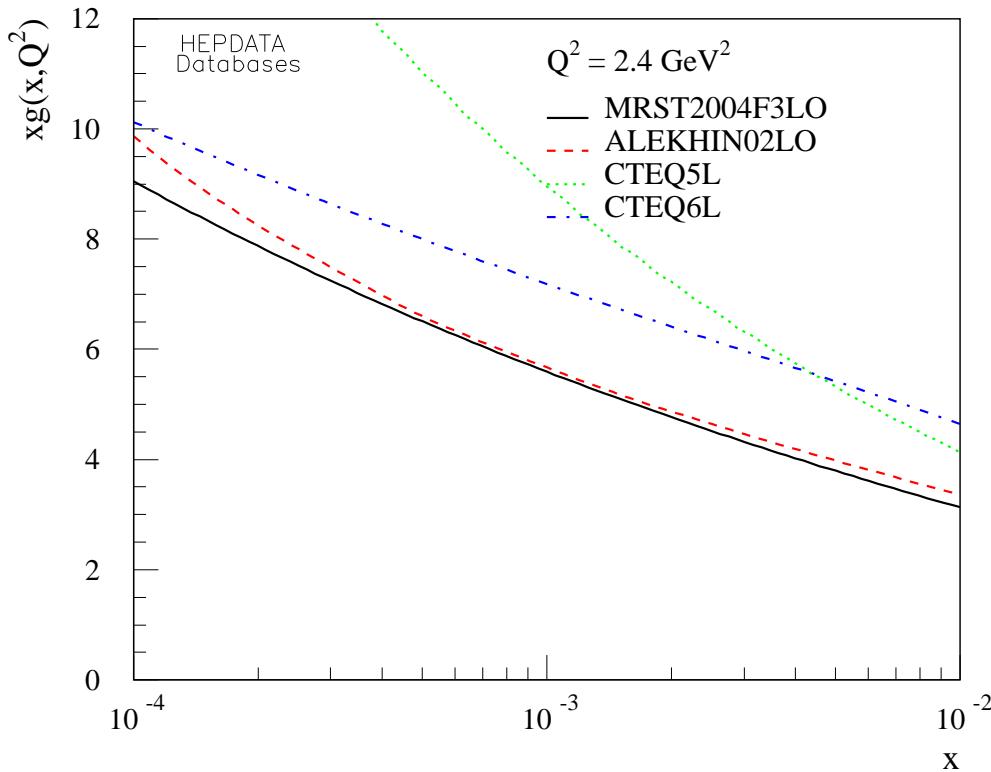
- I. Introduction; the gluon at small  $x$  and low–medium scales
- II. Exclusive  $J/\psi$  production in pQCD
- III. Determining the small  $x$  gluon from diffractive HERA data
- IV. Conclusions/Outlook

Ref/more details: Martin+Nockles+Ryskin+T: *in preparation*

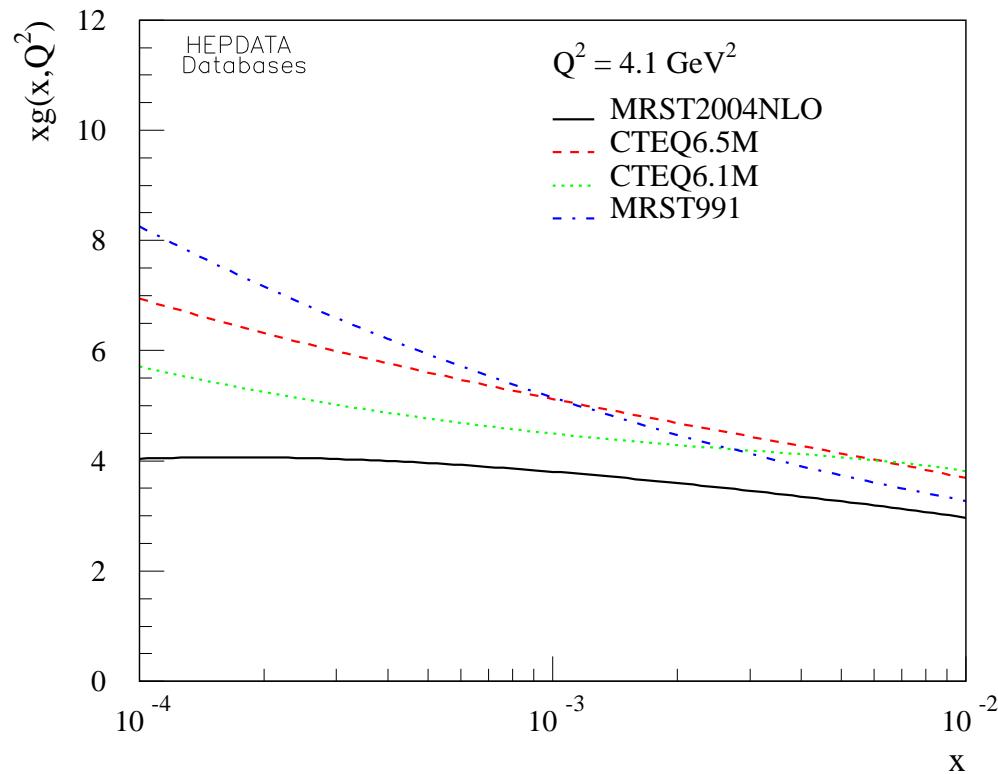
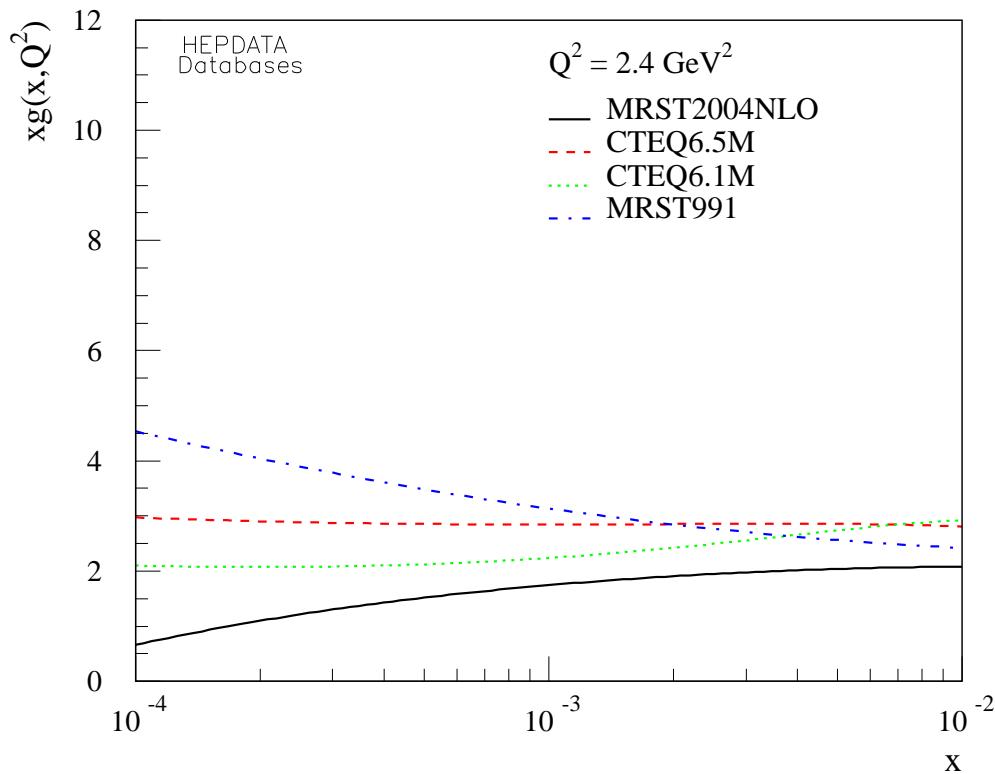
## I. Introduction; the gluon at small $x$

- Global fits constrain the gluon only poorly at small  $x$ :
  - lack of data +  $F_2$  only limited sensitivity to gluon, but needed for
    - ~~ description of semi-hard QCD / high energy scattering
    - ~~ Underlying Events, Multiple Interactions, exclusive Higgs at LHC ...

