Utilizing the causal spectrum of GWs to probe free streaming particles and the cosmological expansion

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arXiv: 2010.03568

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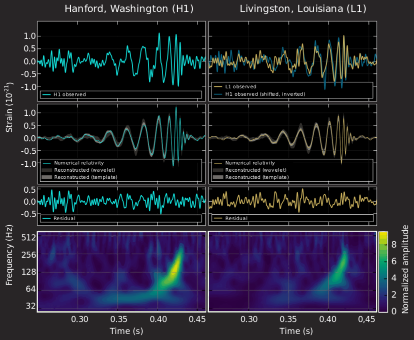
BSM Pandemic

4^{th} Dec. 2020

- 2015 marked the beginning of the era of GW astronomy.
- GW150914: first measurement of GWs from a Binary Black Hole merger!

LIGO

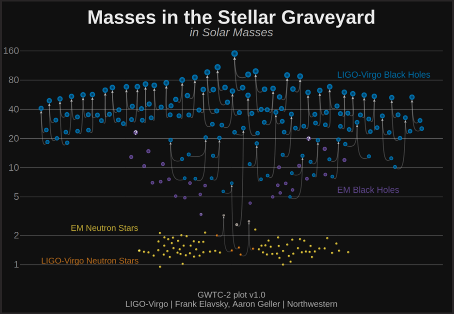
VIRGO

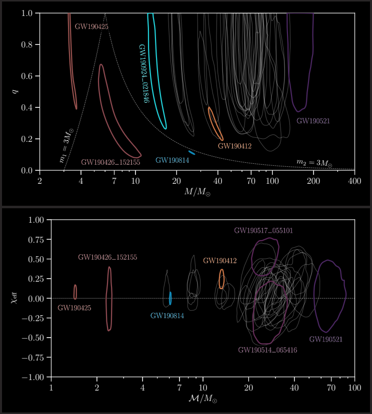


[LIGO, 1602.03837]

The era of Gravitational Wave astronomy

- The detection of many more mergers of compact objects allows us to study the properties of black holes, neutron stars in an unprecedented way.



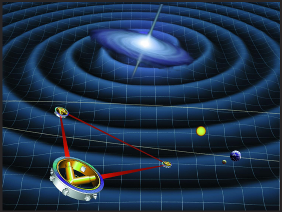


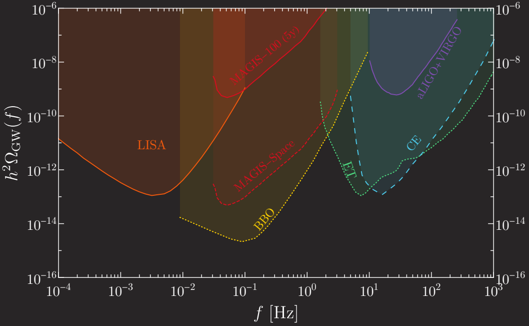
[LIGO-VIRGO O3, 2010.14527]

- Exciting prospects regarding astrophysical BHs, binary formation, H_0 measurements, BH superradiance, astro vs primordial BHs, ...

Stochastic Gravitational Wave Background

- One of the next frontiers in this exciting era is the search for a stochastic GW background, analogous to the CMB.
- Present upper limit from LIGO on stochastic GW background: $\Omega_{\text{GW}}(10 - 100 \text{ Hz}) < 1.7 \cdot 10^{-7}$ [1612.02029]
- Future space-based experiments will extend the reach in GW frequencies down to 0.1 mHz.

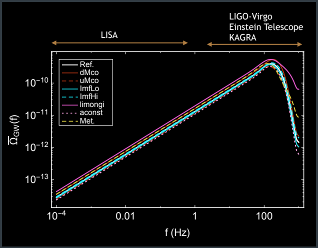




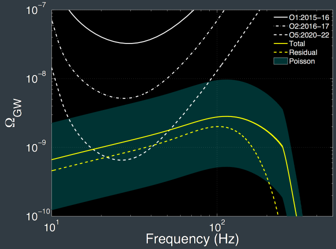
Stochastic GW Backgrounds in SM + ΛCDM

Astrophysical background: binary BH mergers

Unresolved sources lead to a stochastic background.
From stellar-mass BBHs: within reach of LIGO-VIRGO.



