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Bottomonium production  
in the Regge limit of QCD

1. QMRK approach
2. Nonrelativistic QCD
3. Heavy quarkonium production by reggeized gluons
4. Bottomonium production at the Tevatron
5. Heavy quarkonium production at the LHC
6. Conclusion

The unintegrated gluon distribution function  $\Phi(x, |\mathbf{q}_T|^2, u^2)$  is used.

In the QMRK approach,  $q^2 = q_T^2 = -|\mathbf{q}_T|^2 \neq 0$ .

As the theoretical framework of high-energy factorization scheme we consider the quasi-multi-Regge kinematics (QMRK) approach [Lipatov, Kuraev, Fadin].

QMRK is based on effective quantum field theory implemented with the non-abelian gauge-invariant action, as was suggested a few years ago [Lipatov, 1995].

In the high-energy limit ( $x = u/\sqrt{S} \ll 1$  the sum of the large logarithms  $\ln(\sqrt{S}/u)$  in the evolution equation can be more important, we have Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation and  $q_T \neq 0$  for regularized  $t$ -channel gluons.

The QMRK approach

Factorization scale  $u \approx M^2 = \sqrt{M^2 + |\mathbf{p}_T|^2}$

In the collinear Parton Model (PM)  $S > u^2 \gg V_{\text{QCD}}^2$  and one has Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation with large  $\ln(u/V_{\text{QCD}})$  for collinear gluons  $q_T = 0$ .

We have shown that only Kirmser-Watt-Martini-Ryskin undF describes all data for heavy quark and diquarkonium production well.

- 6) Phys. Rev. D **74**, 014024 (2006).
- 5) Phys. Rev. D **73**, 074022 (2006).

B. A. Kniehl, V. A. Saleev and D. V. Vasin:

- 4) Phys. Lett. B **605**, 311 (2005);
- 3) In Proc. of First Int. Workshop "HSQCD 2004", **73** (2004);
- 2) Phys. Atom. Nuccl. **68**, 94 (2005) [Yad. Fiz. **68**, 95 (2005)];
- 1) Phys. Rev. D **68**, 114013 (2003);

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In the stage of the numerical calculations, we have tested different unintegrated distribution functions (undF) for reggeized gluons  $\Phi(x, |\mathbf{q}_T|^2, u_2)$ .

In 2005 [Antonov, Kuraev, Lipatov, Cherednikov] the Feynman rules for the effective theory based on the non-abelian gauge-invariant action [Lipatov, 1995] were derived for the induced and the some important effective vertices.

$$\bar{b}_+^1 = b_+^1 = 0.$$

$$q_2^2 = q_{2T}^2 + \frac{2}{b_+^2} n_- = q_{2T}^2 + x_2^2 P_2^2,$$

$$q_1^2 = q_{1T}^2 + \frac{2}{b_-^2} n_+ = q_{1T}^2 + x_1^2 P_1^2,$$

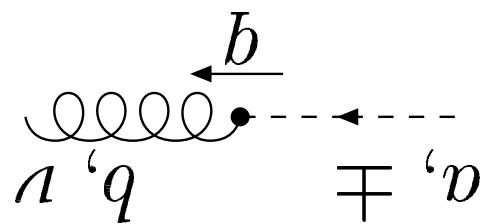
$$k_n : k_\mp = (k_n)$$

$$0 = (u_\mp u_-) = 2, (u_\mp u_+) =$$

$$(u_-)_\alpha = P_\alpha^2/E^2, \quad P_2^2 = E^2(1,0,0,-1)$$

$$(u_+)_\alpha = P_\alpha^1/E^1, \quad P_1^1 = E^1(1,0,0,1)$$

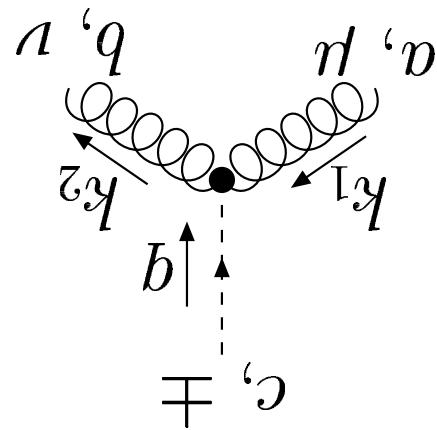
By definition:



$$(1) \quad ,_{\mp} (u_{\mp} b_{\mp}) = (b)_{\mp} I^{q_a}_{\mp}$$

(PR-vertices) has the form:

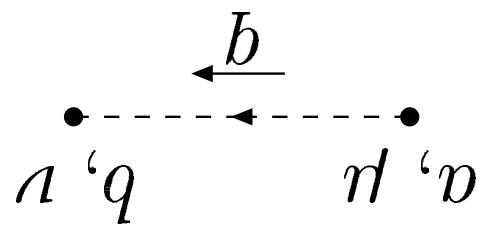
The induced vertices of reggeized gluon transition to Yang-Mills gluon  $R_{\pm} \rightarrow g$



$$\Gamma_{u\bar{u} \rightarrow c\bar{c} b}^{a\bar{a} q\bar{q}}(k_1, k_2, b) = -g_s f_{a\bar{a} q\bar{q}} \cdot (\bar{u} u)(\bar{u} u) \frac{\bar{b}}{b} \delta^{a\bar{a}} \quad (2)$$

(PR-vertices) reads:

The induced interaction vertices of reggeized gluon with two Yang-Mills gluons



$$(3) \quad \cdot \left[ \not{u} (-u)_\alpha (+u) + \not{u} (-u)_\mu (+u) \right] \frac{2g_2}{1} D_{\mu\alpha}^{ab}(b) = -i g_{a\mu}^b$$

The reggeized gluon propagator is specified as follows:

Effective vertex  $R_+ R_- \leftarrow g$ .

$$\begin{aligned}
 & \cdot \left[ u^a \left( \frac{\partial}{\partial k} + q_1^\mu \right) u^b + u^b \left( \frac{\partial}{\partial k} + q_2^\mu \right) u^a \right] \\
 = & V_{q1q2}^{ab}(-q_1, -q_2, k) u^a(-u) u^b(+u) + T_{q1q2}^{ab}(q_1, q_2, k) u^a(+u) u^b(-u) \\
 = & T_{q1q2}^{ab}(q_1, k, q_2)
 \end{aligned}$$

