Active irreversible processes in quantum field theory

Alejandro Jenkins
(U. de Costa Rica & ICTQT Gdańsk)

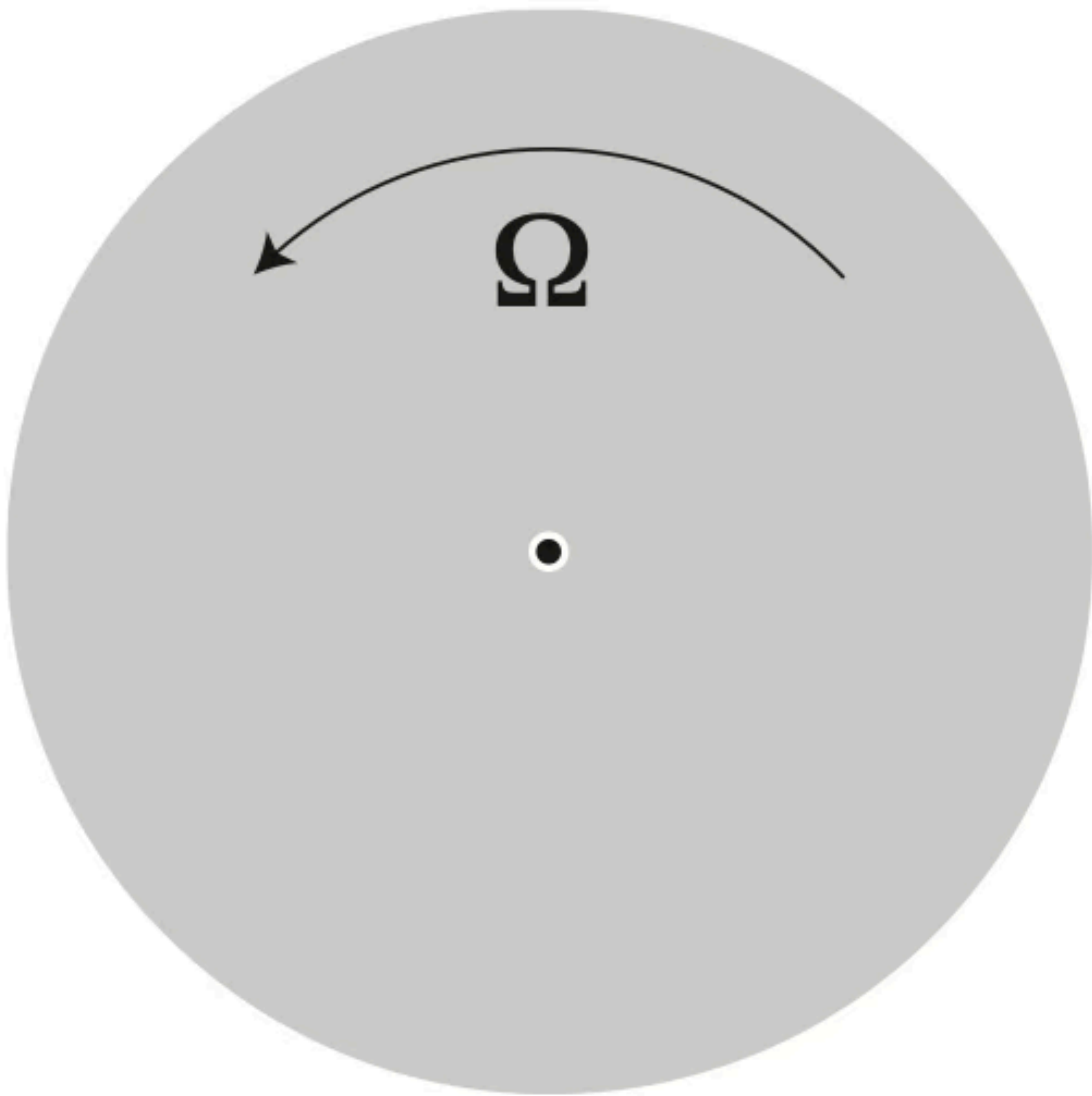
work with Robert Alicki (ICTQT Gdańsk)

I Central American Meeting of High Energy Physics, Cosmology & High Energy Astrophysics (**CAHEP2020**)

2 December 2020

Superradiance I

- Predicted by Zel'dovich in 1971 for rotating black hole (BH)
- Incident electric field: $\mathbf{E} \sim \exp\left[i\left(m\phi - \omega t\right)\right]$



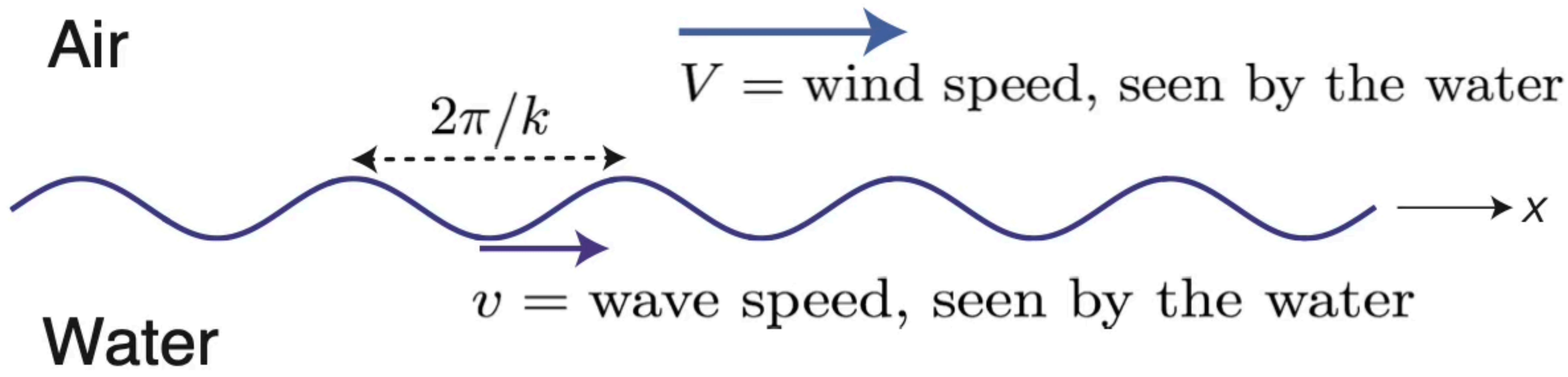
Comoving frame: $\phi' = \phi - \Omega t$

$$\mathbf{E}' \sim \exp\left[im\phi' - i\left(\omega - m\Omega\right)t\right]$$

Superradiance for $m\Omega > \omega$

"Anomalous Doppler shift" in **Ginzburg-Frank theory** of radiation by uniformly moving charges

"What Zel'dovich knew"*



$$\xi \sim \exp\left[ik(x - vt)\right] \qquad \omega = kv$$

Galileo: $$\frac{\partial \xi}{\partial t} \rightarrow \frac{\partial \xi}{\partial t} + V\frac{\partial \xi}{\partial x} = -i\omega\xi + V \cdot ik\xi$$

$$= -ik\xi(v - V)$$

critical wind speed $V = v$

* **Thorne, 2013**

Zel'dovich, JETP Lett. **14**, 180 (1971);
Sov. Phys. JETP **35**, 1085 (1971)

Irreversibility

$$E_{\text{wind}} = \frac{p_{\text{wind}}^2}{2m}$$ Mom. conservation: $\dot{\mathbf{p}}_{\text{wind}} = -\dot{\mathbf{p}}_{\text{wave}}$

$$-\dot{E}_{\text{wind}} = \frac{\mathbf{p}}{m} \cdot \dot{\mathbf{p}}_{\text{wave}} = \mathbf{V} \cdot (f\hbar\mathbf{k})$$

$$\dot{E}_{\text{wave}} = f\hbar\omega = f\hbar v k \; ; \qquad V > v \;\Leftrightarrow\; \left|\dot{E}_{\text{wind}}\right| > \dot{E}_{\text{wave}}$$

$\left|\dot{E}_{\text{wind}}\right| - \dot{E}_{\text{wave}} > 0$ available for **dissipation in the air**

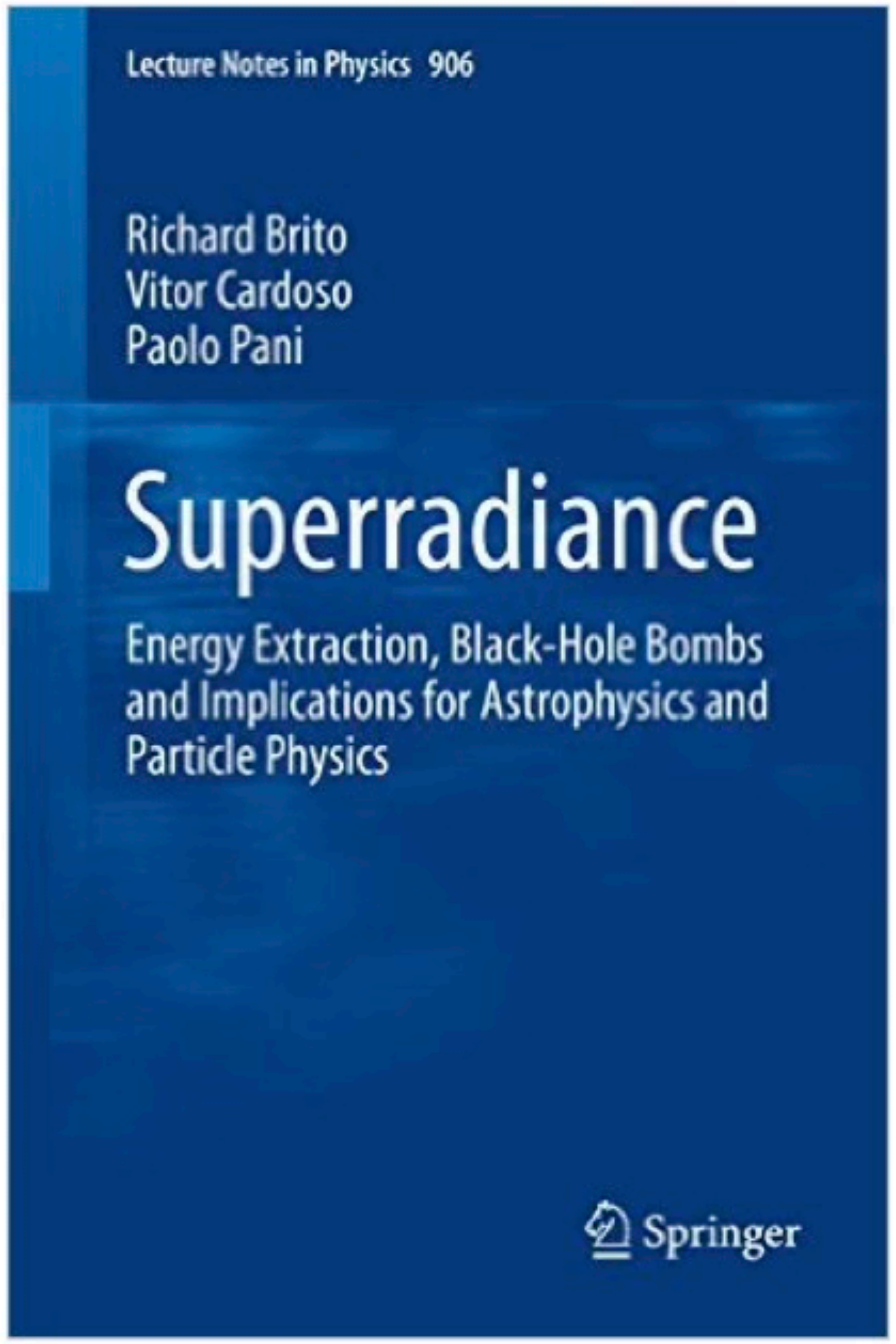
See: R. Alicki & AJ, *Ann. Phys.* (NY) **395**, 69 (2018)

