THE NON-CRITICAL BASELINE FOR FLUCTUATION MEASUREMENTS

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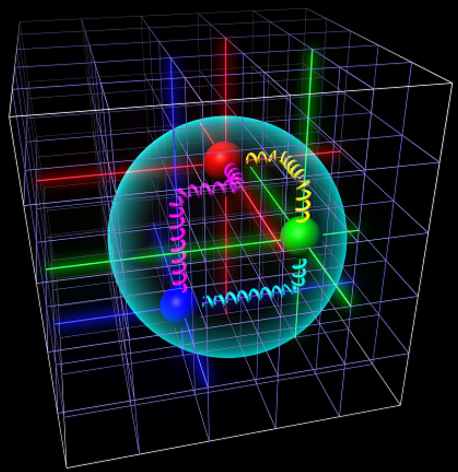
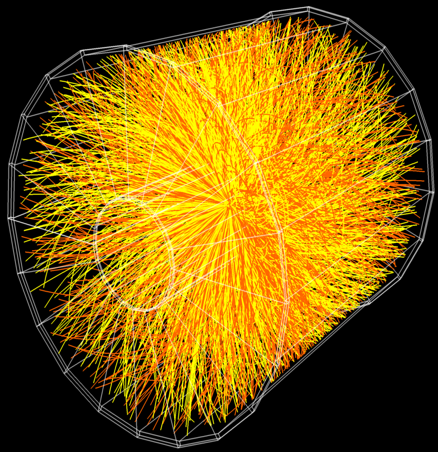


$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3}$$

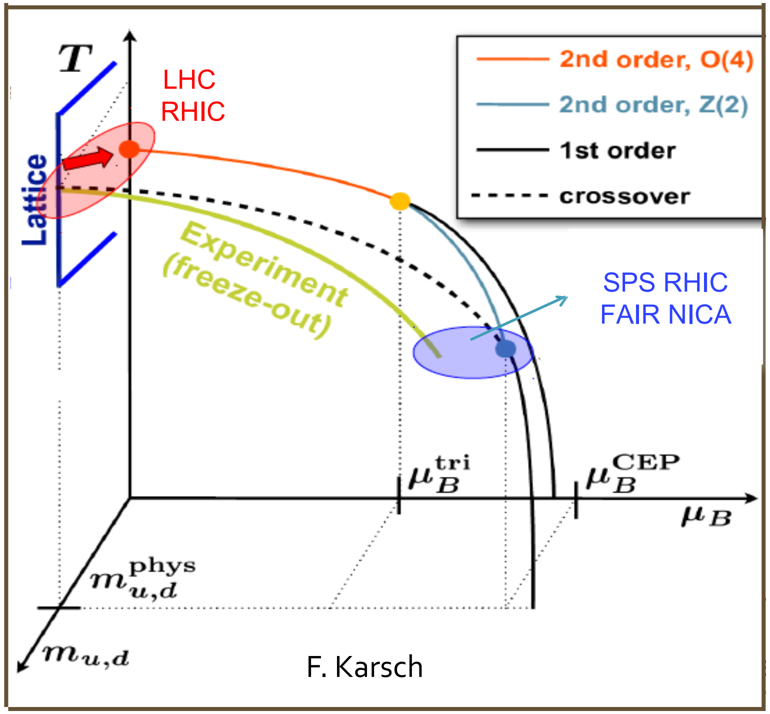


based largely on:
P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel , arXiv:2007.02463

Outline

- Why fluctuations
- The non-critical baseline
  - canonical formulation of higher order cumulants
  - introducing finite acceptances for baryons and anti-baryons
  - comparison to experimental data on net-protons
  - volume fluctuations
- Global vs. local conservation laws, multi-particle correlations
- Summary

Why Fluctuations?



A. Bazavov et al., Phys.Rev. D85 (2012) 054503

To probe the structure of strongly interacting matter

- Locate phase boundaries
- Search for critical phenomena
- ...