The effective 3-vertices, which describes the production of a single gluon with momentum  $k = q_1 + q_2$  and color index  $q$  in the "two reggeons collision"  $R_+ R_- \leftarrow g$  (PR-vertices):

$$x_1 = \frac{q_-^1}{2E_1}, x_2 = \frac{q_+^2}{2E_2}.$$

$$(7) \quad \int d^2\mathbf{q}_{2T} \frac{\pi}{4} \Phi(x_2, |\mathbf{q}_{2T}|_2^2, u_2) \times d\sigma(R + R \leftarrow \mathcal{H}, X + X, \\ \times (\int d^2\mathbf{q}_{1T} \frac{\pi}{4} \Phi(x_1, |\mathbf{q}_{1T}|_2^2, u_2) \int \frac{x}{xp} dx \int = (X + \mathcal{H} \leftarrow d + d) d\sigma$$

are connected as

$$(6) \quad X + \mathcal{H} \leftarrow R + R$$

and the partonic cross section for the reggeized-gluon fusion subprocess

$$(5) \quad X + \mathcal{H} \leftarrow d + d$$

process

In the QMRK approach, the hadronic cross section of quarkonium ( $\mathcal{H}$ ) production in the

$$(4) \quad \lim_{|\mathbf{q}_{1T}|, |\mathbf{q}_{2T}| \rightarrow 0} |A(R + R \leftarrow \mathcal{H} + X)|_2^2 = 0.$$

in the QMRK:

The gauge invariance of the effective theory leads to the following condition for amplitudes

$$(11) \quad \begin{aligned} & (X + \mathcal{H} \leftarrow b + b) \varphi d \\ & \times \int dx^1 G(x^1, u_2) \int dx^2 G(x^2, u_2) \times \\ & = (X + \mathcal{H} \leftarrow d + d) \varphi d \end{aligned}$$

collinear parton model:

So that when  $\mathbf{q}_{1T} = \mathbf{q}_{2T} = 0$  we obtain the conventional factorization formula of the

$$(10) \quad \frac{16|\mathbf{q}_{1T}|_2 |\mathbf{q}_{2T}|_2}{(x^1 x^2 S)^2} = \mathcal{N}$$

$$(6) \quad \begin{aligned} & |A(B + R \leftarrow \mathcal{H} \leftarrow X)d| \\ & \times \frac{2x^1 x^2 S}{\mathcal{N}} = (X + \mathcal{H} \leftarrow Y) \varphi d \end{aligned}$$

The partonic cross section for the two reggeized gluon collision can be presented as follows:

$$(8) \quad x G(x, u_2) = \int d^2 \mathbf{q}_T \frac{\pi}{\mathcal{N}} \Phi(x, |\mathbf{q}_T|_2, u_2),$$

The QMRK approach gives the following:

1. The new set of Feynman's rules for the gauge invariant amplitudes with reggeized gluons (and reggeized quarks).
2. The opportunity to perform (in principal) calculations in the NLO approximation in  $a_s$  with reggeized amplitudes.

The cross section  $d\sigma(a + b \rightarrow \bar{Q}\bar{Q}[n] + X)$  can be calculated in perturbative QCD as an expansion in  $a_s$  using the non-relativistic approximation for the relative motion of the heavy quarks in the  $\bar{Q}\bar{Q}$  pair. The non-perturbative transition of the  $\bar{Q}\bar{Q}$  pair into the physical quarkonium state  $\mathcal{H}$  is described by the NMEs  $\langle \mathcal{O}_{\mathcal{H}}[n] \rangle$ , which can be extracted from experimental data.

$$(12) \quad \langle [n]_{\mathcal{H}} \mathcal{O} \rangle (X + [n] \bar{Q}\bar{Q} \leftarrow q + a) \bar{\sigma} d \sum^u = (X + \mathcal{H} \leftarrow q + a) \bar{\sigma} d$$

In the framework of the NRQCD factorization approach, the cross section of heavy-quarkonium production in a partonic subprocess  $a + b \rightarrow \mathcal{H} + X$  may be presented as a sum of terms in which the effects of long and short distances are factorized as

strong-coupling constant  $a_s$  and the relative velocity  $v$  of the heavy quarks.

NRQCD is organized as a perturbative expansion in two small parameters, the

the effects of long and short distances in heavy-quarkonium production.

The factorization hypothesis of nonrelativistic QCD (NRQCD) assumes the separation of

## NRQCD formalism

$$(14) \quad \langle [{}_{(8)}^1 S]_{{}^3 P} | \Phi_r(0) | {}_2 \rangle = 2N_c(2J+1) \langle \Phi_r(0) | {}_2 \rangle$$

where  $N_c = 3$  and  $J = 1$ .

$$(13) \quad \langle [{}_{(8)}^1 S]_{{}^3 P} | \Phi_r(0) | {}_2 \rangle = 2N_c(2J+1) \langle \Phi_r(0) | {}_2 \rangle$$

which follow to LO in  $u$  from heavy-quark spin symmetry.

$$\langle [{}_{(8)}^1 S]_{{}^3 P} | \Phi_r(0) | {}_2 \rangle = (2J+1) \langle \Phi_r(0) | {}_2 \rangle$$

$$\langle [{}_{(8)}^0 D]_{{}^3 P} | \Phi_r(0) | {}_2 \rangle = (2J+1) \langle \Phi_r(0) | {}_2 \rangle$$

$$\langle [{}_{(8)}^0 D]_{{}^3 P} | \Phi_r(0) | {}_2 \rangle = (2J+1) \langle \Phi_r(0) | {}_2 \rangle$$

NMEs satisfy the multiplicity relations

If  $\mathcal{H} = \mathcal{X}(n_S)$ , and  $n = {}^3 P_J^{(1)}$ ,  ${}^3 S_J^{(8)}$  if  $\mathcal{H} = \chi_{bJ,cJ}(n_P)$ , where  $J = 0, 1$  or  $2$ . Their To leading order in  $u$ , we need to include the  $Q\bar{Q}$  Fock states  $n = {}^3 S_J^{(1)}, {}^3 S_J^{(8)}, {}^1 S_J^0, {}^3 P_J^{(8)}$

can be obtained from the one for an unspecified  $\bar{Q}Q$  state,  $A(a + q \leftarrow \bar{Q}Q)$ , by the application of appropriate projectors.

$$([_{(8)}^f T_{(1,8)}^f] \bar{Q}Q \leftarrow q + a) A$$

The production amplitude

$$N^{\text{pol}} = 2f + 1.$$

where  $N^{\text{col}} = 2N_c$  for the color-singlet state,  $N^{\text{col}} = N_c^2 - 1$  for the color-octet state, and

$$\begin{aligned} & \frac{N^{\text{col}} N^{\text{pol}}}{\langle [_{(8)}^f T_{(1,8)}^f] \bar{Q}Q \rangle} ([_{(8)}^f T_{(1,8)}^f] \bar{Q}Q \leftarrow q + a) \varphi d \\ &= (\mathcal{H} \leftarrow [_{(8)}^f T_{(1,8)}^f] \bar{Q}Q \leftarrow q + a) \varphi d \end{aligned}$$

$$(15) \quad C_1 = \frac{\sqrt{N_c}}{g_{ij}} \text{ and } C_8 = \sqrt{2} T_{ij}^c.$$