- Leading Order: (relevant for MC's)

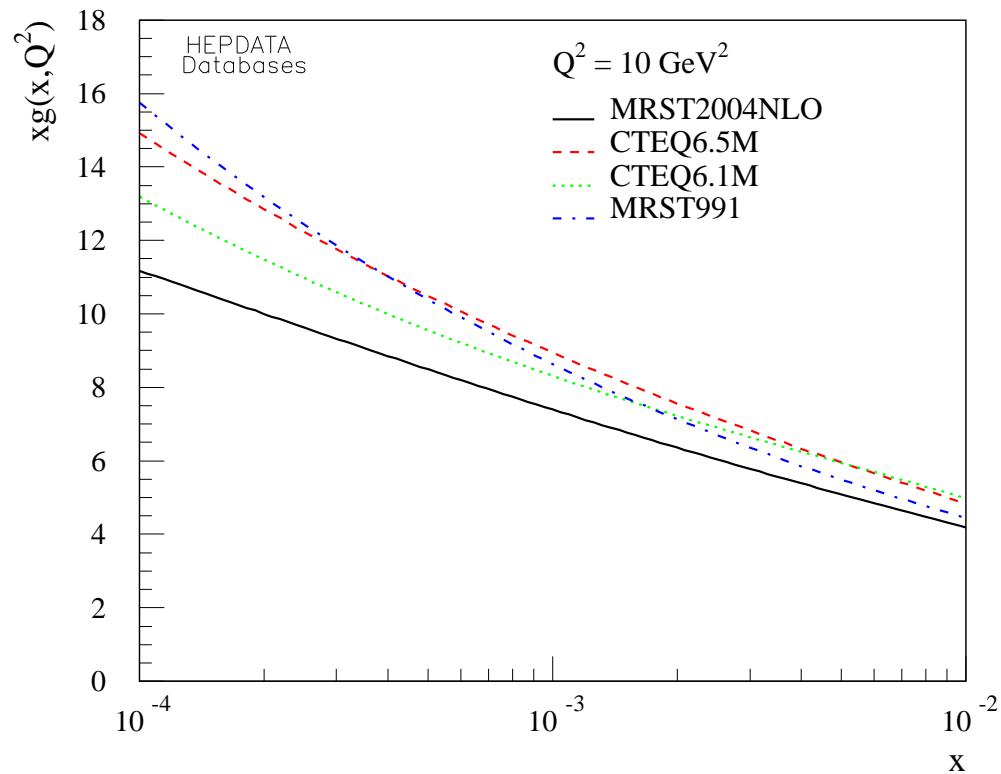
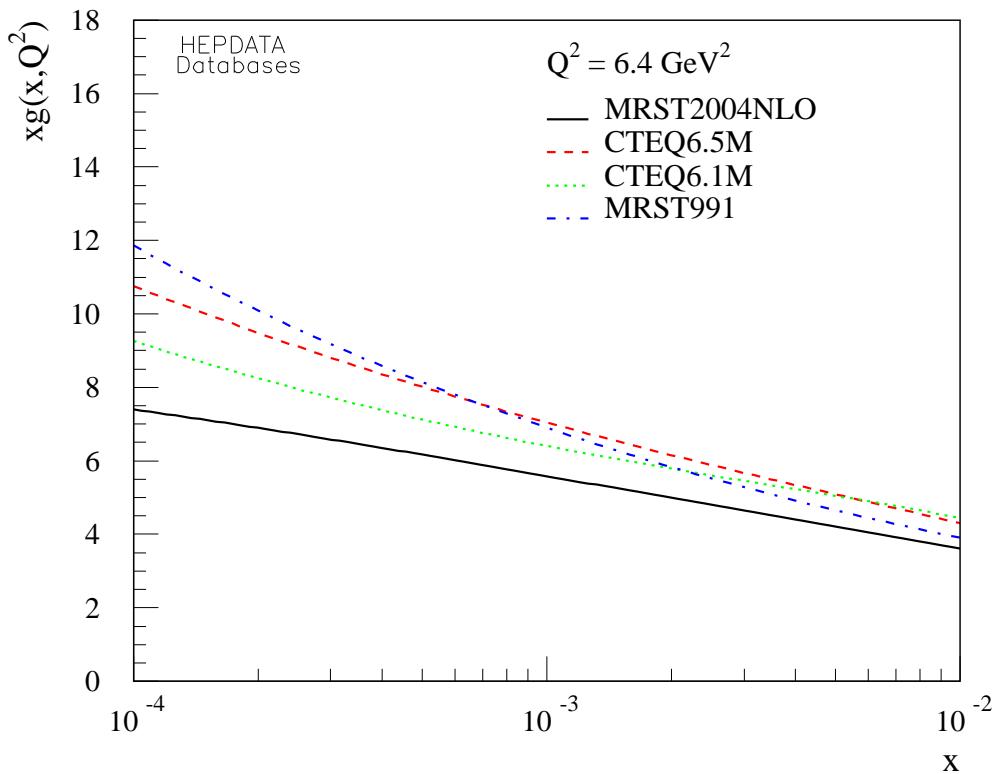


- Next-to-Leading-Order, again at 2.4, 4.1 GeV<sup>2</sup>:



- Big differences between variants; also different shapes
- MRST99 the last ‘physical’ gluon at small  $x$ , used as default in **MartinRyskinTeubner** predictions for diffractive production of  $\rho$ ,  $\phi$ ,  $J/\psi$ ,  $\Upsilon$ .

- NLO, now at 6.4, 10 GeV<sup>2</sup>: still big differences between different fits at small  $x$

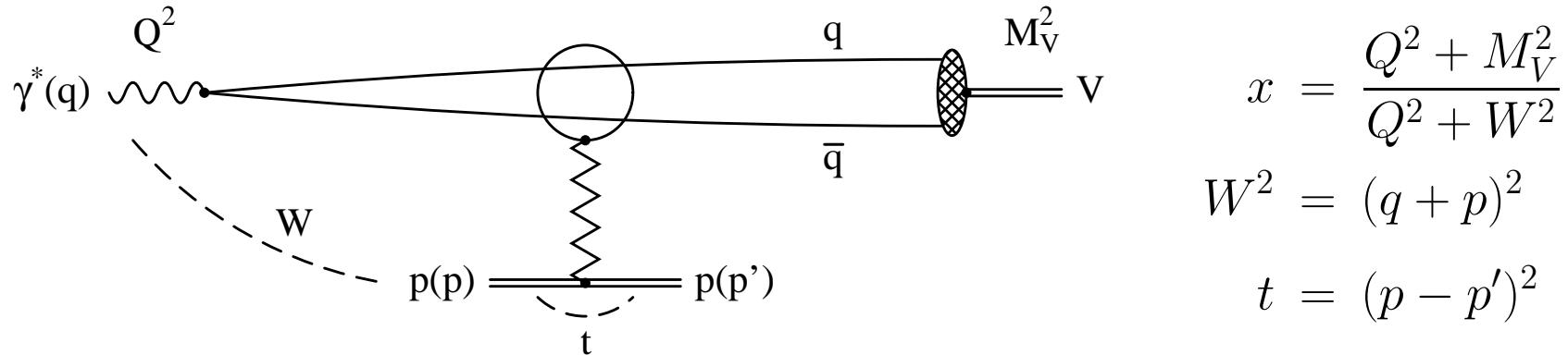


- This is the regime where we have a lot of diffractive data from HERA, in a wide range of the parameter space ( $W, Q^2, M^2, t$ ), e.g. for diffractive Vector Meson production:

$$\sigma(\gamma^* p \xrightarrow{\text{IP}} V p) \sim [xg(x, \text{scale})]^2$$

→ potential to directly constrain the gluon!

