Saturation Model for exclusive processes at HERA, DVCS, F2, including evolution and impact parameter dependence

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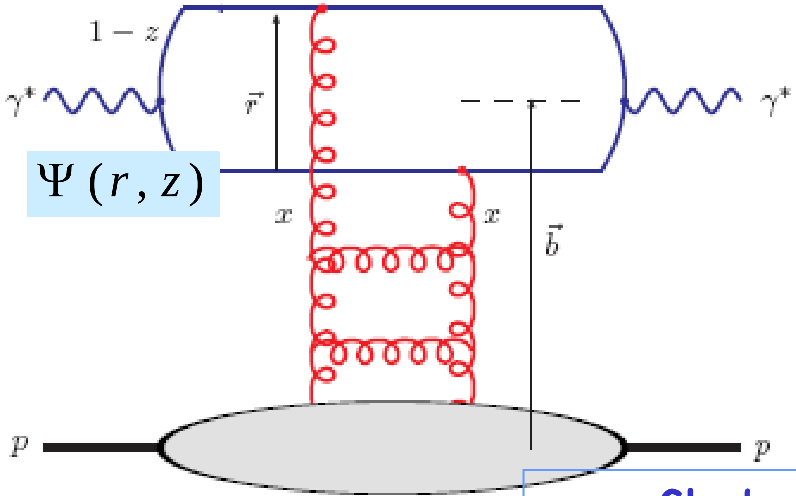
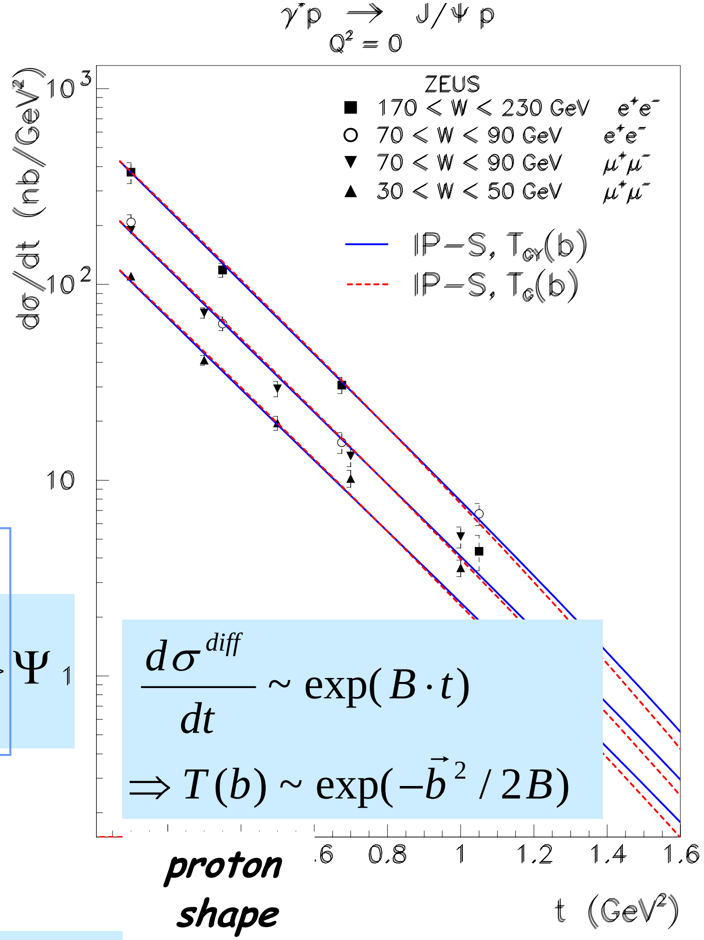
$$\sigma_{tot}^{\gamma p} = \frac{1}{W^2} Im A_{el}(W^2, t=0)$$

Dipole Models

equivalent to LO perturbative QCD for small dipoles

NNPZ, GLM, FKS, GBW, MMS
DGKP, BGBK, IIM, FSS……
KT - Kowalski, Teaney
KMW - Kowalski, Motyka, Watt





Glauber
Mueller

$$\sigma_{tot}^{\gamma^* p} = \int d^2\vec{r} \int_0^1 dz \int d^2 b \Psi^* \cdot 2\left\{1 - \exp(-\frac{\Omega}{2})\right\}\Psi$$

$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)$$

Optical Theorem

$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

$$\Rightarrow T(b) \sim \exp(-\vec{b}^2 / 2B)$$

proton shape

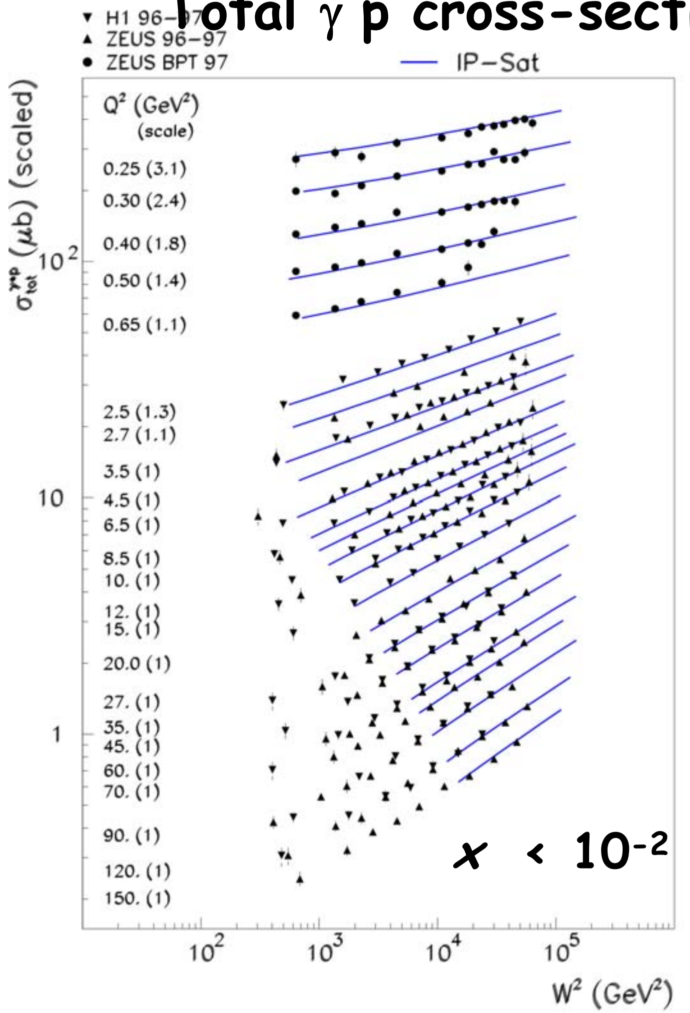
$$\frac{d\sigma_{VM}^{\gamma^* p}}{dt} = \frac{1}{16\pi} \left| \int d^2\vec{r} \int d^2 b e^{-i\vec{b}\cdot\vec{\Delta}} \int_0^1 dz \Psi_{VM}^* \; 2\left\{1 - \exp(-\frac{\Omega}{2})\right\}\Psi \right|^2$$

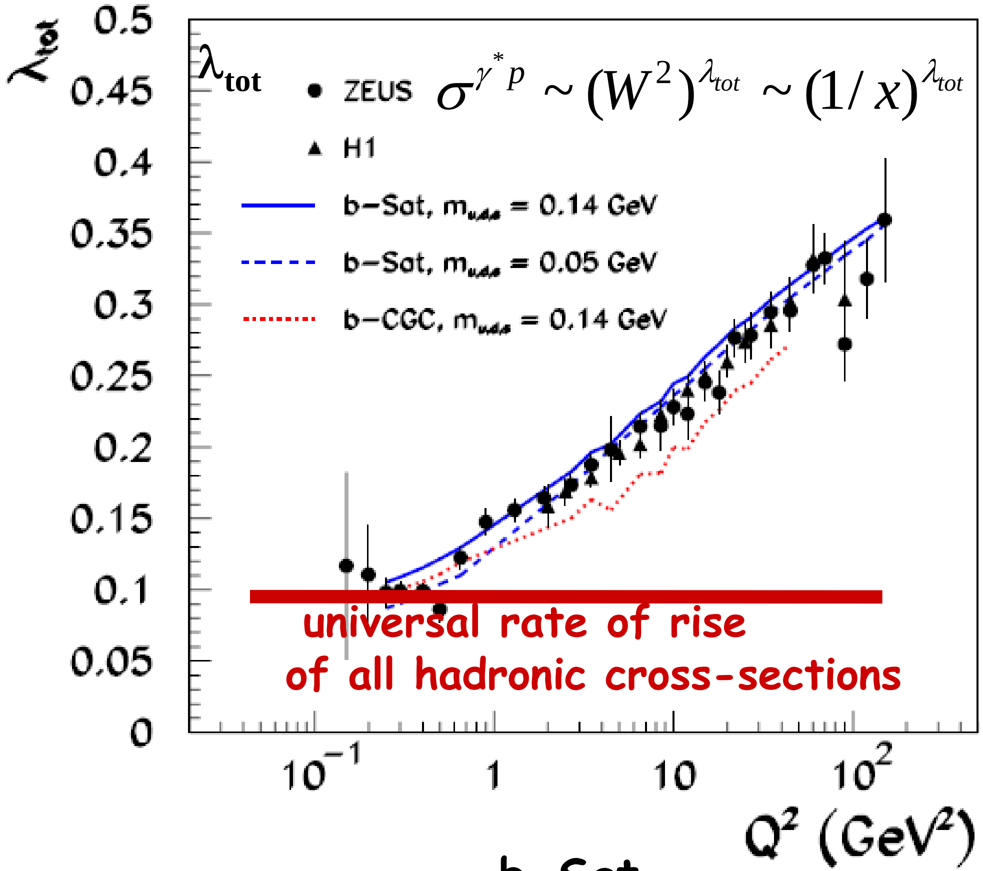
KT

Total γ^*p cross-section

KMW

KT





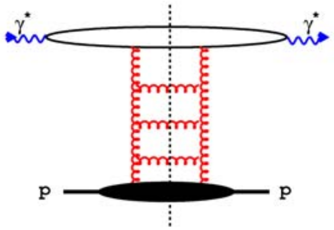
b-Sat

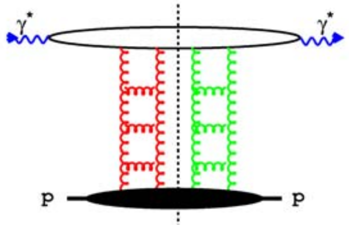
$$\frac{\mathrm{d}\sigma_{q\bar q}}{\mathrm{d}^2\boldsymbol{b}} = 2\left[1-\exp\left(-\frac{\pi^2}{2N_c}r^2\alpha_S(\mu^2)xg(x,\mu^2)T(b)\right)\right] \qquad \mu^2 = \frac{C}{r^2}+\mu_0^2 \qquad xg(x,\mu_0^2) = A_g\left(\frac{1}{x}\right)^{\lambda_g}(1-x)^{5.6}$$

