



CARNEGIE
SCIENCE

Observatories

Collision of Fuzzy Dark Matter Halos

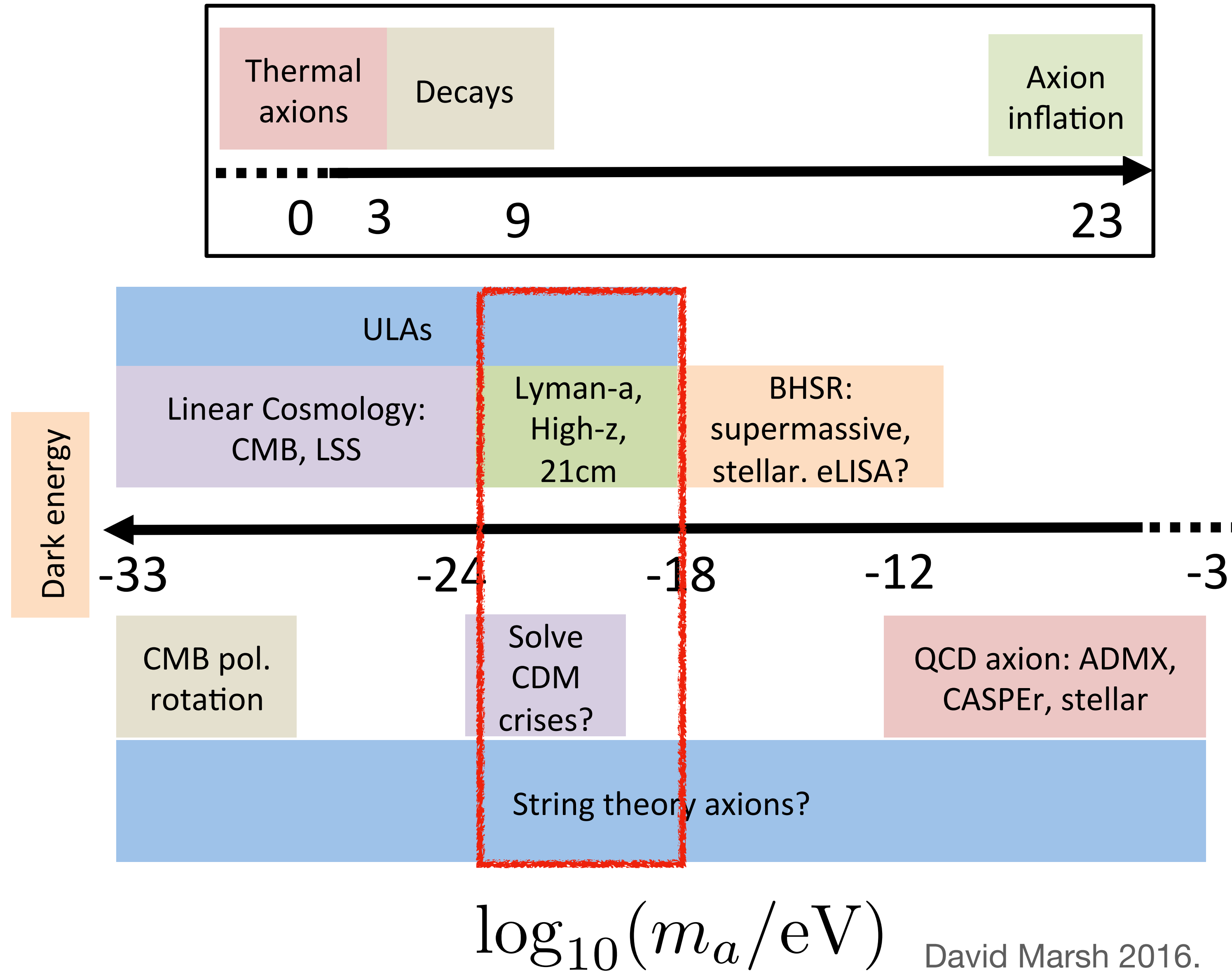
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BSM PANDEMIC Seminars

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Axion-like particles



Fuzzy Dark Matter ($10^{-19} \sim 10^{-22} \text{eV}$)

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{4!} \phi^4 + \dots$$

$$\phi = \text{Re}(\psi e^{imt})$$

Schrödinger-Poisson Equations (non-relativistic limit)

$$i\hbar \frac{\partial}{\partial t} \psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{x}, t) + m\Phi(\mathbf{x})\psi(\mathbf{x}, t)$$

Wave Equation

$$\nabla^2 \Phi(\mathbf{x}, t) = 4\pi G |\psi(\mathbf{x}, t)|^2$$

↓ $\psi = \sqrt{\rho} e^{iS}, v = \frac{\hbar}{m} \nabla S$

$$\dot{\rho} + \nabla \cdot (\rho v) = 0$$

$$\dot{v} + v \cdot \nabla v = -\nabla \Phi + \frac{1}{2m^2} \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \right)$$

Fluid Equation

Simulations of Cosmic Structure Formation

1. Solve the wave equation

(1) Finite difference: Schive et al. 2014a, Bodo et al. 2016

(2) Pseudo-spectral method: Woo et al. 2009, Mocz et al. 2017, Du et al. 2018, Mocz et al. 2019