## II. Exclusive $J/\psi$ production in pQCD



- Factorization of amplitude:  $\mathcal{A}(\gamma^* p \rightarrow V p) = \psi_{q\bar{q}}^\gamma \otimes \mathcal{A}_{q\bar{q}+p} \otimes \psi_{q\bar{q}}^V$   
 → convol. with VM wave function  $\psi_{q\bar{q}}^V$ , non-relat. limit for  $J/\psi$  ok
- ‘Colourless’ strong interaction in LO QCD:  $\mathcal{A}_{q\bar{q}+p} \sim$  two gluon exchange:

$$\frac{d\sigma}{dt} (\gamma^* p \rightarrow V p) \Big|_{t=0} = \frac{\Gamma_{ee}^V M_V^3 \pi^3}{48\alpha} \frac{\alpha_s(\overline{Q}^2)^2}{\overline{Q}^8} [x g(x, \overline{Q}^2)]^2 \left(1 + \frac{Q^2}{M_V^2}\right)$$

with the *effective scale*  $\overline{Q}^2 = (Q^2 + M_V^2)/4$ .

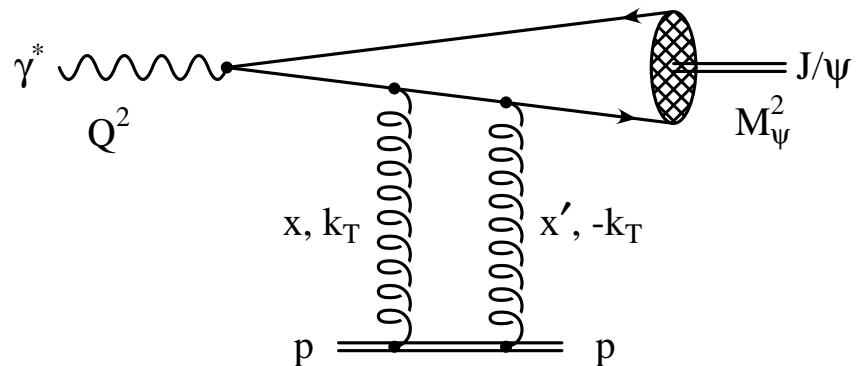
## Corrections beyond the leading $\ln 1/x$ limit:

- $x \neq x' \rightsquigarrow \text{skewing}$ ,  $\mathcal{A} \sim \text{generalized PDF.}$

Correction factor à la Shubaev:

$$R_g = \frac{H_g(x, x' \ll x)}{H_g(x, x)} = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda + \frac{5}{2})}{\Gamma(\lambda + 4)},$$

$\lambda(Q^2)$  the *effective power* of the gluon  $xg \sim x^{-\lambda}$ .



- Contributions from the **real part** of the amplitude through  $\text{Re } \mathcal{A} = \tan(\pi\lambda/2) \text{Im } \mathcal{A}$ .  
(Crossing symmetry + power behaviour  $\text{Im } \mathcal{A} \sim s^\lambda$ .)

## NLO corrections beyond the leading $\ln Q^2$ limit:

$k_T$  factorization using unintegrated gluon  $f(x, k_T^2)$

- In the LLA formula the  $k_T$  of the gluons is neglected ( $k_T^2 \ll \bar{Q}^2$ ):

$$\mathcal{A}^{\text{LLA}} \sim \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} \int^{\bar{Q}^2} \frac{dk_T^2}{k_T^2} f(x, k_T^2) = \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} x g(x, \bar{Q}^2)$$

- Effect numerically important  $\rightsquigarrow$  large corrections in DGLAP when going to (N)NLO
- Use  $k_T$  factorized amplitude:

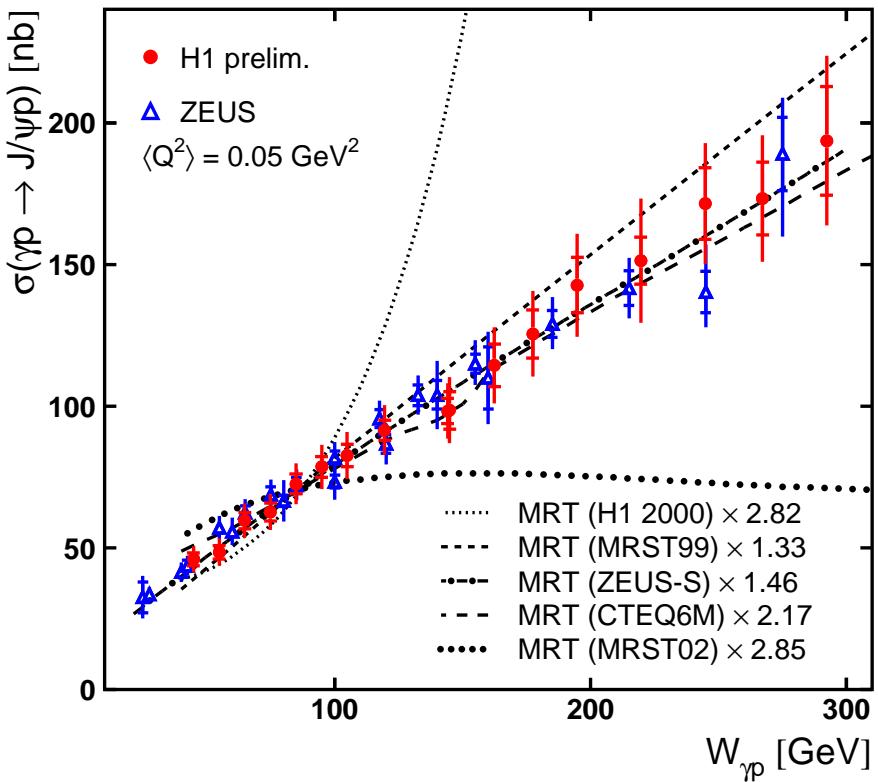
$$\mathcal{A}^{\text{'NLO'}} \sim \frac{\alpha_s(Q_0^2) x g(x, Q_0^2)}{\bar{Q}^4} + \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^2} \int_{Q_0^2}^{\infty} \frac{dk_T^2}{\bar{Q}^2 + k_T^2} \frac{\partial x g(x, k_T^2)}{\partial k_T^2},$$

with small sensitivity on infrared transition parameter  $Q_0^2$  for  $J/\psi$ .

- Small effect when including Sudakov suppression:  $f(x, k_T^2) = \left. \frac{\partial [x g(x, q_0^2) T(q_0^2, \mu^2)]}{\partial \ln q_0^2} \right|_{q_0^2=k_T^2}$   
 with Sudakov factor  $T = \exp\left[\frac{-C_A \alpha_s(\mu^2)}{4\pi} \ln^2 \frac{\mu^2}{q_0^2}\right]$  resums virtual corrections  $\sim$  probability  
 for no gluon emission in the interval  $q_0^2 \dots \mu^2 \sim \bar{Q}^2$ .

# H1 and ZEUS data compared to MRT predictions; gluon fit strategy

Plot thanks to Philipp Fleischmann (H1)



- MRT predictions for VM production based on pQCD able to describe large body of data; however:
  - Huge uncertainty due to input gluon pdf!
  - And: Data much more precise now.
- ~~ This study: use data to *determine* gluon at small  $x$  and low–medium scales via  $\chi^2_{\min}$  fit.