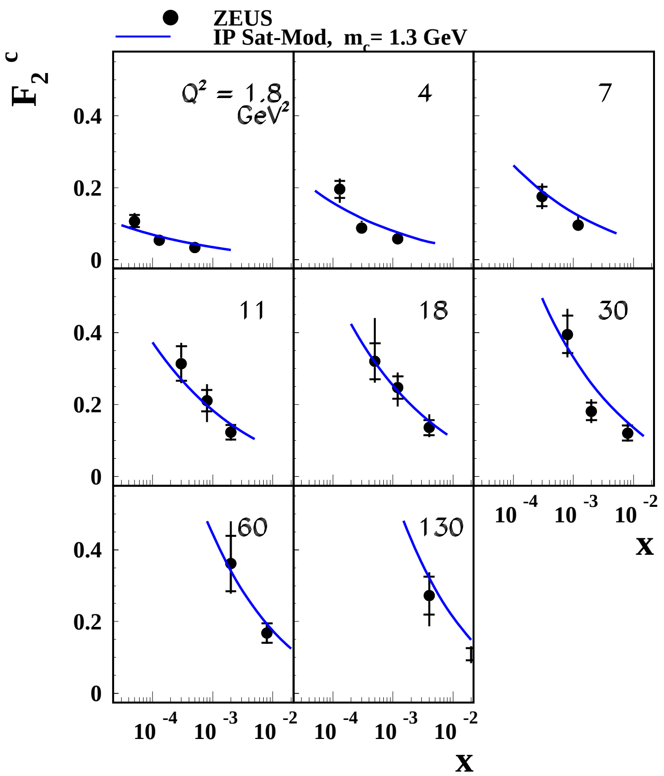
$$\frac{\mathrm{d}\sigma_{q\bar q}}{\mathrm{d}^2\boldsymbol{b}} \equiv 2\mathcal{N}(x,r,b) = 2\times\begin{cases}\mathcal{N}_0\left(\frac{rQ_s}{2}\right)^{2\left(\gamma_s+\frac{1}{\kappa\lambda Y}\ln\frac{2}{rQ_s}\right)} & : \quad rQ_s \le 2 \\ 1-\mathrm{e}^{-A\ln^2(BrQ_s)} & : \quad rQ_s > 2\end{cases}$$

b-CGC

Dipole Model - gluon density convoluted with dipole wave functions simultaneous prediction/description of many reactions

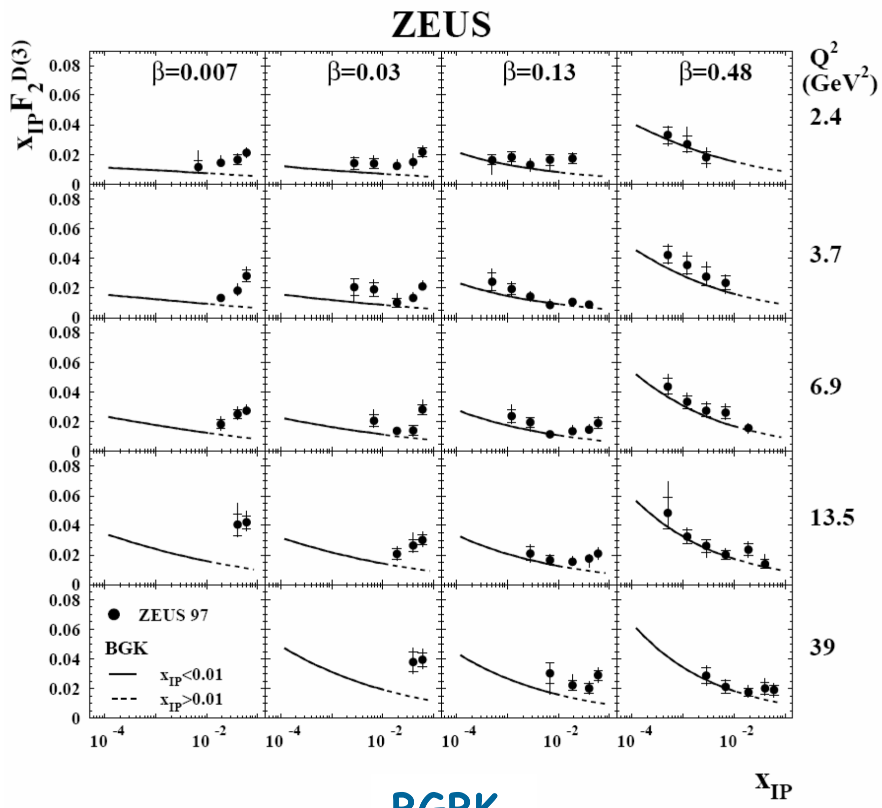






KT

$F_2{}^c$

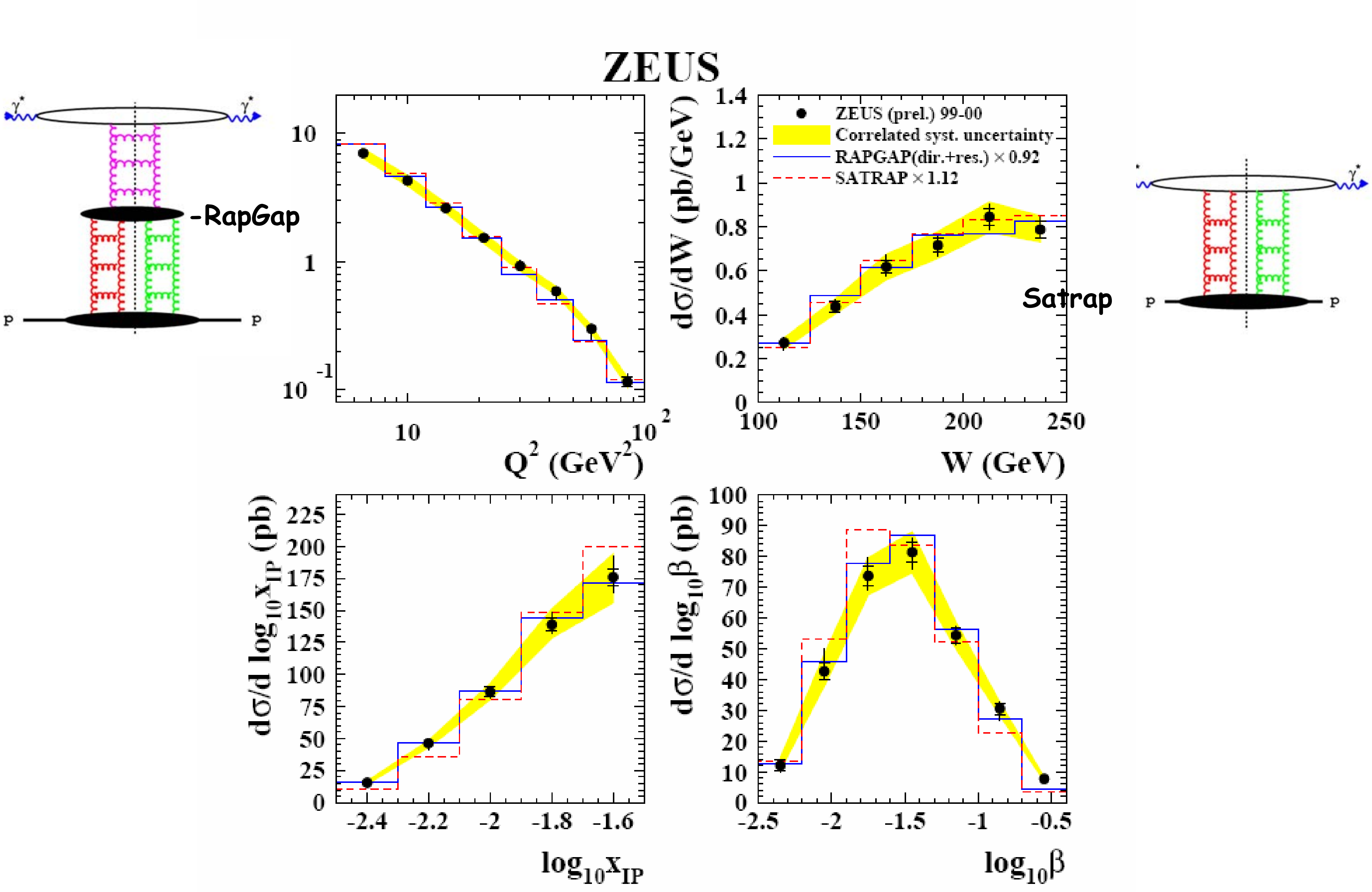


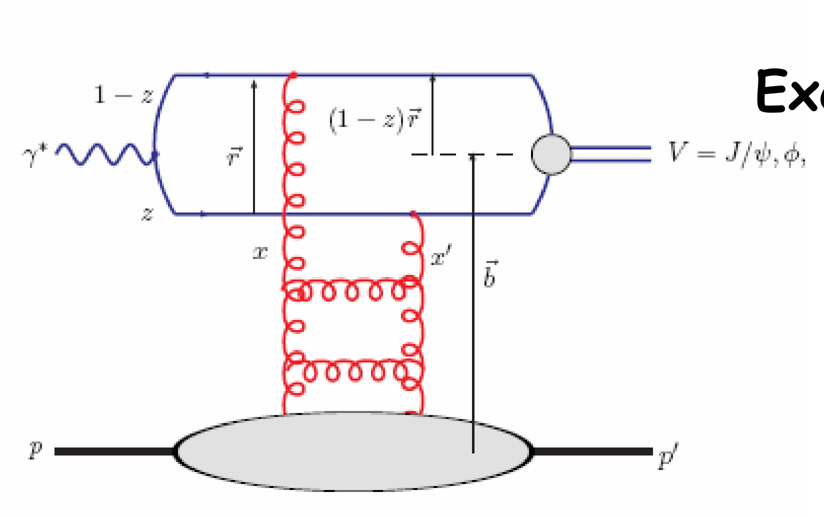
BGBK

Inclusive Diffraction

Diffractive Di-jets

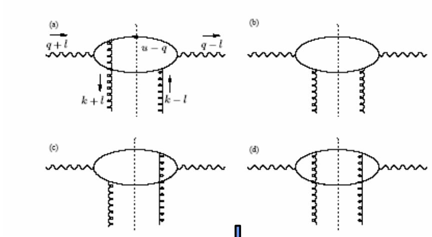
Q^2 > 5 GeV^2





Exclusive Vector Meson Production

H. Kowalski, L. Motyka, G. Watt



Effective modification of Fourier Trans
by Bartels, Golec-Biernat, Peters

$$\frac{\mathrm{d}\sigma_{T,L}^{\gamma^* p\to Vp}}{\mathrm{d}t} = \frac{1}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma^* p\to Vp}\right|^2 = \frac{1}{16\pi}\left|\int \mathrm{d}^2\boldsymbol{r}\int_0^1\frac{\mathrm{d}z}{4\pi}\int \mathrm{d}^2\boldsymbol{b}\;(\Psi_V^*\Psi)_{T,L}\;\mathrm{e}^{-i[\boldsymbol{b}-(1-z)\boldsymbol{r}]\cdot\boldsymbol{\Delta}}\;\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}}\right|^2 (1+\beta^2)$$

$$\beta = \tan(\pi\lambda/2), \quad \text{with} \quad \lambda \equiv \frac{\partial \ln\left(\mathcal{A}_{T,L}^{\gamma^* p\to Vp}\right)}{\partial \ln(1/x)}.$$

Real part correction

$$\boxed{\frac{d\sigma}{d^2 b} = 2(1-\exp(-\Omega/2))}$$

$$\Omega = \frac{\pi^2}{N_C} r^2 \alpha_s(\mu^2)\, x g(x,\mu^2) R_g T(b)$$

Skewedness correction
Martin, Ryskin
Teubner

$$R_g(\lambda) = \frac{2^{2\lambda+3}}{\sqrt{\pi}}\frac{\Gamma(\lambda+5/2)}{\Gamma(\lambda+4)}, \quad \text{with} \quad \lambda \equiv \frac{\partial \ln[xg(x,\mu^2)]}{\partial \ln(1/x)}.$$

Wave Functions

WF Overlaps

$$(\Psi_V^*\Psi)_T = \hat{e}_f e \frac{N_C}{\pi z(1-z)} \left\{ m_f^2 K_0(\epsilon r)\phi_T(r,z) - \left[z^2+(1-z)^2\right]\epsilon K_1(\epsilon r)\partial_r\phi_T(r,z) \right\}$$

$$(\Psi_V^*\Psi)_L = \hat{e}_f e \frac{N_C}{\pi M_V} 2QK_0(\epsilon r)\left\{\left[z(1-z)M_V^2+m_f^2\right]\phi_L(r,z) - \nabla_r^2\phi_L(r,z)\right\},$$

Boosted Gaussian – NNPZ, FKS, FS

Gaussian distribution of quark 3-momentum in the meson rest frame

then boosted to LC

$$p^2 = \frac{k^2+m_f^2}{4z(1-z)} - m_f^2$$

$$\phi_{T,L}(r,z) = \mathcal{N}_{T,L} 4z(1-z)\sqrt{2\pi\mathcal{R}^2}\exp\left(-\frac{m_f^2\mathcal{R}^2}{8z(1-z)} - \frac{2z(1-z)r^2}{\mathcal{R}^2} + \frac{m_f^2\mathcal{R}^2}{2}\right)$$