[Cusin+ '19]



[LIGO 1602.03847]

Distinguishing features: tilt $+2/3$ at low f, and peculiar frequency dep. of anisotropy spectrum. [Bartolo+ '19; Cusin+ '19; Hotinli+ '19]

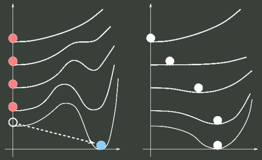
Primordial GWs from inflation

The primordial tensor modes generated by inflation are below $\Omega_{\rm GW} \lesssim 10^{-15}$ for the presently allowed value of r (and flat spectrum) $\Longrightarrow$ unobservable.

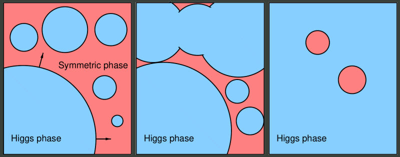
Stochastic GW Backgrounds: BSM

Phase transitions

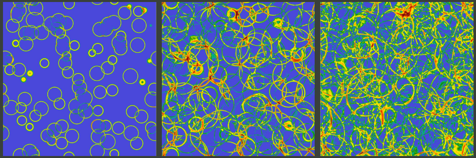
Cosmological phase transitions of the 1^{st} order source GWs.



1^{st} order 2^{nd} order



[Hindmarsh+ '20]



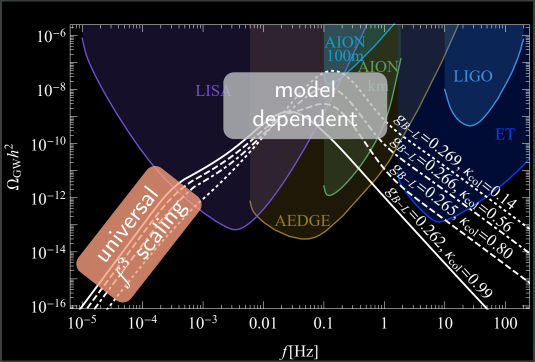
[Hindmarsh+ '15]

In the SM, both EW and QCD phase transitions are of 2^{nd} order, but new physics can generically display 1^{st} order phase transitions.

First Order Phase transitions

GWs can be generated by three sources:

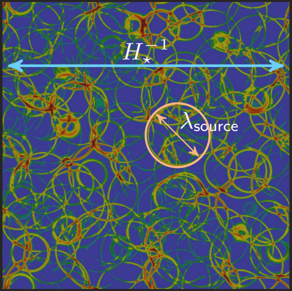
1. bubble collisions (dynamics of the scalar field);
2. sound waves in the plasma;
3. turbulences in the plasma.



[Ellis+ '20]

- The scaling as f^3 at low frequencies is a universal feature (assuming RD). [Caprini+ '09]
- What is the physical origin of this general behaviour?

Causality *prevents local phenomena from being correlated beyond* H^{-1}.



- Source $\Pi_{ij}(x)$ of GWs has a correlation length $\lambda_{\text{source}} \ll H_\star^{-1}$:

$$\langle \Pi(0)\, \Pi(d \gg \lambda_{\text{source}}) \rangle = 0 \;\Rightarrow$$
$$\langle \widetilde{\Pi}(k)\, \widetilde{\Pi}(-k) \rangle \xrightarrow{k \ll \lambda_{\text{source}}^{-1}} \text{constant}$$

- The spectral tilt at low f does not depend on the source.
- Wavelengths which were super-horizon are not sensitive to the details of the generation, but only to the universe expansion and the GW propagation.

- We consider GWs for which

  wavelength $k^{-1} \gg$ corr. length of the source λ_{source}

  period $f^{-1} \gg$ duration of the phase transition β^{-1}

- Eq. of motion for the GW $h_{ij}^{(+,\times)}(k,\tau) \equiv h$:

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 4\mathcal{H}^2 \Pi(k,\tau) = J_\star \; \delta(\tau - \tau_\star)$$

  approximating the source as instantaneous and constant at small k.

- The solution in a radiation-dominated universe is

$$h(\tau) = \frac{a_\star}{a} \underbrace{\frac{1}{k}}_{\text{specific of RD}} J_\star \sin k(\tau - \tau_\star)$$

- The spectrum of Ω_{GW} at low frequencies is

$$\frac{\mathrm{d}\,\Omega_{\text{GW}}}{\mathrm{d}\ln k} \sim \underbrace{k^3}_{\text{phase space}} \cdot \underbrace{k^2}_{\rho_{\text{GW}} \sim h'^2} \cdot \underbrace{\frac{1}{k^2}}_{\text{for RD}} \cdot \underbrace{P_\Pi(k)}_{k \text{ ind. from causality}} \sim k^3 \,.$$

Utilizing the causality-limited spectrum

- Causality (absence of correlation beyond Hubble for local processes) is precisely what makes the source $J_\star$ independent from k for $k \ll \lambda_{\text{source}}^{-1}$.
- The universality of the spectrum at low frequencies for causality-limited processes makes it an exciting tool to study our universe. [Hook, Marques-Tavares, DR 2010.03568]
- Phase transitions are the key example, but also preheating and GWs at 2nd order from peaks in the scalar perturbations are possible causality-limited scenarios.
- What can we extract from it?