- Use of theory described above and a *simple gluon ansatz* with three free parameters:

$$xg(x, \mu^2) = N \cdot x^{a - b \ln(\mu^2 / 0.45 \text{ GeV}^2)}$$

(form used successfully by Martin+Ryskin+Watt, EPJC37,285), ‘QCD anal. of diffr. DIS data’

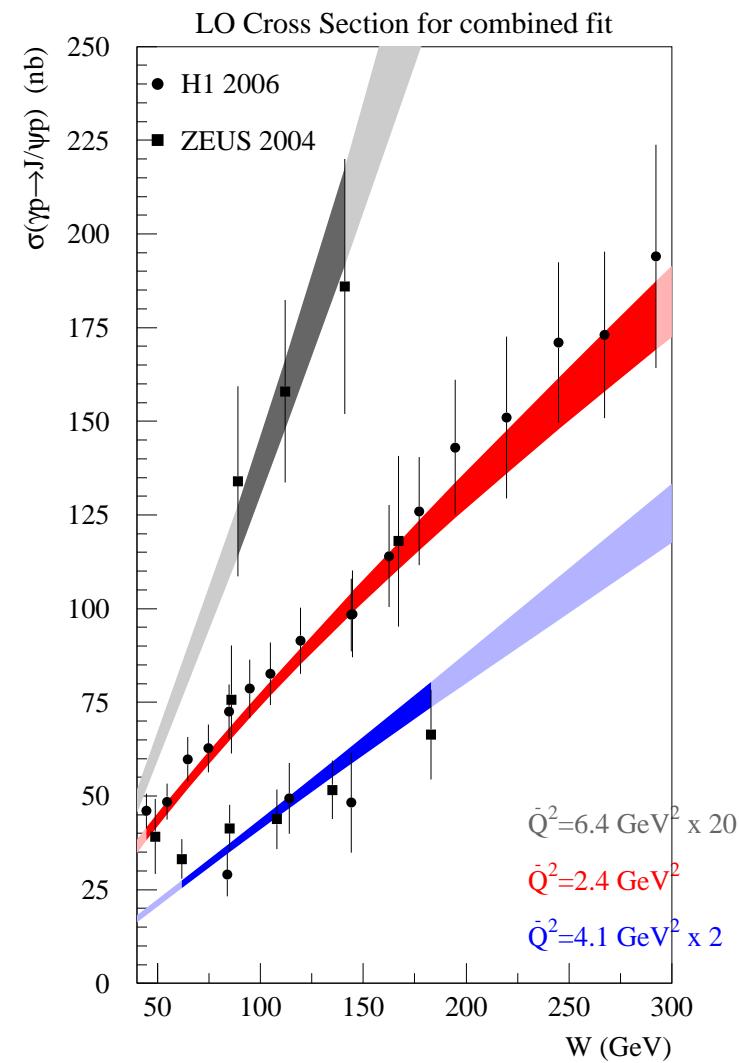
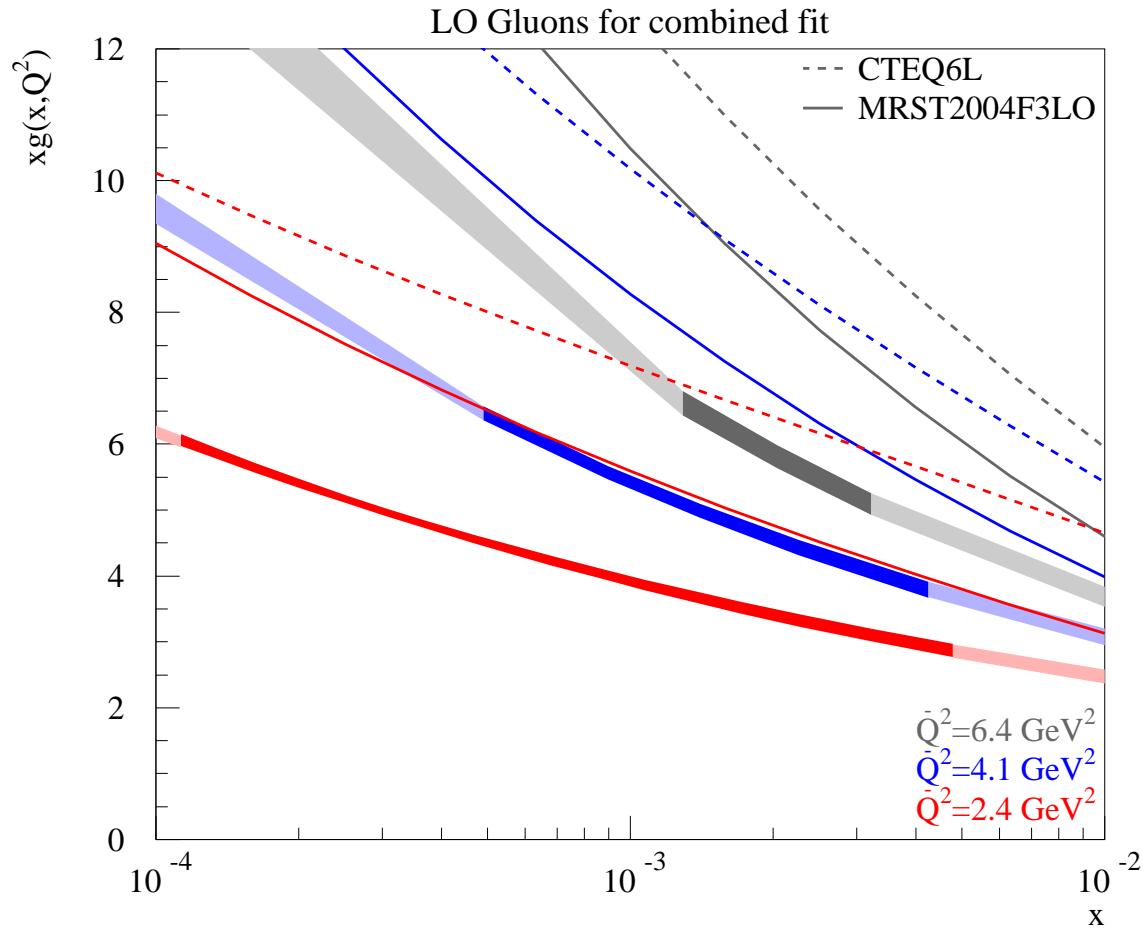
### III. Determining the small $x$ gluon from diffractive HERA data

LO combined fit of H1 and ZEUS  $J/\psi$  data:

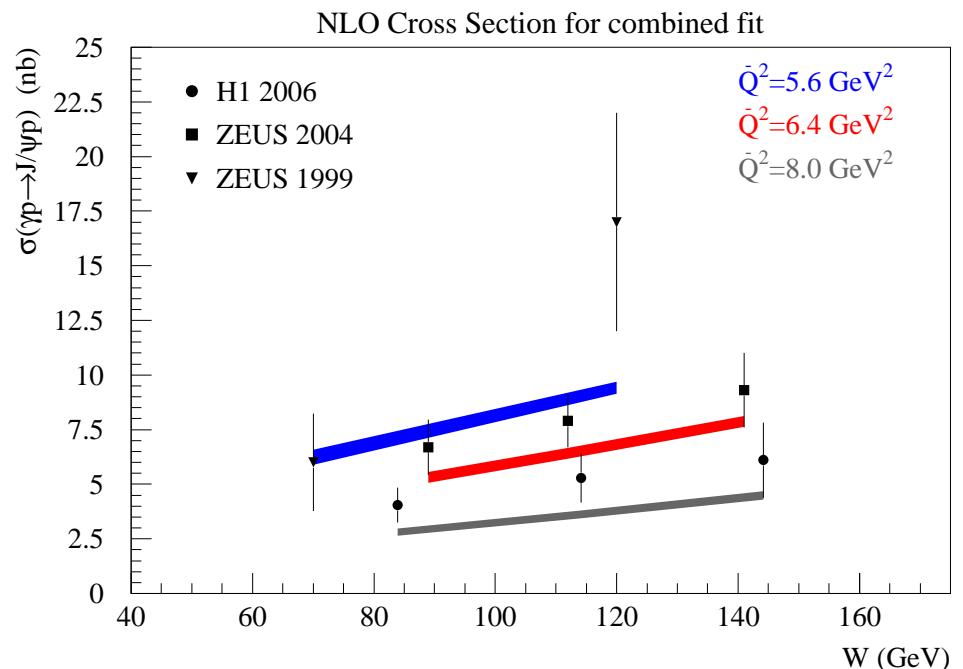
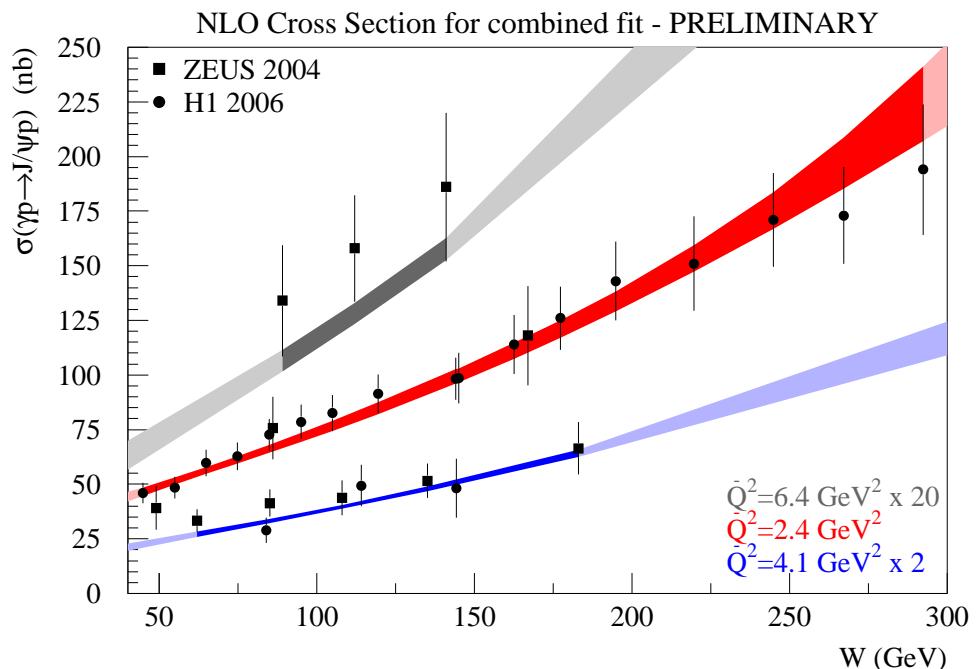
$$\chi^2_{\text{min}}/(d.o.f. = 48) = 0.86$$

$$N = 0.99 \pm 0.09, a = -0.051 \pm 0.012, b = 0.088 \pm 0.005$$

(Error bands including correlations)



# 'NLO' combined fit of H1 and ZEUS $J/\psi$ data: Cross sections



- Good overall fit to available  $J/\psi$  data:  $\chi^2_{\min}/(d.o.f. = 48) = 1.1$
- However tendency to slightly undershoot higher- $Q^2$  data, which are statistically less significant.

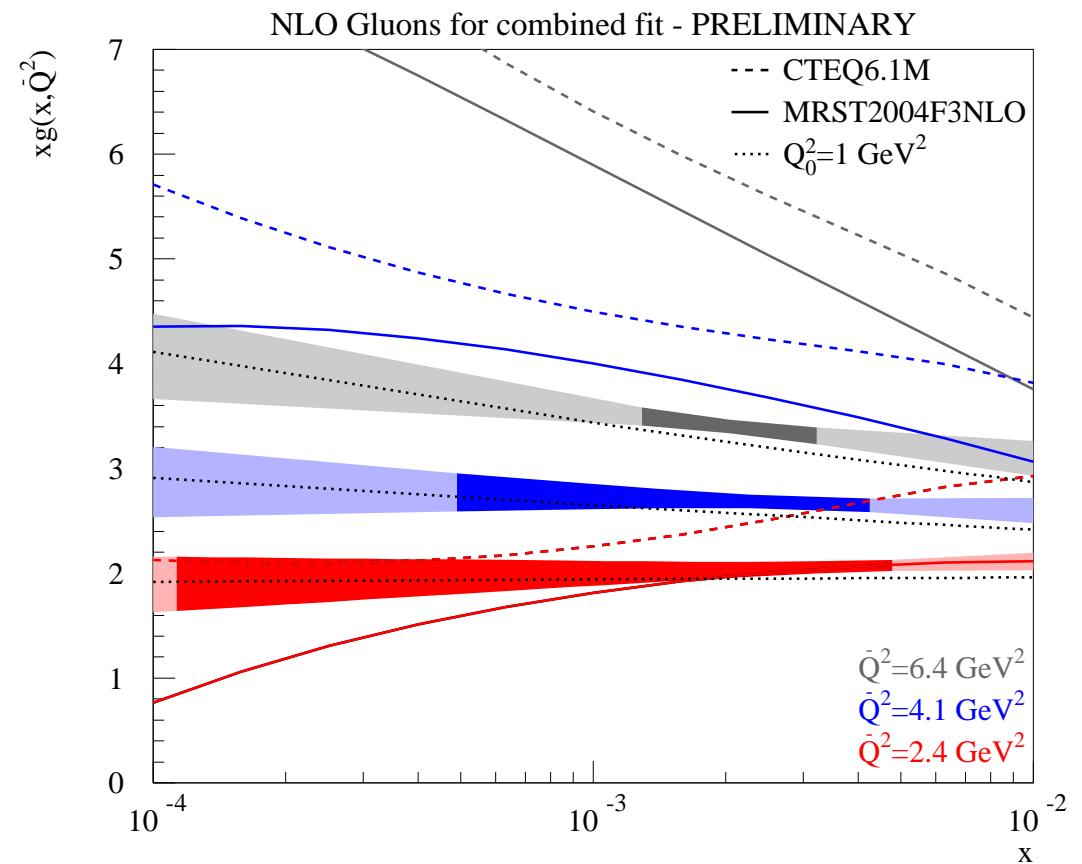
# 'NLO' combined fit of H1 and ZEUS $J/\psi$ data: Gluons

$$N = 2.01 \pm 0.42, a = 0.15 \pm 0.04, b = 0.085 \pm 0.005$$

- Considerable less 'evolution' compared to LO or global fit gluons
- $k_T$ -factorization using *unintegrated* gluons:

$$\mathcal{A} \sim \int_{Q_0^2}^{\infty} dk_T^2 \frac{f(x, k_T^2)}{\bar{Q}^2 + k_T^2}$$

