On the measurements of polarization observables and their application

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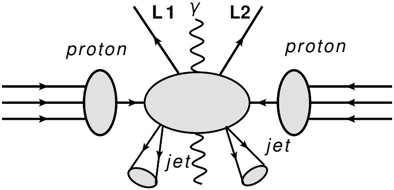
Dilepton plus jets



- Kinematics: 2 leptons + jets + photons
- $p_{\ell\ell} = p_{\ell_1} + p_{\ell_2}$ (e.g. defined in the LAB frame)
- θ, ϕ (of ℓ_1) are defined in the dilepton CMS frame, wrt. a reference axis (e.g. $\vec{p}_{\ell\ell}$)

$$\frac{d\sigma}{\sigma d\cos\theta d\phi} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + A_0\frac{1}{2}(1-3\cos^2\theta) + A_1\sin(2\theta)\cos\phi + A_2\frac{1}{2}\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big].$$

Integrating over ϕ:

$$\frac{d\sigma}{\sigma d\cos\theta} = \frac{3}{8}\Big[(1+\cos^2\theta) + A_0\frac{1}{2}(1-3\cos^2\theta) + A_4\cos\theta\Big].$$

$\rightsquigarrow$ a probe of the dynamics of the lepton-pair creation in a hadronic environment
$A_{3,4,7} = 0$ in QED, A_{0-4}: P-even, A_{5-7}: P-odd (tiny in the SM, in reality?)

Connection to spin-1 theory (I)

[Ref. Gounaris et al IJMPA1993; Aguilar-Saavedra, Bernabeu, arXiv:1508.04592; Aguilar-Saavedra et al, arXiv:1701.03115]

Consider the case of a spin-1 $W^*(m) \to \ell(\lambda_1)\nu_\ell(\lambda_2)$ decay [$\rho_{mm'}$: 3×3 spin-density matrix]:

$$|\mathcal{M}|^2 = \sum_{m,m'} \rho_{mm'} \mathcal{M}_{m\lambda_1\lambda_2}\mathcal{M}^*_{m'\lambda_1\lambda_2},$$

$$= \sum_{m,m'} \rho_{mm'} |a_{\lambda_1\lambda_2}|^2 e^{i(m-m')\phi} d^1_{m\lambda}(\theta) d^1_{m'\lambda}(\theta),$$

$$\mathcal{M}_{m\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} e^{im\phi} d^1_{m\lambda}(\theta), \; \lambda(W^\pm) = \lambda_1 - \lambda_2 = \pm 1,$$

$$d^1_{11}(\theta) = \frac{1+\cos\theta}{2}, \; d^1_{1-1}(\theta) = \frac{1-\cos\theta}{2},$$

$$d^1_{01}(\theta) = \frac{\sin\theta}{\sqrt{2}}, \; d^j_{m'm} = (-1)^{m-m'} d^j_{mm'} = d^j_{-m-m'},$$

θ and ϕ are the polar and azimuthal angles of $\vec{p}_e$ in the W boson rest frame.

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + A_0 \frac{1}{2}(1-3\cos^2\theta) + A_1 \sin(2\theta)\cos\phi$$
$$+ A_2 \frac{1}{2}\sin^2\theta\cos(2\phi) + A_3 \sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5 \sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big]$$

This angular distribution contains all W/Z spin information: 8 (pseudo-)observables!

Connection to spin-1 theory (II)

Relations between the angular coefficients and the spin-density matrix:

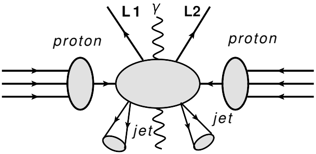
$$A_0 = 2\rho_{00},\; A_1 = \frac{1}{\sqrt{2}}(\rho_{+0} - \rho_{-0} + \rho_{0+} - \rho_{0-}),$$
$$A_2 = 2(\rho_{+-} + \rho_{-+}),\; A_3 = \sqrt{2}b(\rho_{+0} + \rho_{-0} + \rho_{0+} + \rho_{0-}),$$
$$A_4 = 2b(\rho_{++} - \rho_{--}),\; A_5 = \frac{1}{i}(\rho_{-+} - \rho_{+-}),$$
$$A_6 = -\frac{1}{i\sqrt{2}}(\rho_{+0} + \rho_{-0} - \rho_{0+} - \rho_{0-}),\; A_7 = \frac{\sqrt{2}b}{i}(\rho_{0+} - \rho_{0-} - \rho_{+0} + \rho_{-0}), \quad (1)$$

where $b = 1$ for the $W^\pm$ bosons and $b = -c$ for the Z boson, with

$$c = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} = \frac{1 - 4s_W^2}{1 - 4s_W^2 + 8s_W^4} \approx 0.21, \quad s_W^2 = 1 - \frac{M_W^2}{M_Z^2}. \quad (2)$$