"Quantum mechanics helps understand classical mechanics"
— 'Paradoksov', *Sov. Phys. Uspekhi* **9**, 618 (1967)

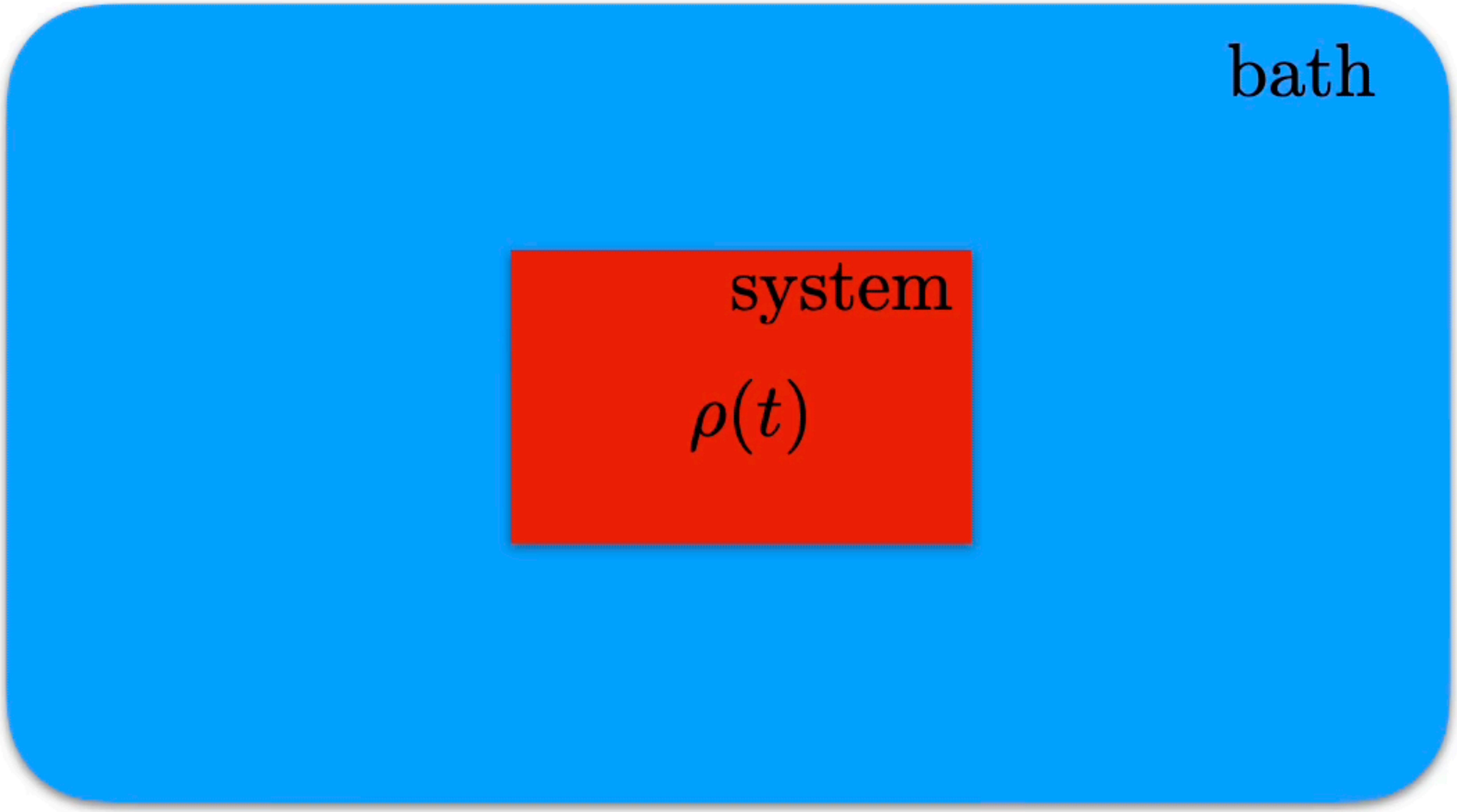
Superradiance II

- Off-equilibrium, unlike Dicke's superradiance (1954)
- field version of **Penrose process**
- Current interest in "BH bombs" (Press & Teukolsky, 1972) for **axion searches**
- also, superradiance of **gravity waves**
- See: Brito, Cardoso & Pani, *Superradiance* (Springer, 2020 [2015]) [arXiv:1501.06570 [gr-qc]] + refs. therein
- holographic superconductors & superfluids from superradiant instability of charged BH's

Hartnoll, Herzog & Horowitz, *PRL* **101**, 031601 (2008) [arXiv:0803.3295 [hep-th]]



Open systems I



Reduced dynamics:

$$\rho(t) = \mathrm{Tr}_{\mathrm{bath}}\left\{U_{\mathrm{full}}(t)\left[\rho(0)\otimes\sigma_{\mathrm{bath}}\right]U^{\dagger}_{\mathrm{full}}(t)\right\}$$

Open systems II

- For weak coupling, correlations in bath decay fast enough to allow Markovian (i.e., history-independent) approximation:

$$\dot{\rho} = -\frac{i}{\hbar}[H_{\rm s}, \rho] + \mathcal{L}\rho$$

- Gorini, Kossakowski & Sudarshan, *J. Math. Phys*. **17**, 821 (1976); Lindblad, *Commun. Math. Phys*. **48**,119 (1976)

- Often called "Lindblad eq." or "GKLS eq."; we'll say "**Markovian Master Equation**" (MME)

- Dynamics of work extraction by quantum system coupled to external disequilibrium of interest in "**quantum thermodynamics**"; see Alicki & Kosloff in *Thermodynamics in the Quantum Regime* (Springer, 2019)

Harmonic oscillator

- Angular frequency ω, set $m = 1$
- Destruction & creation ops.:

$$a = \sqrt{\frac{1}{2\omega\hbar}}(\omega x + ip); \quad a^\dagger = \sqrt{\frac{1}{2\omega\hbar}}(\omega x - ip); \quad [a, a^\dagger] = 1$$

- Number op.: $n = a^\dagger a$
- Hamiltonian: $H = \hbar\,\omega\,(n + 1/2)$
- MME: $$\frac{d}{dt}\rho = -i\omega[n,\rho] + \frac{1}{2}\gamma_\downarrow\big([a, \rho a^\dagger] + [a\rho, a^\dagger]\big) + \frac{1}{2}\gamma_\uparrow\big([a^\dagger, \rho a] + [a^\dagger\rho, a]\big)$$
- (goes back to work by Landau in 1927)
- Damping $\gamma_\downarrow$ & pumping $\gamma_\uparrow$ are Fourier transforms of reservoir auto-correlation functions of observables in oscillator-bath interaction, in bath's stationary state

see, e.g., Alicki & Lendi, *Quantum Dynamical Semigroups & Applications*, 2nd ed., (2007)

KMS

- Kubo-Martin-Schwinger (KMS) condition: if bath is in equilibrium at temp. *T*, then $\boldsymbol{\gamma_\uparrow / \gamma_\downarrow = e^{-\beta \hbar\omega}}$ (where $\boldsymbol{\beta = 1/k_B T}$ is inverse temp.)
- Any initial state $\boldsymbol{\rho(0)}$ thermalizes: $\lim_{t\to\infty} \rho(t) = Z^{-1} e^{-\beta H}$
- $\boldsymbol{\gamma_\uparrow > \gamma_\downarrow}$ corresponds to **population inversion**; negative "local temperature"
- Active state:

$$E(t) = E(0) e^{(\gamma_\uparrow - \gamma_\downarrow)t} + \hbar\omega \frac{\gamma_\uparrow}{\gamma_\uparrow - \gamma_\downarrow} \left[e^{(\gamma_\uparrow - \gamma_\downarrow)t} - 1 \right]$$

- In **QFT** (2nd quantization), each mode appears as simple harmonic oscillator

Anomalous Doppler shift

- Consider mode with wave vector $\boldsymbol{k}$ and angular frequency $\omega(\boldsymbol{k})$
- If coupled to bath with macroscopic velocity $\boldsymbol{v}$, frequency of mode in bath's frame is shifted: $\omega \rightarrow \omega'(\boldsymbol{k}) = \omega(\boldsymbol{k}) - \boldsymbol{k} \cdot \boldsymbol{v}$
- KMS condition becomes: $\gamma_{\uparrow}/\gamma_{\downarrow} = e^{\beta\hbar[\boldsymbol{k} \cdot \boldsymbol{v} - \omega(\boldsymbol{k})]}$
- $\Rightarrow$ for modes satisfying $\omega(\boldsymbol{k}) \leq \boldsymbol{k} \cdot \boldsymbol{v}$, bath acts as **negative-temperature reservoir**
- Irreversible transformation of <u>kinetic energy into coherent radiation</u>