1. How can we physically understand the f^3 scaling?
2. What can alter the propagation of GWs and hence their causality-limited spectrum?
3. How is the expansion history of the Universe influencing the causality-limited spectrum?

Causality-limited spectrum: physical understanding

- We investigate the physical origin of the k scaling of causality-limited GWs:

$$h(\tau) = \frac{a_\star}{a} \underbrace{\frac{1}{k}}_{\text{specific of RD}} J_\star \sin k(\tau - \tau_\star)$$

- The eq. of motion right after $\tau_\star$ is

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 4\mathcal{H}^2 \Pi(k,\tau) = J_\star\,\delta(\tau - \tau_\star)$$

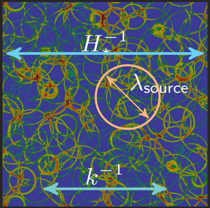
$$\Downarrow$$

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 0\,, \qquad \begin{cases} h(\tau_\star + \epsilon) = 0 \\ \partial_\tau h(\tau_\star + \epsilon) = J_\star \end{cases}$$

- The sudden beat given to the oscillator imprints a velocity and zero displacement to the wave, similarly to a hammer hitting on a string.

Causality-limited spectrum: sub-horizon modes

Sub-horizon modes $\lambda_{\text{source}}^{-1} \gg k \gg \mathcal{H}_\star$



- Modes which are sub-horizon at generation (but still beyond the correlation length of the source) are **under-damped**

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h \boxed{+k^2 h} = 0$$

- The solution is a frictionless oscillation, whose amplitude red-shifts as $1/a$:

$$h(\tau) \approx \frac{a_\star}{a} \underbrace{\boxed{\frac{1}{k}}}_{\text{sub-hor.}} J_\star \sin k(\tau - \tau_\star)$$

- Apart from the redshift, the eq. of state w of the universe does not enter.
- Sub-horizon modes are insensitive to the expansion rate.
- The corresponding Ω_{GW} is

$$\frac{\mathrm{d}\,\Omega_{\text{GW}}}{\mathrm{d}\ln k} \sim \underbrace{k^3}_{\text{phase space}} \cdot \underbrace{k^2}_{\rho_{\text{GW}} \sim h'^2} \cdot \underbrace{\boxed{\frac{1}{k^2}}}_{\text{sub-hor.}} \cdot P_\Pi(k) \sim k^3 \,.$$

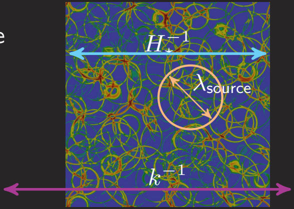
Causality-limited spectrum: super-horizon modes

Super-horizon modes $k \ll \mathcal{H}_\star$

- Super-horizon modes are **over-damped**: Hubble friction prevails.

$$\partial_\tau^2 h + 2\mathcal{H}\,\partial_\tau h + k^2 h = 0$$

$$\begin{cases} h = 0 \\ h' = J_\star \end{cases} \longrightarrow \begin{cases} h = \frac{J_\star}{\mathcal{H}_\star} \\ h' = 0 \end{cases} \longrightarrow \text{frozen}$$



- The dependence on $\mathcal{H}$ along the whole super-horizon phase explains why they are a tool to study the Universe expansion.
- After Hubble crossing at $\mathcal{H}(\tau_k) = k$, h starts redshifting and oscillating:

$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau$$

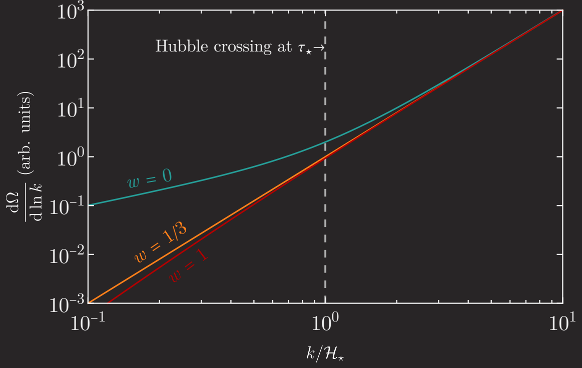
Two effects impact super-horizon modes.