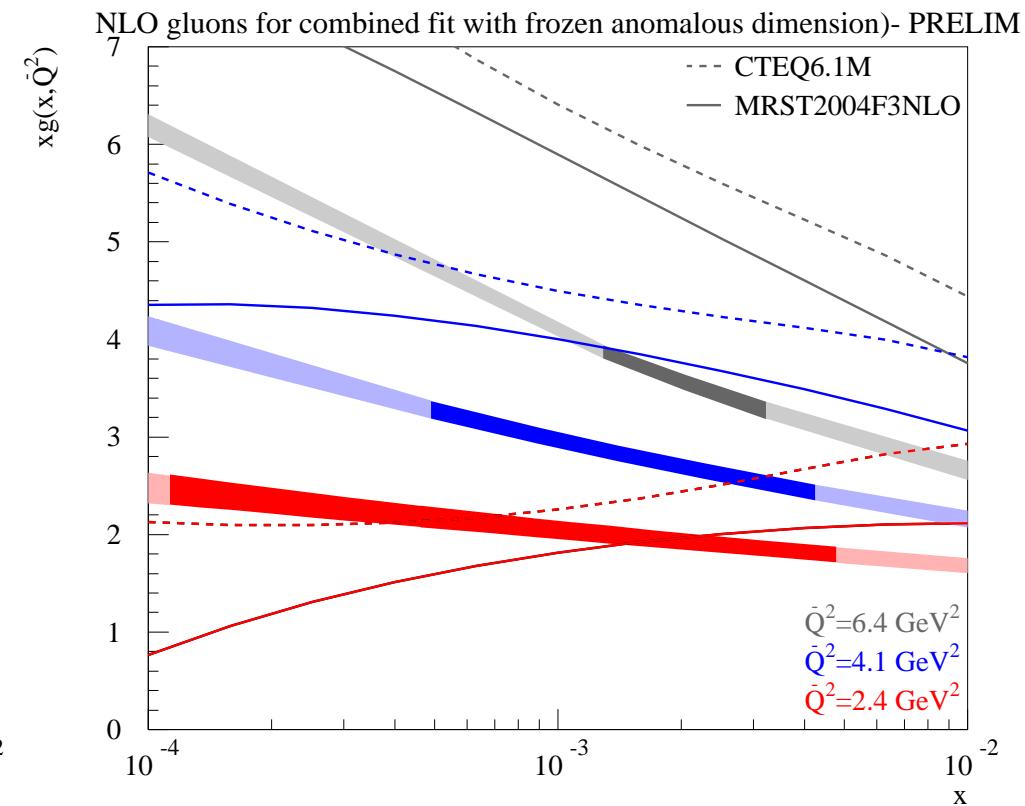
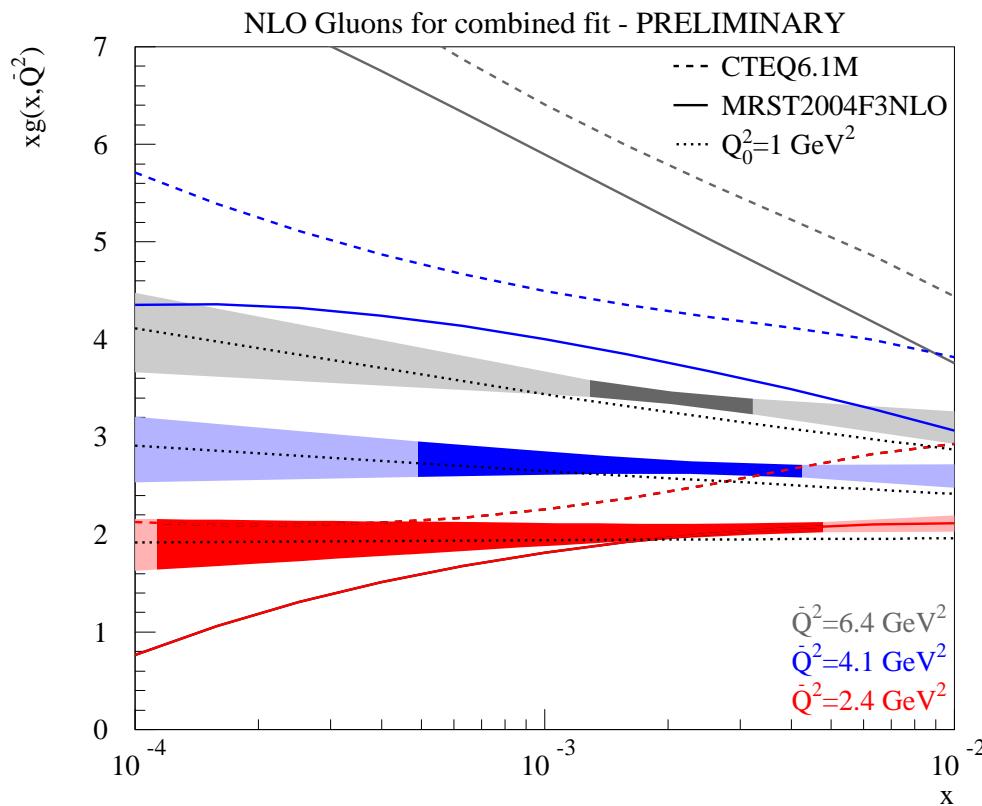
→ significant large-scale contributions due to anomalous dimension rising with  $k_T^2$ .



- In standard DGLAP fits based on collinear factorization strong  $k_T$  ordering and no such large scale contributions from rising anomalous dimension, but corrections captured order by order through coefficient function.

## NLO Gluons

- ‘NLO’ gluon can be used in calculations based on  $k_T$  factorization.
- For applications within DGLAP/collinear factorization framework: mimick DGLAP setup by *freezing* the anomalous dimension of the gluon above scale  $k_T^2 > M_J^2 + Q^2$ :

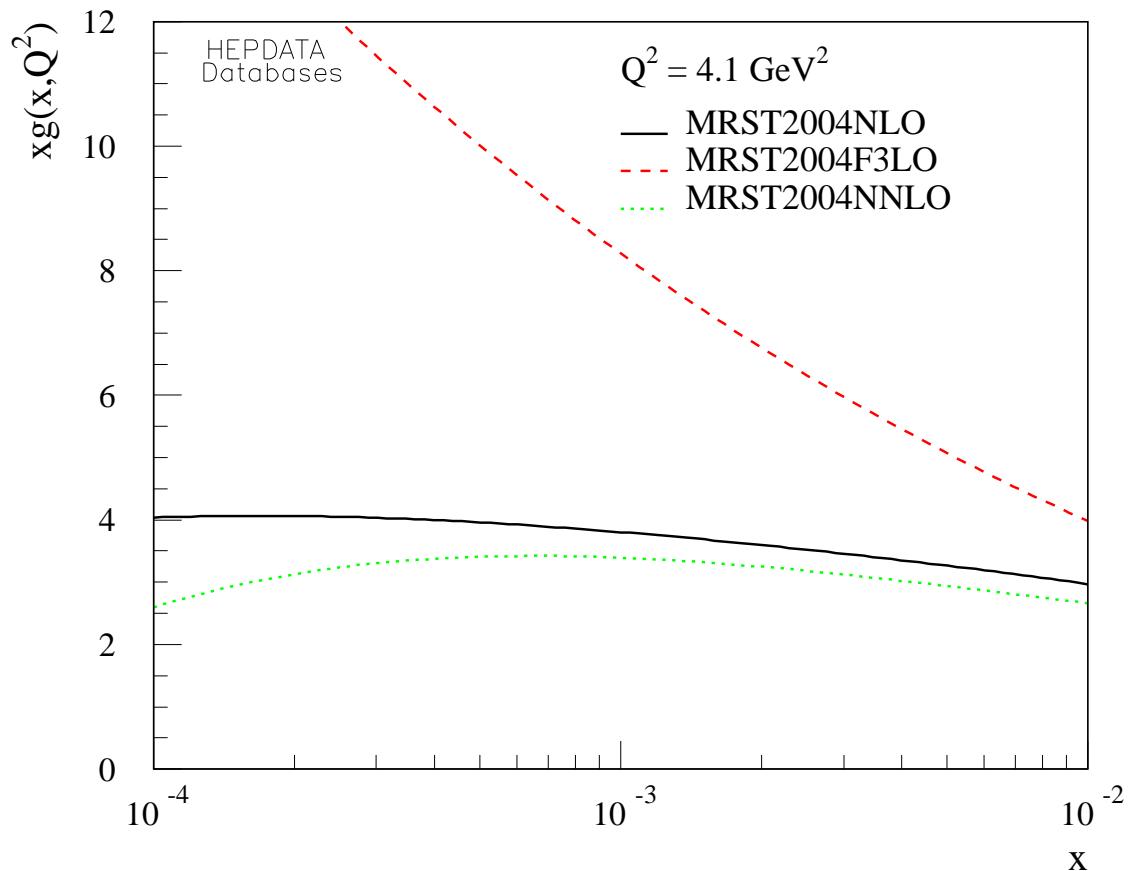
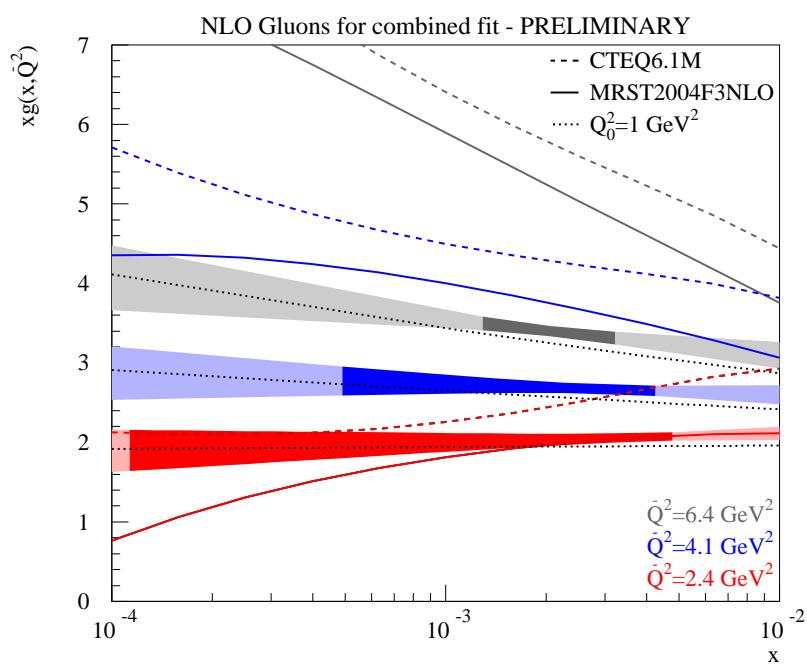