2. N-body+quantum pressure

Mocz et al. 2015, Veltmaat et al. 2016, Nori et al. 2018

3. Hybrid Method

Veltmaat et al. 2018, 2020

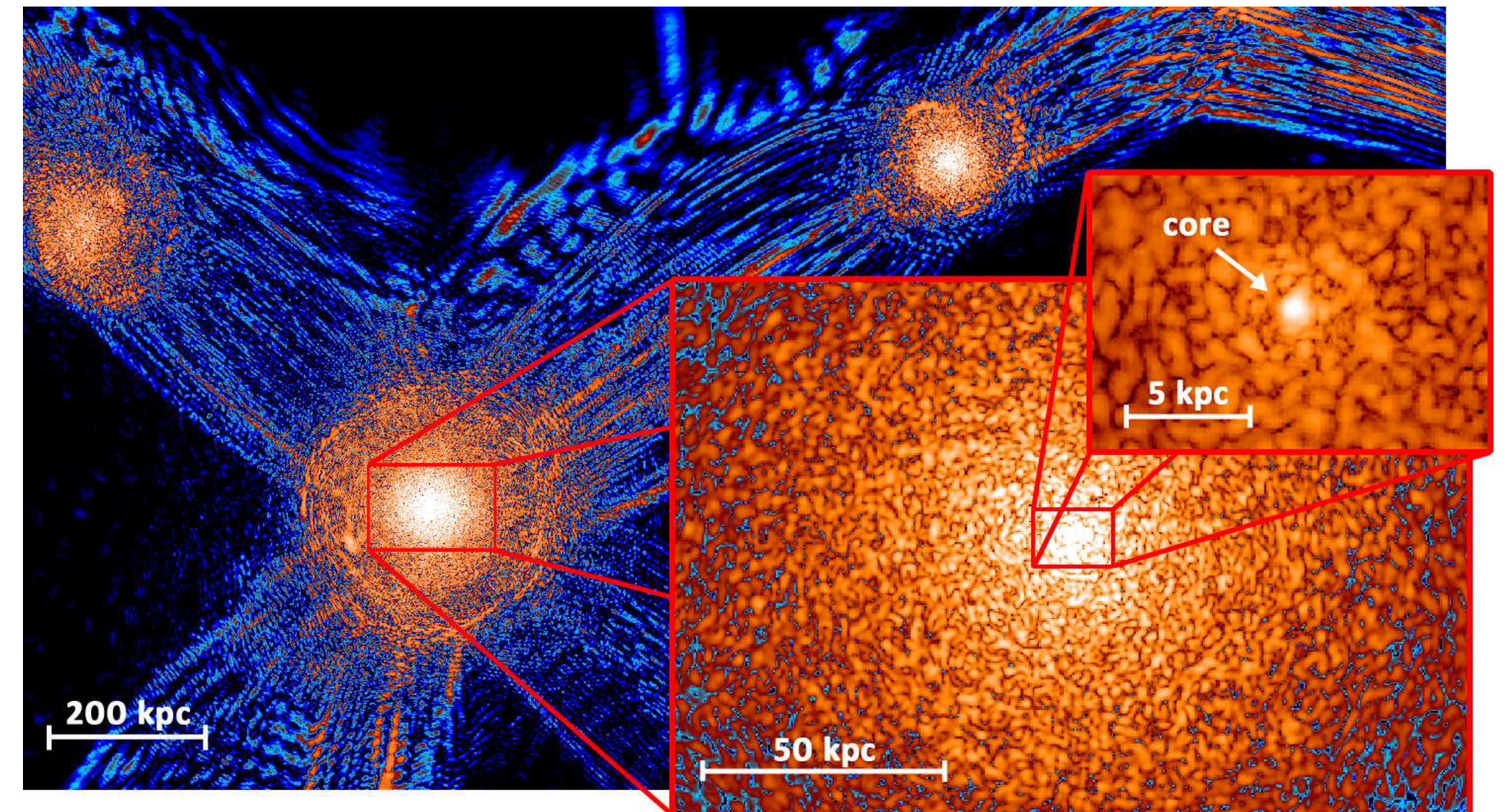
Suppression of small-scale structure

$$m = 10^{-22} eV \rightarrow M_h < 10^9 M_\odot$$

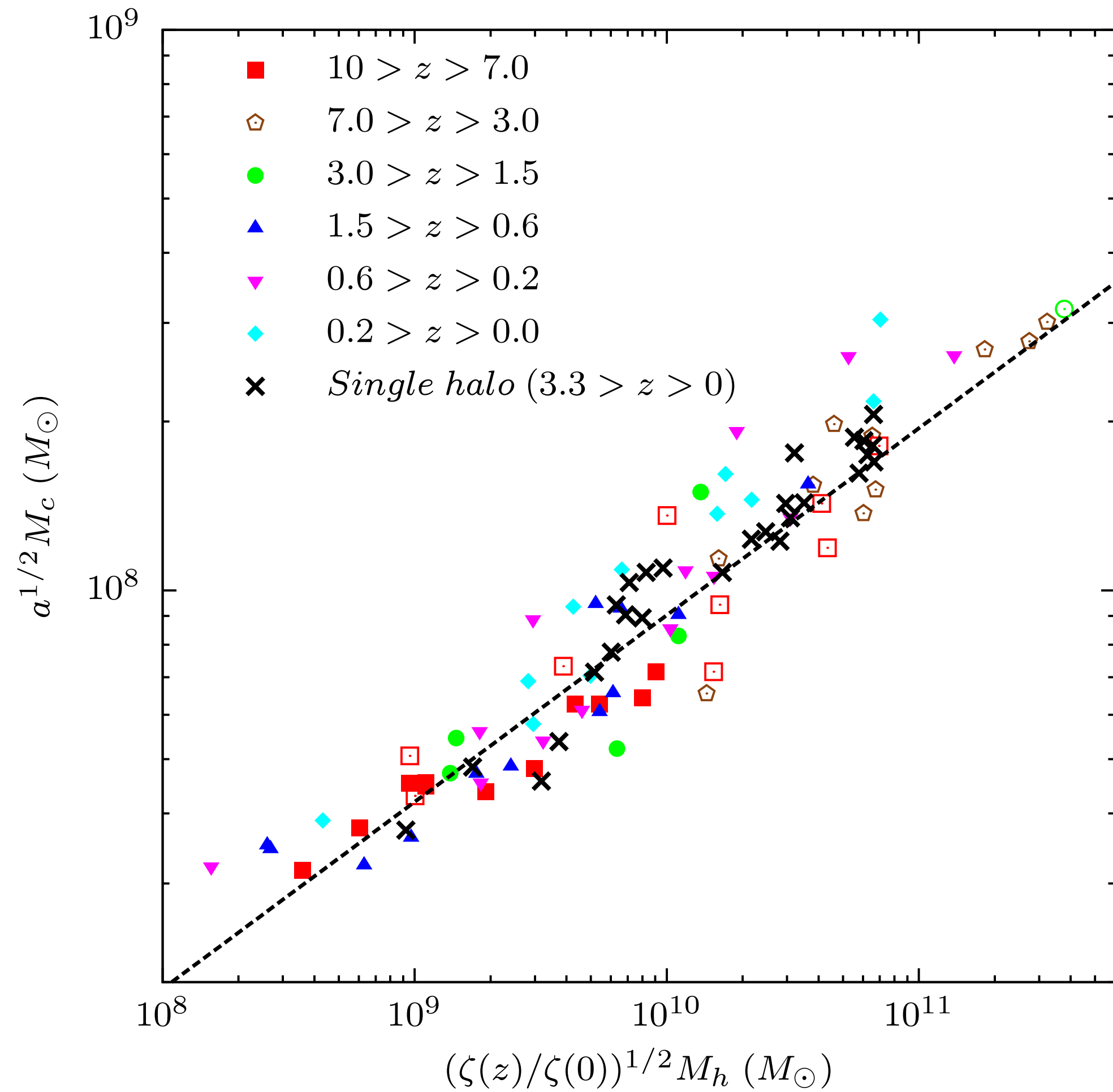
Flat halo core

Soliton solution + NFW envelope

Schive et al. 2014a



Core-halo Mass Relation



Schive et al. 2014b

Soliton (core) density

$$\rho_c(r) = \frac{1.9 a^{-1} (m/10^{-23} \text{eV})^{-2} (r_c/\text{kpc})^{-4}}{[1 + 9.1 \times 10^{-2} (r/r_c)^2]^8} M_\odot \text{pc}^{-3}$$

Core mass - halo mass

$$M_c \propto a^{-1/2} \left(\frac{\xi(z)}{\xi(0)} \right)^{1/6} M_h^{1/3}$$

Uncertainty principle

$$r_c \sigma_h \sim 1 \longrightarrow r_c M_h^{1/3} \sim 1 \xrightarrow{r_c \sim M_c^{-1}} M_c^{-1} M_h^{1/3} \sim 1$$

Valid only for relaxed halos

Collision of Two Halos

1. Initial conditions

$$\psi(x, t) = \sum \sqrt{f(E)} \psi_E(x) e^{-iEt/\hbar}$$

$$f(E) = A \left[e^{-\beta(E-E_c)} - 1 \right], \quad E \leq E_c$$

$$f(E) = 0, \quad E > E_c$$

(King model, Lin et al. 2018)

2. Pseudo-spectral method

$$\psi(x, t + \Delta t) = e^{-i\hat{H}\Delta t} \psi(x, t)$$

$$\exp(-i\hat{H}\Delta t) = \prod_i e^{-it_i K \Delta t} e^{-iv_i W \Delta t}$$

An example (kick-drift-kick, 2-order)

a. gravitation potential

$$\Phi = -i\text{FFT} \left[\frac{4\pi G}{k^2} \text{FFT} [\rho] \right]$$

b. Wave function

Kick: $\psi(x) \leftarrow e^{-\frac{i}{2}\Phi\Delta t} \psi(x)$



Drift: $\psi(k) = \text{FFT}[\psi(x)]$

$$\psi(k) \leftarrow e^{-ik^2\Delta t} \psi(k)$$

$$\psi(x) = i\text{FFT}[\psi(k)]$$



Kick: $\psi(x) \leftarrow e^{-\frac{i}{2}\Phi\Delta t} \psi(x)$

Higher order: **fourth-order time integration**

Two halos with initial velocities v_1 and v_2

$$\psi_{\text{ini}}(x) = \psi_1(x - x_1)e^{\frac{imv_1}{\hbar}x+S_1} + \psi_2(x - x_2)e^{\frac{imv_2}{\hbar}x+S_2}$$