		RD	$a \sim \tau^n,\ n = \begin{cases} 1 & \text{RD} \\ 2 & \text{MD} \end{cases}$
Suppression due to exciting over-damped mode		$\frac{k}{\mathcal{H}_\star}$	$\frac{k}{\mathcal{H}_\star}$
Boost due to mode being frozen while super-horizon		$\frac{\mathcal{H}_\star}{k}$	$\left(\frac{\mathcal{H}_\star}{k}\right)^n$
Super-horizon	$\lvert h\rvert \approx$	$\frac{a_\star}{a}\frac{J_\star}{k}$	$\frac{a_\star}{a}\frac{J_\star}{k}\left(\frac{\mathcal{H}_\star}{k}\right)^{n-1}$
Sub-horizon	$\lvert h\rvert \approx$	$\frac{a_\star}{a}\frac{J_\star}{k}$	$\frac{a_\star}{a}\frac{J_\star}{k}$

$\Omega_{\text{GW}} \sim k^3$ (sub-horizon)

$$\Omega_{\text{GW}} \sim k^{5-2n} = \begin{cases} k^3 & \text{RD} \\ k & \text{MD} \end{cases} \text{(super-horizon)}$$

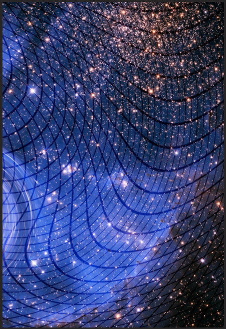
- To confirm these estimates, we solve the full eq. of motion, getting Bessel functions $j_{n-1}(k\tau)$, $y_{n-1}(k\tau)$.
- Notice the change in slope appearing at $k = \mathcal{H}_\star$, the conformal Hubble at the phase transition.



Propagation of GWs: effect of relativistic free-streaming particles

2. What can alter the propagation of GWs and hence their causality-limited spectrum?

- An important effect for the GW spectrum, also known as Weinberg damping [Weinberg '04], concerns the impact of free-streaming (FS) particles on the GW propagation.



- GWs are sourced by the anisotropic component π_{ij} of the stress tensor:

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 4\mathcal{H}^2 \pi_{ij}$$

- FS particles travel distances $\sim H^{-1}$ along geodesics, and are affected by passing GWs.
- In turn, FS particles react on the GWs by acting as a small friction term: $\pi_{ij} \propto h'_{ij}$.
- The effect is active as soon as $h' \neq 0$ and decreases in time as the particles' momenta redshift.
- Only relativistic FS particles have an impact, $T_{ij} \sim p_i p_j$.

Weinberg damping in the SM

- In the SM, the only FS species are neutrinos after their decoupling around $T \sim 2$ MeV, and they contribute with $f_\nu = \frac{\rho_\nu}{\rho_{\text{tot}}} = 0.4$.
- In the case of primordial GWs, they were frozen $(h' = 0)$ until horizon-entry. The damping is effective as the mode crosses the horizon and starts oscillating.
- The eq. of motion is [Weinberg '04]

$$h'' + 2\mathcal{H}h' + k^2 h = -24 f_\nu \mathcal{H}^2 \int_{\tau_0}^{\tau} K\Big(k(\tau - \tilde{\tau})\Big) h'(\tilde{\tau}) \, \mathrm{d}\tilde{\tau}$$

$$K(s) = \frac{3 \sin s}{s^5} - \frac{3 \cos s}{s^4} - \frac{\sin s}{s^3}$$

- In the SM, this effect is frequency independent and reduces the GW amplitude by 0.8:

$$\Omega_{\text{GW}}(k) \longrightarrow 0.64\, \Omega_{\text{GW}}(k)$$

Weinberg damping for phase transitions

- For phase transitions (fast GW source), there is a further effect *at generation*, if some new FS particles (f_{FS}) are present at early times ($T_\star \gg$ MeV).
- Initial condition for fast sources: $\begin{cases} h(\tau_\star) = 0 \\ h'(\tau_\star) = J_\star \end{cases}$
- In the super-horizon limit $k \to 0$, the eq. of motion simplifies: $K(s) \to \frac{1}{15}$, and the integral is solved to

$$h'' + 2\mathcal{H}h' + \left(k^2 + \frac{8 f_{\mathsf{FS}}}{5}\mathcal{H}^2\right) h = 0$$

- Sub-horizon modes $k \gg \mathcal{H}_\star$ at generation: both Hubble friction and Weinberg damping are negligible.

$$h'' + 2\mathcal{H}h' + \left(k^2 + \frac{8 f_{\mathsf{FS}}}{5}\mathcal{H}^2\right) h = 0$$

These modes are unaffected by damping $\Rightarrow$ feature at $k = \mathcal{H}_\star$.