The projection operators on the color-singlet and color-octet states read:

$$\Pi^1_a = \frac{1}{\sqrt{8m^3}} \left( u + \frac{\not{p}}{2} \right) \gamma_a \left( u - \frac{\not{p}}{2} - m \right) \cdot \left( u + \frac{\not{p}}{2} \right) \gamma_5 \left( u - \frac{\not{p}}{2} + m \right)$$

$$\Pi^0 = \frac{1}{\sqrt{8m^3}} \left( u + \frac{\not{p}}{2} \right) \gamma_5 \left( u - \frac{\not{p}}{2} - m \right) \cdot \left( u + \frac{\not{p}}{2} \right) \gamma_a \left( u - \frac{\not{p}}{2} + m \right)$$

The projectors on the spin-0 and spin-1 states read:

$${}_{0=b}^0 | \left[ (d)^{\alpha\beta} (\bar{Q}Q \leftarrow q + a) A \right] \text{Tr} [C_{1,8} \Pi_a^1 A(a + q) \bar{Q}Q \leftarrow q + a] A = ( [{}_{(8)} P_{1,8}^f] \bar{Q}Q \leftarrow q + a) A$$

$${}_{0=b}^0 | \left[ (d)^{\alpha\beta} (\bar{Q}Q \leftarrow q + a) A \right] \text{Tr} [C_{1,8} \Pi_a^1 A(a + q) \bar{Q}Q \leftarrow q + a] A = ( [{}_{(8)} S_{1,8}^1] \bar{Q}Q \leftarrow q + a) A$$

$${}_{0=b}^0 | \left[ (\bar{Q}Q \leftarrow q + a) A \right] \text{Tr} [C_{1,8} \Pi^0 A(a + q) \bar{Q}Q \leftarrow q + a] A = ( [{}_{(8)} S_{1,8}^0] \bar{Q}Q \leftarrow q + a) A$$

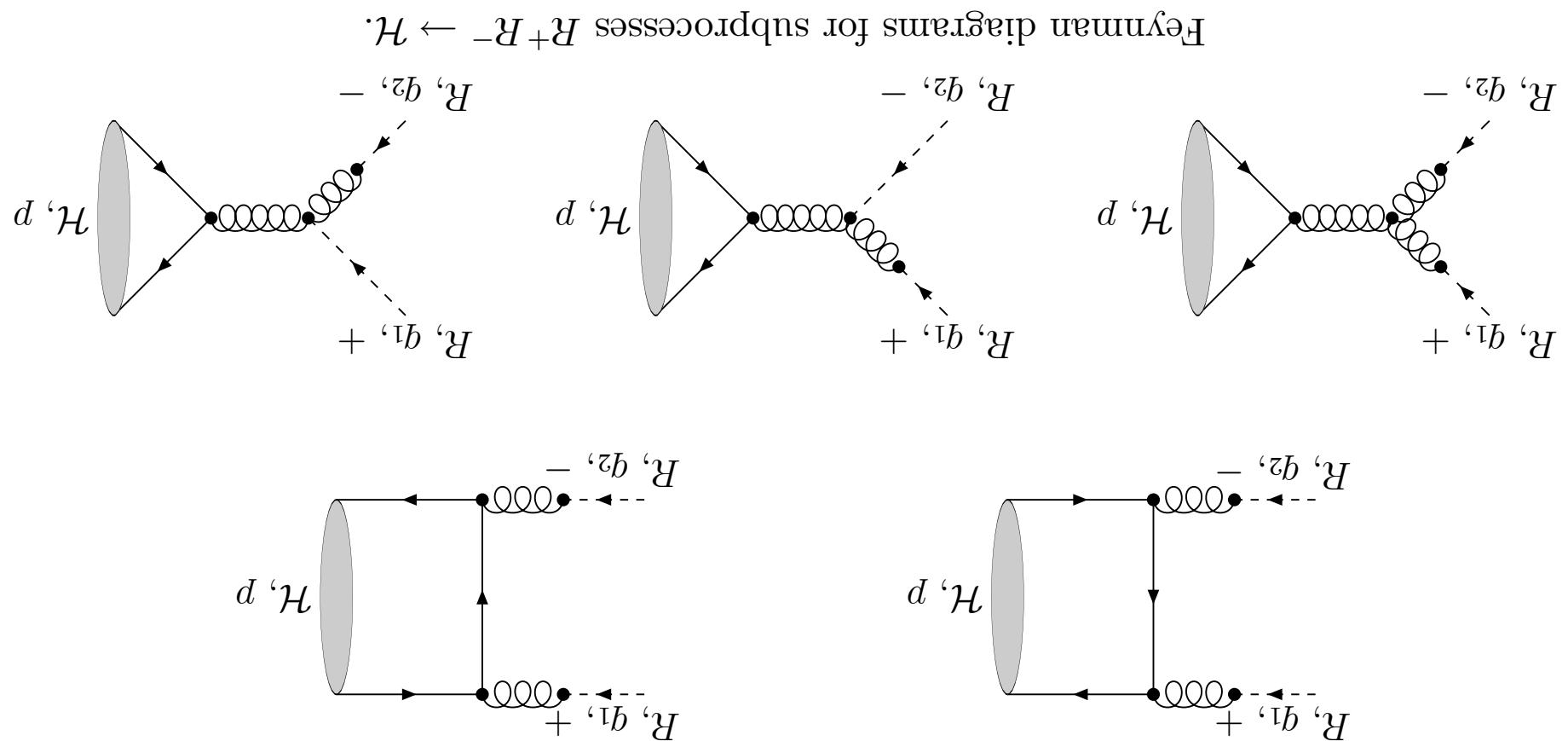
To obtain the projection on the state with orbital-angular-momentum quantum number  $L$ , we need take  $L$  times the derivative with respect to  $a$  and then put  $b = 0$ .

$$R + R \leftarrow \mathcal{H}[\epsilon S_{(1)}^1] + g,$$

$$R + R \leftarrow \mathcal{H}[\epsilon D_{(1)}^f, \epsilon S_{(8)}^0, \epsilon S_{(8)}^1, \epsilon D_{(8)}^f],$$

In this section, we obtain the squared amplitudes for inclusive quarkonium production via the fusion of two reggeized gluons in the framework of the NRQCD. We work at LO in  $\alpha_s$  and consider the following partonic subprocesses:

## Heavy quarkonium production by reggeized gluons



$$\begin{aligned}
& \frac{|A(R + R \rightarrow H[3P_{(8)}^2]||^2}{|A(R + R \rightarrow H[3P_{(8)}^2(t_1, t_2, \phi)]||^2} = \frac{\frac{3}{4}\pi^2\alpha_s^2 \langle O_H[3P_{(8)}^2] \rangle}{M_b^5 F[3P_{(8)}^2(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3P_{(8)}^1]||^2}{|A(R + R \rightarrow H[3P_{(8)}^1(t_1, t_2, \phi)]||^2} = \frac{10\pi^2\alpha_s^2 \langle O_H[3P_{(8)}^1] \rangle}{M_b^5 F[3P_{(8)}^1(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3P_{(8)}^0]||^2}{|A(R + R \rightarrow H[3P_{(8)}^0(t_1, t_2, \phi)]||^2} = \frac{5\pi^2\alpha_s^2 \langle O_H[3P_{(8)}^0] \rangle}{M_b^5 F[3P_{(8)}^0(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[1S_{(8)}^0]||^2}{|A(R + R \rightarrow H[1S_{(8)}^0(t_1, t_2, \phi)]||^2} = \frac{12\pi^2\alpha_s^2 \langle O_H[1S_{(8)}^0] \rangle}{M_b^3 F[1S_{(8)}^0(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3S_{(8)}^1]||^2}{|A(R + R \rightarrow H[3S_{(8)}^1(t_1, t_2, \phi)]||^2} = \frac{2\pi^2\alpha_s^2 \langle O_H[3S_{(8)}^1] \rangle}{M_b^3 F[3S_{(8)}^1(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3P_{(1)}^2]||^2}{|A(R + R \rightarrow H[3P_{(1)}^2(t_1, t_2, \phi)]||^2} = \frac{45\pi^2\alpha_s^2 \langle O_H[3P_{(1)}^2] \rangle}{M_b^5 F[3P_{(1)}^2(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3P_{(1)}^1]||^2}{|A(R + R \rightarrow H[3P_{(1)}^1(t_1, t_2, \phi)]||^2} = \frac{16\pi^2\alpha_s^2 \langle O_H[3P_{(1)}^1] \rangle}{M_b^5 F[3P_{(1)}^1(t_1, t_2, \phi)]}, \\
& \frac{|A(R + R \rightarrow H[3P_{(1)}^0]||^2}{|A(R + R \rightarrow H[3P_{(1)}^0(t_1, t_2, \phi)]||^2} = \frac{8\pi^2\alpha_s^2 \langle O_H[3P_{(1)}^0] \rangle}{M_b^5 F[3P_{(1)}^0(t_1, t_2, \phi)]}.
\end{aligned}$$

We have obtained

$$\begin{aligned}
& F_{[S^1]}(t_1, t_2, \phi) = \\
& \frac{16t_1t_2}{(M^2 + t_1 + t_2)^2} \left[ (M^2 + |D_T|^2)(t_1 + t_2) \right. \\
& \quad \left. + M^2(t_1 + t_2 - 2\sqrt{t_1t_2} \cos \phi) \right], \\
& F_{[S^0]}(t_1, t_2, \phi) = \\
& \frac{32M^2t_1t_2 \sin^2 \phi}{(M^2 + t_1 + t_2)^2} \left[ (M^2 + t_1 + t_2) \right. \\
& \quad \left. + 2\sqrt{t_1t_2} \right]^2, \\
& F_{[P^0]}(t_1, t_2, \phi) = \\
& \frac{32M^2t_1t_2}{(M^2 + t_1 + t_2)^2} \left[ (3M^2 + t_1 + t_2) \cos \phi + \right. \\
& \quad \left. 9(M^2 + t_1 + t_2)^{\frac{3}{2}} \right] \\
& F_{[P^1]}(t_1, t_2, \phi) = \\
& \frac{32M^2t_1t_2}{(M^2 + t_1 + t_2)^2} \left[ (3M^2 + t_1 + t_2) \right. \\
& \quad \left. + M^2(t_1 + t_2 - 2\sqrt{t_1t_2} \cos \phi) \right], \\
& F_{[P^2]}(t_1, t_2, \phi) = \\
& \frac{3(M^2 + t_1 + t_2)^{\frac{3}{2}}}{16M^2t_1t_2} \left[ (3M^2 + t_1 + t_2)W^2 + \right. \\
& \quad \left. + (t_1 + t_2)^2 \cos^2 \phi + 4t_1t_2 + \right. \\
& \quad \left. + 2\sqrt{t_1t_2} [3M^2 + 2(t_1 + t_2) \cos \phi] \right]
\end{aligned}$$

$$\frac{|A(g + g \rightarrow H[2S+1]_L^{J_f}(1,8))]|^2}{\int_{2\pi}^{2\pi} d\phi_1 \int_{2\pi}^{2\pi} d\phi_2} = \lim_{t_1, t_2 \rightarrow 0} \int_0^0 \frac{2\pi}{2\pi} \frac{2\pi}{2\pi} |A(R + R \rightarrow H[2S+1]_L^{J_f}(1,8))]|^2.$$

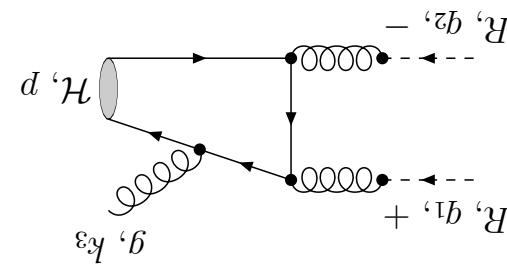
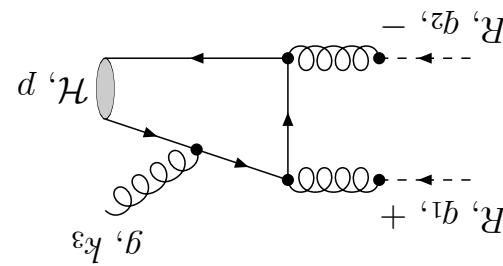
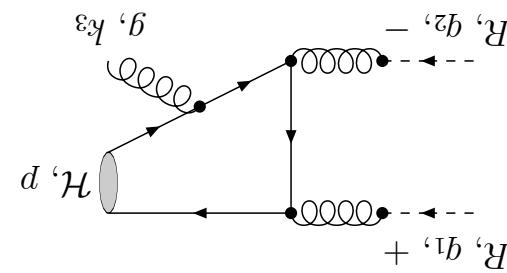
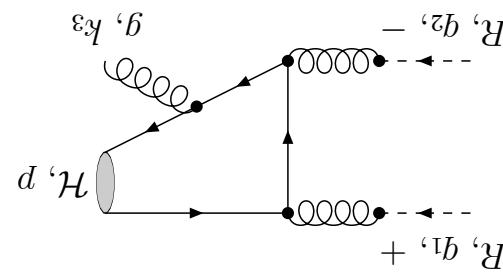
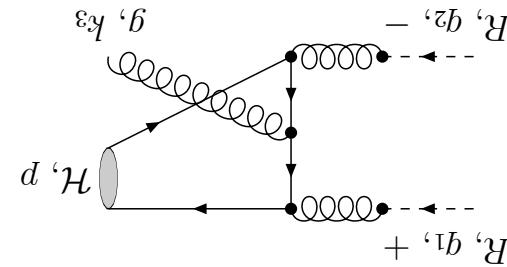
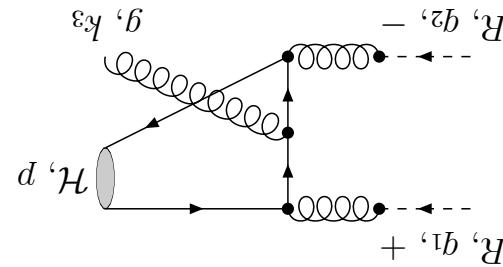
$$|\mathbf{p}_T|^2 = t_1 + t_2 + 2\sqrt{t_1 t_2} \cos \phi$$