Bosons & fermions

- Independent birth-death process for each mode *k* of quantum field coupled to heat bath:

$$\dot{P}_n(k;t) = \gamma_\downarrow(k)(n+1)P_{n+1}(k;t) + \gamma_\uparrow(k)\left[1 \pm (n-1)\right] P_{n-1}(k;t) \\ - \left[\gamma_\downarrow(k)n + \gamma_\uparrow(k)(1 \pm n)\right] P_n(k;t)$$

- **Stimulated emission (+)** of bosons gives <u>positive</u> feedback
- **Pauli exclusion (-)** makes feedback <u>negative</u>
- $\boldsymbol{\gamma}_\downarrow$ and $\boldsymbol{\gamma}_\uparrow$ can be computed from **Fermi golden rule**

<u>See:</u> Alicki, *Int. J. Theor. Phys.* **16**, 351 (1977); Alicki & Lendi, (2007), I, 1.3

Non-thermal radiation

- *Ω* appears *only* through Doppler shift: $\gamma^{\Omega}_{kk}(x) = \gamma^{0}_{kk}(x + m\Omega)$
- Effective $H_{\mathrm{b}}^{\mathrm{eff}} = H_{\mathrm{b}}^{0} - \Omega L_{\mathrm{b}}^{z}$ leads to **population inversion** for modes with $m\Omega > \omega$
- Bosonic occupation numbers grow exponentially, due to **stimulated emission**
- Analogous to **laser action**
- Knowing $\rho(t)$, we may compute entropy production, heat current, and work
- Well-behaved classical limit; see Ginzburg, *Prog. Optics* **32**, 267 (1993)

What about fermions?

- Doppler shift induces **population inversion** for fermionic modes with $m\Omega + \mu > \omega$
- No stimulated emission, but if pumped fermion decays into **second bath**, *active current* can be sustained
- Qualitatively novel prediction; <u>requires quantum treatment</u>
- Provides plausible explanation of **triboelectricity**

Van de Graaf

"Charge carriers can be moved **against the field** if they are stuck to a nonconducting belt. They are stuck so tightly that they can't slide backward along the belt in the generally downward electric field […] We need not consider here the means for putting charge on and off the belt."

– Purcell, *Electricity & Magnetism*, 3rd ed., (2013 [1963]), sec. 4.9

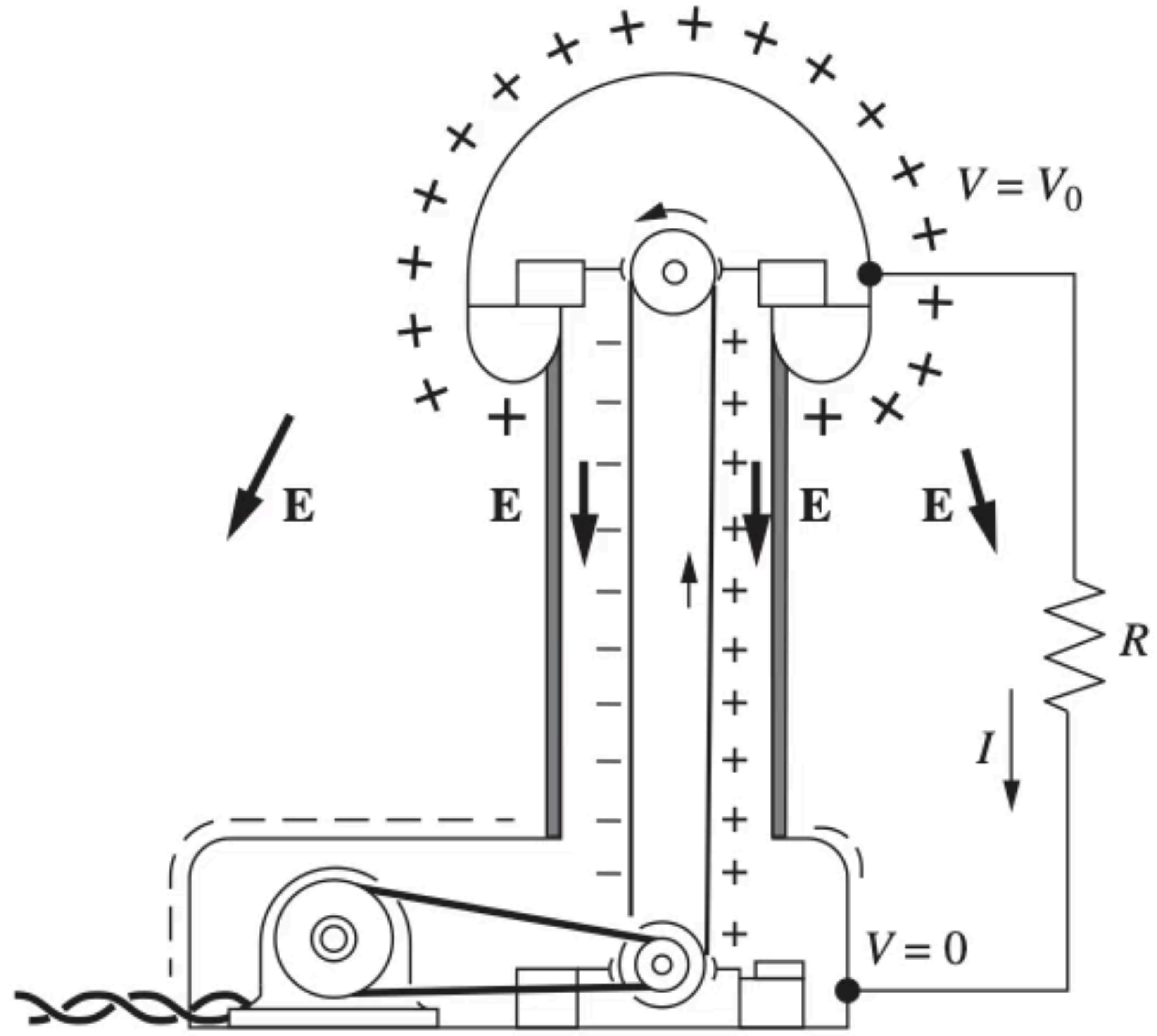
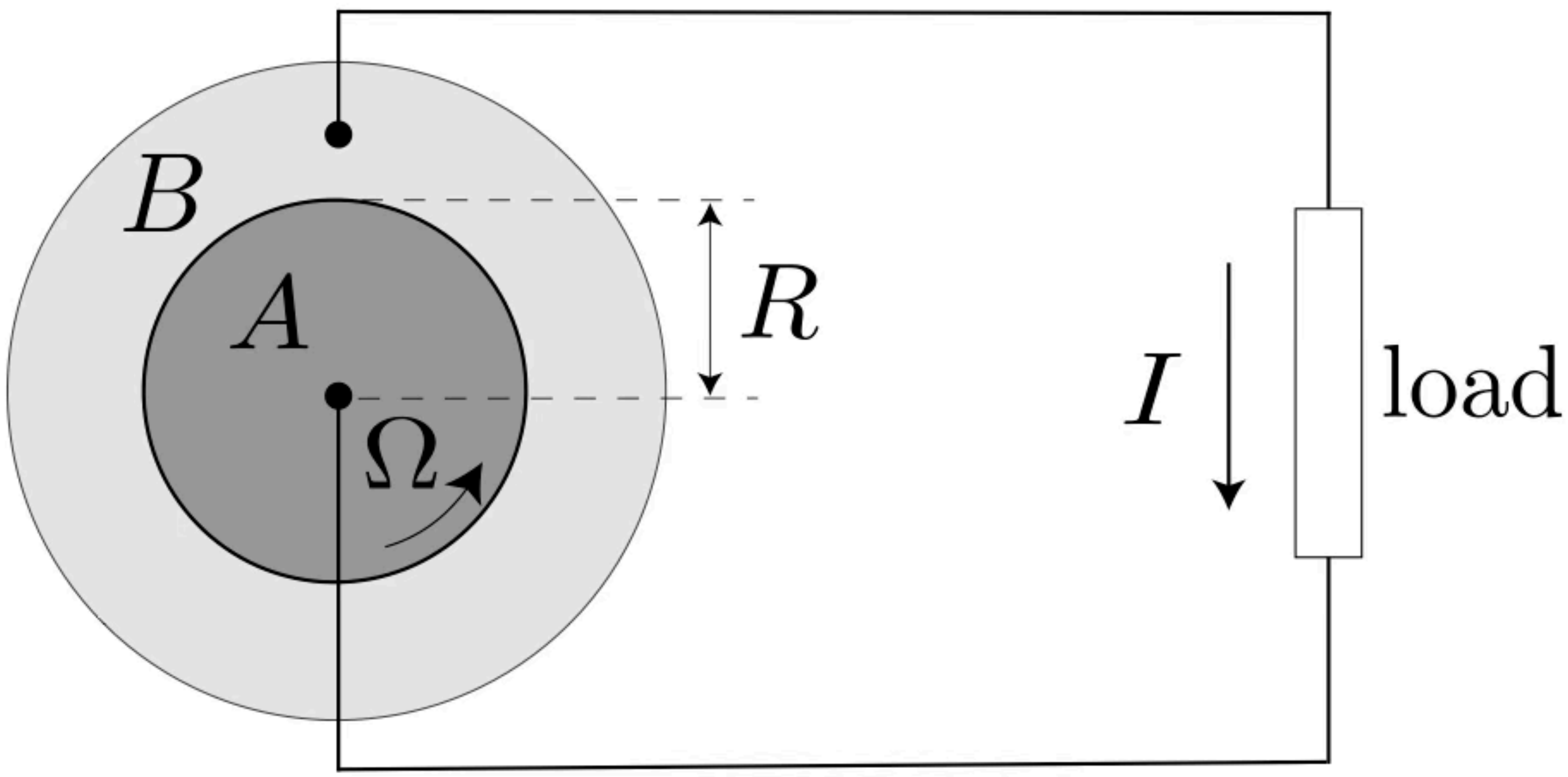


Figure 4.19.
In the Van de Graaff generator, charge carriers are mechanically transported in a direction opposite to that in which the electric field would move them.