- Super-horizon modes $k \ll \mathcal{H}_\star$: these modes are damped by Hubble friction, and the Weinberg term determines whether they are over- or under-damped.

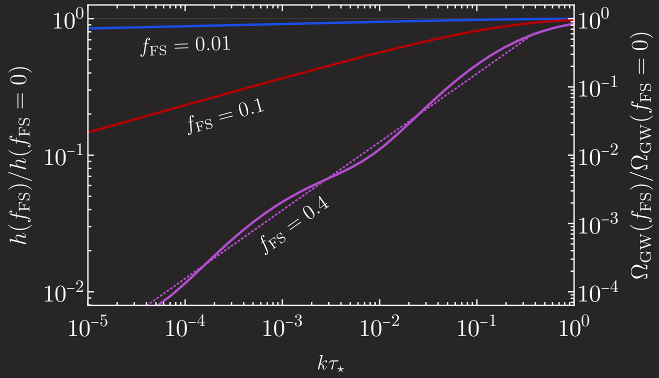
$$h'' + \underbrace{2\mathcal{H}h'}_{\text{friction}} + \underbrace{\left(k^2 + \frac{8 f_{\text{FS}}}{5}\mathcal{H}^2\right)}_{\text{mass term}} h = 0$$

- **Over-damped**: $(\text{friction})^2 \gg (\text{mass})^2$, or $f_{\text{FS}} \ll 1$. The mode does not oscillate while super-horizon, and its amplitude is dampened compared to the case $f_{\text{FS}} = 0$.
- **Under-damped**: $(\text{friction})^2 < (\text{mass})^2$, or $f_{\text{FS}} > 16\%$. The Weinberg term is so large to induce oscillations while super-horizon. On top of the damping, oscillations appear in the spectrum.

- In presence of rel. FS particles at the phase transition, the GW spectrum changes tilt below $k < \mathcal{H}_\star$:

$$\frac{\Omega_{\mathsf{GW}}^{(f_{\mathsf{FS}})}}{\Omega_{\mathsf{GW}}} \sim \begin{cases} k^{\frac{16 f_{\mathsf{FS}}}{5}} & f_{\mathsf{FS}} < \frac{5}{32} \\ k\left[c_1 + c_2 \sin\left(\sqrt{\frac{32}{5} f_{\mathsf{FS}} - 1}\, \ln(k\tau_\star) + c_3\right)\right] & f_{\mathsf{FS}} > \frac{5}{32} \end{cases}$$

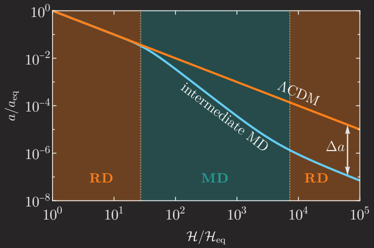
- Current bounds $\Delta N_{\mathsf{eff}} < 0.3$ allow for $f_{\mathsf{FS}} \sim 9\%$ at early times.



Probing the cosmological expansion with causal GWs

3. How is the expansion history of the Universe influencing the causality-limited spectrum?

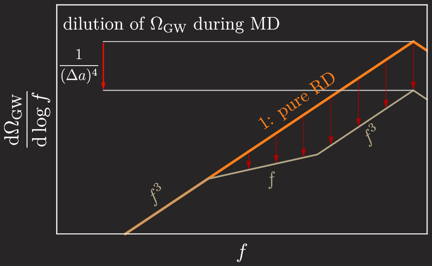
- Alternative expansion histories imply two modifications:
  1. Change the shape of the GW spectrum for modes which are super-horizon at generation;
  2. Change the rescaling between comoving modes k and physical frequencies $f = k/a$.