Gauss LC - KT

Gaussian distribution of quark 2-momentum in LC, factorization of r, z components

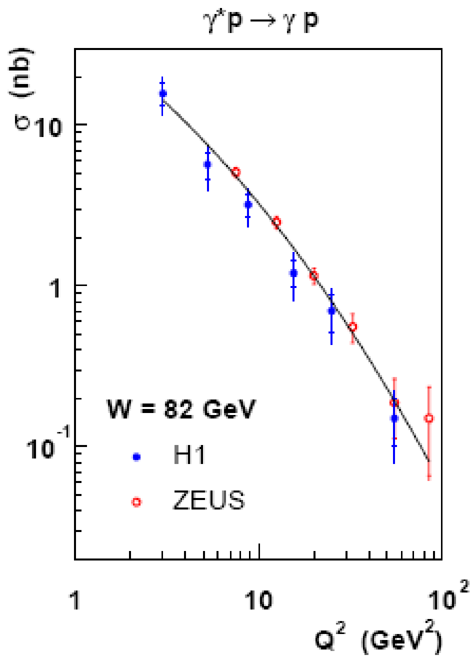
- strong endpoint suppression in ϕ_T

$$\varphi_L(r,z) = \frac{N}{2\pi R_L^2} z(1-z)\exp\left(-\frac{r^2}{2R_L^2}\right) \qquad \varphi_T(r,z) = \frac{N}{2\pi R_T^2} z^2(1-z)^2\exp\left(-\frac{r^2}{2R_T^2}\right)$$

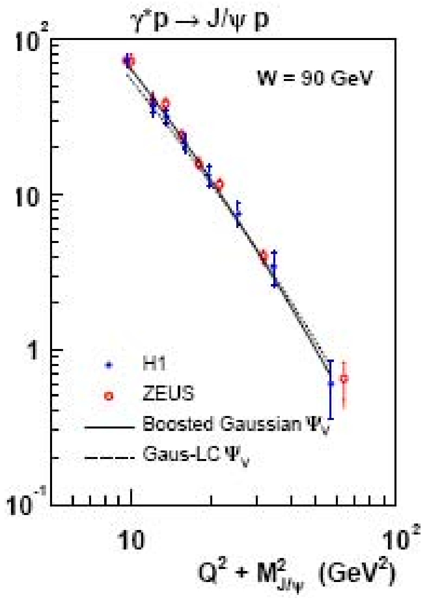
Parameters of WF fixed by normalization conditions and the values of mesons decay constant, f_V

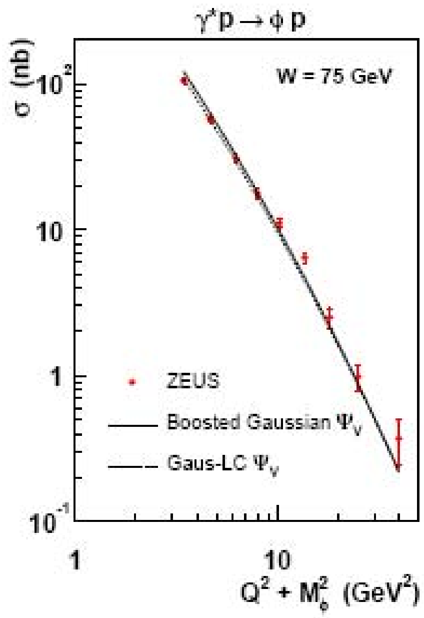
from gluon density convoluted with dipole wave functions we obtain simultaneous prediction/description of many reactions

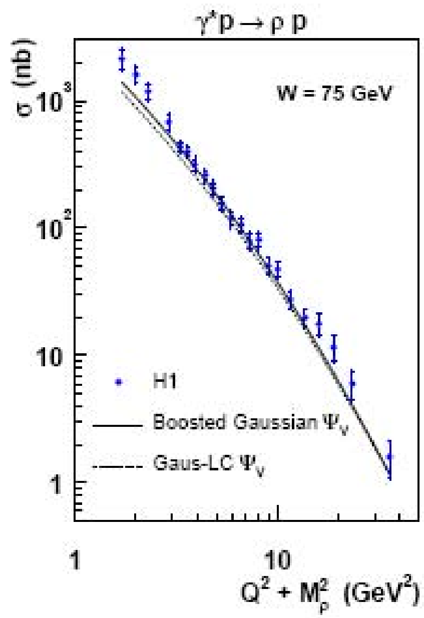
DVCS



Vector Mesons



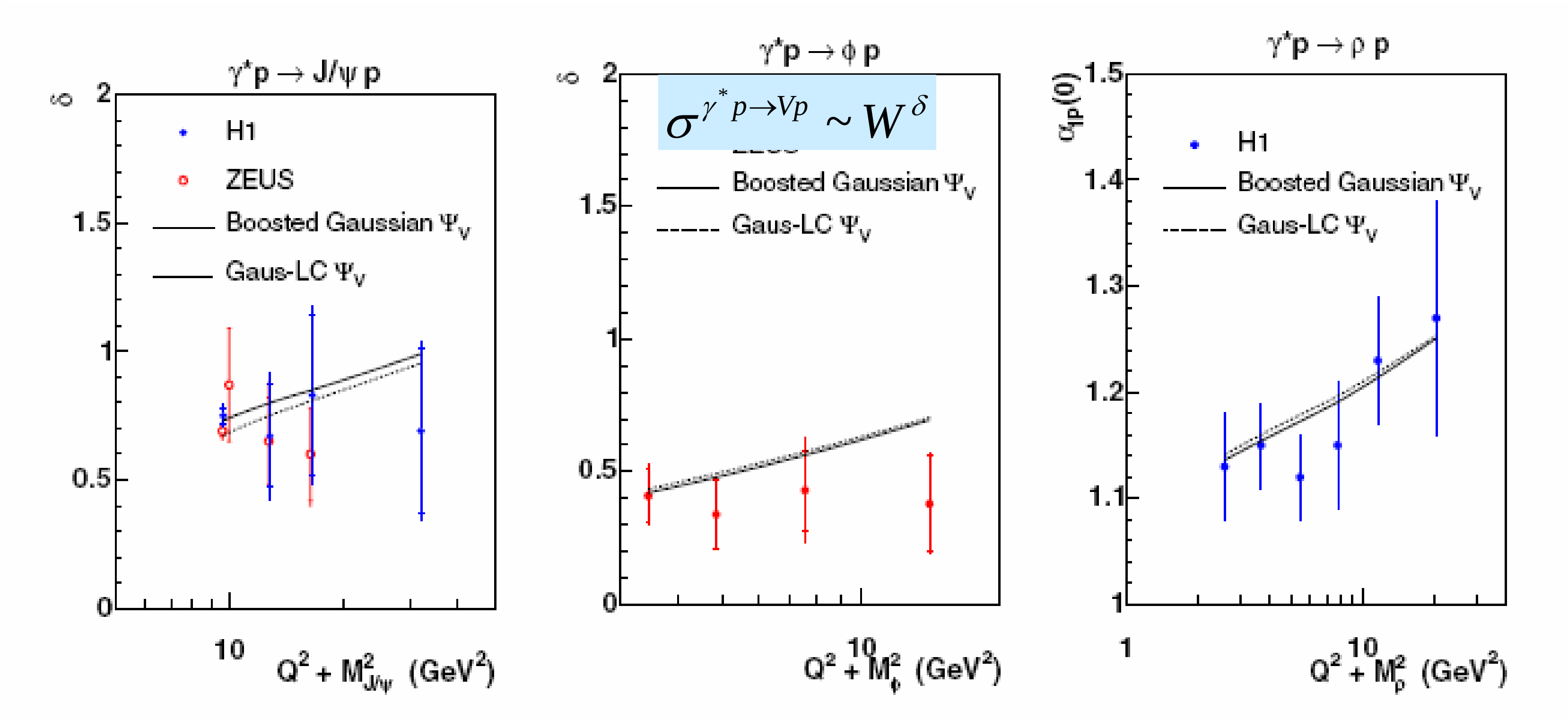




Note: educated guesses for VM wf work surprisingly well

evolution of a saturated gluon density

Rates of rise of the VM cross-sections

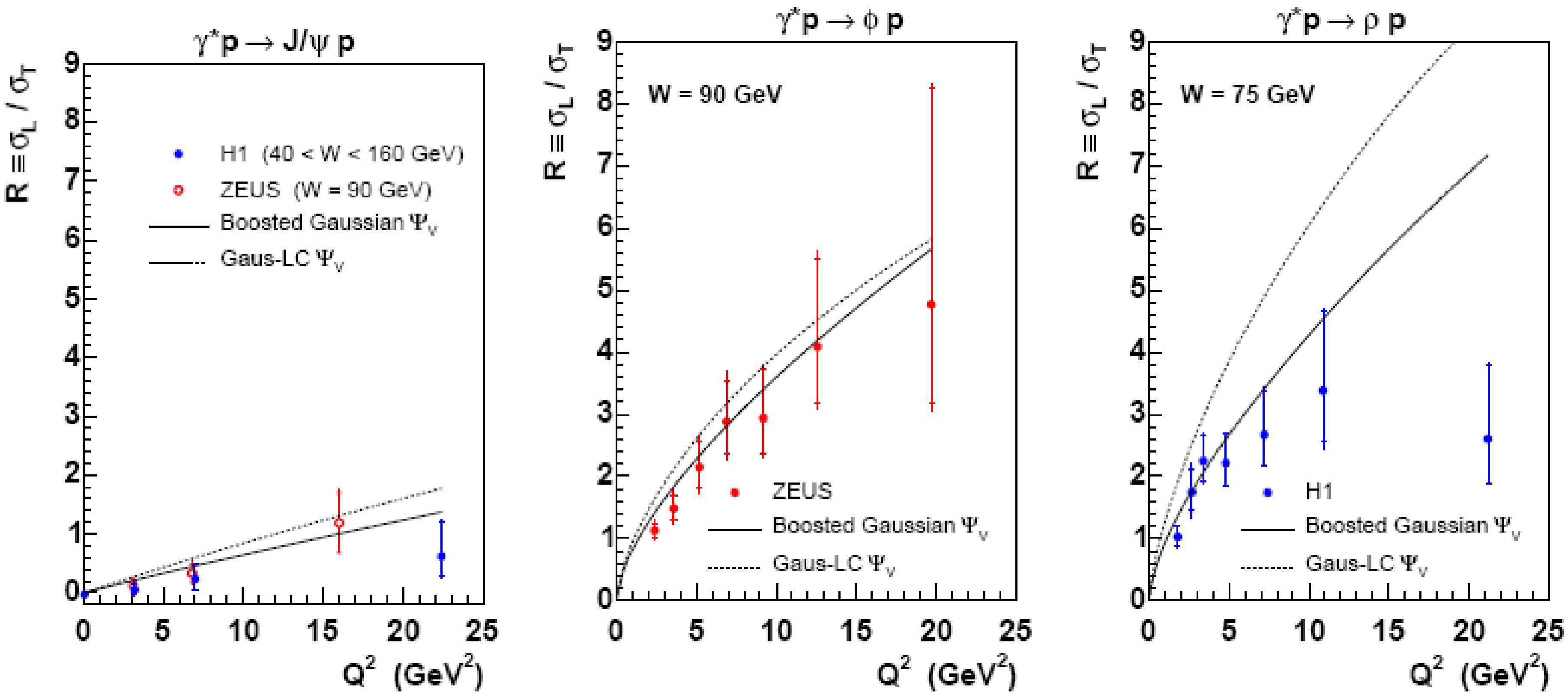


At EIC it should be possible to reduce the errors by a large factor, $O(100)$, ➔ study of t-dependent rate of rise
➔
study of b-dependent Pomeron evolution
- direct insight into saturation inside the proton or nuclei