- The above simple relations between A_i and the ρ_{ij} were proven for LO decays.
- A_5, A_6, A_7 come from the imaginary parts of ρ_{ij} (absorptive part of scattering amplitudes) $\rightsquigarrow$ expected to be very small.
- $A^Z_{3,4,7} \propto c$, originated from the L-R asymmetry in the $Z^* \to \ell^+\ell^-$ decay. Vanish for the photon.

Master equation



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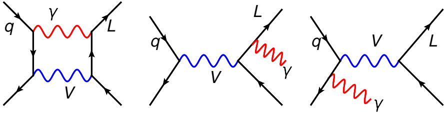
$$\frac{d\sigma}{\sigma d\cos\theta d\phi} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + A_0\frac{1}{2}(1-3\cos^2\theta) + A_1\sin(2\theta)\cos\phi + A_2\frac{1}{2}\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big].$$



- Correct to all oders?
- What is p_{ℓ_i}? bare ($m_\ell > 0$), dressed, or Born-like ($p_V = p_{\ell_1} + p_{\ell_2} + k_\gamma = p^B_{\ell_1} + p^B_{\ell_2}$) leptons?
- Born-like was justified in [Ebert et al 2006.11382], but their argument is only based on FSR, and how can we know if k is FSR or ISR in experiment ?

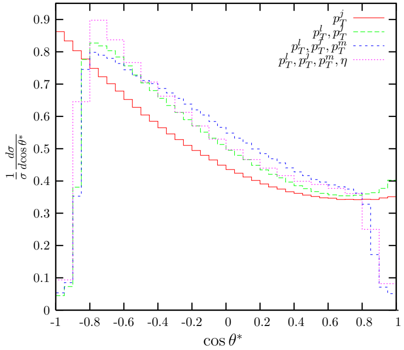
What to do ?

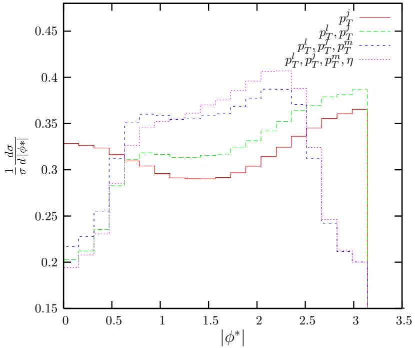
- IMO, using dressed leptons is the best choice (used in most NLO EW calculations, bare leptons lead to large corrections and theoretical uncertainties)
- Experiments: Besides providing the coefficients A_i, the full angular distributions $d\sigma/(d\cos\theta d\phi)$ and $d\sigma/(d\cos\theta)$ should also be provided. After all, all information is encoded in these distributions. They should be therefore measured as precisely as possible. Extracting the coefficients A_i is a separate issue.
- Precision measurements require selection cuts on the individual decay leptons, but these cuts (p_ℓ^T, η_ℓ) lead to shape distortion of the angular distributions.

Effects of lepton cuts

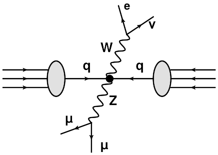
W^+ + 1jet [Stirling, Vryonidou 1204.6427]:

$$p_{T,j} > 20\,\text{GeV},\ p_{T,\ell} > 20\,\text{GeV},\ E_{T,m} > 20\,\text{GeV},\ |\eta_{\ell,j}| < 2.5.$$





Extracting coefficients A_i: WZ production



$$\frac{d\sigma}{\sigma d\cos\theta_e} \equiv \frac{3}{8}\left[(1 \mp \cos\theta_e)^2 f_L^{W^\pm} + (1 \pm \cos\theta_e)^2 f_R^{W^\pm} + 2\sin^2\theta_e f_0^{W^\pm}\right]$$

$$f_L^{W^\pm} = \frac{1}{4}(2 - A_0 \mp A_4),\ f_R^{W^\pm} = \frac{1}{4}(2 - A_0 \pm A_4),\ f_0^{W^\pm} = \frac{1}{2}A_0$$

[ATLAS WZ 1902.05759]: $f_{L,0,R}$ are fitted from fiducial $d\sigma/(d\cos\theta)$ using three templates. These templates are generated using Monte-Carlo event generators, assuming the SM.