$$\Delta S = S_1 - S_2$$

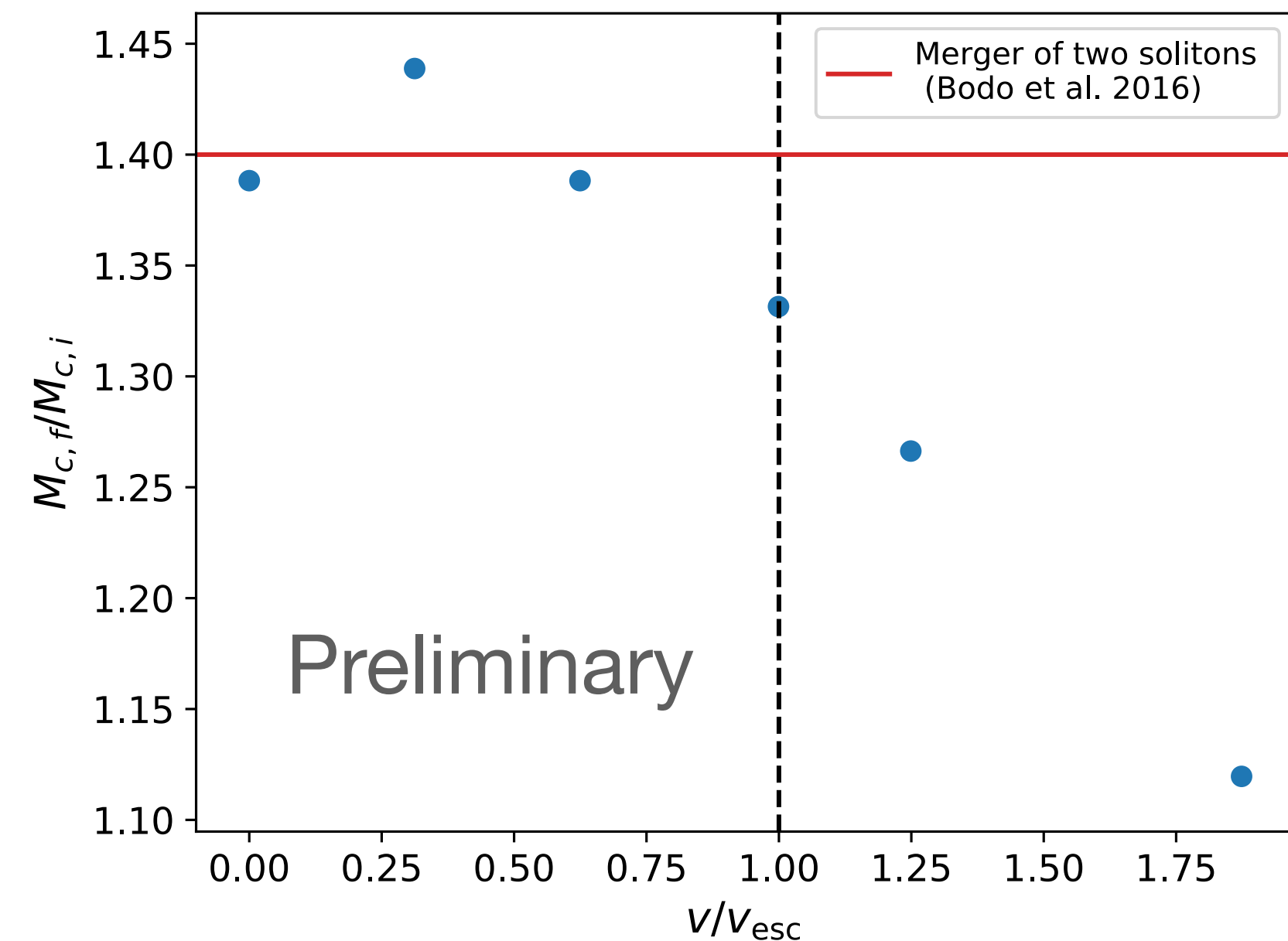
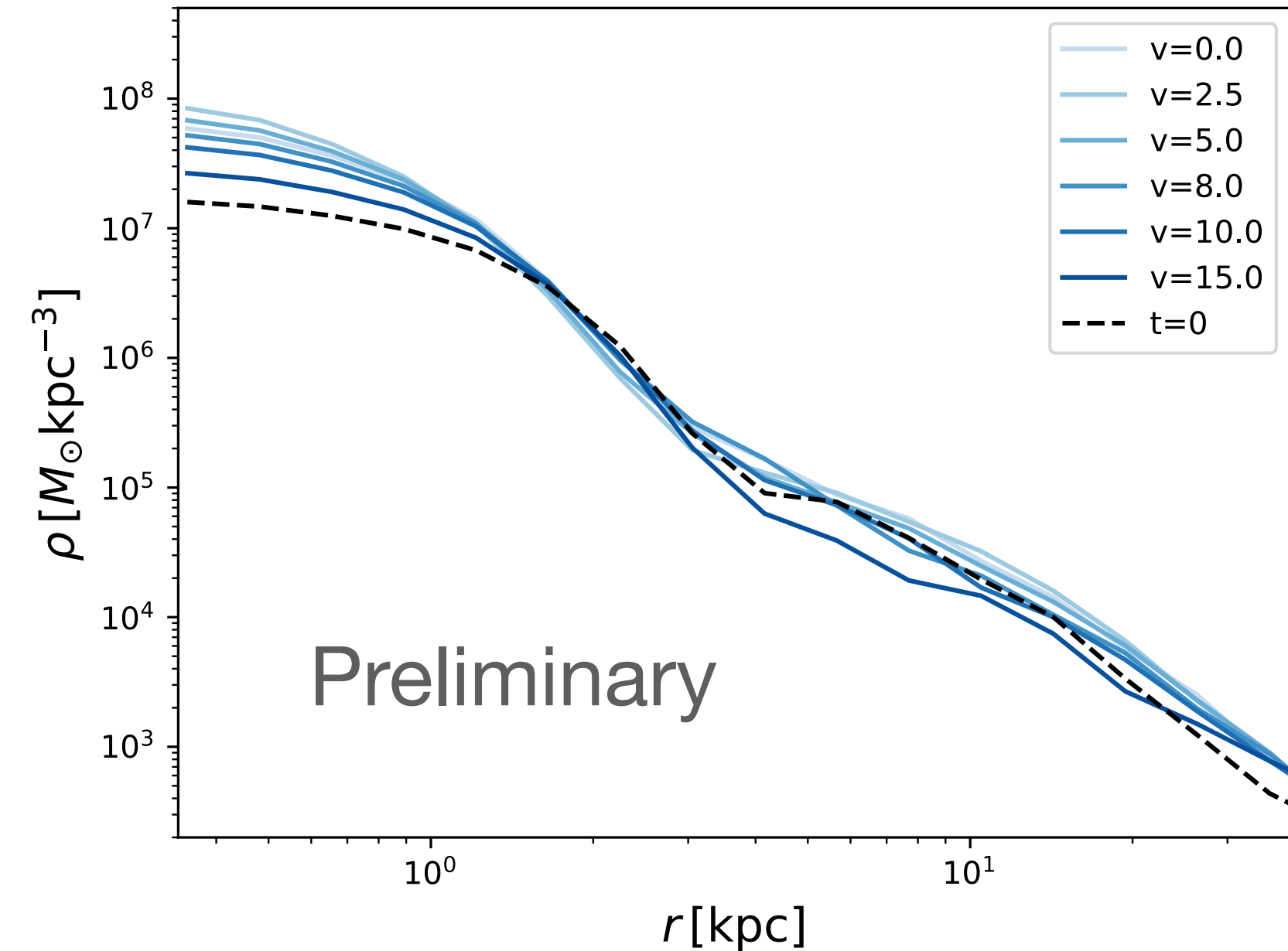
Consider two halos with equal mass
Escape velocity