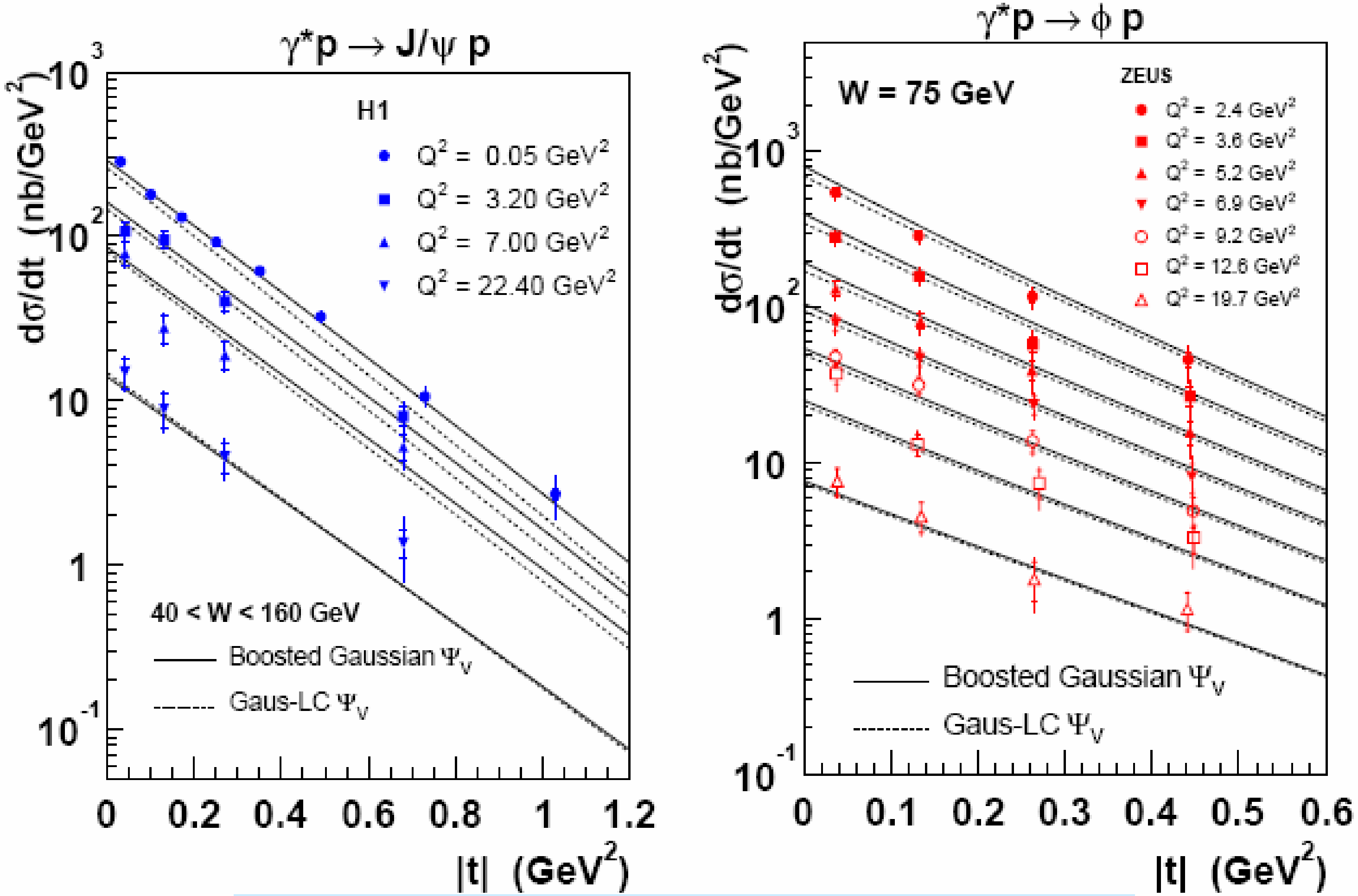
KMW

properties of wave functions

Ration of longitudinal/transverse x-sections



Exponential fall of t-distributions



$$\frac{d\sigma^{diff}}{dt} \sim \exp(B_D \cdot t) \qquad \Rightarrow T(b) \sim \exp(-\vec{b}^{\,2} / 2B_G)$$

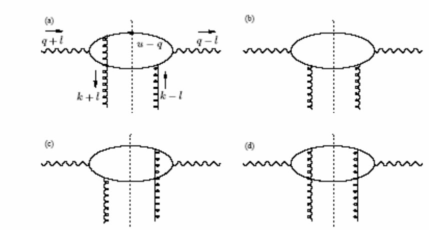
➔ **gaussian shape of the proton in the impact parameter b**

Description of the size of interaction region B_D

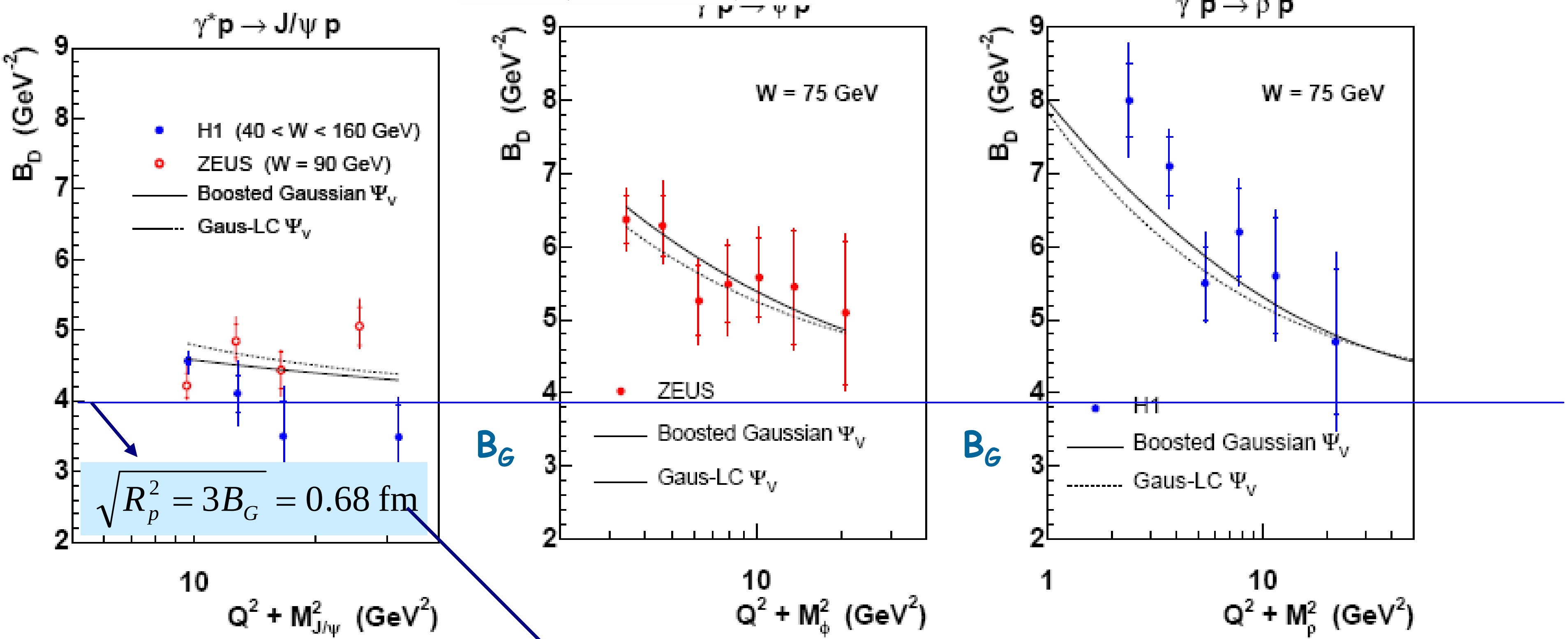
$$\frac{d\sigma^{diff}}{dt} \sim \exp(B_D \cdot t) \qquad \Rightarrow T(b) \sim \exp(-\vec{b}^{\,2}/2B_G)$$

Modification by Bartels, Golec-Biernat, Peters

$$e^{i\vec{b}\cdot\vec{\Delta}} \to e^{i(\vec{b}+(1-z)\vec{r})\cdot\vec{\Delta}}$$



KMW

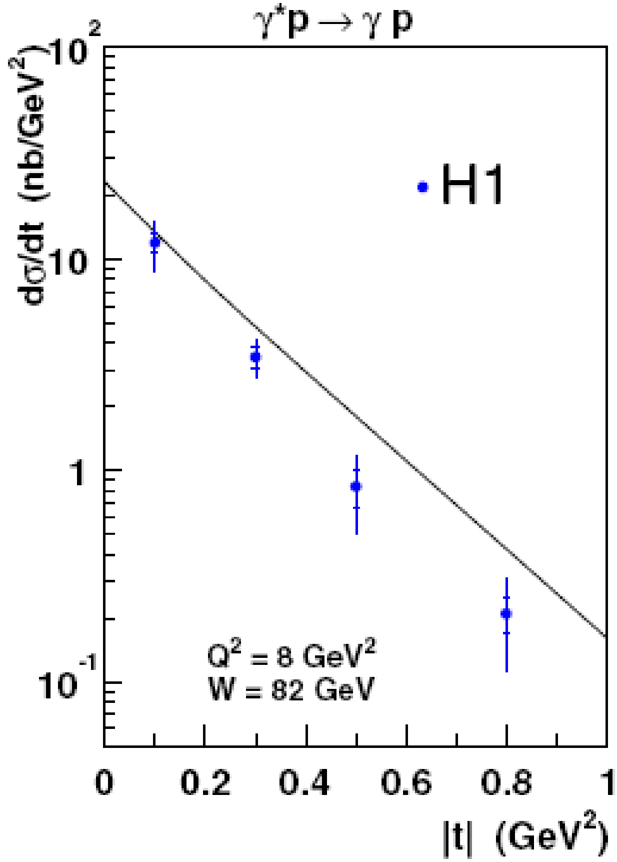


$$\sqrt{R_p^2 = 3B_G} = 0.68\ \mathrm{fm}$$

$$R_p = 0.870 \pm 0.008\ \mathrm{fm}$$
$$\Rightarrow B_G = 6.48\ \mathrm{GeV}^2$$