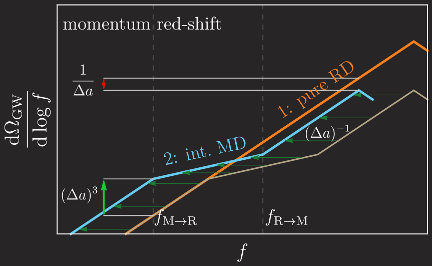


- We consider an intermediate MD era:
  - case 1: only RD
  - case 2: RD $\to$ intermediate MD $\to$ RD.
- The transition from RD to MD happens due to some non rel. species taking over.
- This species later decays into radiation.
- The scale factor has an overall difference

$$\Delta a = \left(\frac{T_{\mathsf{R}\to\mathsf{M}}}{T_{\mathsf{M}\to\mathsf{R}}}\right)^{1/3} > 1$$

Intermediate phase of MD



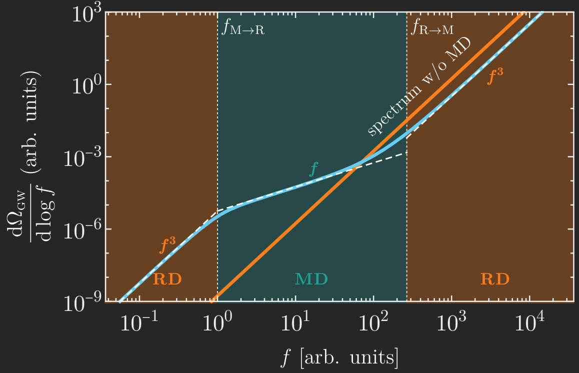


- GW modes that were sub-horizon during MD redshift more in case 2: a expands more.
- $\Rightarrow$ suppression $(\Delta a)^{-4}$ at high f.
- Modes which enter after M→R have the same evolution in the two cases.
- The intermediate range interpolates between the two, with $\Omega_{\text{GW}} \sim k$.

- The physical frequency $f = k/a$ is moved to lower values in case 2, because of the larger redshift: $f \to f/(\Delta a)$.
- Given the tilt f^3, this implies that **low frequency modes have an overall boost** of $(\Delta a)^3$.
- The net effect for high frequencies is a suppression $(\Delta a)^{-1}$.

Intermediate phase of MD

- The numerical solution confirms these scalings.
- The low-frequency range (which could be the only one potentially accessible for GWs from reheating) is made more visible by a MD phase.



Measuring $w(\tau)$?

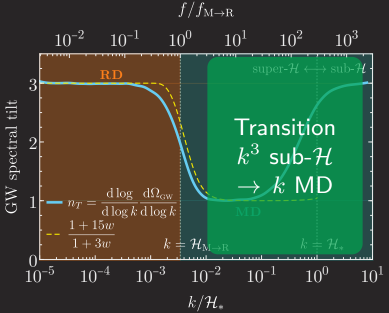
- The GW spectrum for super-$\mathcal{H}$ modes and *constant* w is

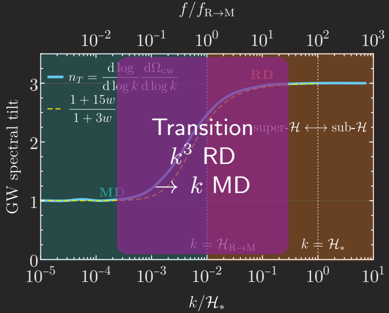
$$\Omega_{\rm GW}(k) \sim k^{5-2n} = k^{\frac{1+15w}{1+3w}}$$

- $w(\tau)$: for each k we identify w with its value at Hubble crossing. If $w'(\tau) \ll \mathcal{H}$,

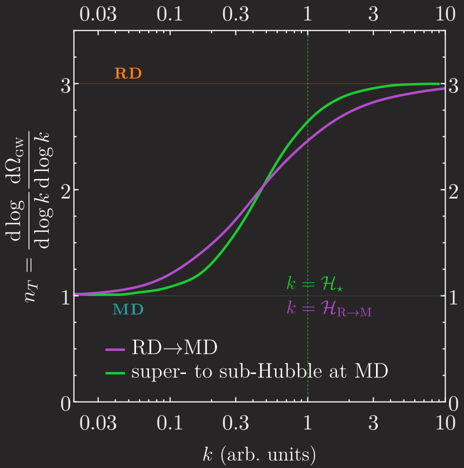
$$\text{GW tilt } = \frac{{\rm d}\log\Omega_{\rm GW}(k)}{{\rm d}\log k} \approx \frac{1+15w(\tau)}{1+3w(\tau)}\,.$$

- The agreement ends up being quite good, although approximate.





Measuring $w(\tau)$?



- It seems difficult in practice, but it could still be possible.