$$v_{\text{esc}} = \sqrt{GM_h/R_{\text{ini}}}$$

(1) Head on collision $\Delta S = 0$

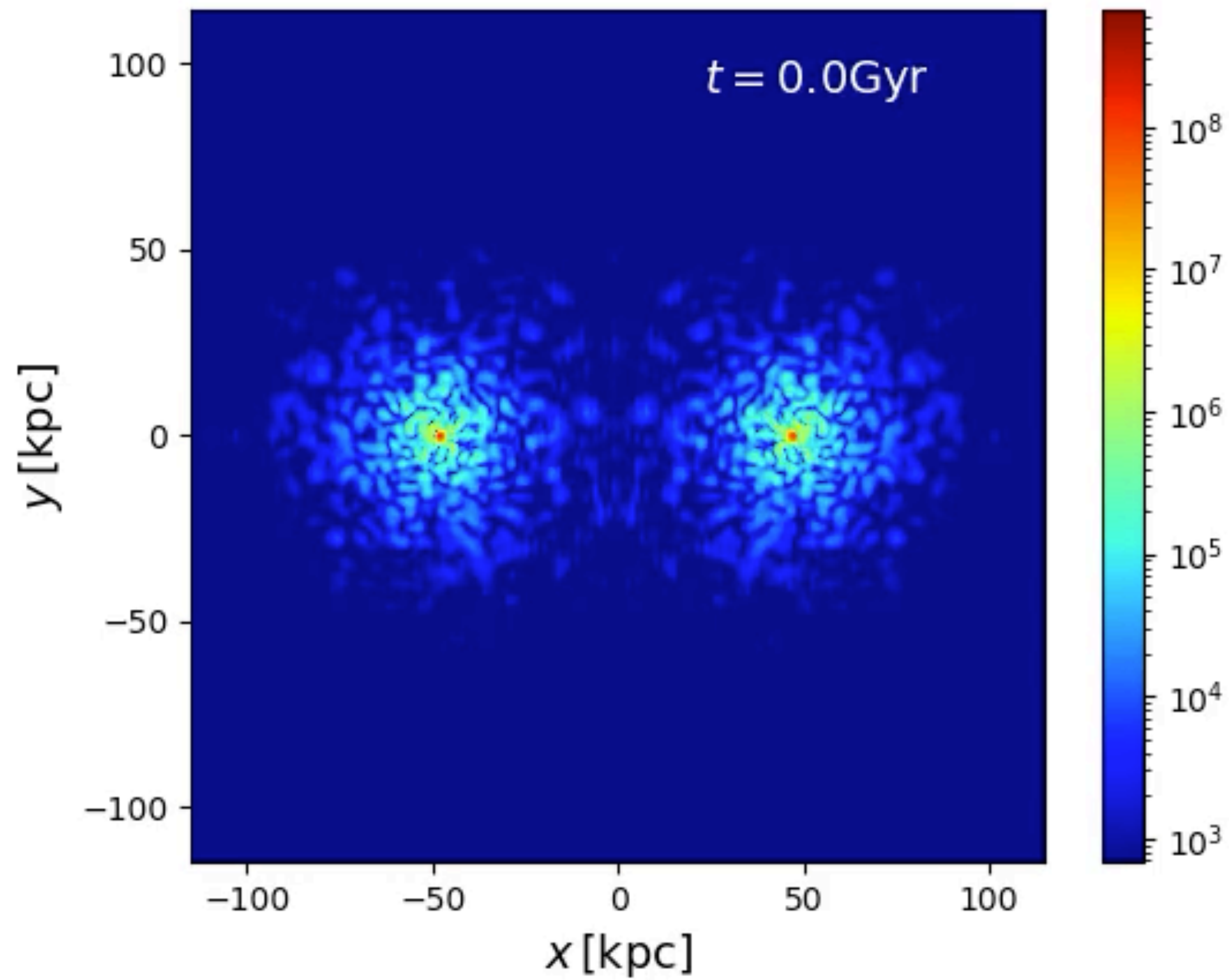
$$M_h = 10^9 M_{\odot}, R_{\text{vir}} = 12.8 \text{ kpc}$$