Here  $\mathbf{p}_T = \mathbf{q}_{1T} + \mathbf{q}_{2T}$ ,  $t_{1,2} = |\mathbf{q}_{1,2T}|^2$ , and  $\phi = \phi_1 - \phi_2$  is the angle enclosed between  $\mathbf{q}_{1T}$  and  $\mathbf{q}_{2T}$ , so that

$$\begin{aligned}
 & \cdot \frac{\frac{3}{4} \pi^2 \alpha_s^2 \langle O_{\mathcal{H}} [{}^3P_{(8)}^2] \rangle}{\langle O_{\mathcal{H}} [{}^3D_{(8)}^2] \rangle} = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(8)}^2])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(8)}^2])|^2} \\
 & 0 = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(8)}^1])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(8)}^1])|^2} \\
 & , \quad \frac{5 \pi^2 \alpha_s^2 \langle O_{\mathcal{H}} [{}^3S_{(8)}^0] \rangle}{\langle O_{\mathcal{H}} [{}^1S_{(8)}^0] \rangle} = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(8)}^0])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^1S_{(8)}^0])|^2} \\
 & \cdot \frac{12 \pi^2 \alpha_s^2 \langle O_{\mathcal{H}} [{}^1S_{(8)}^1] \rangle}{\langle O_{\mathcal{H}} [{}^3S_{(8)}^1] \rangle} = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(8)}^1])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^1S_{(8)}^1])|^2} \\
 & 0 = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(1)}^2])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(1)}^2])|^2} \\
 & , \quad \frac{32 \pi^2 \alpha_s^2 \langle O_{\mathcal{H}} [{}^3P_{(1)}^2] \rangle}{\langle O_{\mathcal{H}} [{}^3D_{(1)}^2] \rangle} = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(1)}^2])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(1)}^2])|^2} \\
 & 0 = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(1)}^1])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(1)}^1])|^2} \\
 & , \quad \frac{8 \pi^2 \alpha_s^2 \langle O_{\mathcal{H}} [{}^3P_{(1)}^0] \rangle}{\langle O_{\mathcal{H}} [{}^3D_{(1)}^0] \rangle} = \frac{|A(g + g \leftarrow \mathcal{H} [{}^3P_{(1)}^0])|^2}{|A(g + g \leftarrow \mathcal{H} [{}^3D_{(1)}^0])|^2}
 \end{aligned}$$

In this way, we recover the well-known results:

Feynman diagrams for subprocesses  $R_+ + R_- \leftarrow \mathcal{H}_3 S_{(1)}^1 g$ .



$$(16) \quad \Omega_{prompt}(\Upsilon(u)) = \Omega_{direct}(\Upsilon(u)) + ((S u) \Upsilon \leftarrow (u) \Upsilon) \Omega \sum^u + ((S u) \Upsilon \leftarrow (, u) \Upsilon) \Omega \sum^u + ((S u) \Upsilon(u))$$

TeV; for prompt  $J/\psi$  at the  $\sqrt{S} = 1.96$  TeV.  
 TeV; for direct  $J/\psi$ , for  $J/\psi$  from  $\psi'$  decays, for  $J/\psi$  from  $\chi_{cJ}$  decays at the  $\sqrt{S} = 1.8$   
 $\sqrt{S} = 1.8$  TeV and for prompt  $\Upsilon(1S)$  in the different intervals of rapidity at the  $\sqrt{S} = 1.96$   
 Nowadays Tevatron CDF data incorporate  $p_T$ -spectra for prompt  $\Upsilon(1S, 2S, 3S)$  at the

## Heavy quarkonium production at the Tevatron

$$\xi_1 = \frac{2E_1}{p_0 + p_3}, \quad \xi_2 = \frac{2E_2}{p_0 - p_3}.$$

We use also following variables

respectively.

$$y = \frac{1}{2} \ln \frac{p_0 - p_3}{p_0 + p_3}, \quad \eta = \frac{1}{2} \ln \frac{|\mathbf{p}| - p_3}{|\mathbf{p}| + p_3},$$

$d^u = (p_0, \mathbf{p}_T, p_3)$  are given by

The rapidity and pseudorapidity of a heavy quarkonium state with four-momentum

$$\cdot \frac{(x_2 - \xi_2) S}{1} \left( (\mathbf{b}_{1T} + \mathbf{b}_{2T} - \mathbf{d}_T)_2^2 - M_2^2 - |x_2 \xi_1 S|_2^2 \right)$$

where

$$\times \frac{\Phi(x_1, |\mathbf{b}_{1T}|_2, u_2) \Phi(x_2, |\mathbf{b}_{2T}|_2, u_2) |A(R + B \rightarrow \mathcal{H})|_2}{128\pi^3 \int d^2 \mathbf{b}_{1T} \int d^2 \mathbf{b}_{2T} \int dx_2^2} \times$$

$$= \frac{dy |d\mathbf{d}_T| p}{(X + \mathcal{H} \leftarrow d + d) dp}$$

For the  $2 \rightarrow 2$  subprocess, we have

$$\times \Phi(\xi_2, |\mathbf{b}_{2T}|_2, u_2) \delta(\mathbf{b}_{1T} + \mathbf{b}_{2T} - \mathbf{d}_T) |A(R + B \rightarrow \mathcal{H})|_2 \cdot$$

$$\times \Phi(\xi_1, |\mathbf{b}_{1T}|_2, u_2) \Phi(\xi_2, |\mathbf{b}_{2T}|_2, u_2) |A(R + B \rightarrow \mathcal{H})|_2 \times$$