E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi_T}{V} \qquad \chi_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

direct link to the EoS

$$\langle N^2\rangle - \langle N\rangle^2 = \kappa_2(N) = T^2\frac{\partial^2 lnZ}{\partial\mu^2}$$

probing the response of the system to external perturbations

Understanding the QCD phase transition

Freeze-out at the phase boundary

$$T_{fo}^{ALICE} = 156.5 \pm 1.5\text{MeV} \pm 3\text{ MeV(sys)}$$

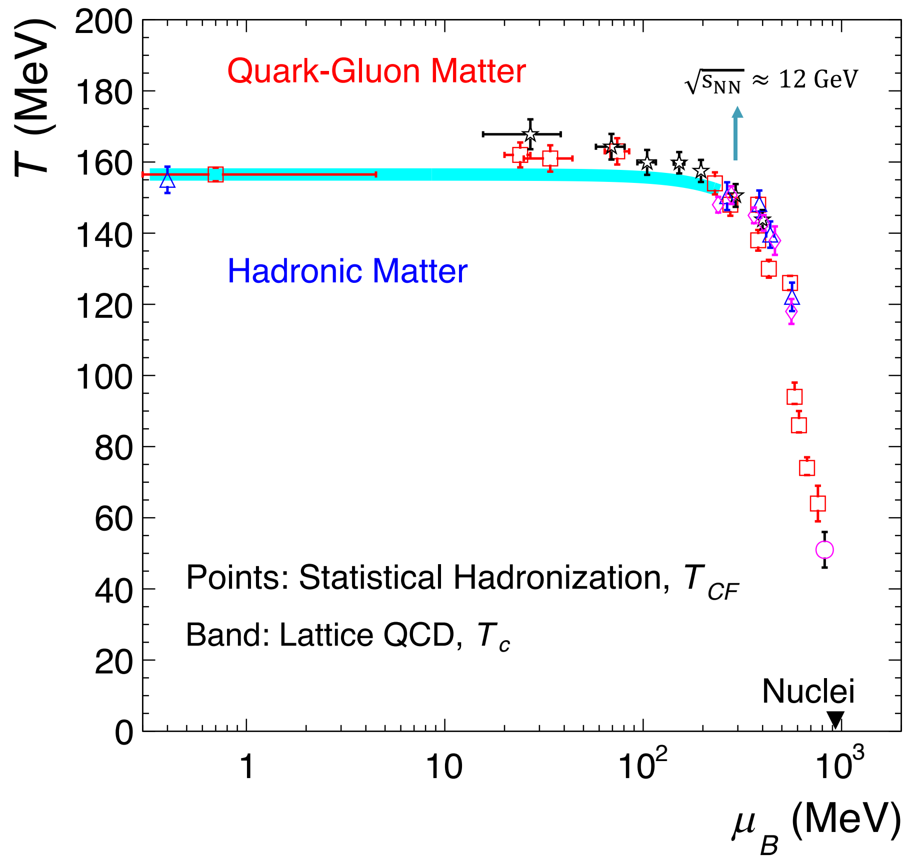
$$T_{c}^{LQCD} = 156.5 \pm 1.5\ MeV$$

Experimental plan:

- measuring fluctuations of net-baryons along the QCD phase boundary

Open questions:

- the order of the phase transition
- existence of the critical endpoint
- ...



A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature 561, 321–330 (2018)
A. Bazavov et al., Phys.Rev. D85 (2012) 054503

Theory vs. experiment

for a thermal system in a fixed volume V within the Grand Canonical Ensemble (CGE)

$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} \equiv \frac{\kappa_2(\Delta N_B)}{VT^3}$$

$$\hat{\chi}_n^B = \frac{1}{VT^3} \frac{\partial^n lnZ(V,T,\mu_{B,Q,S})}{\partial(\mu_B/T)^n}$$

Assumptions in theory:

- *Volume is fixed in each event*
- ***Conservations are imposed on the averages***

Reality in experiments:

- *Volume fluctuates from E-to-E*
- ***Conservations depend on acceptance***

artefact of volume fluctuations

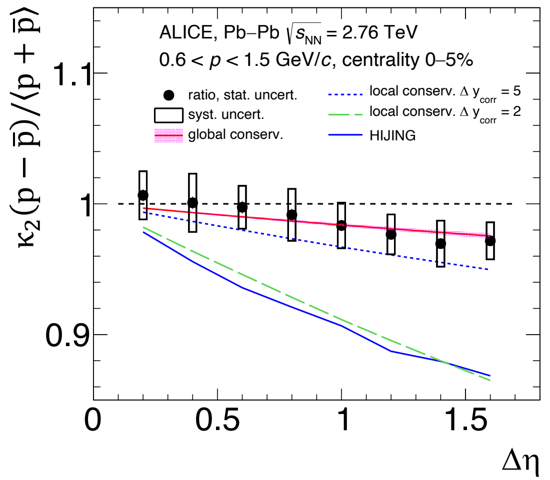
$$\frac{\kappa_4^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \qquad \frac{\kappa_3^{exp}(\Delta N_B)}{\kappa_2^{exp}(\Delta N_B)} \neq \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$

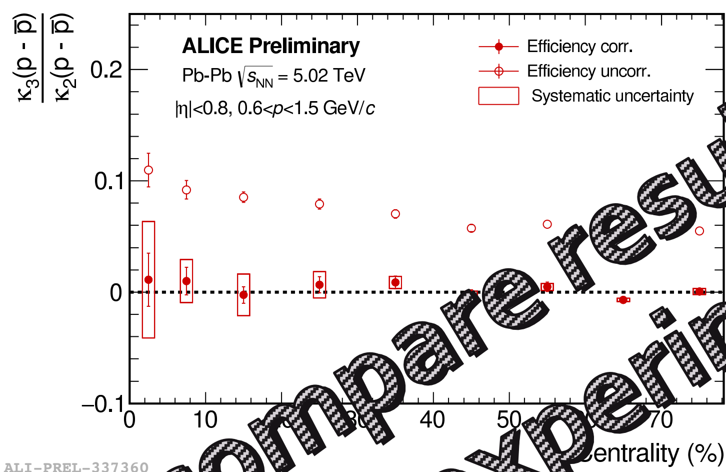
not discussed in this presentation

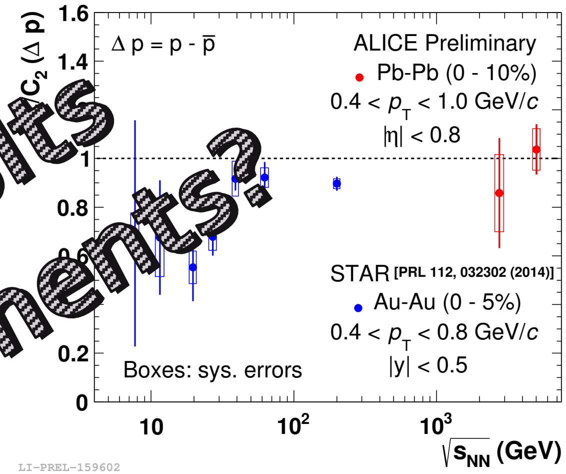
outlined in:

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911
M. I. Gorenstein, M. Gazdzicki, PRC 84 (2011) 014904