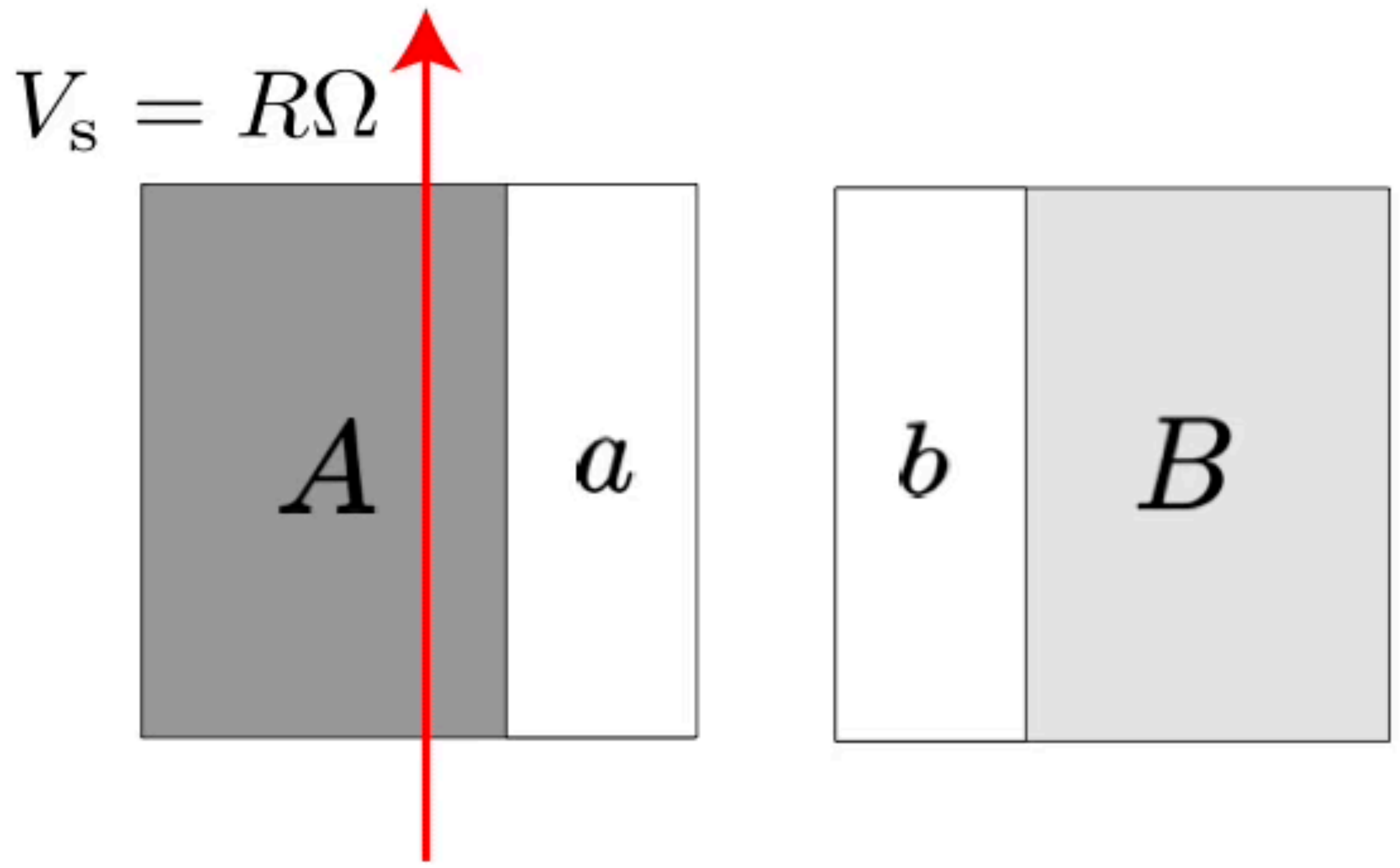
Minimal tribo-generator



Faraday-Maxwell: $\mathcal{E} = \oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a} = 0$

See: Alicki & AJ, *Phys. Rev. Lett.* **125**, 186101 (2020)

Open system



system: surfaces *a* + *b*

baths: bulks *A* & *B*

At rest:

$$H_0^x = \sum_{\sigma,m} \omega_x(\sigma,m) c_x^\dagger(\sigma,m) c_x(\sigma,m) \quad \text{for} \quad x = a, b$$

$$H_0^X = \sum_{\kappa,m} \omega_X(\kappa,m) c_X^\dagger(\kappa,m) c_X(\kappa,m) \quad \text{for} \quad X = A, B$$

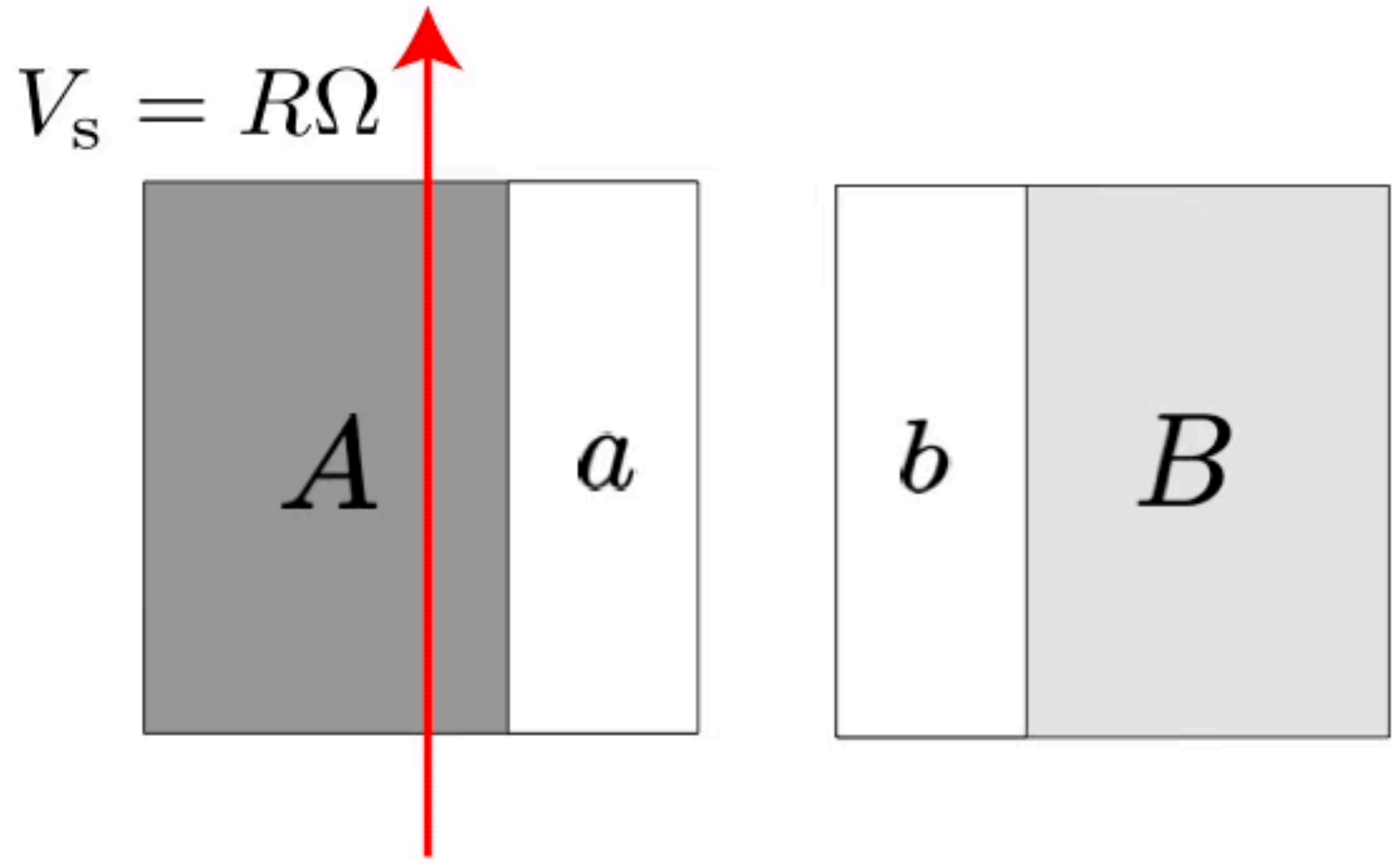
In motion:

(Doppler shift)

$$H_\Omega^a = \sum_{\sigma,m} \left[\omega_a(\sigma,m) - m\Omega\right] c_a^\dagger(\sigma,m) c_a(\sigma,m)$$

$$H_\Omega^A = \sum_{\kappa,m} \left[\omega_A(\kappa,m) - m\Omega\right] c_A^\dagger(\kappa,m) c_A(\kappa,m)$$

Open system, cont.