Conclusions

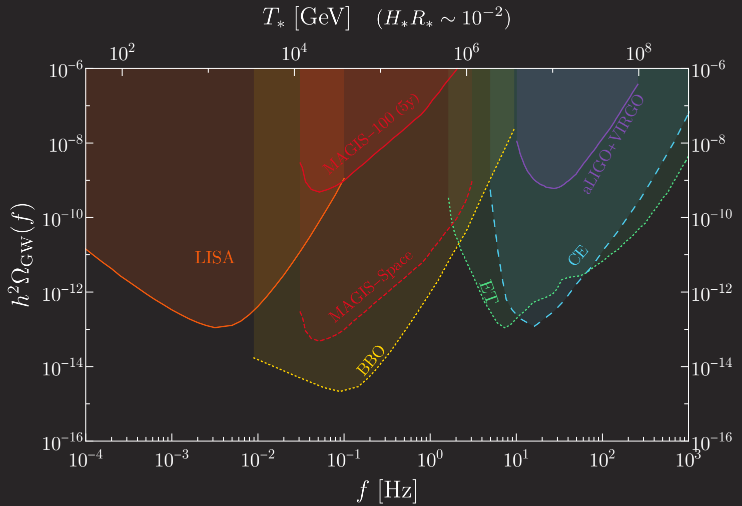
- GWs generated by causal phenomena (uncorrelated beyond H^{-1}), such as a phase transition, are insensitive to the details of the generation.
- The universal behaviour of the causal spectrum makes it an attracting tool to explore the cosmology of the early universe.
- Deviations from the prediction of f^3 would robustly signal new physics.
- Causal modes can be understood in simple physical terms, which highlight the impact of modifications of the cosmological model.

1. The presence of extra free-streaming species could be read off from the GW spectrum, and cross-checked with measurements of ΔN_{eff}.
2. Intermediate phases of MD, which can arise in modifications of ΛCDM, amplify the GW signal at low frequencies.
3. Various phenomena could imprint a change of tilt around $k = \mathcal{H}_\star$, potentially allowing to measure the conformal Hubble rate around the phase transition.

Thanks for your attention!

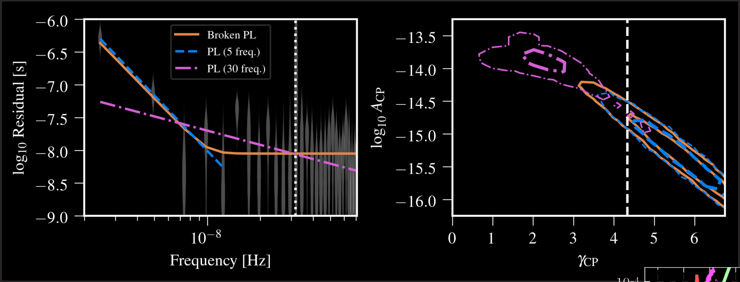
1. BACKUP SLIDES

Temperature of the phase transition

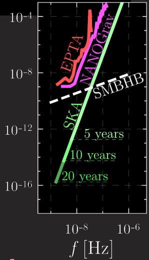


Stochastic Gravitational Wave Background

- At present there is a very interesting result from the NANOGrav collaboration (using Pulsar Timing). [NANOGrav 2009.04496]



- It's soon to tell the origin of this red-noise process:
  1. Improperly modelled source of systematic noise;
  2. SGWB from the mergers of supermassive BHs;
  3. SGWB from new physics.
- Regardless, we could witness in the near future the discovery of a stochastic background of GWs!



[Breitbach+ '18]

Causality-limited spectrum: scaling during RD

Radiation domination

- Super-horizon modes: $\mathcal{H} \sim \frac{1}{\tau} \sim \frac{1}{a}$ and they enter the horizon at $\mathcal{H}(\tau_k) = k$, so

$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau = \frac{a_\star}{a} \frac{J_\star}{k} \sin k\tau$$

- They match precisely the sub-horizon solution!
- The reason is that two competing effects precisely cancel during RD:
  1. Suppression $\dfrac{k}{\mathcal{H}_\star}$ due to exciting over-damped mode;
  2. Boost of $\dfrac{a(\tau_k)}{a_\star}$ due to mode being frozen while super-horizon $\overset{\text{RD}}{\to} \dfrac{\mathcal{H}_\star}{k}$.
- As a result, for the standard case of a phase transition during RD, there are no features around $k \sim \mathcal{H}_\star$.
- All modes have an amplitude $\dfrac{1}{k}$, and $\Omega_{\text{GW}} \sim k^3$.