Templates are obtained by reweighting of Powheg+Pythia MC events according to:

$$w_{L,R,0} = \frac{1}{U}\left[(1 \mp \cos\theta_e)^2, (1 \pm \cos\theta_e)^2, 2\sin^2\theta_e\right],$$
$$U = (1 \mp \cos\theta_e)^2 f_L^{\text{gen}} + (1 \pm \cos\theta_e)^2 f_R^{\text{gen}} + 2\sin^2\theta_e f_0^{\text{gen}}$$

- The reweighting is done in the fiducial phase space (with lepton cuts)
- f_i^{gen} are determined as functions of p_T^V and y_V in the total phase space (better in $p_T^V, \cos\theta_V, m_V$)

I tried to simplify it by setting $|\mathcal{A}|^2 = 1$, $PDFs = 1$, $U = 1 \rightsquigarrow$ the fit does not work for fiducial cuts (OK for total phase space cut $66 < m_{\mu^+\mu^-} < 116$ GeV).

Question: Are the templates model/generator independent?

A_i without fitting

Ideal case: Inclusive cut (no cuts on individual leptons)

$$\frac{d\sigma}{\sigma d\cos\theta d\phi} = \frac{3}{16\pi}\Big[(1+\cos^2\theta) + A_0\frac{1}{2}(1-3\cos^2\theta) + A_1\sin(2\theta)\cos\phi$$
$$+ A_2\frac{1}{2}\sin^2\theta\cos(2\phi) + A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5\sin^2\theta\sin(2\phi) + A_6\sin(2\theta)\sin\phi + A_7\sin\theta\sin\phi\Big]. \tag{3}$$

Projections:

$$< f(\theta) > = \int_{-1}^{1} d\cos\theta f(\theta)\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta},$$
$$< f(\theta,\phi) > = \int_{-1}^{1} d\cos\theta\int_{0}^{2\pi} d\phi f(\theta,\phi)\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta d\phi}.$$

Inclusive polarization observables:

$$A_0 = 4 - \langle 10\cos^2\theta\rangle,\ A_1 =< 5\sin 2\theta\cos\phi >,\ \ldots$$

Fiducial polarization observables

Note: $d\sigma/(d\cos\theta d\phi)$ is now calculated using the full matrix elements with arbitrary cuts on the individual leptons as usual. Radiative decays and interference effects are all included.

Projections:

$$< f(\theta) > = \int_{-1}^{1} d\cos\theta f(\theta) \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta},$$

$$< f(\theta,\phi) > = \int_{-1}^{1} d\cos\theta \int_{0}^{2\pi} d\phi f(\theta,\phi) \frac{1}{\sigma} \frac{d\sigma}{d\cos\theta d\phi}.$$

Fiducial (arbitrary cuts on leptons) polarization observables:

$$A_0 = 4 - \langle 10\cos^2\theta \rangle,\ A_1 =< 5\sin 2\theta \cos\phi >,\ \ldots$$

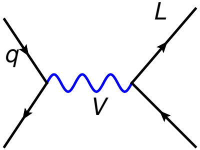
Comparison to the inclusive polarizations:

- Same definition using projections (same characteristics: tiny A_{5-7}, f_0 decreases with p_T, ...).
- The 8 fiducial coefficients A_i are no longer enough to describe the $g = d\sigma/(\sigma d\cos\theta)$ distribution, since some information is lost after the projections. E.g. $< \cos\theta >= \int \cos\theta(g + \cos^{2n}\theta) d\cos\theta$.

On-shell approximation

For the case of fiducial cuts, theorists can do the template fit as experimentalists do to extract the coefficients from the lepton angular distribution. This guarantee that we are comparing the same thing. This requires an extra effort to generate the templates.

There is another method to get useful estimates.



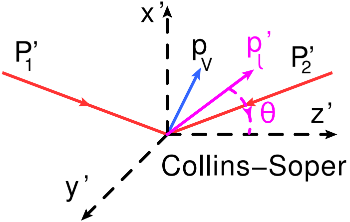
$$\mathcal{A}^V_{\rm OS} = \sum_{\lambda=1}^{3} \mathcal{A}^V_\lambda \quad \text{(on-shell approximation)}$$