Surface-bulk tunneling:

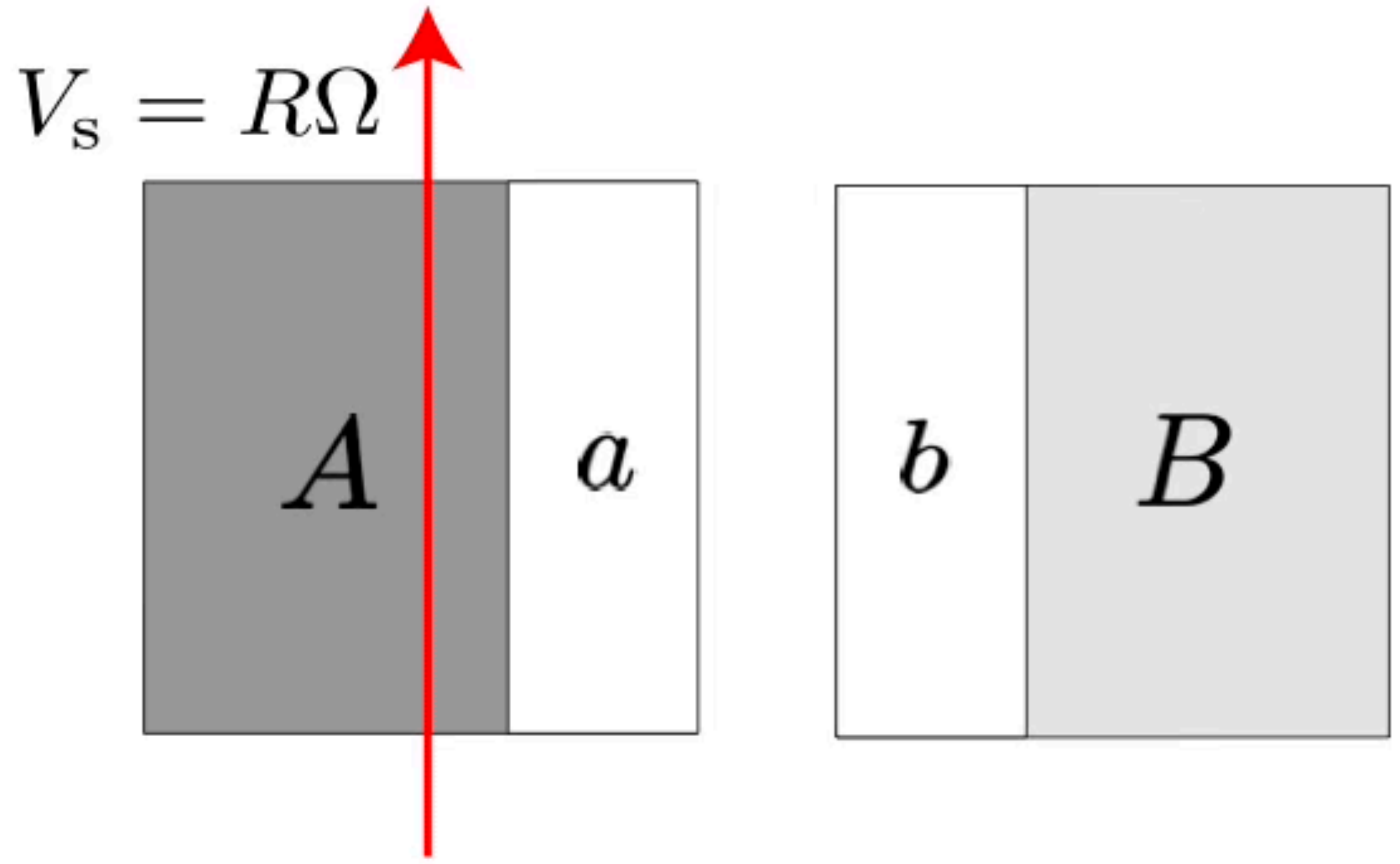
$$H_X^x = \sum_{\kappa,\sigma,m} g_X^x(\kappa,\sigma,m) c_X^\dagger(\kappa,m) c_x(\sigma,m) + \mathrm{h.c.}$$

Surface states localized along transport direction

⇒ *ab* interaction gives only hybridization, absorbable into modified wave functions

Total Hamiltonian: $H_{\mathrm{tot}} = H_\Omega^a + H_0^b + H_\Omega^A + H_0^B + H_A^a + H_B^a + H_A^b + H_B^b$

Kinetic eqs.



Kinetic eq.: $\dot{n}_x = \gamma_\uparrow^{xA} + \gamma_\uparrow^{xB} - \left(\gamma_\downarrow^{xA} + \gamma_\downarrow^{xB} + \gamma_\uparrow^{xA} + \gamma_\uparrow^{xB}\right) n_x$

KMS:

$$\gamma_\uparrow^{aA}(\sigma, m) = e^{-\beta(\omega_a(\sigma,m)-\mu_A)}\gamma_\downarrow^{aA}(\sigma, m)$$

$$\gamma_\uparrow^{aB}(\sigma, m) = e^{-\beta(\omega_a(\sigma,m)-m\Omega-\mu_B)}\gamma_\downarrow^{aB}(\sigma, m)$$

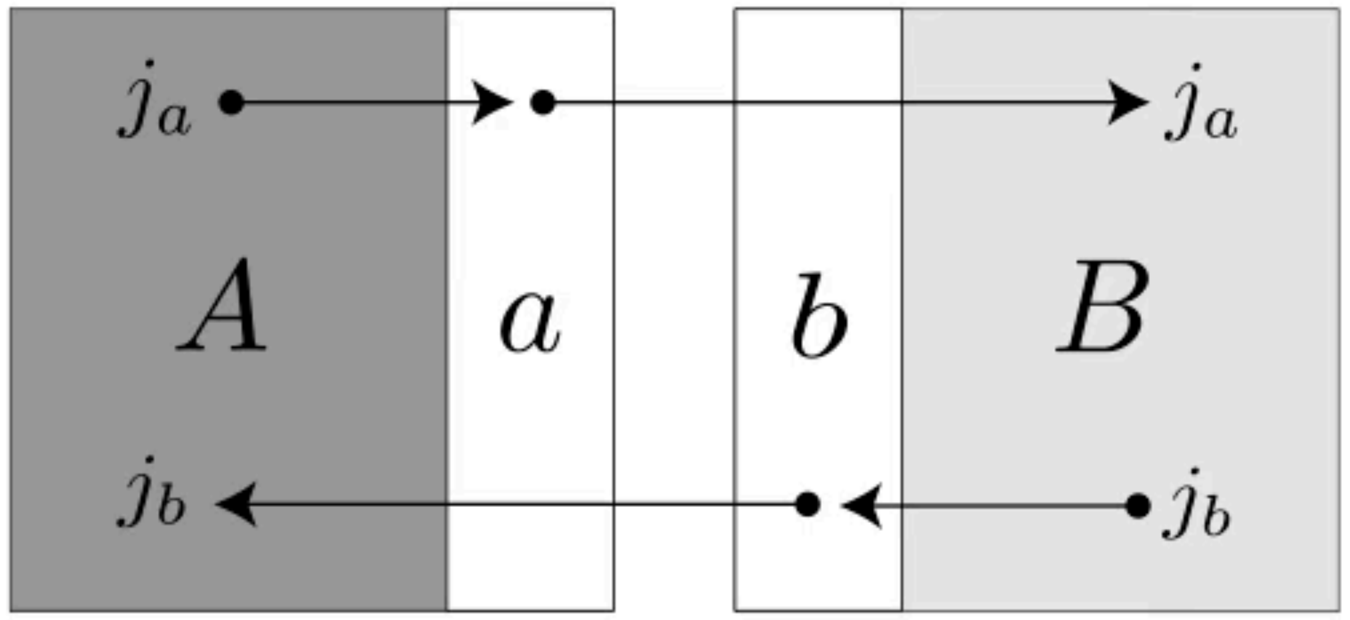
Steady state:

$$n_a = \bar{n}_a \equiv \left(\gamma_\uparrow^{aA} + \gamma_\uparrow^{aB}\right)/\Gamma^a = \text{const.}$$

$$n_b = \bar{n}_b \equiv \left(\gamma_\uparrow^{bA} + \gamma_\uparrow^{bB}\right)/\Gamma^b = \text{const.}$$

$$\Gamma^a \equiv \gamma_\uparrow^{aA} + \gamma_\downarrow^{aA} + \gamma_\uparrow^{aB} + \gamma_\downarrow^{aB} \;; \qquad \Gamma^b \equiv \gamma_\uparrow^{bA} + \gamma_\downarrow^{bA} + \gamma_\uparrow^{bB} + \gamma_\downarrow^{bB}$$

Tribocurrents I



$$j_a = \gamma_\uparrow^{aA} - \left(\gamma_\downarrow^{aA} + \gamma_\uparrow^{aA}\right)\bar{n}_a = \frac{\gamma_\uparrow^{aA}\gamma_\downarrow^{aB}\left[1 - e^{\beta(m\Omega+\mu_B-\mu_A)}\right]}{\Gamma^a}$$

$$j_b = \gamma_\uparrow^{bB} - \left(\gamma_\downarrow^{bB} + \gamma_\uparrow^{bB}\right)\bar{n}_b = \frac{\gamma_\downarrow^{bA}\gamma_\uparrow^{bB}\left[1 - e^{-\beta(m\Omega+\mu_B-\mu_A)}\right]}{\Gamma^b}$$

Net current: $J = -e\left(\sum_{\sigma,m} j_a(\sigma,m) + \sum_{\sigma',m} j_b(\sigma',m)\right)$