$$\{N, a, b\} : 2.01 \pm 0.42, 0.15 \pm 0.04, 0.085 \pm 0.005$$

$$1.14 \pm 0.1, 0.086 \pm 0.02, 0.1 \pm 0.006$$

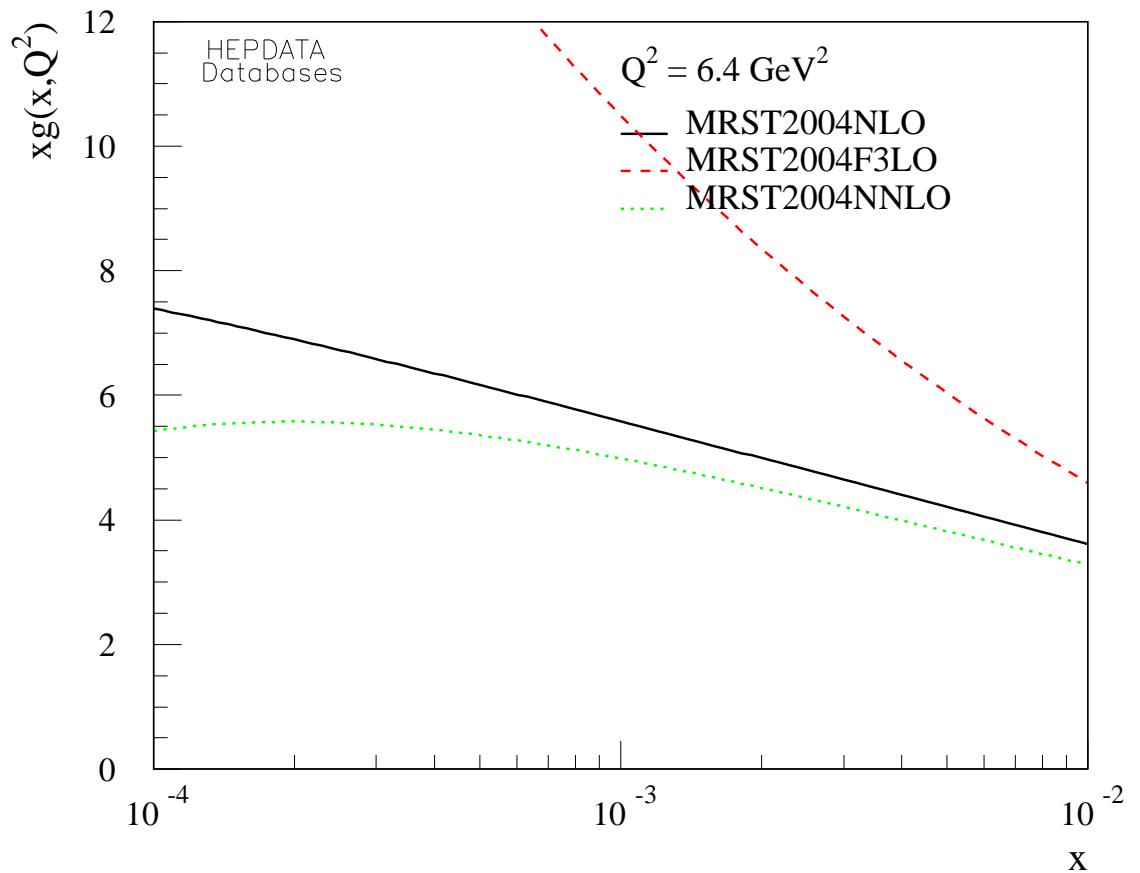
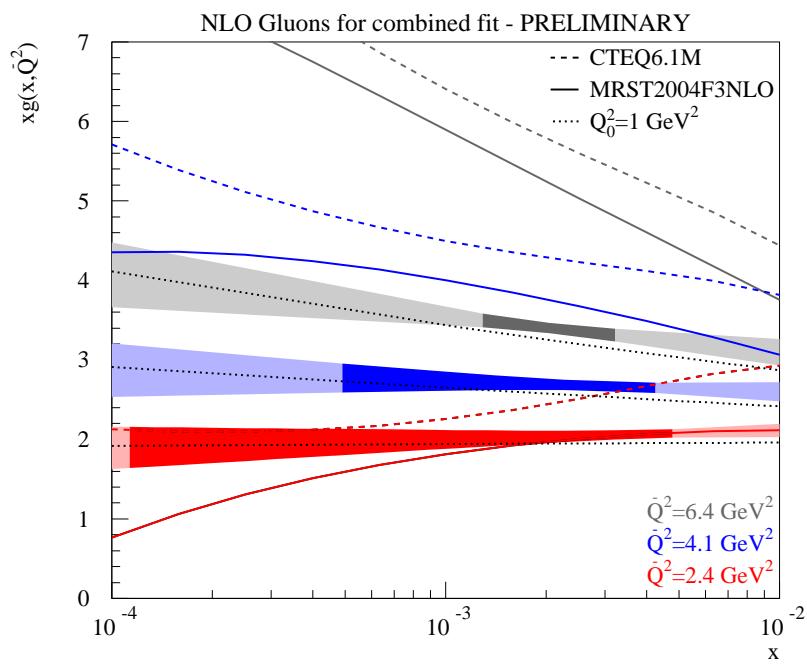
## NLO Gluons

- In DGLAP further flattening of the gluon from NLO to NNLO; we are capturing some of the NNLO corrections



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## IV. Conclusions/Outlook

- Current PDF fits do not constrain the gluon well at small  $x$  and low–medium scales.
- Diffractive Vector Meson production is very sensitive to the gluon in this regime.
- We have used the pQCD approach to determine the LO gluon pdf from exclusive  $J/\psi$  data from H1 and ZEUS.  
Our gluon fit is in fair agreement with global fits at  $x \sim 10^{-2}$  but shows a slightly flatter small  $x$  behaviour.
- Within  $k_T$  factorization a fit of a ‘NLO’ gluon turns out to be less steep compared to DGLAP NLO global fits, as expected from the different framework.
- Gluons applicable for predictions at LO and within  $k_T$  factorization.
- Further studies ongoing to quantify the correlation.  
Our goal is to use diffractive data to better constrain global fit gluons.