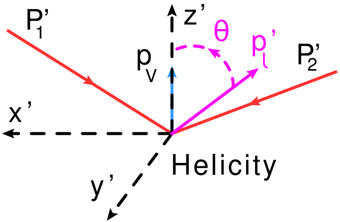
$$d\sigma^{\rm OS} = C \sum_{\lambda,\lambda'=1}^{3} \mathcal{A}^V_\lambda (\mathcal{A}^V_{\lambda'})^* = \sum_{i=1}^{9} d\sigma_i^{\rm OS}$$

$$A_i^{\rm OS} = \frac{d\sigma_i^{\rm OS}}{d\sigma^{\rm OS}} \quad \text{: can be calculated with fiducial cuts!}$$

Note: $A_i^{\rm tem} \neq A_i^{\rm OS}$ (diff. by off-shell, interference and kinematic effects)
Note: $d\sigma_i^{\rm OS}$ are not Lorentz invariant, hence reference frame dependent.
$A_i^{\rm tem}$ are coordinate system dependent.

Coordinate systems





- Collins-Soper coordinate system [CS, 1977]: z' is the bisector of $\vec{P}'_1$ and $-\vec{P}'_2$, points into the hemisphere of $\vec{p}_V$ (in the lab frame).
- Helicity c.s [Bern et al, arXiv:1103.5445]: $z' = \vec{p}_{V,Lab}$.
- f_0 is the longitudinal fraction in the helicity coordinate systems, but not in the Collins-Soper system.

WZ production

13 TeV, ATLAS fiducial cuts (arXiv:1902.05759):

$$p_{T,e} > 20\,\text{GeV}, \quad p_{T,\mu} > 15\,\text{GeV}, \quad |\eta_\ell| < 2.5,$$
$$|m_{\mu^+\mu^-} - M_Z| < 10\,\text{GeV}, \quad \Delta R(\mu^+, \mu^-) > 0.2, \quad \Delta R(e^+, \mu^\mp) > 0.3,$$
$$m_{T,W} = \sqrt{2 p_{T,\nu} p_{T,e}[1 - \cos\Delta\phi(e,\nu)]} > 30\,\text{GeV}$$

Inclusive cut:

$$66 < m_{\mu^+\mu^-} < 116\,\text{GeV}$$

W^+ coefficients in W^+Z production: P-odd coefficients

ATLAS fiducial [Baglio, Ninh 1810.11034]:

Method	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
HE D-LO	1.023	−0.326	−1.404	−0.156	−0.445	−0.0001[3]	0.00001[39]	0.0001[3]
HE F-LO	1.026	−0.286	−1.315	−0.251	−0.447	−0.0021[3]	−0.0006[4]	−0.0036[3]
CS D-LO	1.459	0.299	−0.971	−0.073	−0.544	−0.00001[38]	0.00003[38]	−0.0001[3]
CS F-LO	1.397	0.229	−0.945	0.0025[3]	−0.613	−0.0002[4]	0.0021[4]	0.0036[3]

ATLAS inclusive [Baglio, Ninh 1910.13746]:

Method	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
HE D-LO	0.439	−0.023	−0.154	0.053	−0.757	−0.0001[2]	−0.0001[2]	0.00003[15]
HE F-LO	0.478	0.011	−0.117	−0.161	−0.660	−0.0032[6]	−0.0013[3]	0.0029[4]
CS D-LO	0.574	0.295	−0.019	−0.466	−0.605	−0.0001[1]	0.0001[2]	−0.00002[16]
CS F-LO	0.543	0.219	−0.052	−0.230	−0.682	−0.0013[3]	0.0034[4]	−0.0029[4]

- Watch: A_5, A_6, A_7 are zero in DPA, independent of cuts or coordinate system
- A_5, A_6, A_7 are non-zero for the full matrix elements. This is probably a finite-width effect (i.e. sensitive to the absortive parts of the amplitudes) $\rightsquigarrow$ precision measurements of P-odd observables!

Summary

There are still many open issues, but I think measuring the lepton angular distributions and the coefficients A_i for a single gauge boson is very interesting. Not discussed here, but the helicity correlations between the two gauge bosons in VV, $VVjj$ production are also very interesting.

- Open issues: the validity of the master equation to all orders? Dressed leptons should be used?
- The angular distributions and the coefficients A_i are two separate things. Experimentalists should provide them both.
- Providing also the projections A_i will be useful and make the results more accessible to theorists.
- Very difficult, but proving the P-odd $A_{5,6,7} \neq 0$ will be a probe to the absortive parts of the scattering amplitudes. Do not set them to zero in the fit. The projections A_i are better for this purpose (smaller errors). See e.g. [Frederix, Hagiwara, Yamada, Yokoya 1407.1016 for W + jets].

Thank you for your attention!