the gluonic proton radius smaller than the quark radius

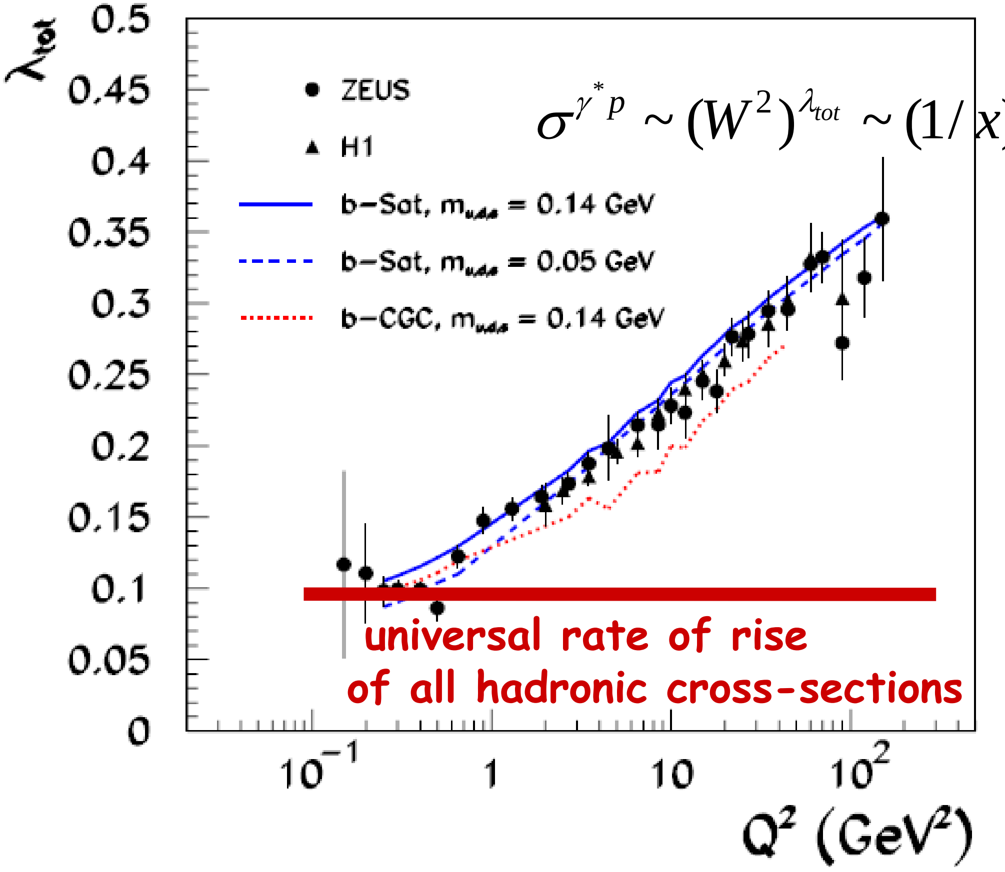
t-distributions of DVCS



What have we learnt from HERA about small-x

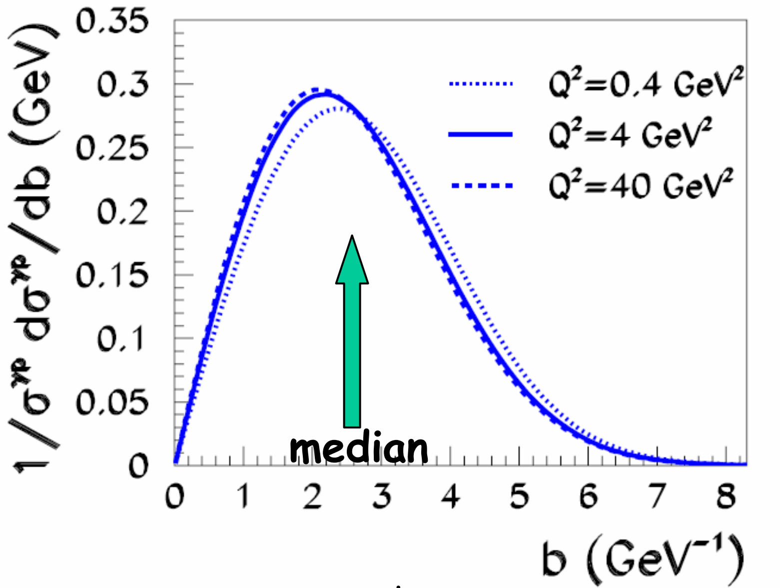
Rate of rise of the $\gamma^* p$ cross-section

$$\sigma^{\gamma^* p} \sim (W^2)^{\lambda_{tot}} \sim (1/x)^{\lambda_{tot}}$$



In the impact parameter dipole models of DIS with the gaussian proton shape the fit of the rate of rise of $\sigma^{\gamma^* p}$ requires QCD evolution which is DGLAP-like

DGLAP-like ←→ strong interdependence between λ and Q^2



Large fraction of $\sigma^{\gamma^* p}$ comes from the region of large *b* where matter density is low $T(b) \sim \exp(-b^2_{MEDIAN} / 2 \cdot B_G) \approx 40\%$

Saturation scale

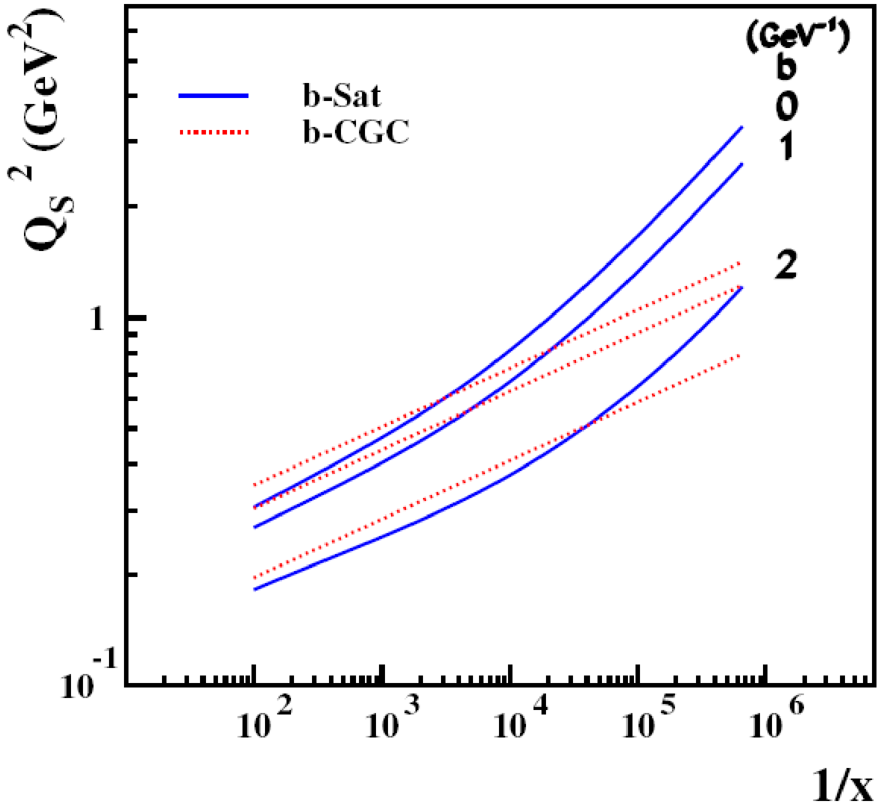
(a measure of gluon density at which gluon re-scattering starts to be substantial)

$$Q_S^2 = \frac{2}{r_S^2}$$

$$\frac{d\sigma_{qq}(x,r)}{d^2b} = 2\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3} r^2 \alpha_s x g(x, C/r^2 + Q_0^2) T(b)\right)\right\}$$

$$(Q_S^2)_g = \frac{N_C}{C_F}(Q_S^2)_q = \frac{9}{4}(Q_S^2)_q$$

HERA



RHIC

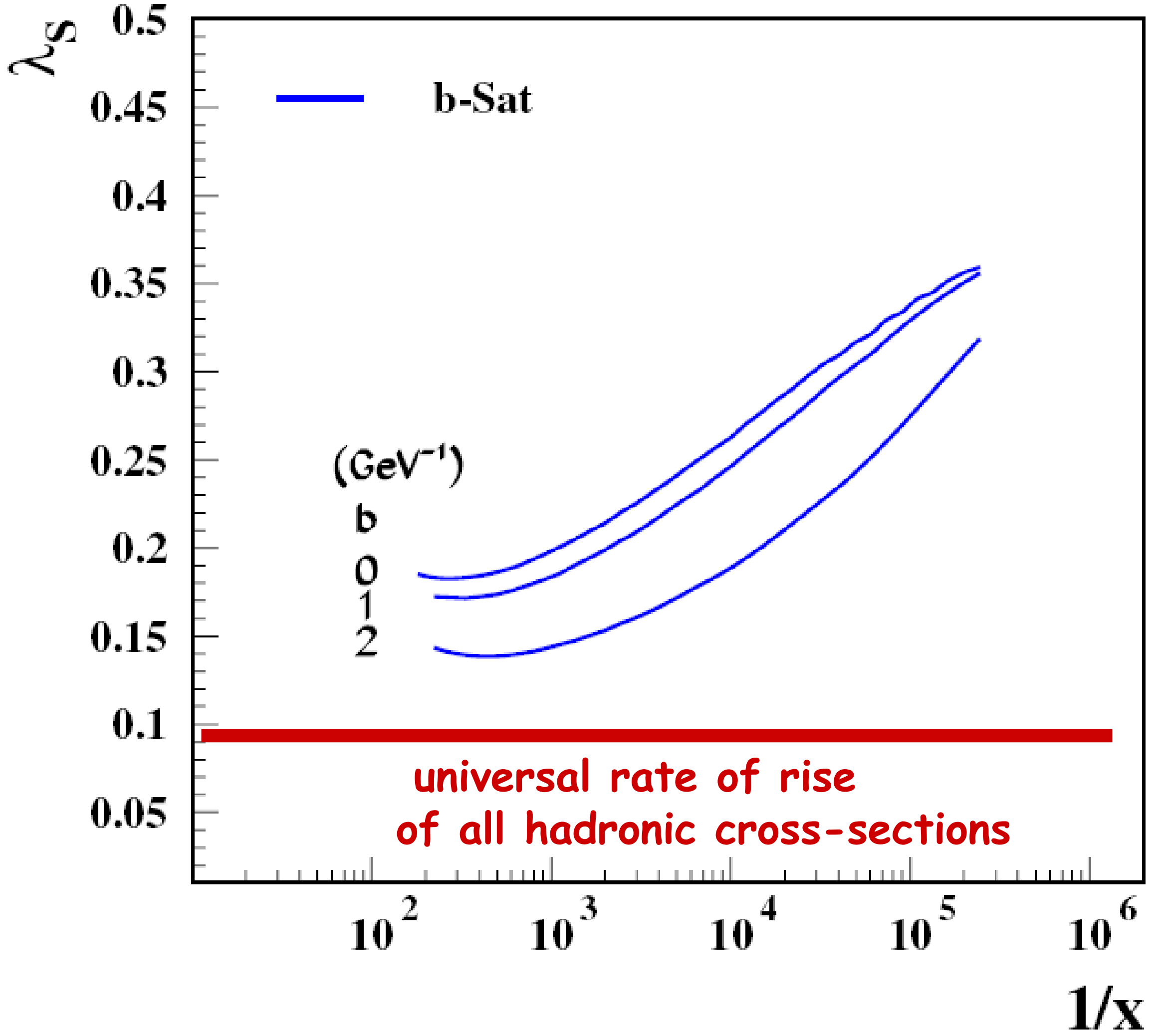
$$Q_S^2 = \frac{4\pi^2 \alpha_s N_C}{N_C^2 - 1}\frac{1}{\pi R^2}\frac{dN}{dy}$$

$$\frac{dN}{dy} \approx 1000 \qquad R \approx 7\ \mathrm{fm}$$

$$Q_S^2 \approx 1.3\ \mathrm{GeV}^2$$

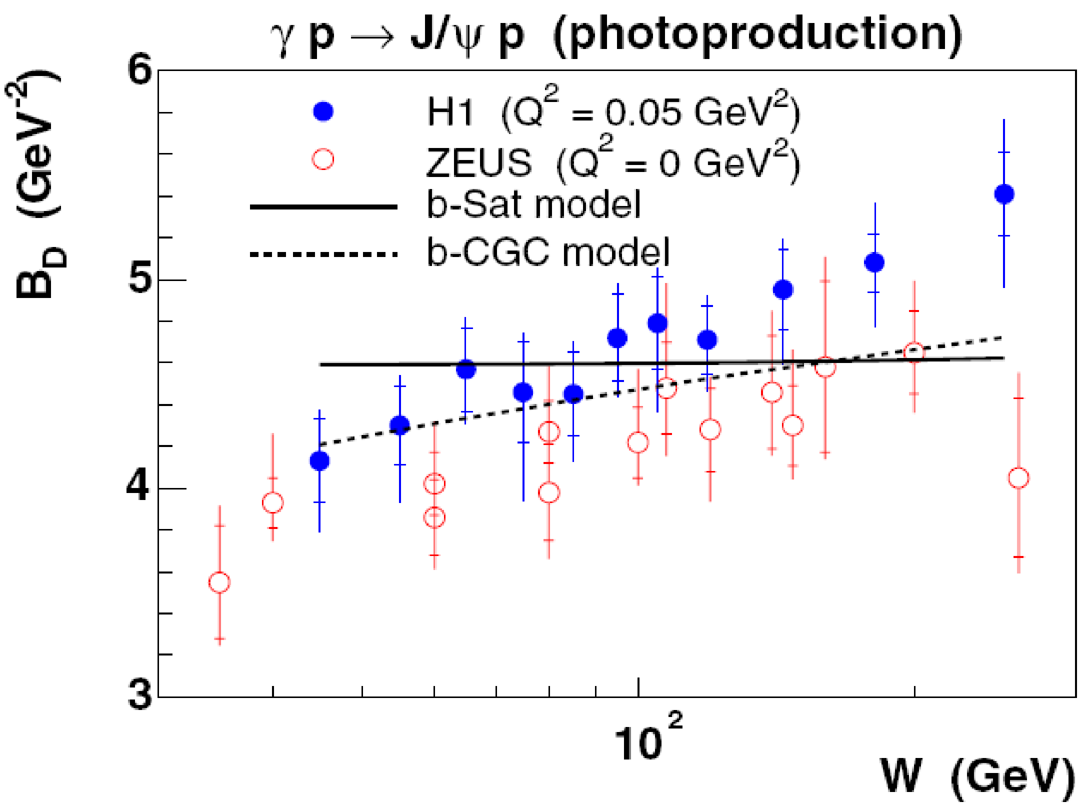
Q_S^{RHIC} $(x=10^{-2})$ ~ $Q_S^{HERA}(x=10^{-4})$

Is saturated state observed at HERA perturbative?