Fermi-Dirac: $n_X(x) \equiv \dfrac{1}{e^{\beta(x-\mu_X)}+1}$

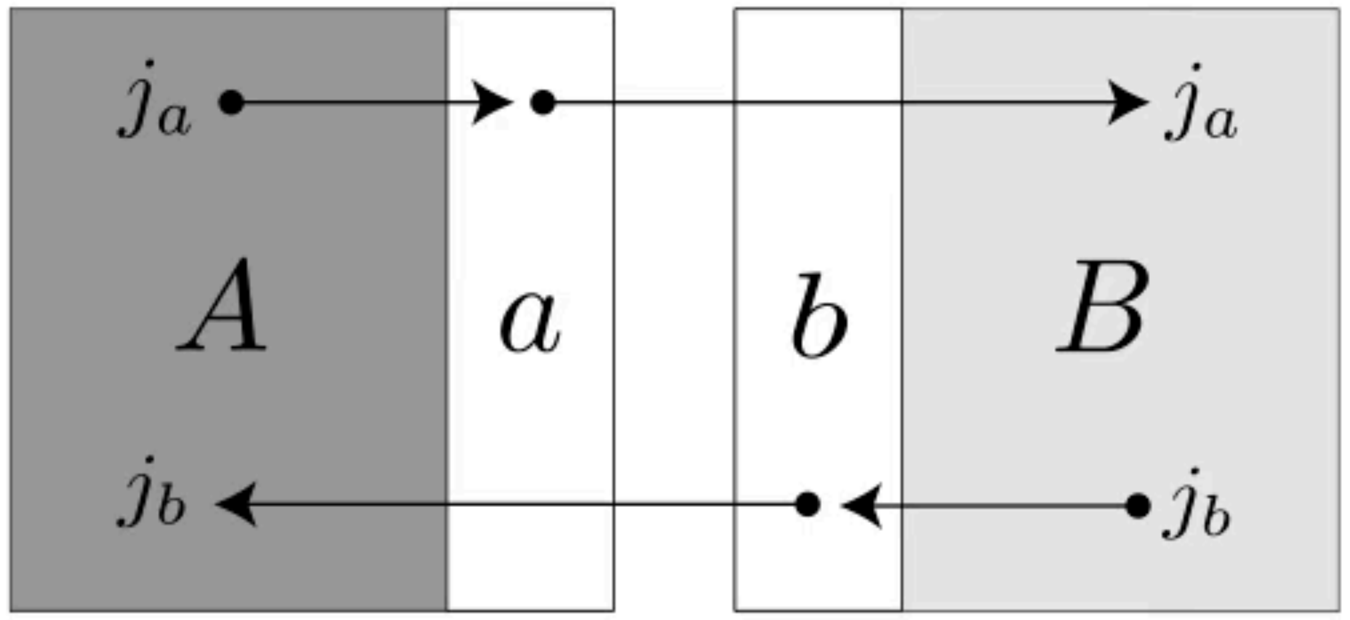
$$j_a(\sigma,m) \sim \gamma_\uparrow^{aA}\gamma_\downarrow^{aB} \sim n_A(\omega_a(\sigma,m))[1 - n_B(\omega_a(\sigma,m) - m\Omega)]$$

For $\beta\mu \gg 1$, $\gamma_\uparrow^{aA}\gamma_\downarrow^{aB} \sim \chi_{[\mu_B+m\Omega,\, \mu_A]}(\omega_a(\sigma,m))$

Only surface modes satisfying $m\Omega < \mu_A - \mu_B$ contribute to j_a

$\Rightarrow j_a > 0$

Tribocurrents II

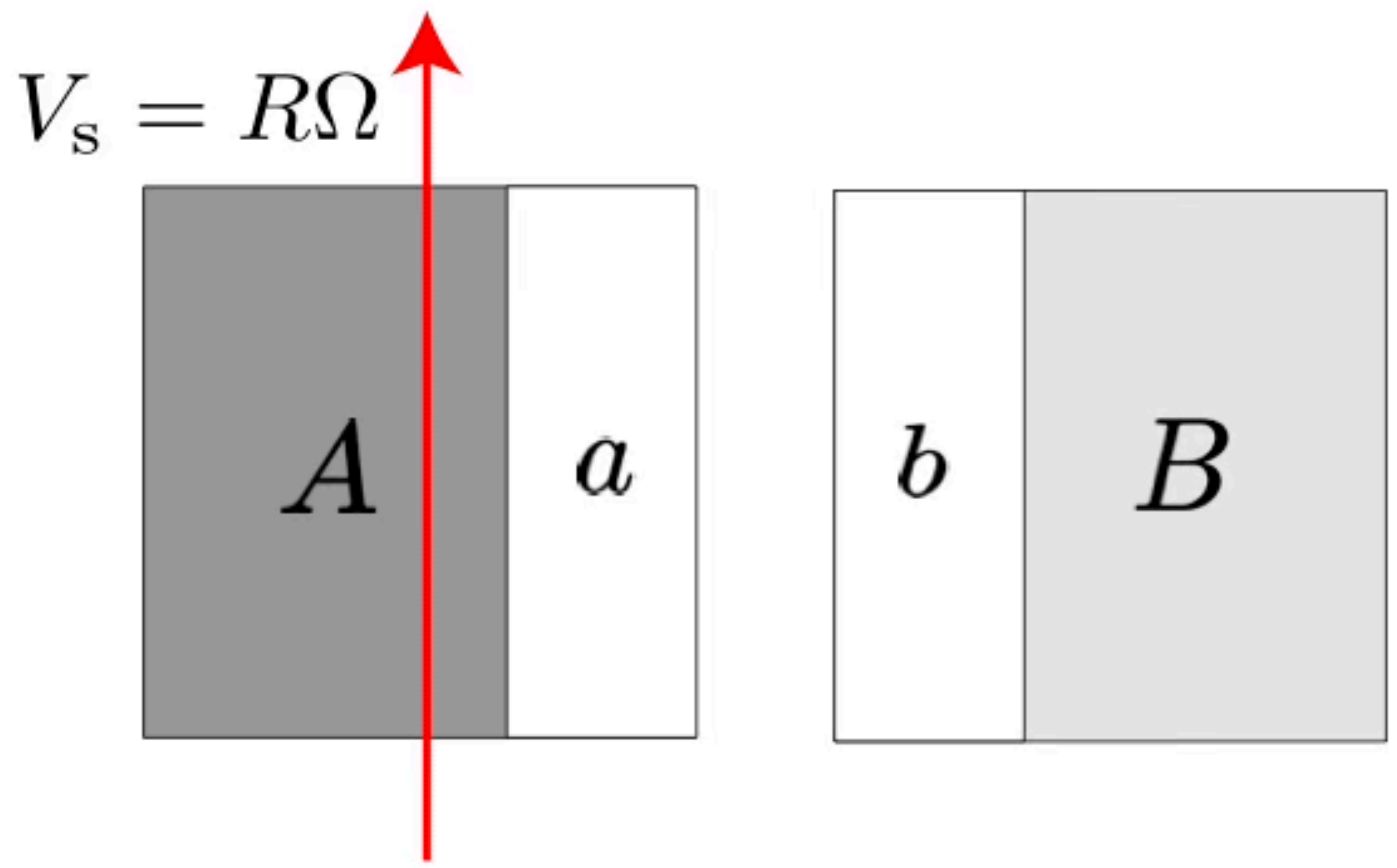


$$j_a = \gamma_\uparrow^{aA} - \left(\gamma_\downarrow^{aA} + \gamma_\uparrow^{aA}\right)\bar{n}_a = \frac{\gamma_\uparrow^{aA}\gamma_\downarrow^{aB}\left[1 - e^{\beta(m\Omega+\mu_B-\mu_A)}\right]}{\Gamma^a}$$

$$j_b = \gamma_\uparrow^{bB} - \left(\gamma_\downarrow^{bB} + \gamma_\uparrow^{bB}\right)\bar{n}_b = \frac{\gamma_\downarrow^{bA}\gamma_\uparrow^{bB}\left[1 - e^{-\beta(m\Omega+\mu_B-\mu_A)}\right]}{\Gamma^b}$$

- Similarly, only surface modes satisfying $\boldsymbol{m\Omega > \mu_A - \mu_B}$ contribute to j_b
- $\Rightarrow j_b > 0$
- As charging increases $|\mu_A - \mu_B|$, fewer modes contribute to corresponding j_x, and more modes to the **opposing** j_x
- Sign of J depends on relative magnitudes of $\gamma_\uparrow^{aA}\gamma_\downarrow^{aB}/\Gamma^a$ and $\gamma_\downarrow^{bA}\gamma_\uparrow^{bB}/\Gamma^b$

Basic phenomenology



- In cylindrical coords. $\boldsymbol{k} = (\boldsymbol{k_m}, \boldsymbol{k_z})$

- Fermi wave vector $k_\mathrm{F} \geq \sqrt{k_m^2 + k_z^2}$

$$|m\Omega| = |k_m V_\mathrm{s}| \leq k_\mathrm{F} V_\mathrm{s}$$

- Bound on open-circuit tribo-voltage:

$$e\phi_\mathrm{oc} = |\mu_A - \mu_B|_\text{at zero current} \lesssim \hbar k_\mathrm{F} V_\mathrm{s}$$

Pheno., cont.

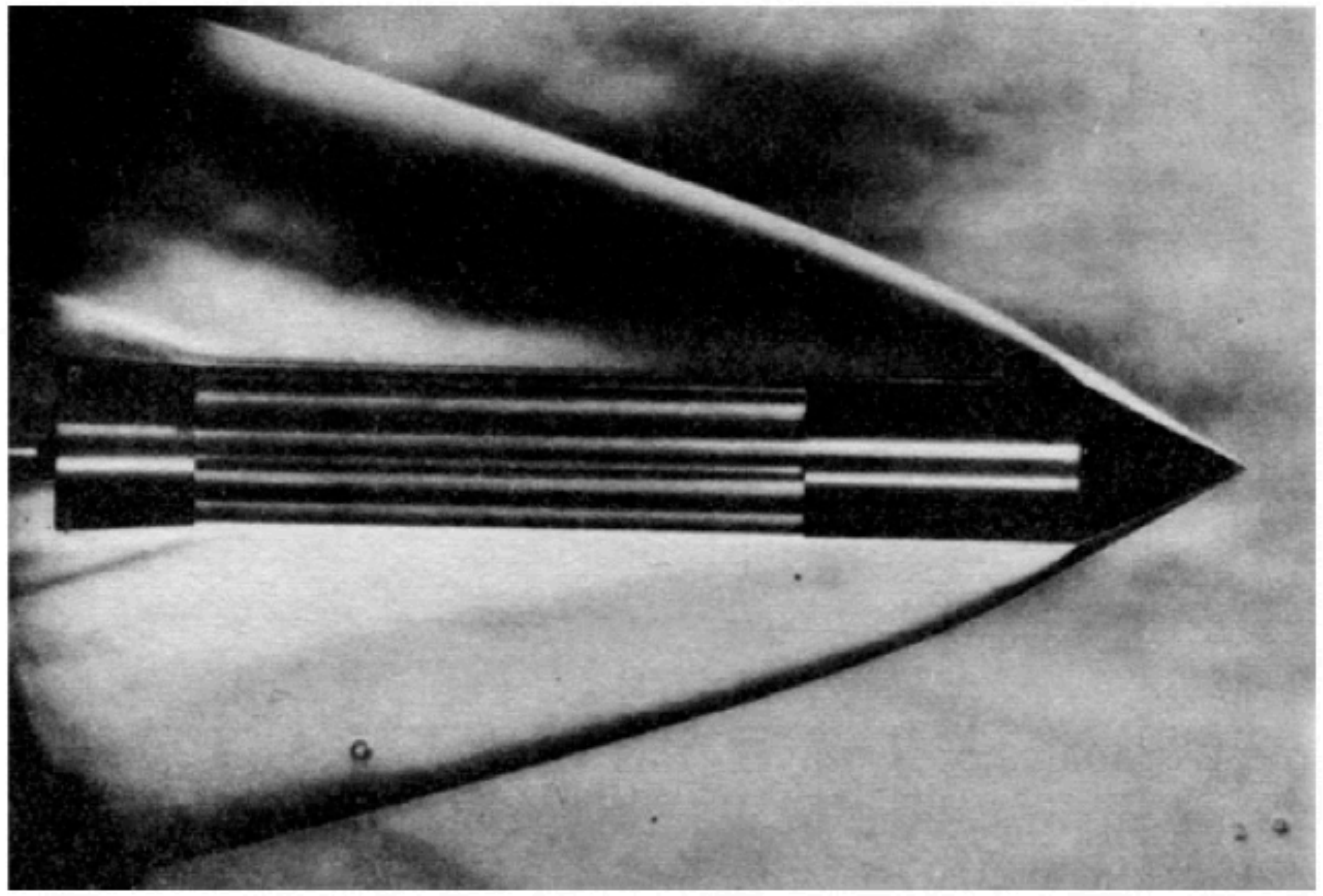
- For $k_F \simeq 10^{10}$ $\mathbf{m^{-1}}$ (**inter-atomic**) and $V_S \simeq 1$ $\mathbf{m/s}$, we get $\phi_{oc} \lesssim 10^{-5}$ $\mathbf{V}$
- Voltage enhanced by **mechanical separation** of electrified surfaces
- Increase from inter-atomic to 1 m scale gives $\phi_{oc} \lesssim 10^{5}$ $\mathbf{V}$, as in Van de Graaf generator
- Expect "triboelectric series" correlated with materials' work functions