$$= \frac{dy |d\mathbf{d}_T| p}{(X + \mathcal{H} \leftarrow d + d) dp}$$

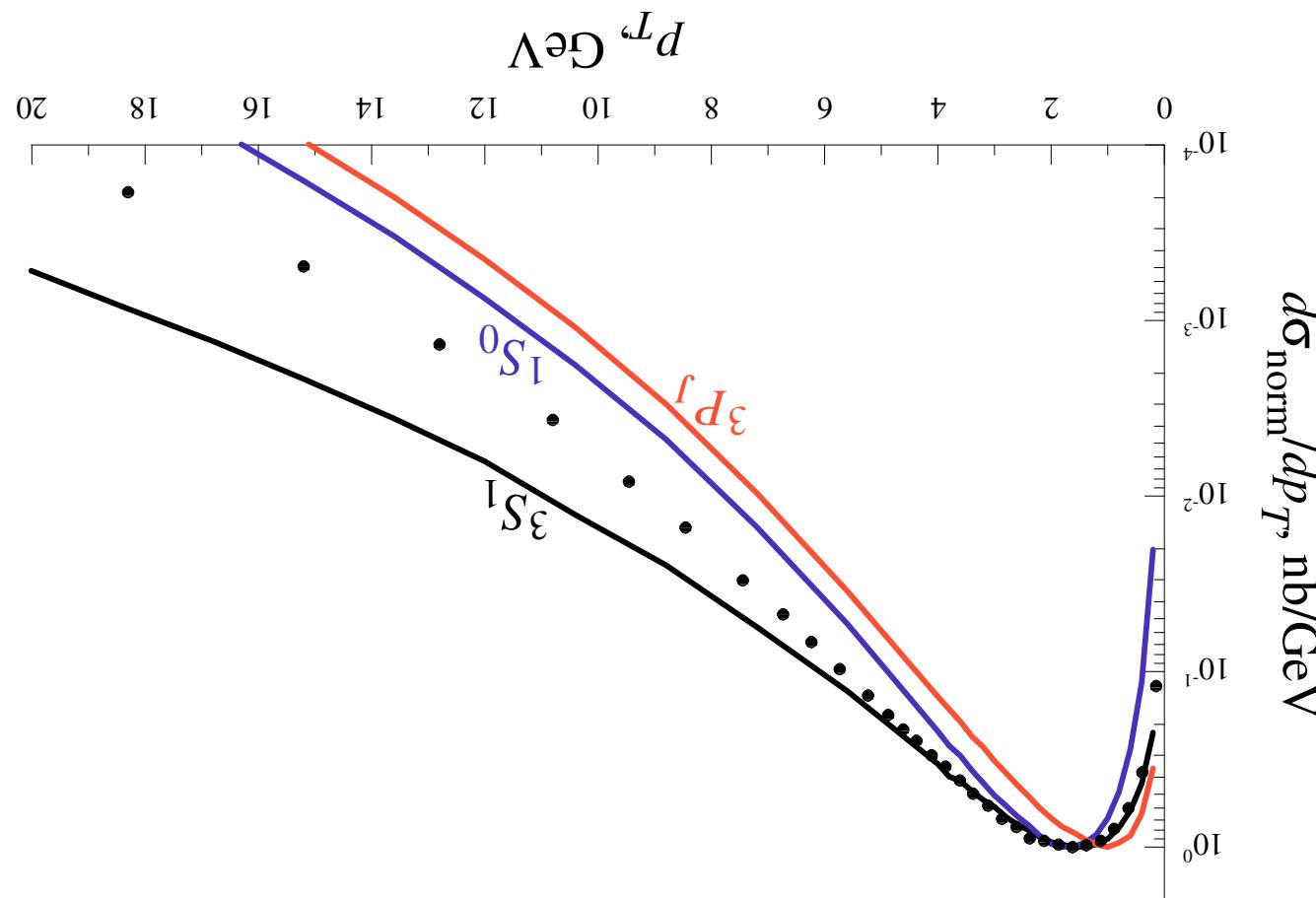
In the case of the  $2 \rightarrow 1$  subprocesses, we obtain

By contrast, QMRK fit allow us to determine  $\langle Q_H [^1S_{(8)}^0] \rangle$  and  $\langle Q_H [^3P_{(8)}^0] \rangle$  separately, which is due to the different  $|\mathbf{p}_T|$  dependence of the respective contributions for  $|\mathbf{p}_T| < 8(4)$  GeV.

$$(17) \quad \langle Q_H [^1S_{(8)}^0] \rangle + \langle Q_H [^3P_{(8)}^0] \rangle = M_H$$

In previous fit of CDF data were used for the region of large  $|\mathbf{p}_T| > 8(4)$  GeV only, and the linear combination

Contributions to the  $p_T$  distribution of direct  $\Upsilon(1S)$  hadroproduction in  $p\bar{p}$  scattering with  $\sqrt{S} = 1.8$  TeV and  $|y| < 0.4$  from the relevant color-octet states. All distributions are normalized on unit in their peak values.



In\Out	$\Upsilon(3S)$	$\chi_{b2}(2P)$	$\chi_{b1}(2P)$	$\chi_{b0}(2P)$	$\Upsilon(2S)$	$\chi_{b2}(1P)$	$\chi_{b1}(1P)$	$\chi_{b0}(1P)$	$\Upsilon(1S)$
$\Upsilon(3S)$	1	0.114	0.113	0.054	0.106	0.007208	0.00742	0.004028	0.102171
$\chi_{b2}(2P)$	—	1	—	—	0.162	0.011016	0.01134	0.006156	0.129565
$\chi_{b1}(2P)$	—	—	1	—	0.21	0.01428	0.0147	0.00798	0.160917
$\chi_{b0}(2P)$	—	—	—	1	0.046	0.003128	0.00322	0.001748	0.0167195
$\Upsilon(2S)$	—	—	—	—	1	0.068	0.07	0.038	0.319771
$\chi_{b2}(1P)$	—	—	—	—	—	—	1	—	0.22
$\chi_{b1}(1P)$	—	—	—	—	—	—	—	1	0.35
$\chi_{b0}(1P)$	—	—	—	—	—	—	—	1	0.06
$\Upsilon(1S)$	—	—	—	—	—	—	—	—	1

Table: Inclusive branching fractions for transitions between spin-triplet bottomonium states.