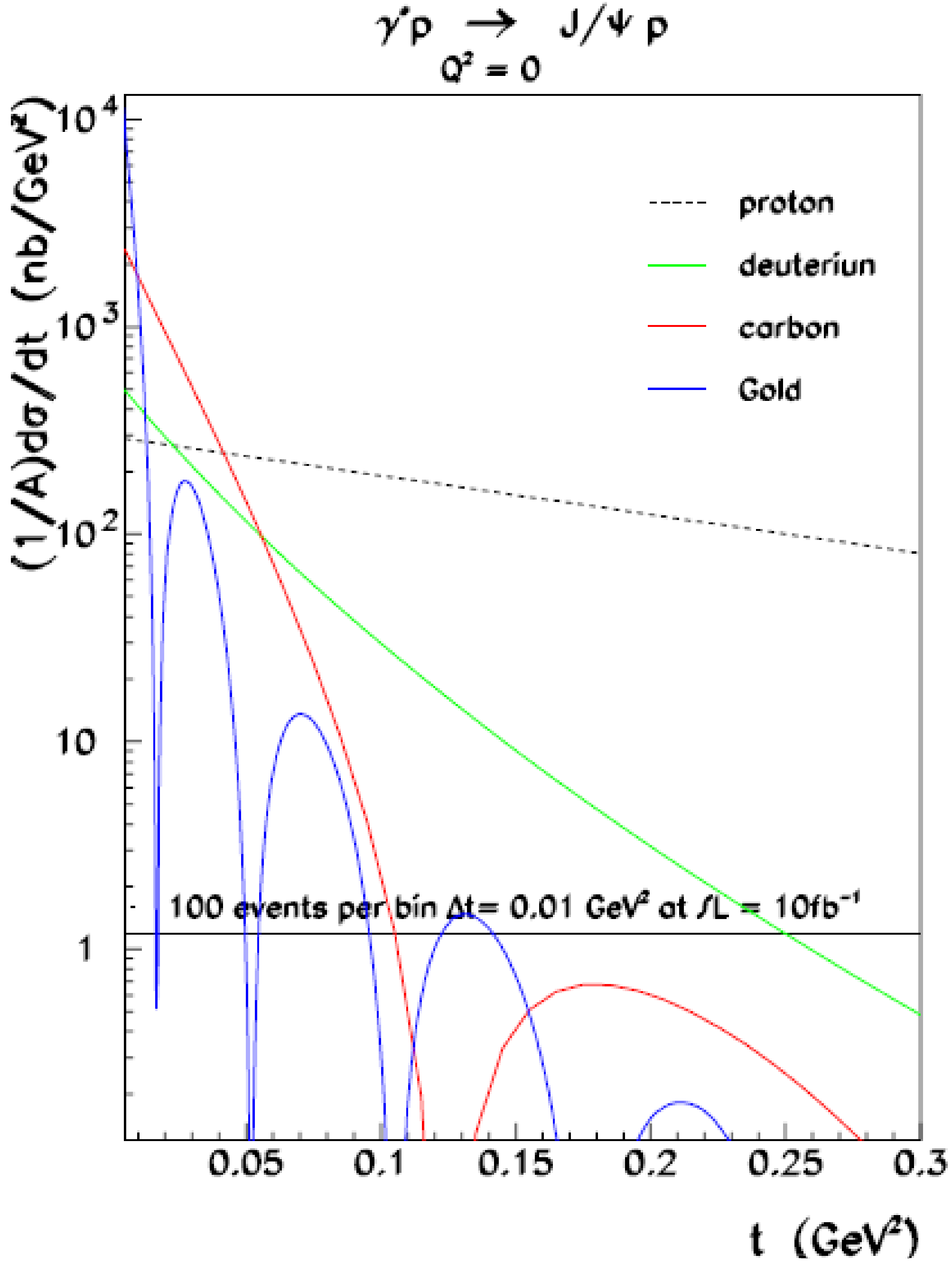
properties gluon density

Random walk in the impact parameter



Study of the Glue

t-distributions
for exclusive diffractive
meson production
on proton and nuclei
at EIC

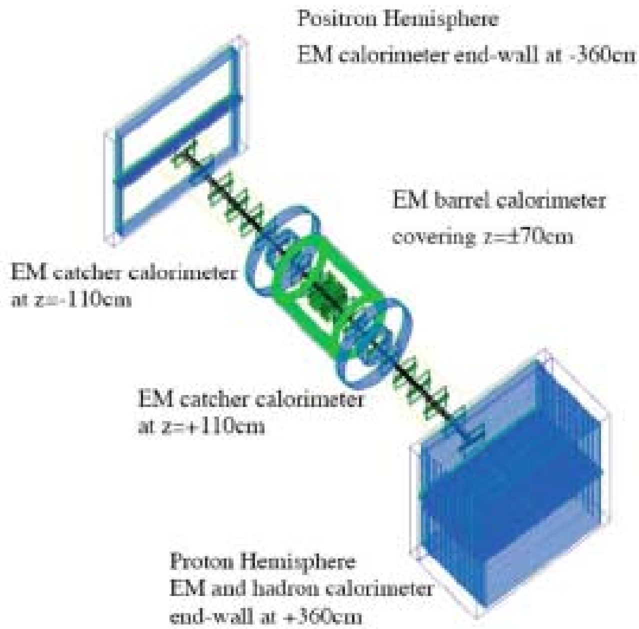


first estimate of the expected
measurement precision:

$\Delta p_T < 30$ MeV, $t \sim p_T^2$
$\Delta t < 0.01$ GeV^2
for proton and light nuclei

What are the lessons/open questions for EIC from exclusive diffraction at HERA

- **eRHIC**
  - Variable beam energy
  - P-U ion beams
  - Light ion polarization
  - Huge luminosity

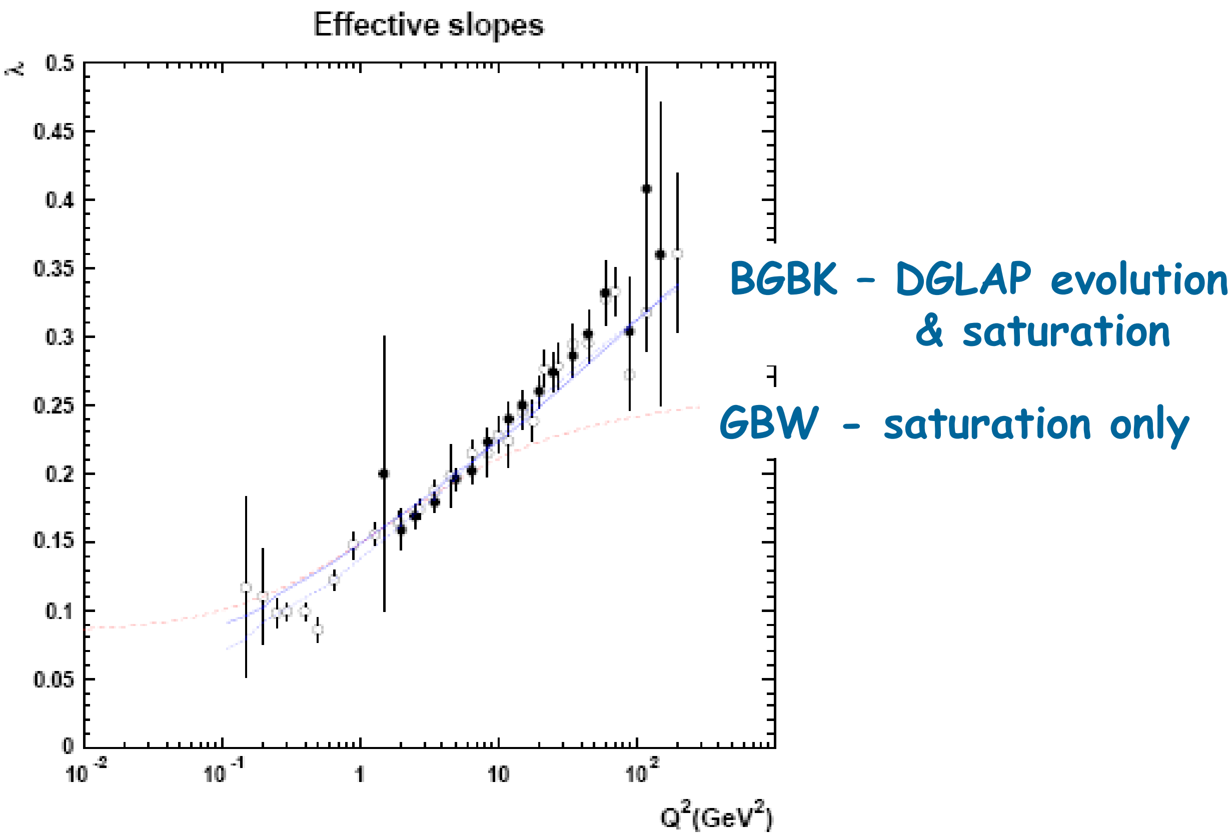


Precise study of gluon density, tested with dipoles of various sizes, measure gluon diffusion evolution effects

Diffractive vector mesons scattering - an excellent probe of nuclear matter,

why is the gluonic radius smaller than the quark radius??

>>>>> Measure *t* distribution on (polarized) nuclei <<<<<<
>>>>> Obtain holographic picture of nuclei !!!! <<<<<<

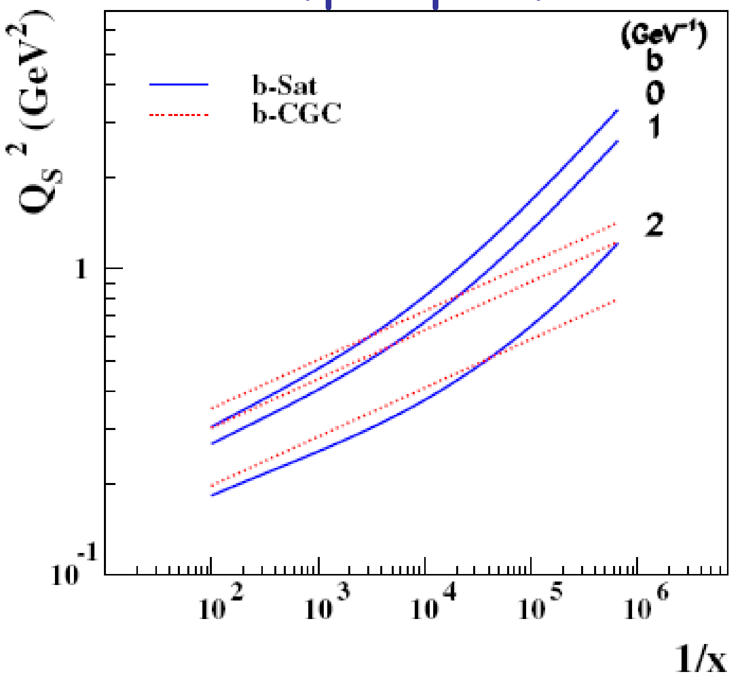

Effective slopes
λ
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
10 -2
10 -1
1
10
10 2
Q2(GeV2)
BGBK – DGLAP evolution
& saturation
GBW - saturation only