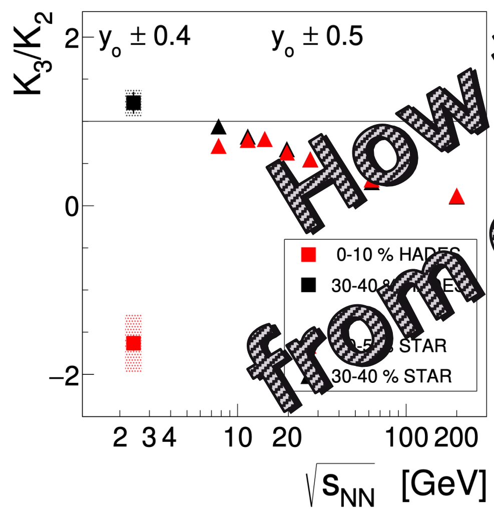
Experimental results on net-protons

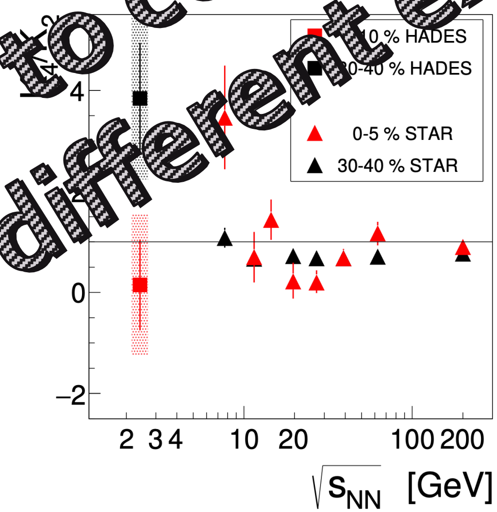


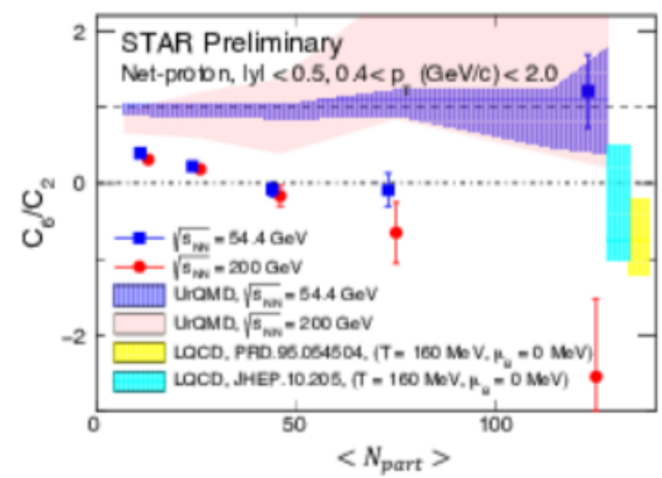




How to compare results from different experiments?







Note different notations: $\kappa_n \equiv C_n \equiv K_n$

ALICE: PLB 807 (2020) 135564, M. Arslandok, QM19, N. Behera, QM18
HADES: PRC 102 (2020) 024914
STAR: arXiv:2001.02852, A. Pandav, QM19

establishing the non-critical baseline

- canonical formulation of higher order cumulants
- introducing finite acceptances for baryons and anti-baryons
- comparison to experimental data

considering acceptances for both baryons and anti-baryons

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel , arXiv:2007.02463

using one single acceptance for baryons and anti-baryons

A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA982, (2019), 307-310

The formalism

Canonical partition function in a finite volume V at temperature T

$$Z_B(V,T) = \sum_{N_B=0}^{\infty} \sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z_B)^{N_B}}{N_B!} \frac{(\lambda_{\bar{B}} z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B) = \left(\frac{\lambda_B z_B}{\lambda_{\bar{B}} z_{\bar{B}}}\right)^{\frac{B}{2}} I_B\left(2z\sqrt{\lambda_B \lambda_{\bar{B}}}\right)$$

K. Redlich and L. Turko, Z. Phys. C5 (1980) 201, V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA982, (2019), 307-310

- B net-baryon number, conserved in each event
- I_B modified Bessel function of the first kind
- $z_B, z_{\bar{B}}$ single particle partition functions for baryons, antibaryons
- $\lambda_B, \lambda_{\bar{B}}$ auxiliary parameters for calculating mean number of baryons, antibaryons

$$z = \sqrt{z_B z_{\bar{B}}} = \sqrt{\langle N_B\rangle_{GCE} \langle N_{\bar{B}}\rangle_{GCE}}$$

$\langle N_B\rangle_{GCE}, \langle N_{\bar{B}}\rangle_{GCE}$ are in GCE, **experiments measure canonical multiplicities** $\langle N_B\rangle$, $\langle N_{\bar{B}}\rangle$

$$\langle N_B\rangle = \lambda_B \frac{\partial \ln Z_B}{\partial \lambda_B}\bigg|_{\lambda_B=\lambda_{\bar{B}}=1} = z\frac{I_{B-1}(2z)}{I_B(2z)}$$

$$\langle N_{\bar{B}}\rangle = \lambda_{\bar{B}} \frac{\partial \ln Z_B}{\partial \lambda_{\bar{B}}}\bigg|_{\lambda_B=\lambda_{\bar{B}}=1} = z\frac{I_{B+1}(2z)}{I_B(2z)}$$