Pheno., cont.

- Why current flows in opposite directions at Van de Graaf generator's two terminals
- Compatible with **mosaic** of charging at the $\sim 1\ \mu\mathbf{m}$ roughness scale; Baytekin *et al.*, *Science* **333**, 308 (2011)
- Approx. symmetric interval of triboelectric charge densities $[-\boldsymbol{\sigma}_{\mathbf{max}}, \boldsymbol{\sigma}_{\mathbf{max}}]$ reported in Zou *et al.*, *Nat. Comm.* **11**, 2093 (2020)
- Qualitatively different from any *CP*-violation in high-energy theory; irreversibility implies *T*-violation

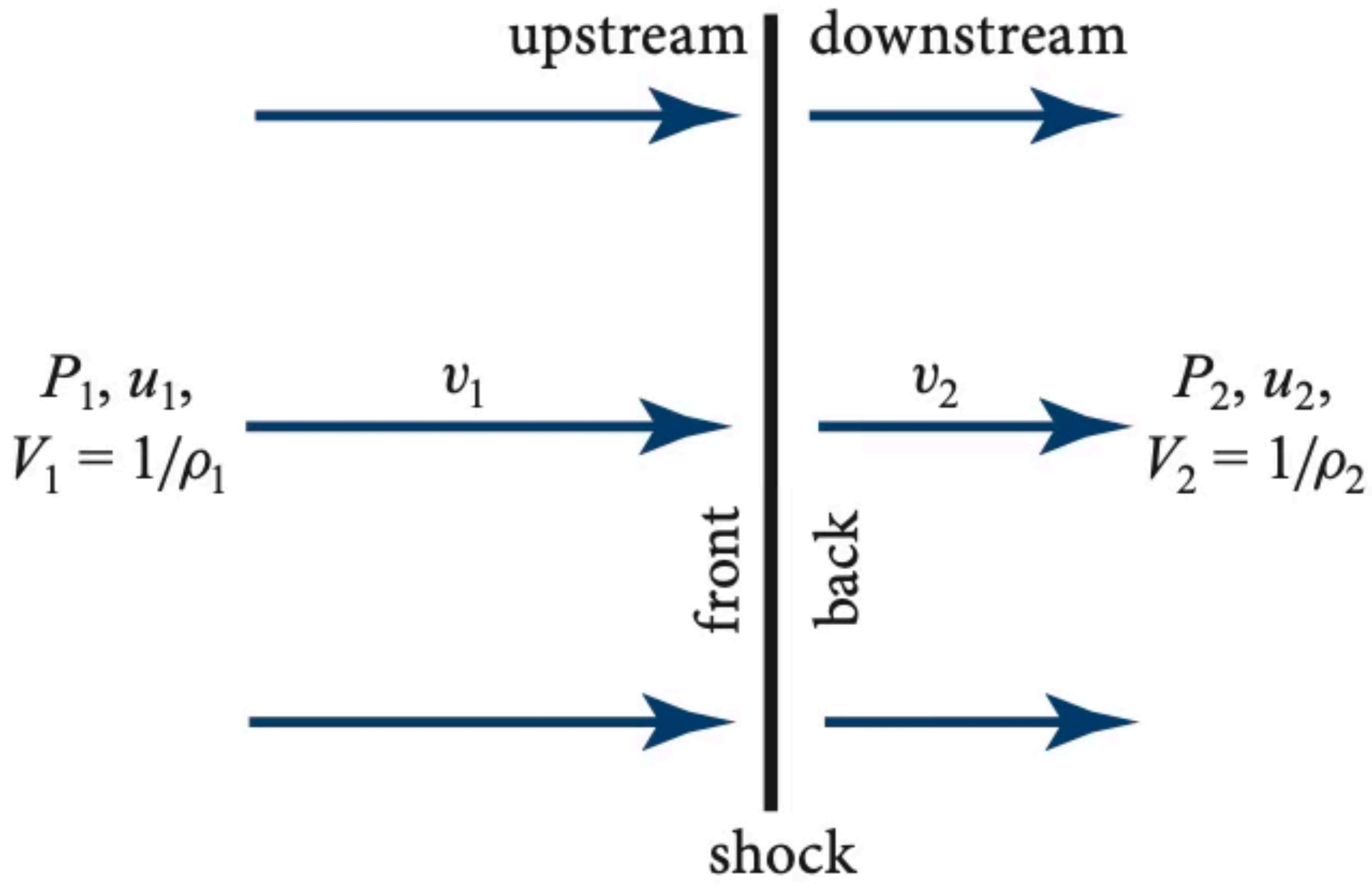
Shocks

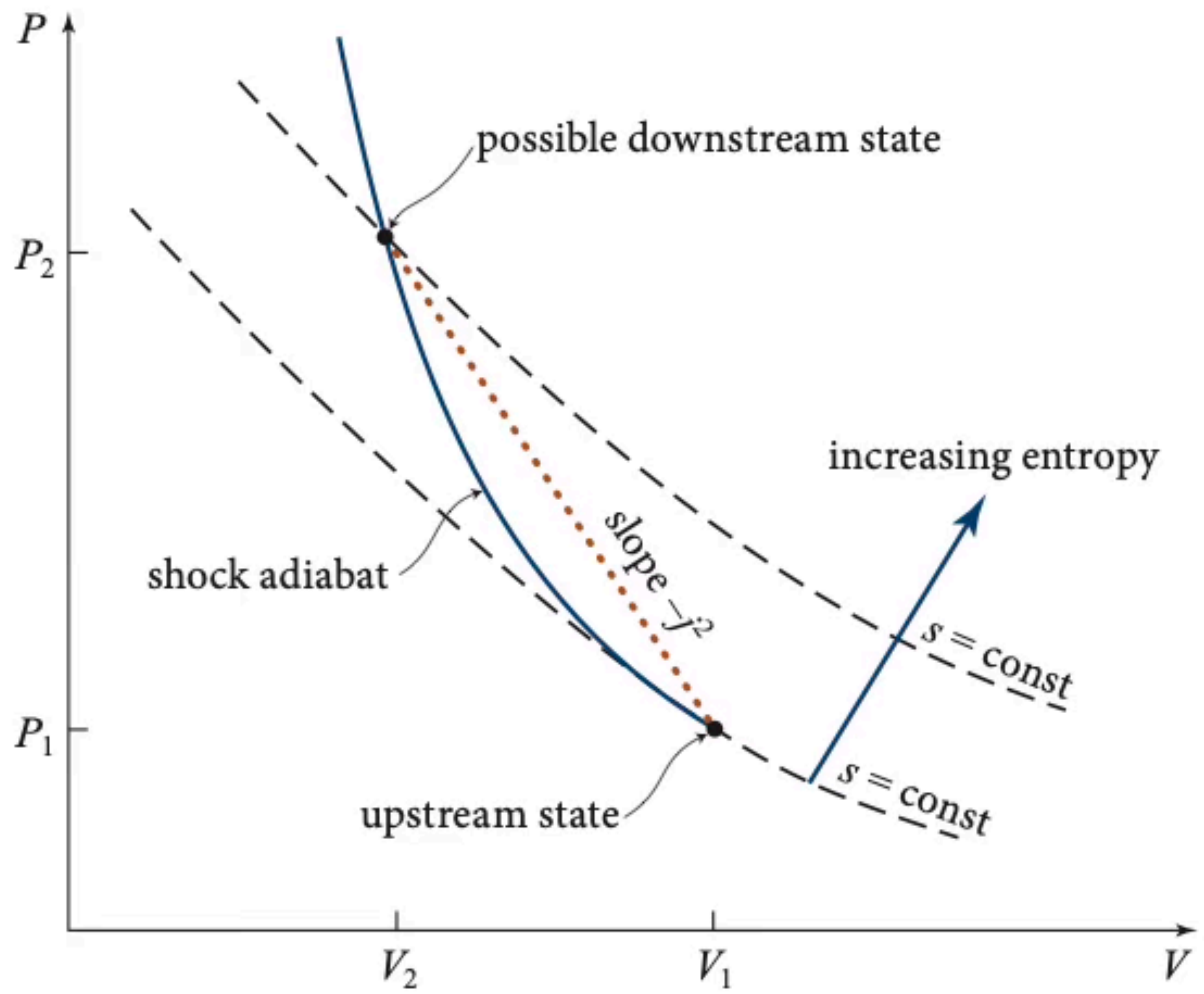
- "It turns out, very interestingly, that once the object is moving faster than the speed of sound, it will *make* sound. It is not necessary that it have a certain tone vibrational character."
- Has "counterparts in acoustics, chromodynamics and, strictly speaking, **in any field theory**" — Ginzburg, *Prog. Optics* **32**, 267 (1993)
- "Radiation during the uniform motion of various sources is a **universal phenomenon** rather than an eccentricity" — Ginzburg, *Phys. Usp.* **39**, 973 (1996)