## Other NLO corrections beyond the leading $\ln Q^2$ limit:

- Relativistic corrections  $\mathcal{O}(v^2/c^2)$  from the wave function:

If treated properly including higher Fock component  $c\bar{c}g$  states of the  $J/\psi$ , and with use of experimentally measured  $\Gamma_{ee}$ , these corrections were shown to be small,  $\mathcal{O}(4\%)$  [Hoodbhoy]

- $q$  contributions suppressed for small  $x$  production of  $J/\psi$ :

$$\text{estimate: } \mathcal{A}_{\text{sea}} \sim \mathcal{A}_{\text{gluon}} \frac{1}{6} \frac{R_q}{R_g} \frac{\alpha_s}{4\pi}, \quad R_q/R_g \sim 2 - 3$$

- Genuine ‘hard’ NLO corrections to the  $\gamma^* gg \rightarrow VM$  impact factor:

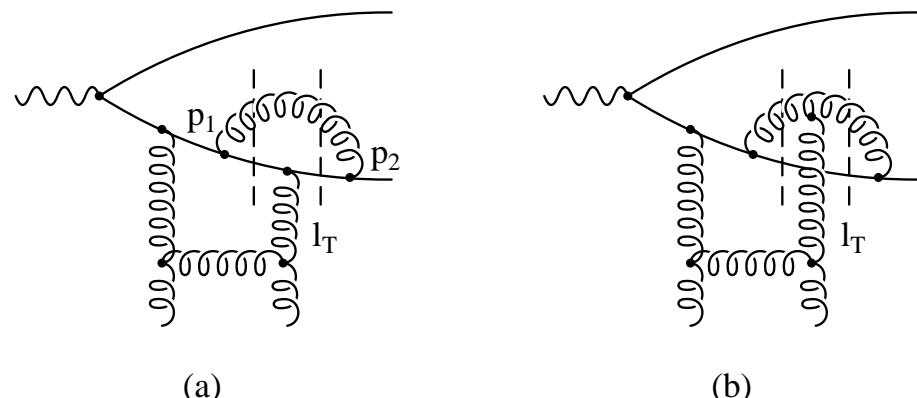
Still not available  $\rightsquigarrow$  additional  $K$  factor  $\rightsquigarrow$  normalization uncertainty

However: large logarithmic contributions captured with  $k_T$  factorization scheme and scale choice.

Some more details of the MRT approach:

## $K$ factor:

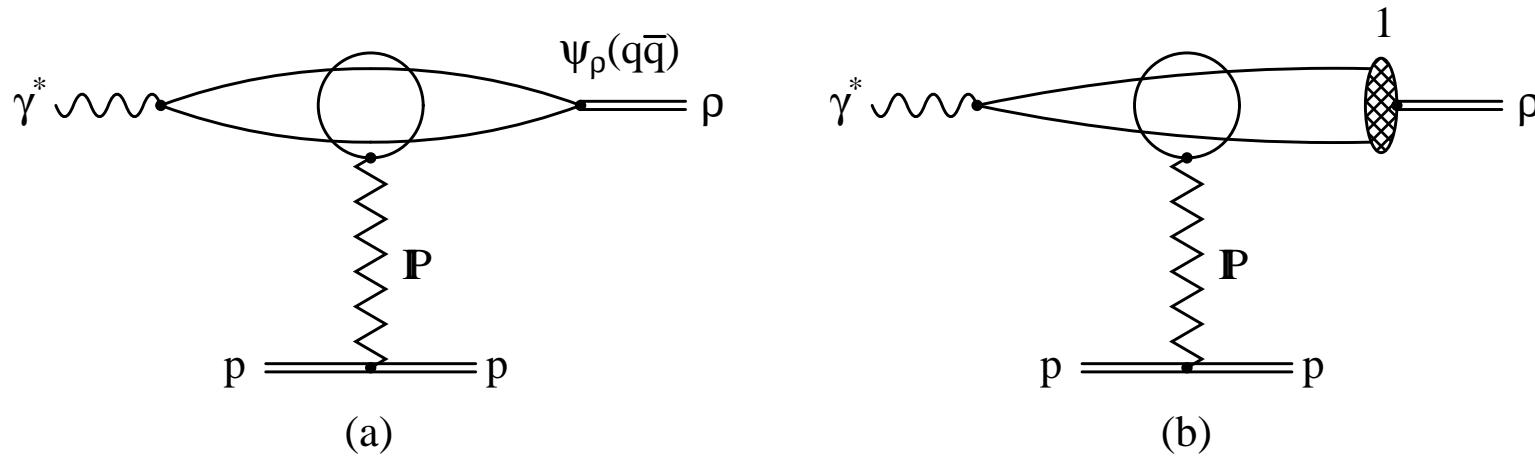
Important missing ingredient for a full NLO prediction: One loop corrections to the  $[(q\bar{q})(2g)]$  vertex



- Typically lead to a significant enhancement in the normalization of QCD processes  $\rightarrow K$  factor (may also be fitted from data)
- Up to now no full calculation within  $k_T$  factorization
- MRT estimate the  $K$  factor from  $\pi^2$  enhanced terms, analogous to the well known corrections in Drell-Yan  $\rightsquigarrow \sigma = \sigma^0 \exp[\pi^2 C_F \alpha_s(..)/\pi].^*$
- First results for diagrammatic calculation for  $\mathcal{A}^L$  by D.Yu. Ivanov et al.

\* Exp. of the double logarithmic Sudakov form factor  $\sim \ln^2(-M^2)$ ,  $\ln(-M^2) = \ln M^2 + i\pi$

Alternative to use of VM wave function: Parton Hadron Duality:



Assumption (case of  $\rho$ ):  $\gamma^* \rightarrow q\bar{q}$  cross section in the region  $M_{q\bar{q}} \sim M_\rho$  saturated by  $\rho$  (up to  $\sim 10\%$  for  $\omega$ ) when integrated over a *suitable* (universal?!) mass interval  $\Delta M$ :

$$\sigma(\gamma^* p \rightarrow \rho p) \simeq 0.9 \sum_{q=u,d} \int_{M_{min}^2}^{M_{max}^2} \frac{d\sigma(\gamma^* p \rightarrow (q\bar{q})p)}{dM^2} dM^2$$

- + Projection of  $q\bar{q}$  state on the correct VM Quantum Numbers  $J^P = 1^-$ .  
(→ Suppression of IR divergencies for contr. from transverse photon!)

## Structure of the calculation à la Martin+Ryskin+Teubner:

Contributions to  $\sigma(\gamma_{L,T}^* p \rightarrow V p)|_{t=0}$  from Re, Im for L, T, numerically ('straightforward', no iterative procedure for effective scales):

$$PHD: \int dM^2 \left[ \text{Projection: } \int dk_T^2 \left( \text{Skewed A's w. K fact.; Re: } \int dl_T^2 \right) \right]^2$$