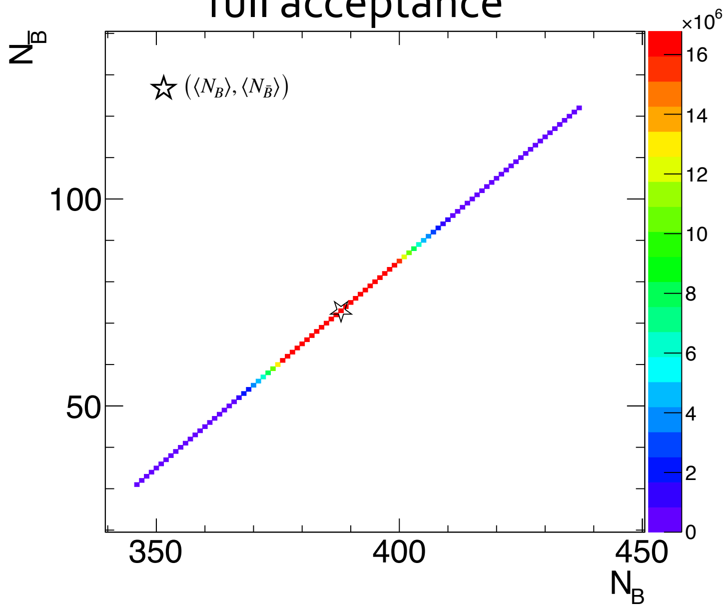
we recalculate z by solving Eq. for $\langle N_B\rangle$ or $\langle N_{\bar{B}}\rangle$

Full vs. limited acceptance

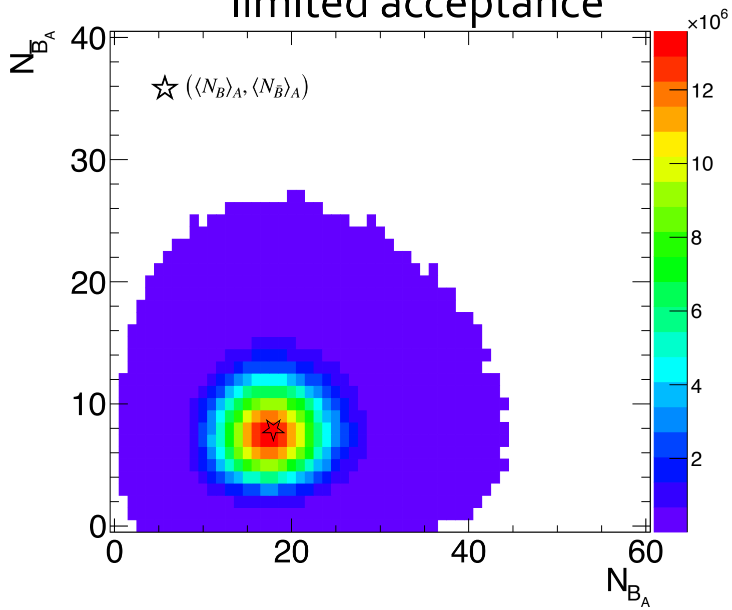
the underlying PDF for anti-baryons

$$P_B(N_{\bar{B}}) = \frac{1}{I_B(2z)} \frac{z^B z^{2N_{\bar{B}}}}{(N_{\bar{B}} + B)!\, N_{\bar{B}}!}, \qquad N_B = N_{\bar{B}} + B$$

full acceptance



limited acceptance



$$\alpha_B = \frac{\langle N_B \rangle_A}{\langle N_B \rangle}$$

$$\alpha_{\bar{B}} = \frac{\langle N_{\bar{B}} \rangle_A}{\langle N_{\bar{B}} \rangle}$$

fluctuations of net-baryons appear only inside limited acceptance

net-proton is a proxy of net-baryon; if and only if the isospin correlations are negligible