— *Feynman Lectures*, "Waves" vol. I, no. 51,

Rankine-Hugoniot





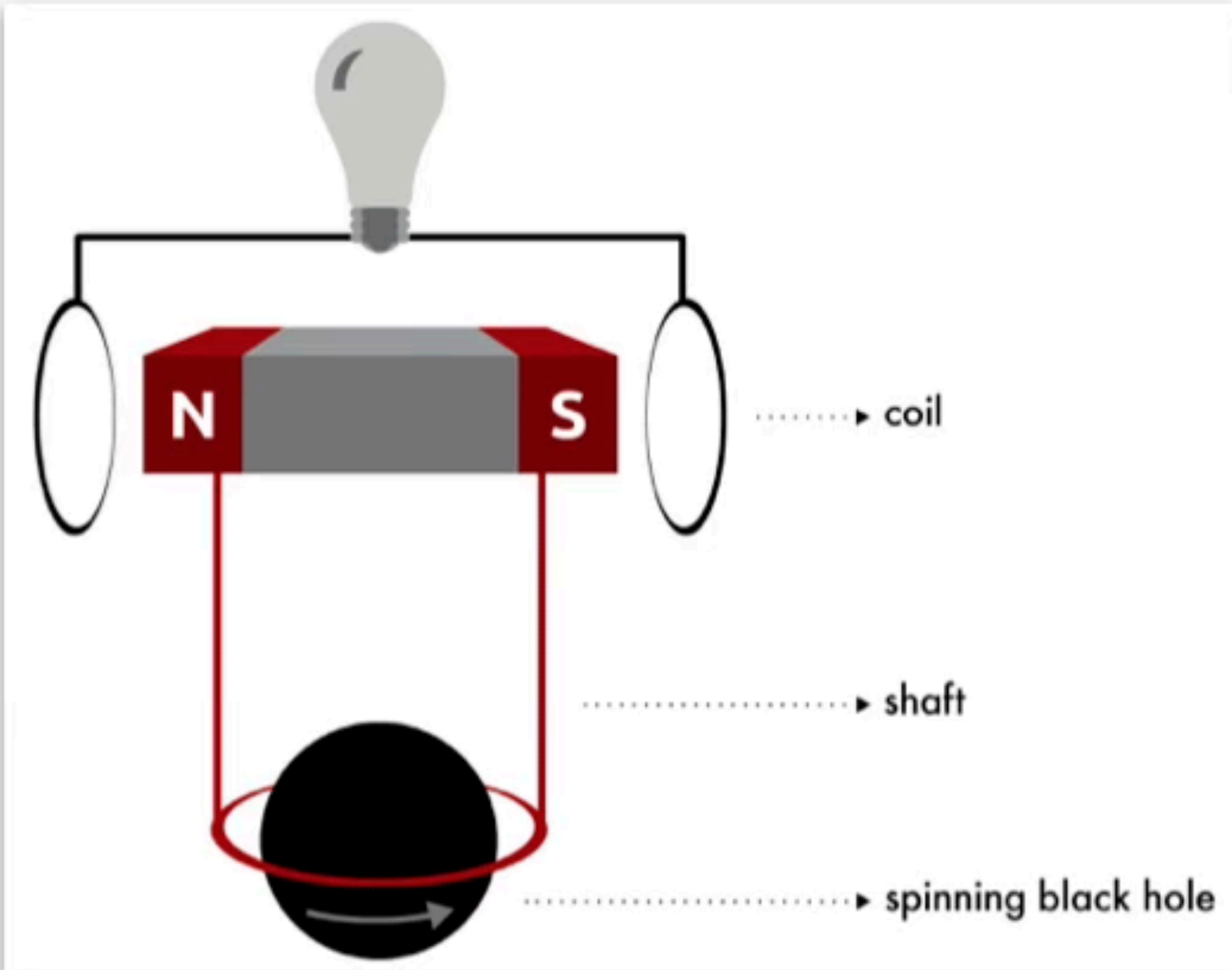
Stokes "was persuaded by his former student Rayleigh and others that such discontinuities were impossible, because they would violate energy conservation. With a deference that professors traditionally show their students, Stokes believed Rayleigh." — Thorne & Blandford, *Modern Classical Physics* (2017)

"It seems to be a good principle that the prediction of a **singularity** by a physical theory indicates that the theory has broken down." — Hawking & Ellis (1973)

Active galactic nucleus (AGN)

Artist's conception of super-massive black hole at center of NGC 1365 spiral galaxy. Source: NASA

Blandford-Znajek (1977)



<u>Fuente</u>: Brito, Cardoso & Pani, p. 48

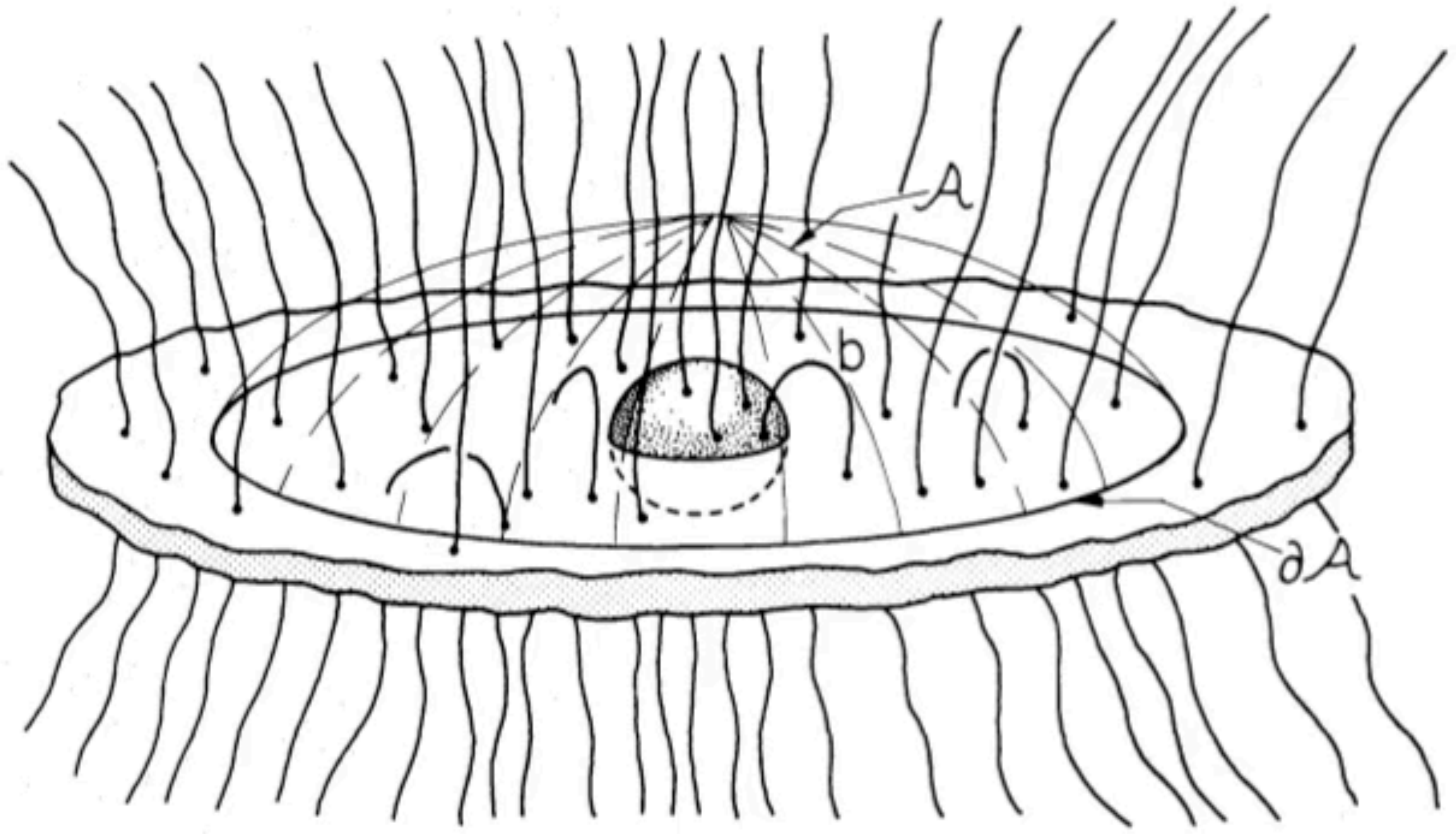


Figure 1. Accretion disc around a black hole, with magnetic field lines threading it. Although the disc is shown thin, nothing anywhere in our analysis constrains it to be so. The surface $\mathscr{A}$ and its boundary $\partial\mathscr{A}$ are used in the mathematical discussion of magnetic flux conservation in Section 3.

<u>Fuente</u>: D. Macdonald & K. S. Thorne, "Black-hole electrodynamics: an absolute-space/universal-time formulation", *Mon. Not. R. astr. Soc.* **198**, 345 (1982)

Summary & outlook

- **Superradiance**: If object can damp a degree of freedom, it must *also* be able —in supercritical regime— to anti-damp it

- Active, irreversible process (non-thermal radiation)

- Fermions don't superradiate, but active states may set up macroscopic currents

- Approach should be extended to other **active processes** in quantum field theory, including generation of shock waves