Causality-limited spectrum: scaling for generic w

Generic equation of state $a \sim \tau^n$

- Generic equation of state: $a \sim \tau^n$ where $n = \frac{2}{1+3w}$ is 1 for RD, 2 for MD.
- Super-horizon modes: $\mathcal{H} \sim \dfrac{1}{\tau} \sim \dfrac{1}{a^{1/n}}$ so

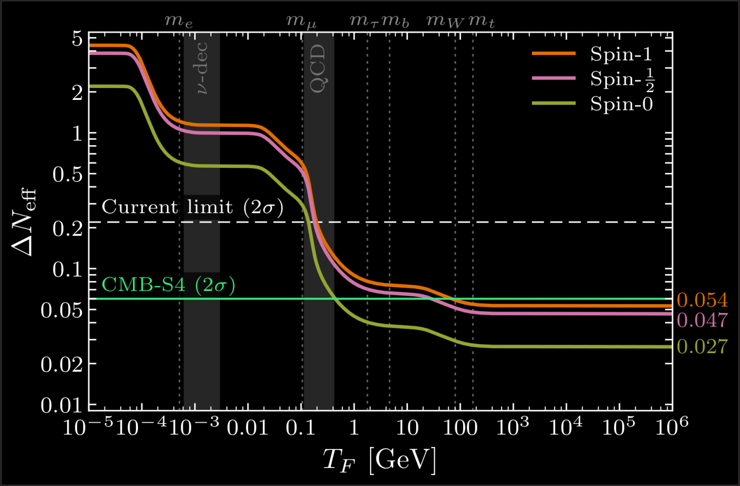
$$h \approx \frac{a(\tau_k)}{a} \frac{J_\star}{\mathcal{H}_\star} \sin k\tau = \frac{a_\star}{a} \left(\frac{\mathcal{H}_\star}{k}\right)^{n-1} \frac{J_\star}{k} \sin k\tau$$

- For $n \neq 1$ the scaling is not $1/k$ like sub-horizon modes.
- Physically, the boost in amplitude due to the mode being frozen is $\left(\frac{\mathcal{H}_\star}{k}\right)^n$, which for MD is larger than the suppression $\frac{k}{\mathcal{H}_\star}$ due to over-damping.
- The conformal time before horizon-entry is the same (from $\mathcal{H}_\star$ to k), but the expansion of a during that time is different.
- The spectral tilt is then

$$\Omega_{\text{GW}} \sim k^3 \ \text{(sub-horizon)}$$

$$\Omega_{\text{GW}} \sim k^{5-2n} = \begin{cases} k^3 & \text{RD} \\ k & \text{MD} \end{cases} \ \text{(super-horizon)}$$

Measurement of ΔN_{eff}



[CMB-S4 Science report 1907.04473]

Schematic derivation of Weinberg damping

[Weinberg '04; Watanabe, Komatsu '06]

- GWs are sourced by the anisotropic component π_{ij} of the stress tensor:

$$h''_{ij} + 2\mathcal{H}h'_{ij} + k^2 h_{ij} = 4\mathcal{H}^2\pi_{ij}$$

$$T_{ij} = p\, g_{ij} + a^2\pi_{ij}\,, \qquad T_{ij}^{(\nu)} = \frac{1}{\sqrt{-g}}\int \frac{\mathrm{d}^3 q}{q^0} q_i q_j F^{(\nu)}(q)$$

- The ν phase space distribution $F(x,p)$ is obtained from the collisionless Boltzmann (i. e. Vlasov) equation.
- By decomposing $F(x,p) = \overline{F}(p) + \delta F(x,p)$ where $\overline{F}(p)$ is the equilibrium distribution, and keeping 1st order terms in perturbation theory:

$$0 = \frac{\mathrm{d}F}{\mathrm{d}t} = \frac{\partial F}{\partial \tau} + \frac{\mathrm{d}x^i}{\mathrm{d}t}\frac{\partial F}{\partial x^i} + \frac{\mathrm{d}p^0}{\mathrm{d}t}\frac{\partial F}{\partial p^0}$$

- The last term is obtained from the geodesic equation:

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\lambda} = -\Gamma^\mu_{\alpha\beta}p^\alpha p^\beta \implies \frac{1}{p^0}\frac{\mathrm{d}p^0}{\mathrm{d}t} = -H - \frac{1}{2}\frac{\partial h_{ij}}{\partial t}\frac{p^i p^j}{(p^0)^2}$$

  As ν's propagate in a FRW universe with GWs, they lose (or gain) energy depending on the sign of h'.
- δF is computing by integrating the Boltzmann eq. over time, and the result is

$$h'' + 2\mathcal{H}h' + k^2 h = -24 f_\nu \mathcal{H}^2 \int_{\tau_0}^{\tau} \mathrm{d}\tau' \frac{j_2[k(\tau-\tau')]}{k^2(\tau-\tau')^2} h'(\tau')$$