$$M_{c,i} = 5.3 \times 10^7 M_{\odot}$$

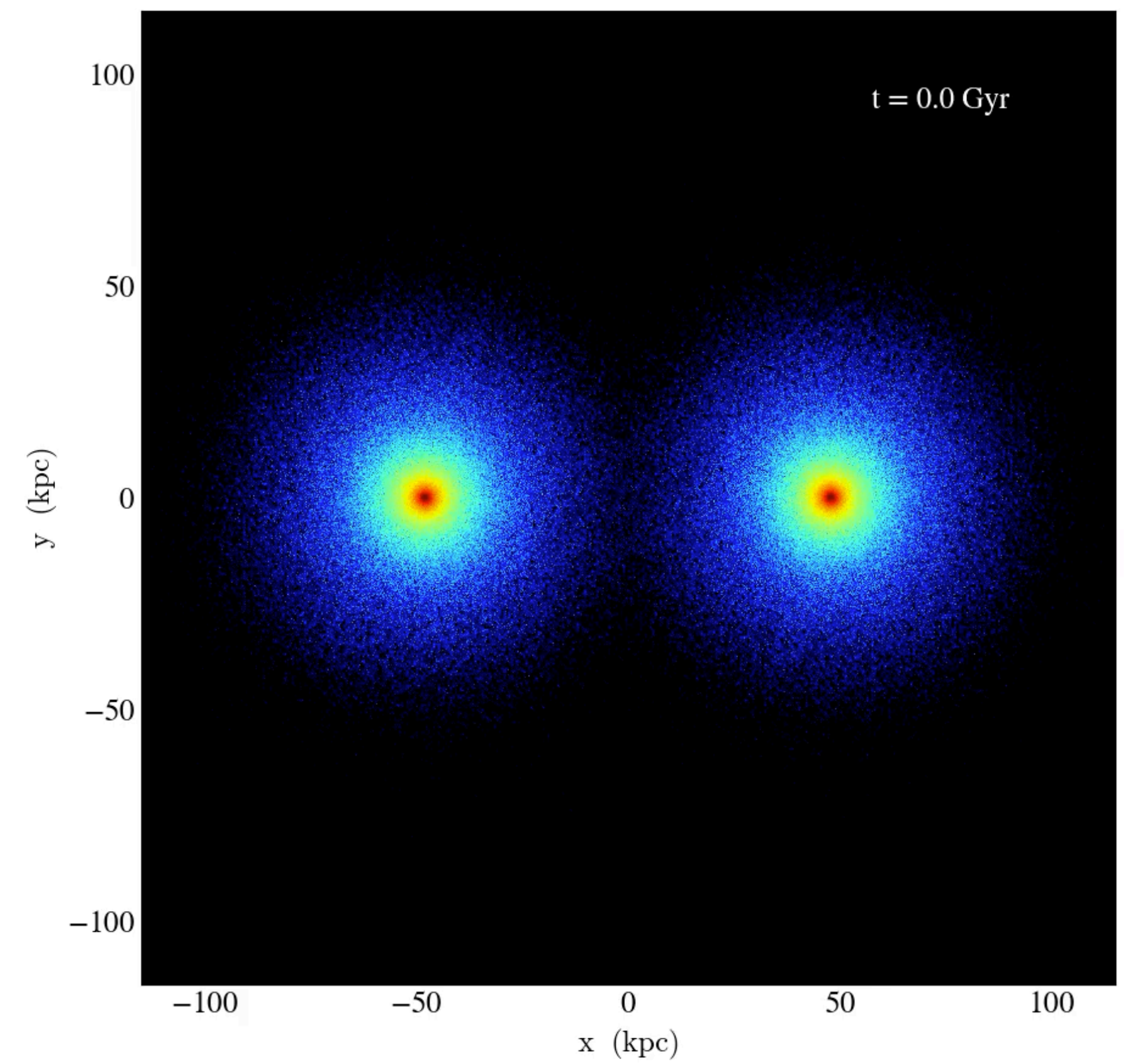


$$M_h = 10^{9.5} M_{\odot}, v_{\text{ini}} = 1.68 v_{\text{esc}}$$

Fuzzy DM



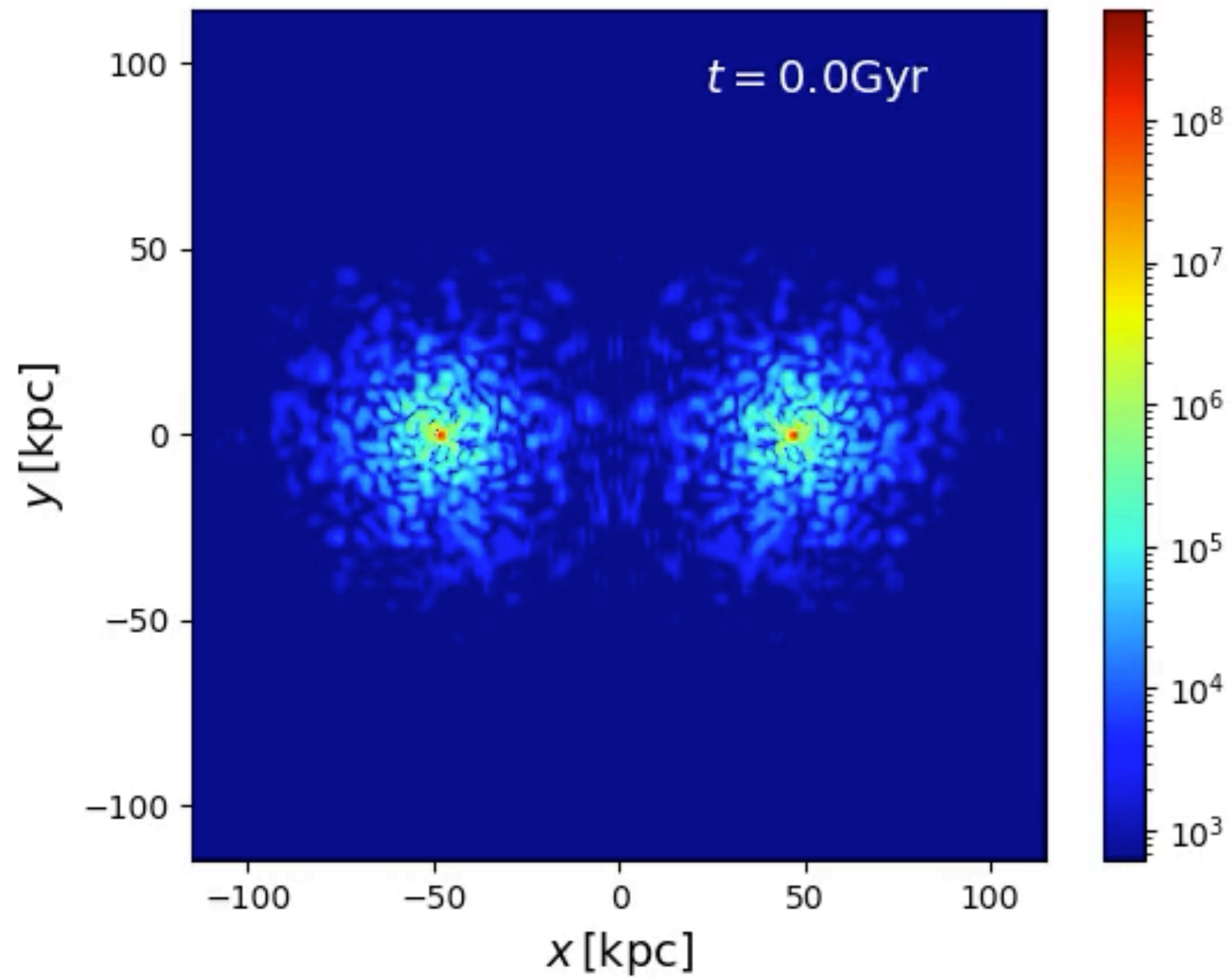
Collisionless Cold DM



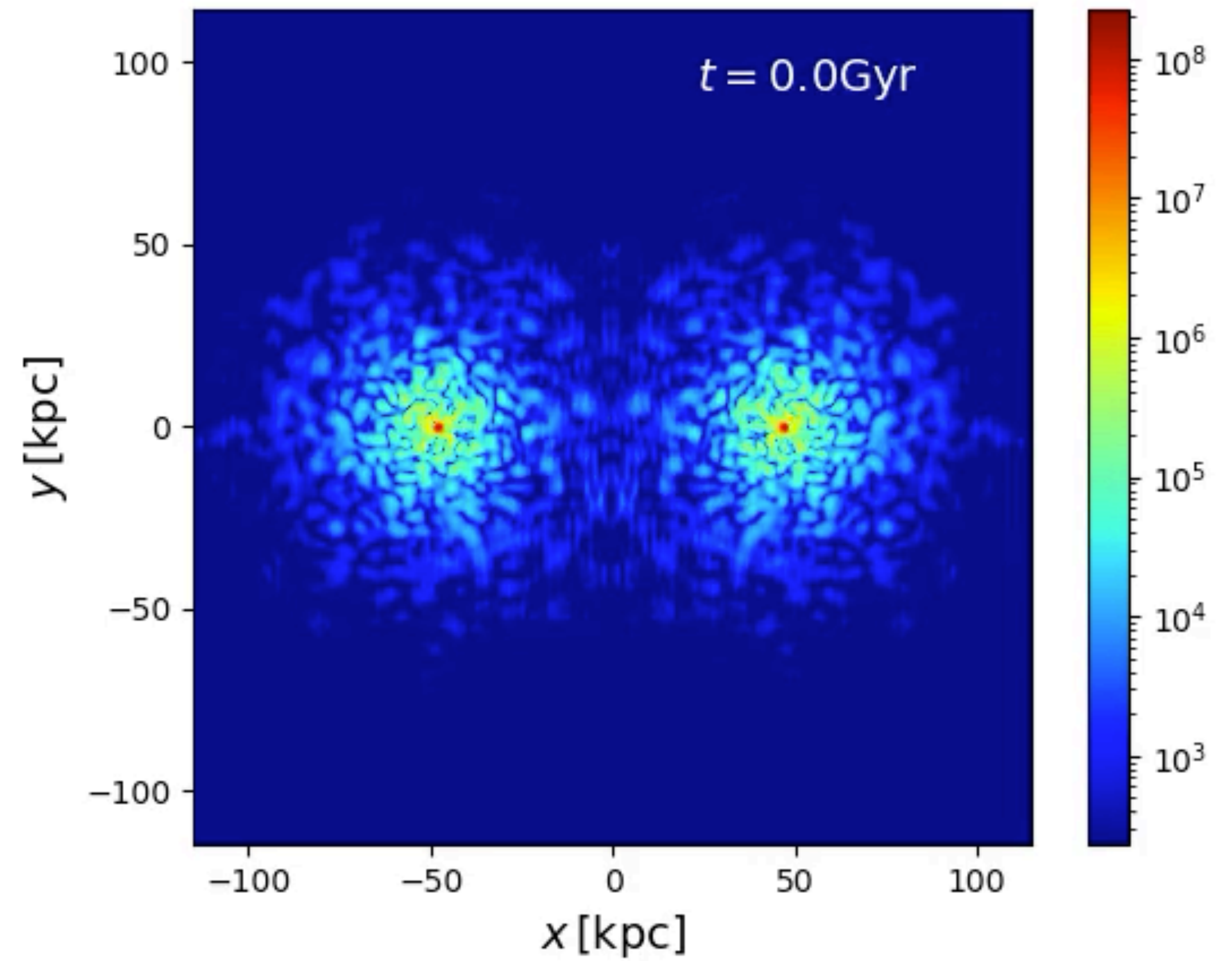
Core stays at the center

(2) Head on collision $\Delta S \neq 0$

$$\Delta S = \frac{\pi}{2}$$

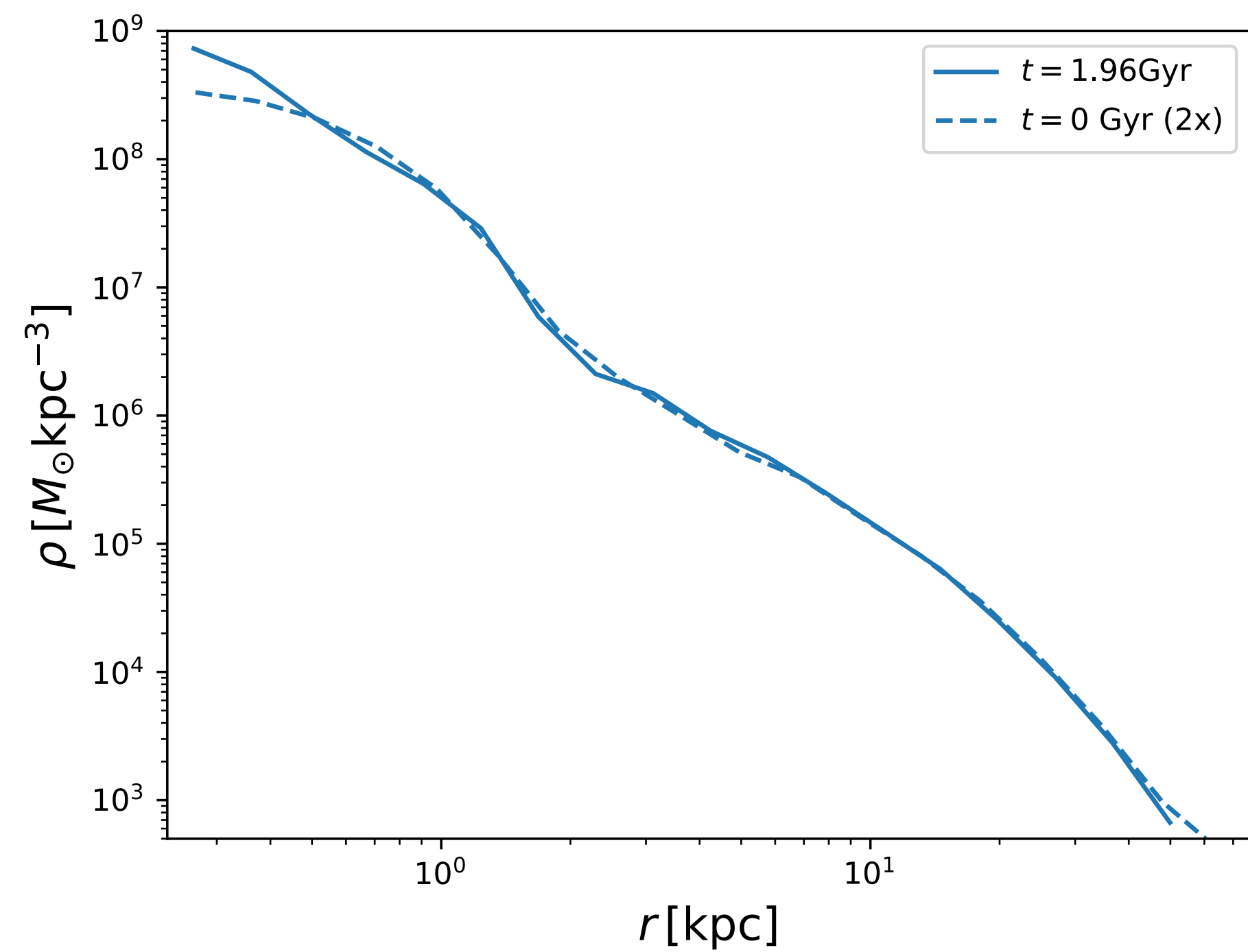


$$\Delta S = \pi$$

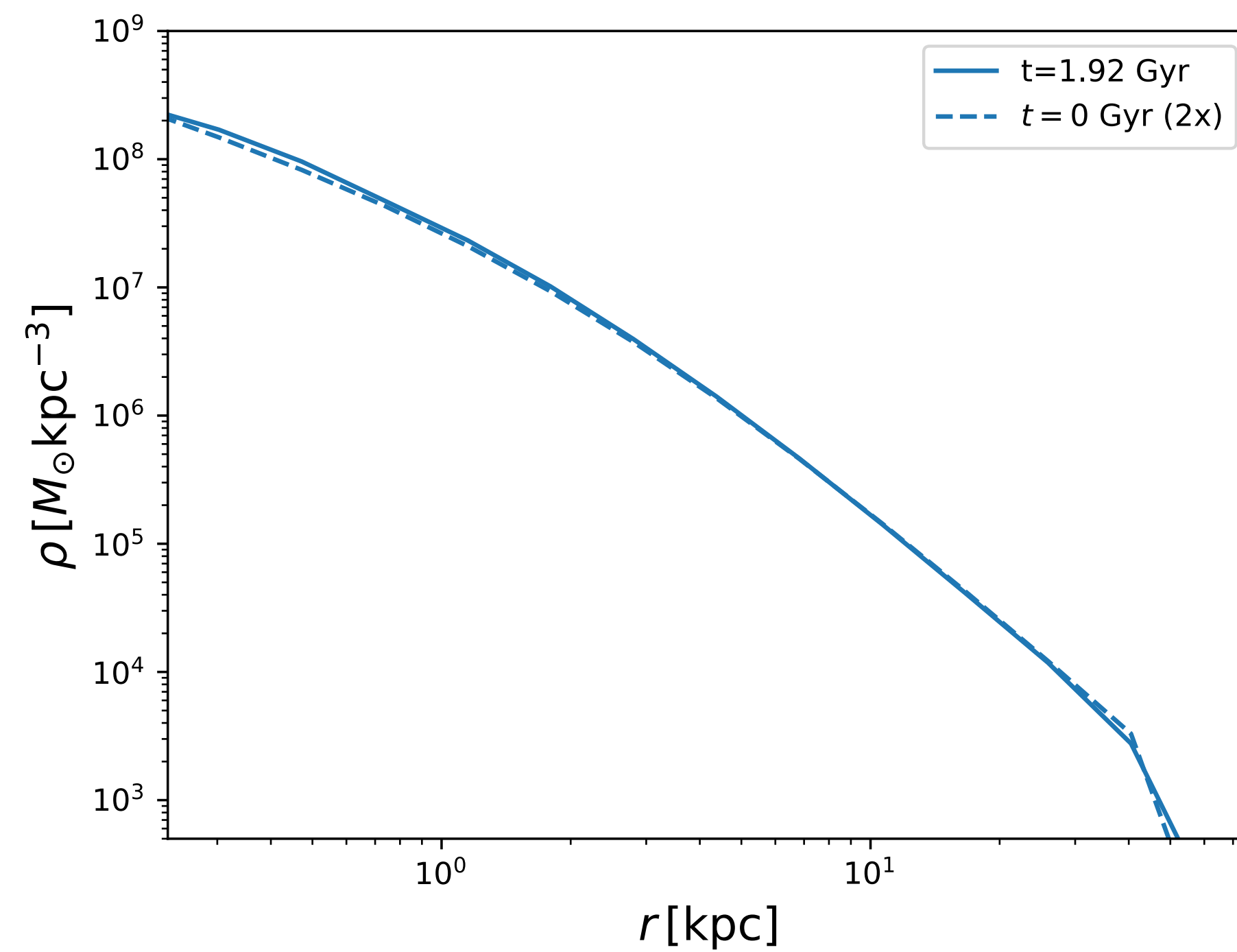


Density profile when two halos overlap

Fuzzy DM

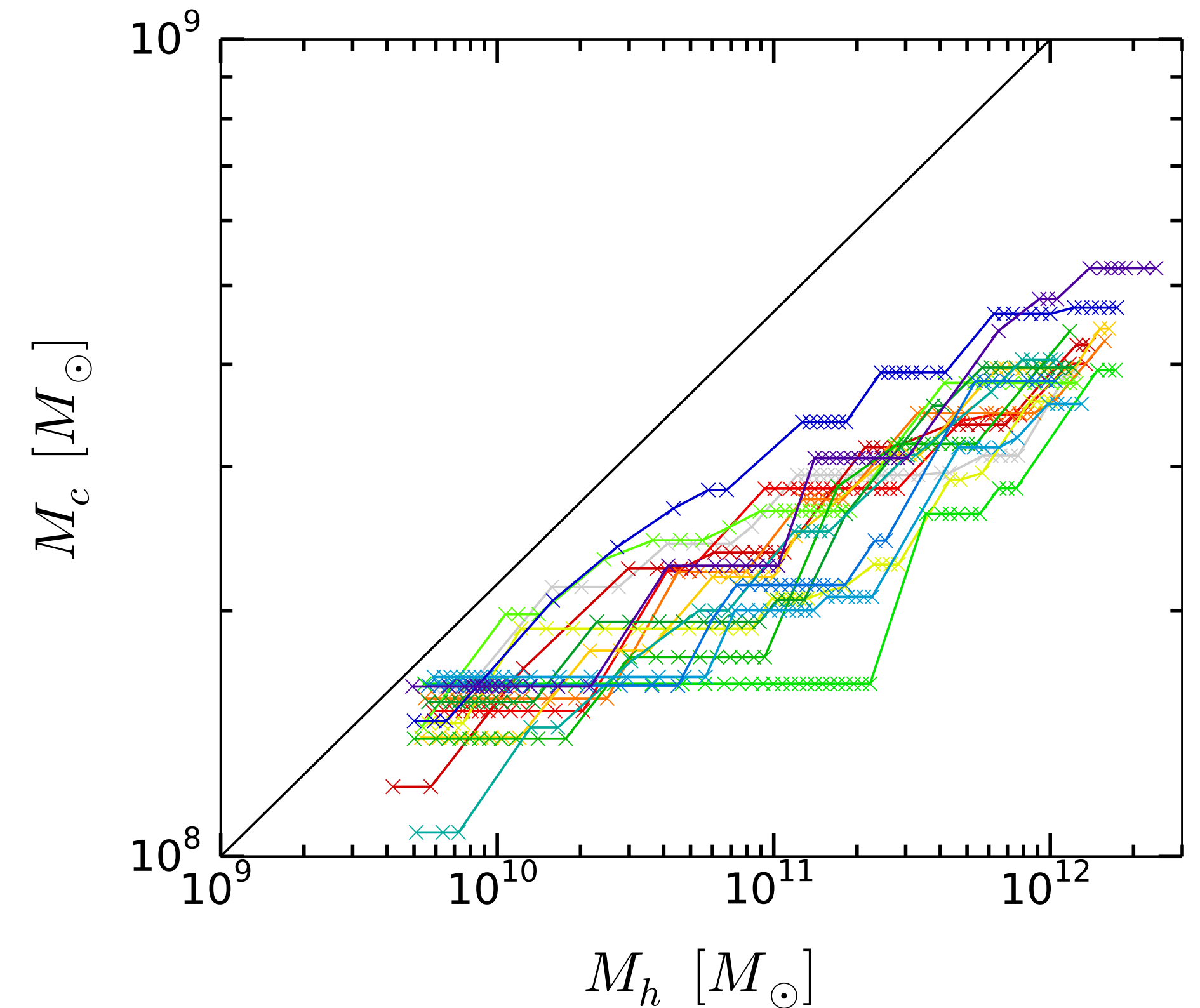


Collisionless Cold DM



Semi-analytic Model

1. Build merger trees (extended Press-Schechter formalism)
2. Models for the core evolution
 - (a) core growth by major mergers
 - (b) core mass loss due to tidal stripping
3. Models for the evolution of halos
 - (a) dynamical friction
 - (b) tidal effects
 - (c) dynamical heating



Du et al. 2016

Summary

The collision of Fuzzy Dark Matter halos behaves differently on small scales

- Interference pattern in the density

$$\rho = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_1 \psi_2^*$$

Observational features in merging galaxies? (Paredes et al. 2016)

- The solitonic cores are easier to merge

Orphan cores?