$$v_2^{cc} \approx 0.3, \quad v_2^{bb} \approx 0.1$$

Color Octet Contribution  $\ll 1$   
Color Singlet Contribution

n / n	PM	Fit JB	Fit JS	Fit KMR	$\chi^2/\text{d.o.f}$	—	2.9	2.7 · 10⁻¹	4.9 · 10⁻¹
$\langle \Omega \chi(1S)[1S(8)], \text{GeV}^3 \rangle$	$1.4 \cdot 10^{-1}$	0.0	0.0	$1.1 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$	$9.5 \cdot 10^{-2}$	$\langle \Omega \chi(1S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(1S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(1S)[1S(1)], \text{GeV}^3 \rangle$	$1.1 \cdot 10^{-1}$	0.0	0.0	$1.1 \cdot 10^{-1}$	$2.0 \cdot 10^{-2}$	$5.3 \cdot 10^{-3}$	0.0	$\langle \Omega \chi(1S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(1S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(1S)[3S(1)], \text{GeV}^3 \rangle$	$1.1 \cdot 10^{-1}$	0.0	0.0	$1.1 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$	0.0	0.0	$\langle \Omega \chi(1S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(1S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(2S)[1S(8)], \text{GeV}^3 \rangle$	0.0	0.0	0.0	$1.1 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	$1.6 \cdot 10^{-1}$	0.0	$\langle \Omega \chi(2S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(2S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(2S)[1S(1)], \text{GeV}^3 \rangle$	0.0	0.0	0.0	$1.1 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	0.0	$\langle \Omega \chi(2S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(2S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(2S)[3S(8)], \text{GeV}^3 \rangle$	0.0	0.0	0.0	$1.1 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	0.0	$\langle \Omega \chi(2S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(2S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(2P)[1S(8)], \text{GeV}^3 \rangle$	$8.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	0.0	0.0	$2.6 \cdot 10^{-2}$	$8.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	$\langle \Omega \chi_{b0}(2P)[3P(1)], \text{GeV}^5 \rangle$	$\langle \Omega \chi_{b0}(2P)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(2P)[1S(1)], \text{GeV}^3 \rangle$	$8.0 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	0.0	0.0	$2.6 \cdot 10^{-2}$	$2.6 \cdot 10^{-3}$	$0.0$	$\langle \Omega \chi_{b0}(2P)[3P(1)], \text{GeV}^5 \rangle$	$\langle \Omega \chi_{b0}(2P)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(3S)[1S(8)], \text{GeV}^3 \rangle$	$5.4 \cdot 10^{-2}$	0.0	0.0	$1.1 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	$5.9 \cdot 10^{-3}$	$\langle \Omega \chi(3S)[3S(1)], \text{GeV}^3 \rangle$	$\langle \Omega \chi(3S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(3S)[1S(1)], \text{GeV}^3 \rangle$	$5.4 \cdot 10^{-2}$	0.0	0.0	$1.1 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$	$3.4 \cdot 10^{-3}$	$\langle \Omega \chi(3S)[3S(1)], \text{GeV}^3 \rangle$	$\langle \Omega \chi(3S)[3S(8)], \text{GeV}^3 \rangle$
$\langle \Omega \chi(3S)[3S(8)], \text{GeV}^3 \rangle$	$5.4 \cdot 10^{-2}$	0.0	0.0	$1.1 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$	$4.3 \cdot 10^{-2}$	$3.4 \cdot 10^{-3}$	$\langle \Omega \chi(3S)[3P(8)], \text{GeV}^5 \rangle$	$\langle \Omega \chi(3S)[3P(8)], \text{GeV}^5 \rangle$
$\langle \Omega \chi(3P)[1S(8)], \text{GeV}^3 \rangle$	$2.4 \cdot 10^{-1}$	—	—	—	—	—	—	$\chi^2/\text{d.o.f}$	$\chi^2/\text{d.o.f}$

Table: NMEs for  $\chi(1S, 2S, 3S)$ , and  $\chi_{bJ}$

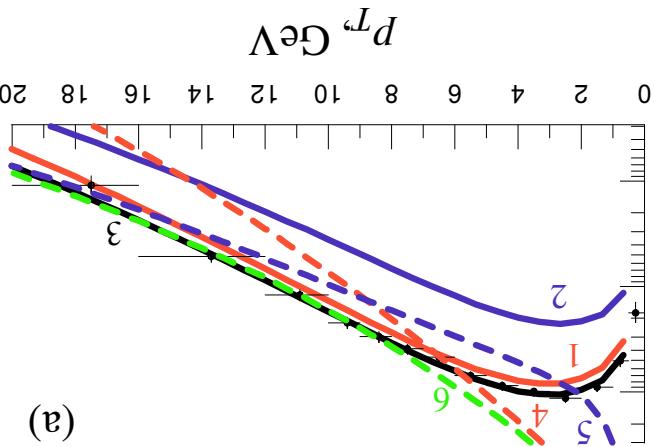
$\chi^2/\text{d.o.f}$	4.1	3.0	2.2 (*)	-	PM	Fit JB	Fit JS	Fit KMR	NME
$\langle Q_{\chi_{c0}}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.4 \cdot 10^{-3}$	$0$	$2.2 \cdot 10^{-4}$	$4.7 \cdot 10^{-5}$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$	$8.9 \cdot 10^{-2}$	$\langle Q_{\chi_{c0}}[{}^3P_0^{(1)}] \rangle / \text{GeV}^5$
$\langle Q_{\phi'}[{}^3P_0^{(8)}] \rangle / \text{GeV}^5$	$3.9 \cdot 10^{-3}$	$0$	$0$	$0$	$6.9 \cdot 10^{-3}$	$0$	$0$	$0$	$\langle Q_{\phi'}[{}^1S_0^{(8)}] \rangle / \text{GeV}^3$
$\langle Q_{\phi'}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.2 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$8.3 \cdot 10^{-4}$	$6.5 \cdot 10^{-1}$	$6.5 \cdot 10^{-1}$	$3.0 \cdot 10^{-4}$	$1.5 \cdot 10^{-3}$	$8.3 \cdot 10^{-4}$	$\langle Q_{\phi'}[{}^3S_1^{(1)}] \rangle / \text{GeV}^3$
$\langle Q_{J/\psi}[{}^3P_0^{(8)}] \rangle / \text{GeV}^5$	$2.8 \cdot 10^{-2}$	$0$	$0$	$0$	$4.3 \cdot 10^{-2}$	$6.6 \cdot 10^{-3}$	$9.0 \cdot 10^{-3}$	$1.4 \cdot 10^{-2}$	$\langle Q_{J/\psi}[{}^1S_0^{(8)}] \rangle / \text{GeV}^3$
$\langle Q_{J/\psi}[{}^3S_1^{(8)}] \rangle / \text{GeV}^3$	$4.4 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$	$2.7 \cdot 10^{-3}$	$1.3$	$1.3$	$1.3$	$1.3$	$1.3$	$\langle Q_{J/\psi}[{}^3S_1^{(1)}] \rangle / \text{GeV}^3$

B. A. Kniehl, V. A. Saleev and D. V. Vasin, Phys. Rev. D **73**, 074022 (2006).

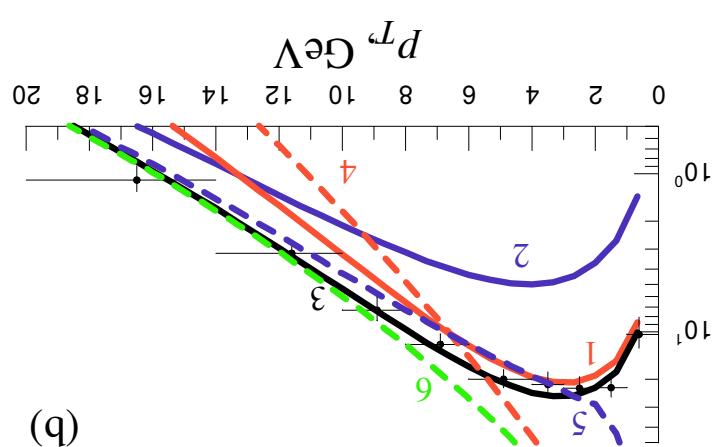
Table: NMEs for  $J/\psi$ ,  $\phi'$  and  $\chi_{cJ}$