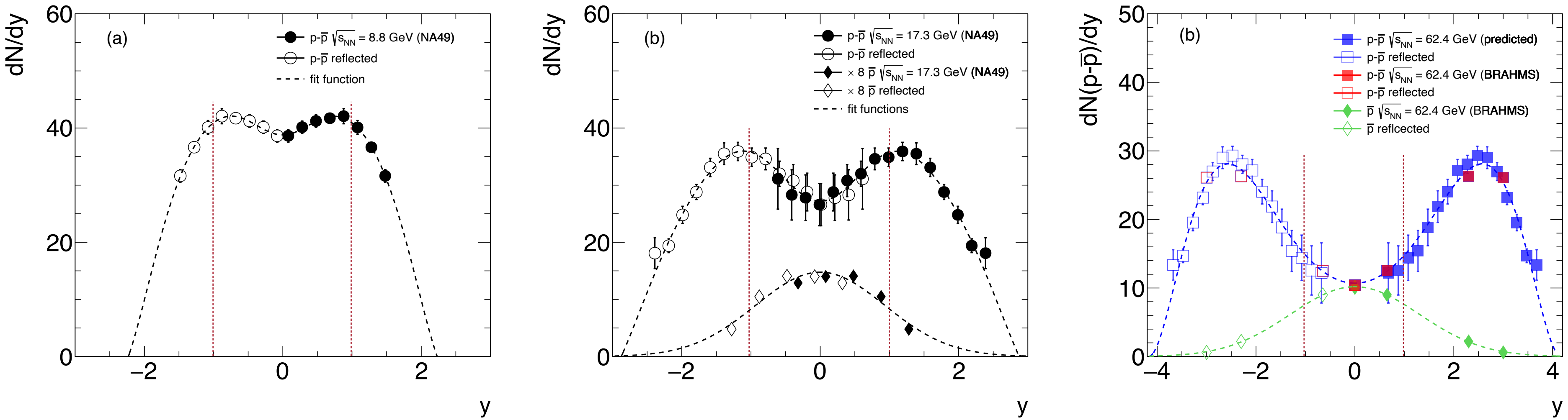
net-protons fluctuate even in full acceptance

M. Kitazawa, M. Asakawa, PRC86 (2012) 024904

Introducing finite acceptances

in experiments the finite acceptance is introduced by applying cuts on y and/or $p_\perp$

NA49 Pb-Pb, $\sqrt{s_{NN}}$=8.8 GeV   **NA49** Pb-Pb, $\sqrt{s_{NN}}$=17.3 GeV   **BRAHMS** Au-Au, $\sqrt{s_{NN}}$=62.4 GeV



we consider acceptances for both protons and anti-protons

Note

there are only 4 measurements from BRAHMS (■, ♦)

the full distribution for BRAHMS (blue dashed curve) is our prediction

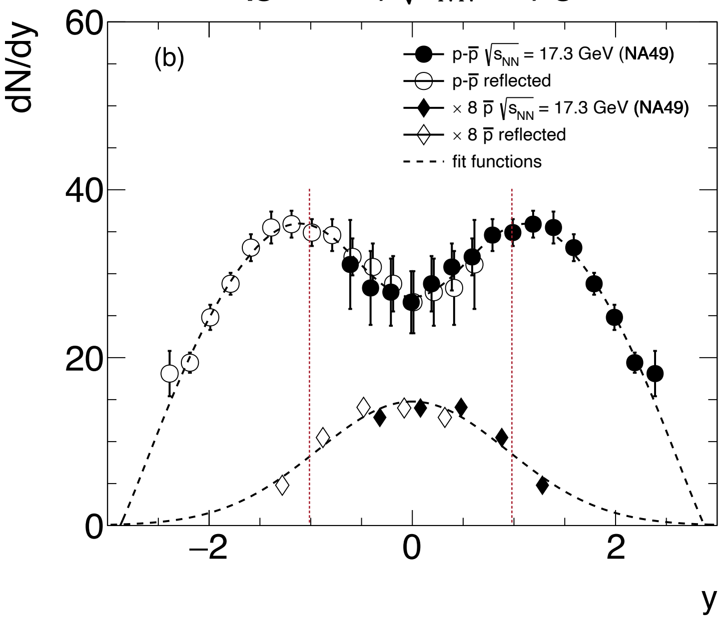
NA49: PRL. 82 (1999) 2471-2475, PRC 83 (2011) 014901