Build a well-calibrated semi-analytic model

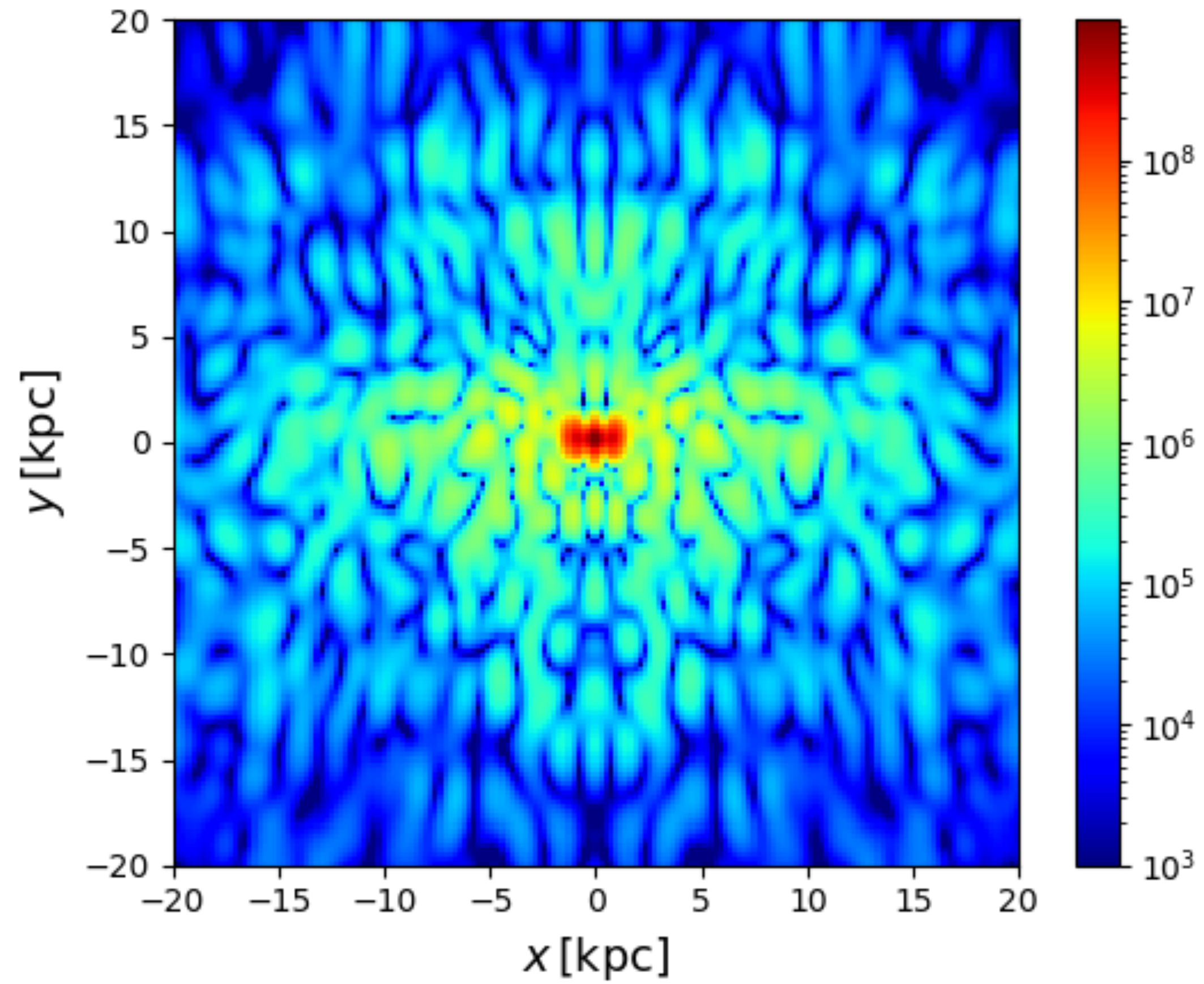
Gravitational lensing, a large number of realizations needed

Milky Way-size halos (currently can not be simulated directly)

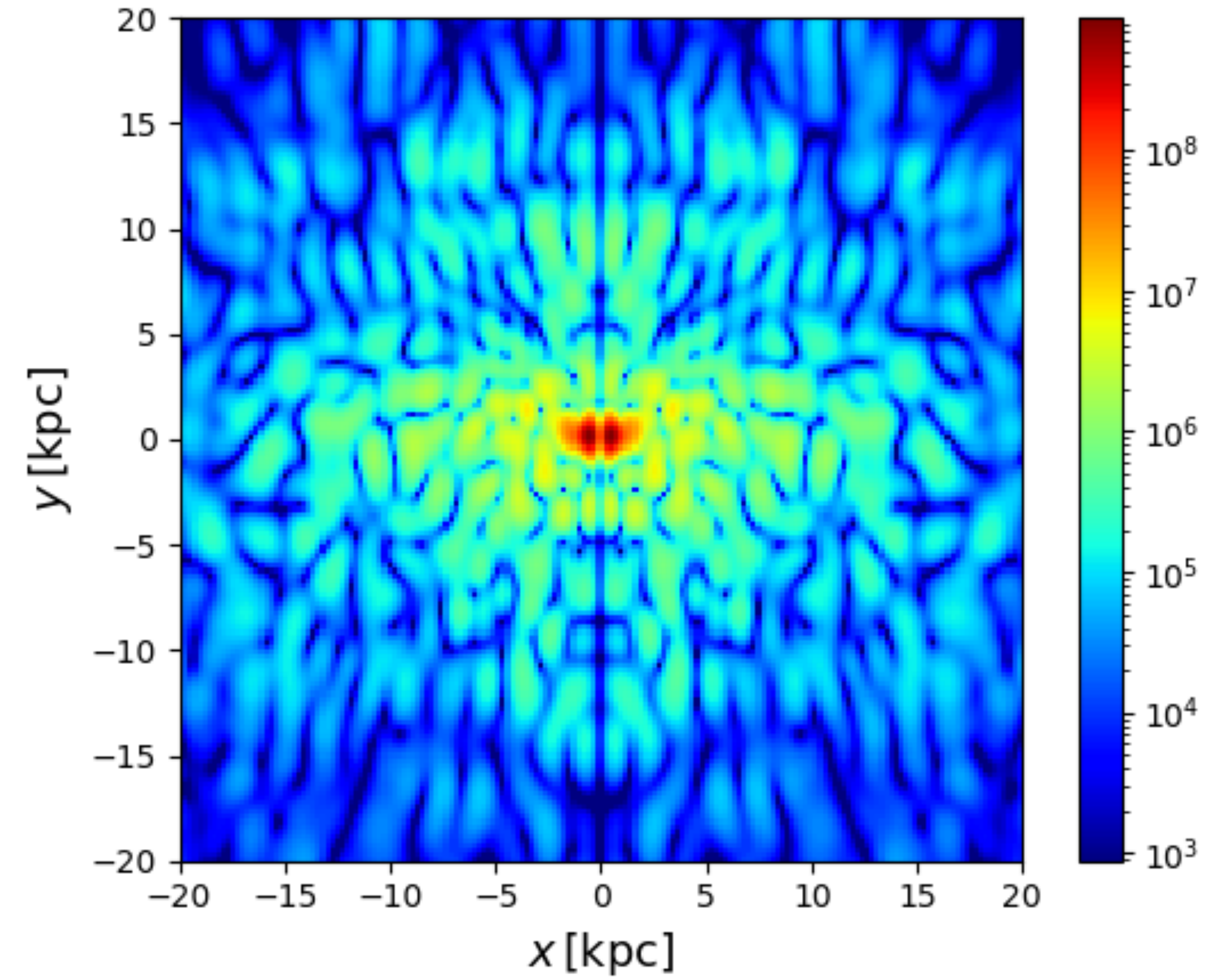
Thanks!