Prompt  $\Upsilon(nS)$   $p_T$ -spectra.  $\Upsilon(1S)$  - a,  $\Upsilon(2S)$  - b,  $\Upsilon(3S)$  - c, KMR distribution function

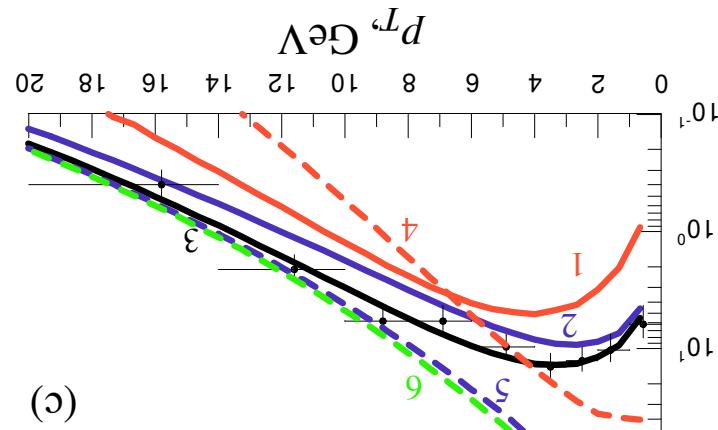
$$B(\Upsilon(nS) \rightarrow \mu^+ \mu^- d\sigma/dp_T dy)_{|y| < 0.4}, \text{ pb/GeV}$$



(a)

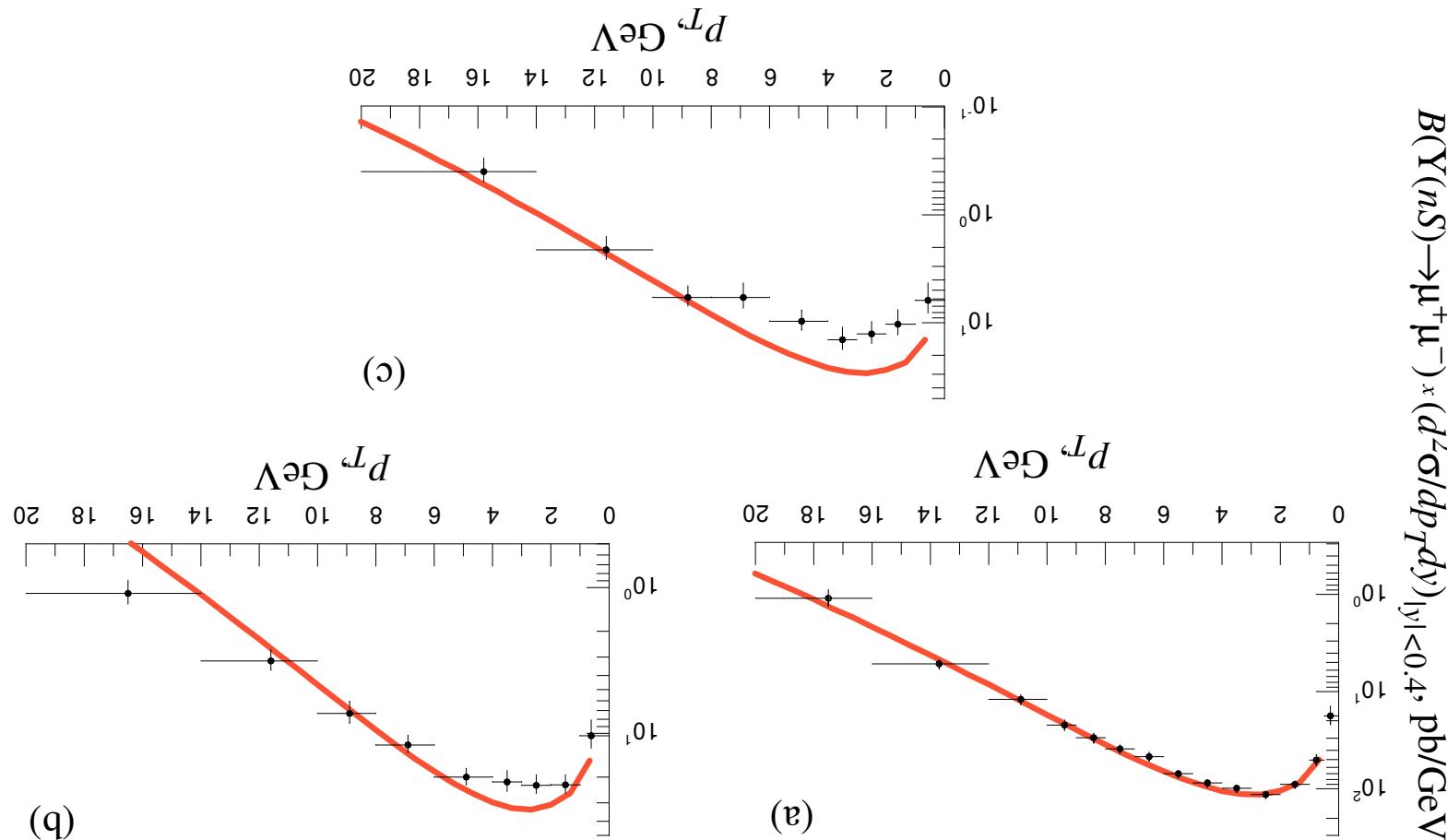


(b)



(c)

Prompt  $\Upsilon(nS)$   $p_T$ -spectra.  $\Upsilon(1S)$  - a,  $\Upsilon(2S)$  - b,  $\Upsilon(3S)$  - c, KMR distribution function, Color Singlet Model with  $\chi_{bJ}(3P)$  contribution.



- Conclusions
1. Working at LO in the QMRK plus NRQCD approach, we analytically evaluated the squared amplitudes of heavy quarks and direct heavy quarkonium production in two reggeized gluon collisions.
  2. We extracted the relevant color-octet NMES,  $\langle O_H [^1S_0]_{(8)} \rangle$ , and  $\langle O_H [^3P_0]_{(8)} \rangle$  for  $H = \chi(1S, 2S, 3S)$ ,  $J/\psi$ ,  $\psi'$ ,  $\chi_{cJ}(1P)$  and  $\chi_{bJ}(1P, 2P)$ , through fits to  $p_T$  distributions measured by the CDF Collaboration in  $p\bar{p}$  collisions at the Tevatron with  $\sqrt{s} = 1.8$  TeV and 1.96 TeV. Our fit to the Tevatron CDF data turned out to be satisfactory with the KMR unintegrated gluon distribution function in the proton.
  3.  $\nabla S \approx \nabla L \approx 0$ .
  4.  $\langle O_{(bb)} [^2S+1T_f]_{(8)} \rangle \gg \langle O_{(cc)} [^2S+1T_f]_{(8)} \rangle$
  5. We have demonstrated phenomenological application of the QMRK approach for heavy quarkonium production at high energies.