More about saturation

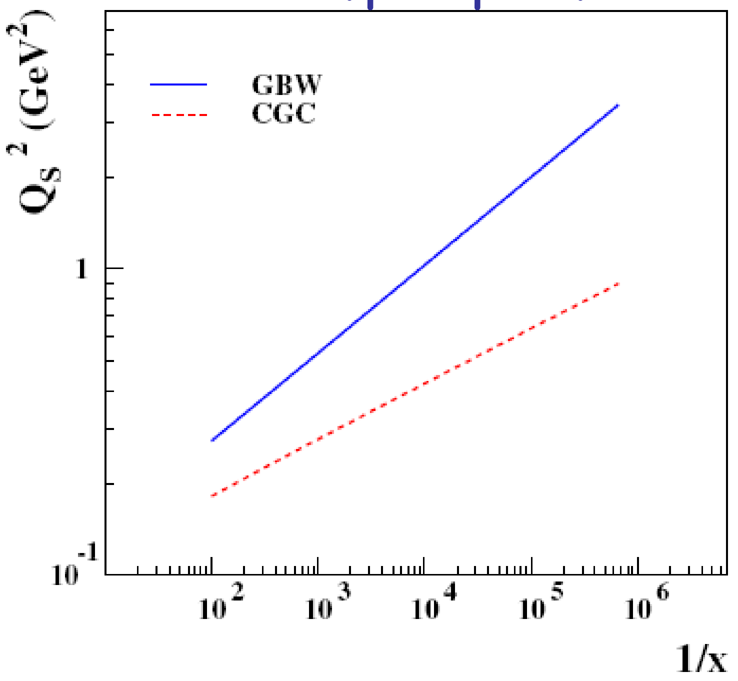
$$Q_S^2 \equiv 2/r_S^2$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} \equiv 2\mathcal{N}(x, r, b) \qquad \mathcal{N}(x, r_S, b) = 1 - \mathrm{e}^{-(1/2)}$$

with impact parameter

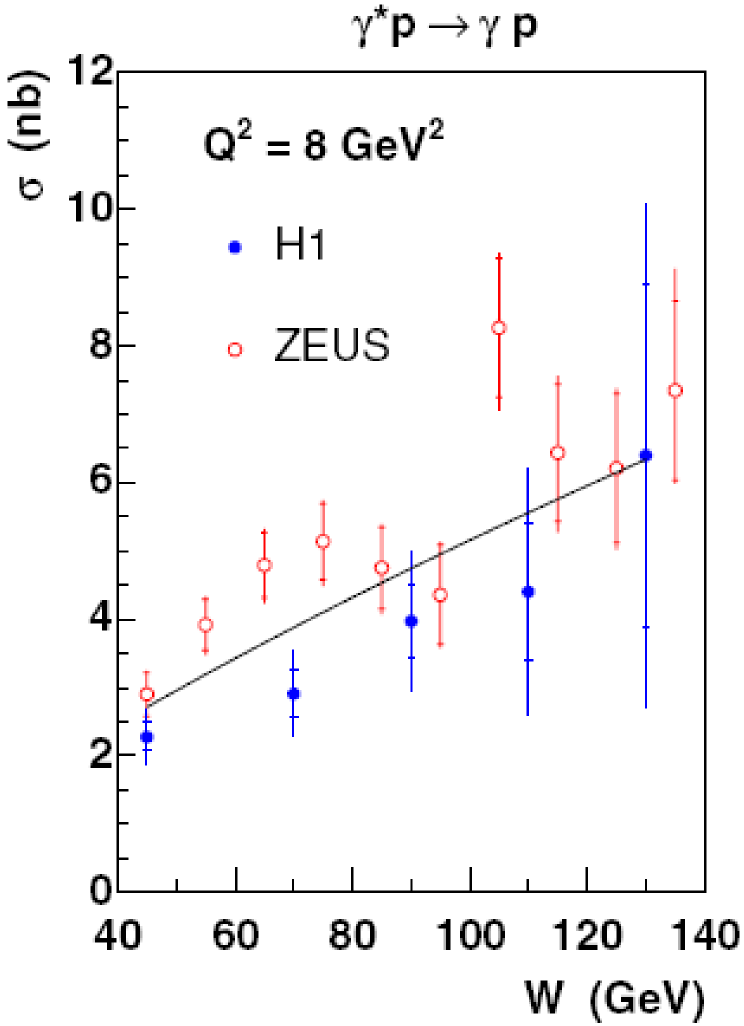


without impact parameter

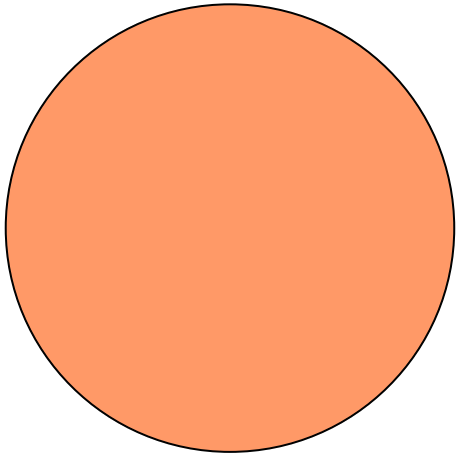


properties of the gluon density

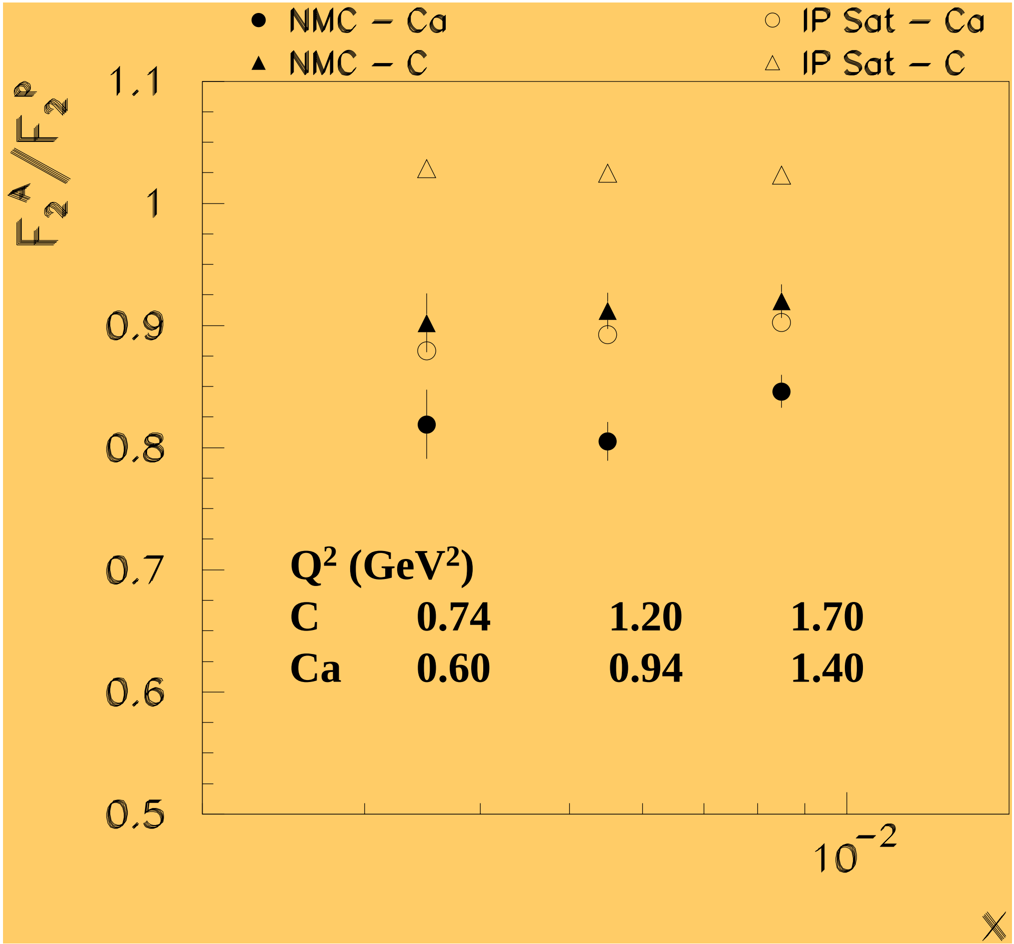
Rise of the DVCS cross-sections

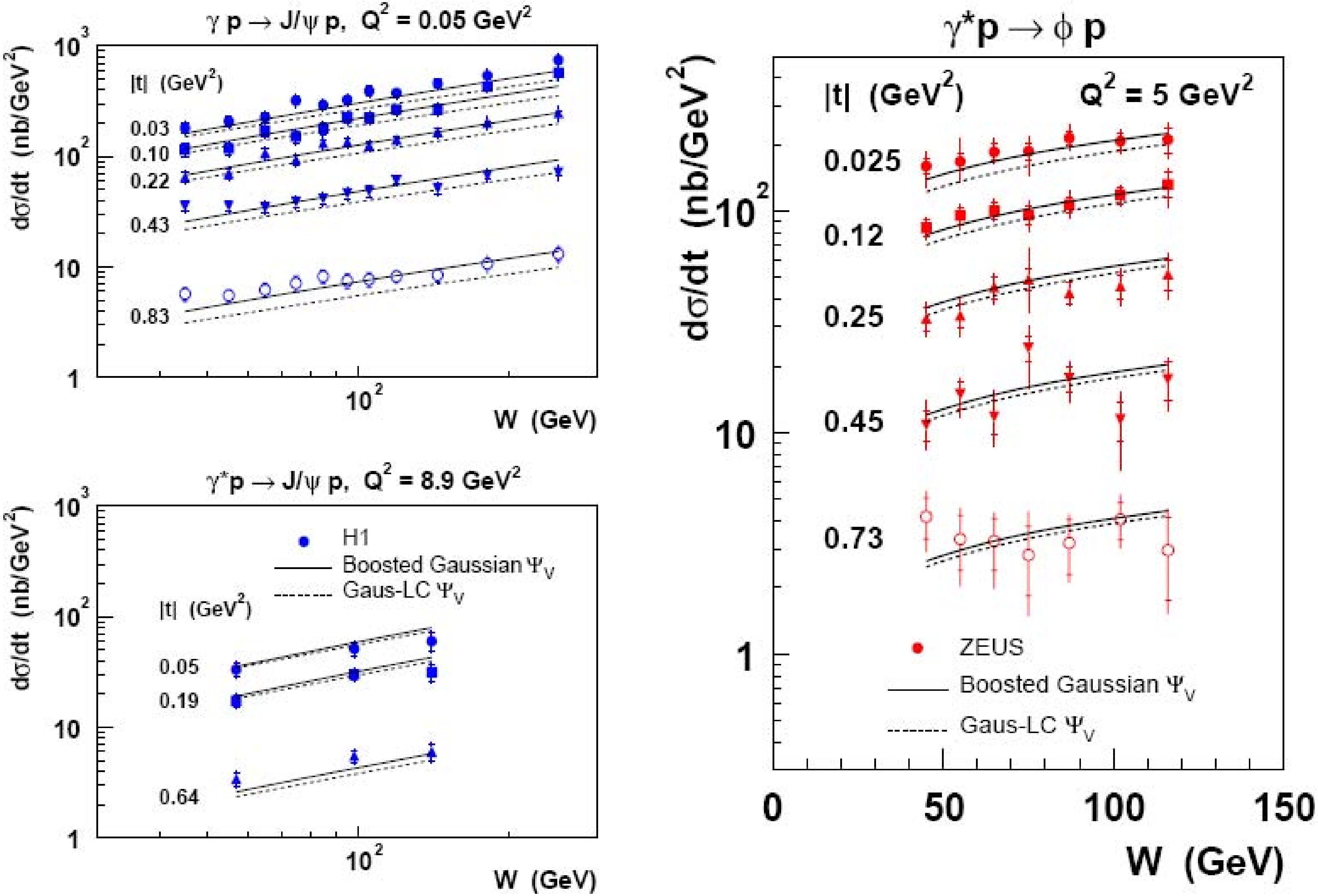


$$\frac{d\sigma_{qq}^{A}(x,r)}{d^2b} = \frac{2}{A}\cdot\left\{1-\exp\left(-\frac{\pi^2}{2\cdot 3}r^2\alpha_s xg(x,\mu^2)AT_{WS}(b)\right)\right\}$$