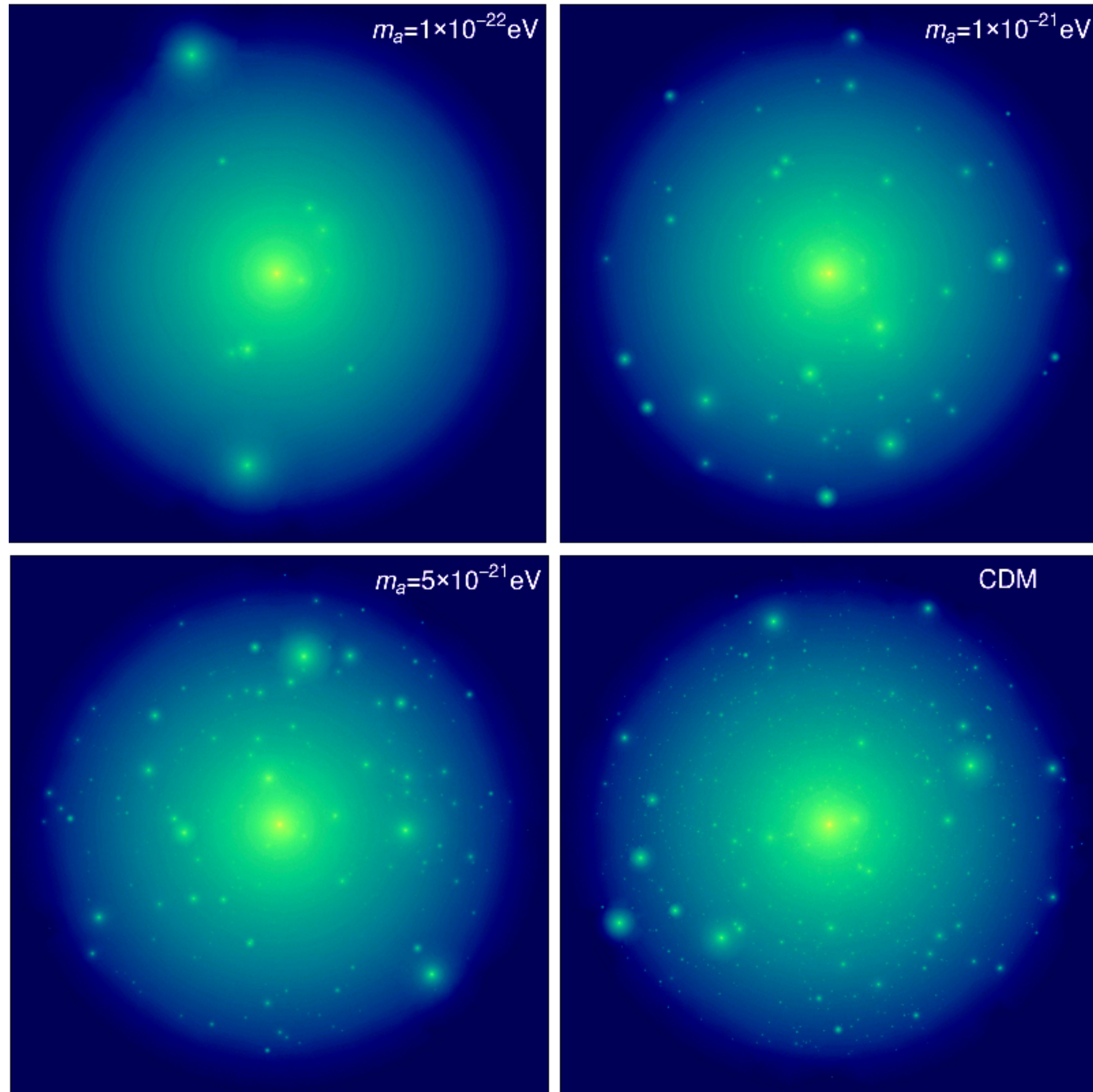
Backup slides

$$\Delta S = 0$$



$$\Delta S = \pi$$

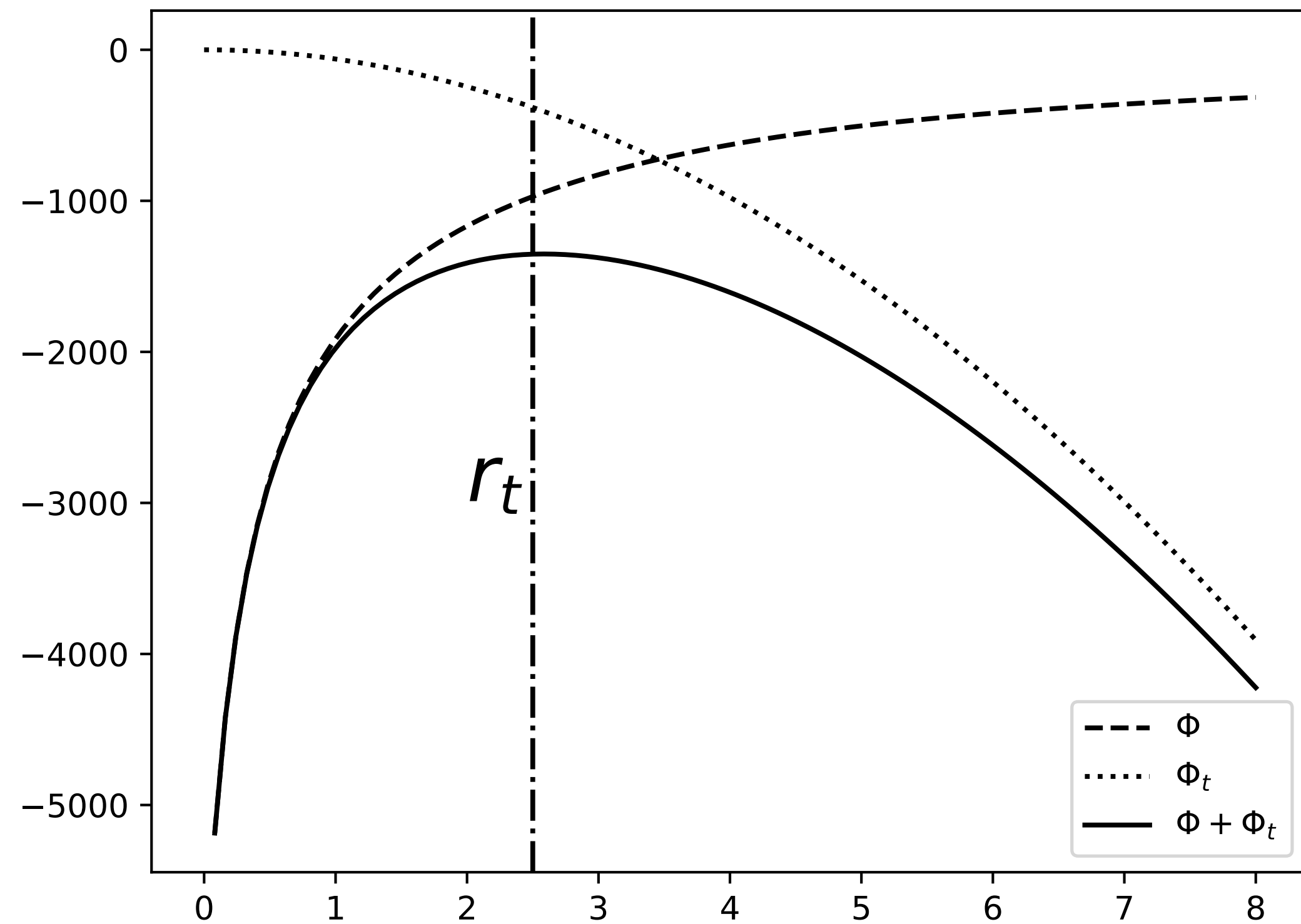




Milky Way-sized halos from Semi-analytic models

Evolution in tidal field (subhalos)

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}, t) + m[\Phi(\mathbf{x}) + \Phi_t(\mathbf{x})]\psi(\mathbf{x}, t)$$



$$\Phi_t(r) = -\frac{GM_h(R)}{R^3}r^2$$

Tunneling effects

(Hui et al. 2017, Du et al. 2018)