Smooth Gluon Cloud

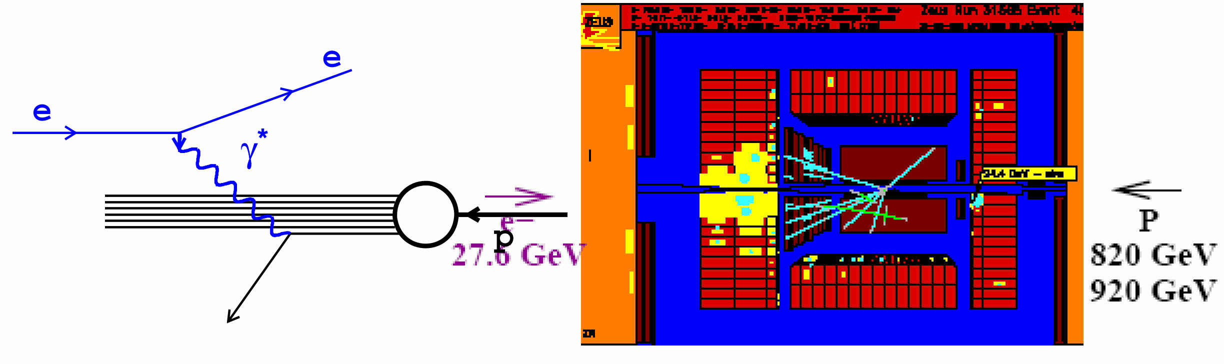




$\gamma\, p \to J/\psi\, p,\ Q^2 = 0.05\ GeV^2$
$d\sigma/dt\ (nb/GeV^2)$
$|t|\ (GeV^2)$
0.03
0.10
0.22
0.43
0.83
W (GeV)
$\gamma^* p \to J/\psi\, p,\ Q^2 = 8.9\ GeV^2$
H1
Boosted Gaussian Ψ_V
Gaus-LC Ψ_V
0.05
0.19
0.64
$\gamma^* p \to \phi\, p$
$Q^2 = 5\ GeV^2$
0.025
0.12
0.25
0.45
0.73
ZEUS
Boosted Gaussian Ψ_V
Gaus-LC Ψ_V
W (GeV)

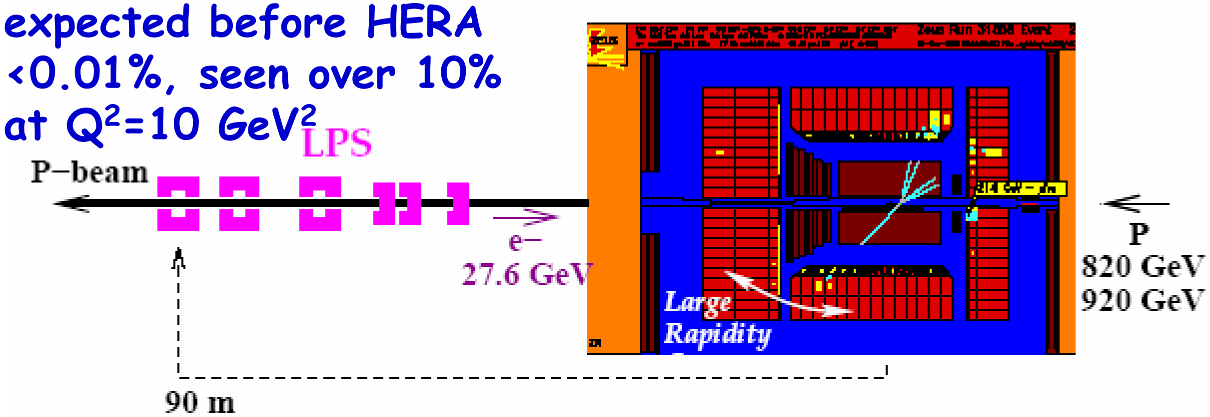
Hard Diffraction – the HERA surprise

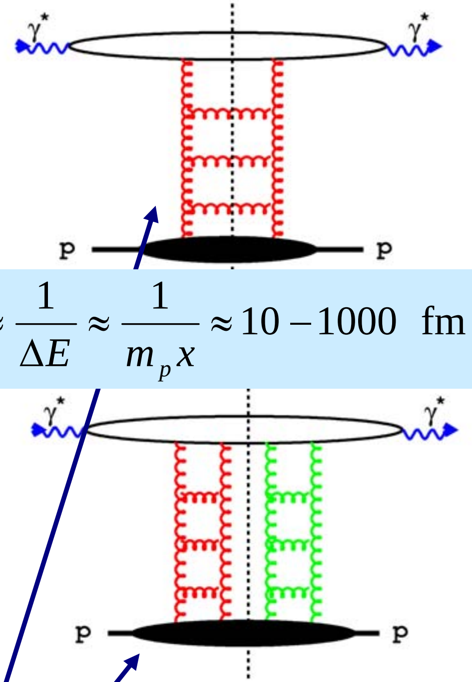
Non-Diffractive Event



Diffractive Event

expected before HERA
<0.01%, seen over 10%
at Q^2=10 GeV^2



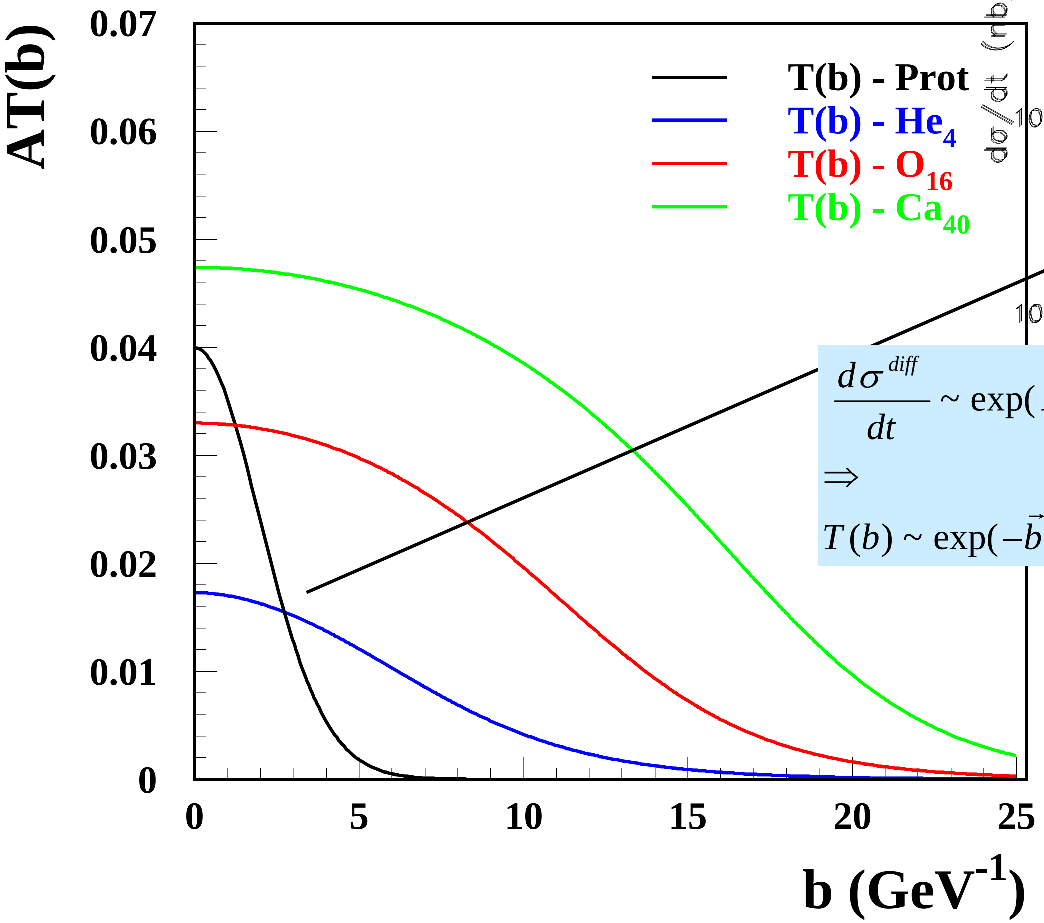


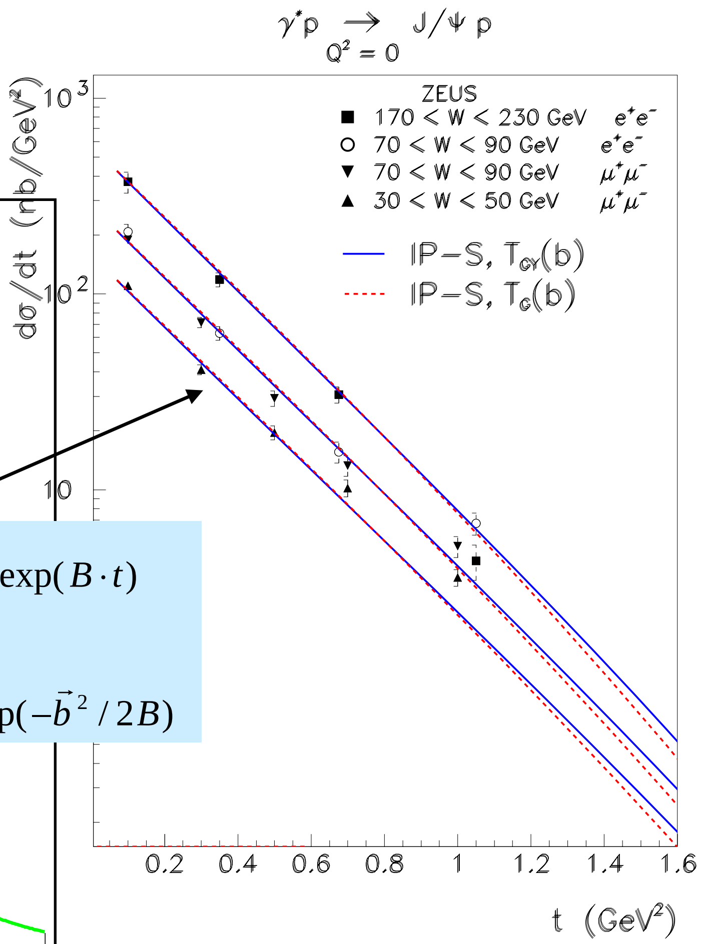
$$\tau_{qq} \approx \frac{1}{\Delta E} \approx \frac{1}{m_p x} \approx 10 - 1000 \text{ fm}$$

Diffraction at HERA is so large because it is a shadow of DIS (i.e. inelastic processes) ➔ dipole picture

$$\sigma_{tot}^{\gamma^* p} = \frac{1}{W^2} Im A_{el}(W^2, t = 0)$$

A glimpse into nuclei





$$\frac{d\sigma^{diff}}{dt} \sim \exp(B \cdot t)$$

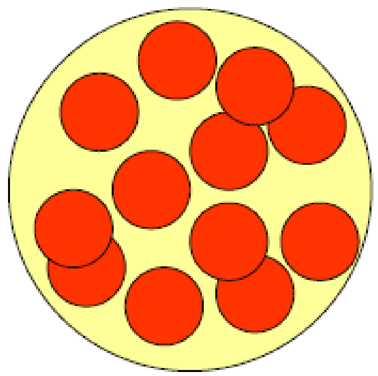
$$\Rightarrow$$

$$T(b) \sim \exp(-\vec{b}^{\,2} / 2B)$$

Naïve assumption for T(b):
Wood-Saxon like, homogeneous, distribution of nuclear matter

DIS on Nuclei

$$\frac{d\sigma_{qq}^{A}(x,r)}{d^{2}b} = \frac{2}{A}\cdot\left\{1-\left(1-T_{WS}(b)\sigma_{qq}(x,r)/2\right)^{A\cdot}\right\}$$



Lumpy Gluon Cloud