BRAHMS: Phys.Lett.B 677 (2009) 267-271

Important caveat

- The **STAR** data contain $\boldsymbol{p_\perp}$ cuts (in addition to $\boldsymbol{y}$ cuts) and contributions from feed-down
- The **NA49** $\boldsymbol{y}$ distributions are $\boldsymbol{p_\perp}$ integrated and are corrected for feed-down effects

NA49 Pb-Pb, $\sqrt{s_{NN}}$ = 17.3 GeV



NA49: PRL. 82 (1999) 2471-2475, PRC 83 (2011) 014901

empirical solution

$$\gamma_p \int_{-0.5}^{0.5} \frac{dN_p}{dy} dy = \langle n_p \rangle \qquad \gamma_{\bar{p}} \int_{-0.5}^{0.5} \frac{dN_{\bar{p}}}{dy} dy = \langle n_{\bar{p}} \rangle$$

$\frac{dN_p}{dy}, \frac{dN_{\bar{p}}}{dy}$ rapidity distributions from **NA49, BRAHMS**

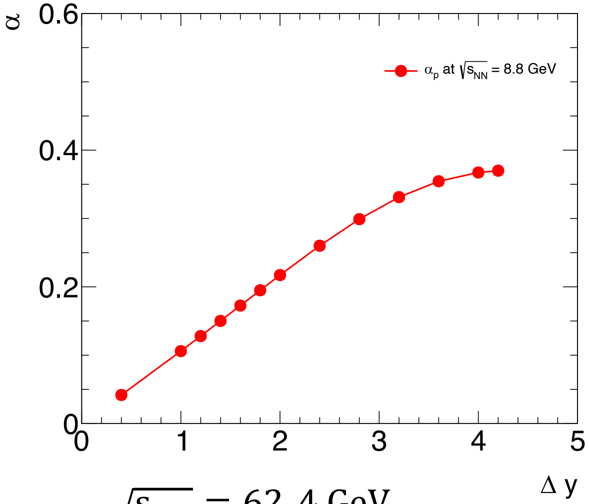
$\langle n_p \rangle, \langle n_{\bar{p}} \rangle$ mean numbers from STAR cumulant analysis

$$\alpha_p = \frac{\gamma_p \int_{y_{min}}^{y_{max}} \frac{dN_p}{dy} dy}{\langle N_B \rangle} \qquad \alpha_{\bar{p}} = \frac{\gamma_{\bar{p}} \int_{y_{min}}^{y_{max}} \frac{dN_{\bar{p}}}{dy} dy}{\langle N_{\bar{B}} \rangle}$$

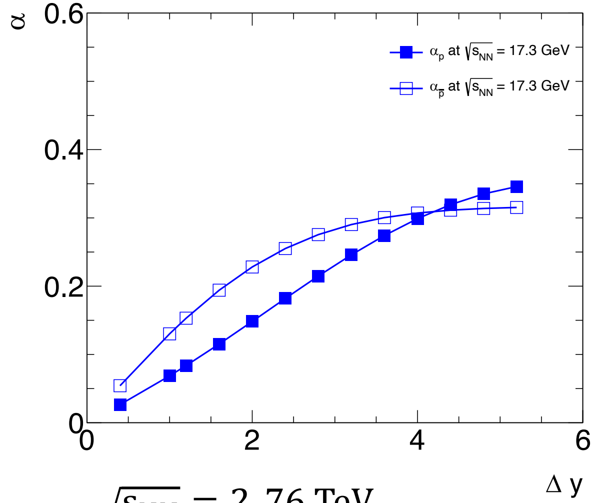
$$|y_{max} - y_{min}| = \Delta y$$

Accepted protons and anti-protons

$\sqrt{s_{NN}} = 8.8$ GeV



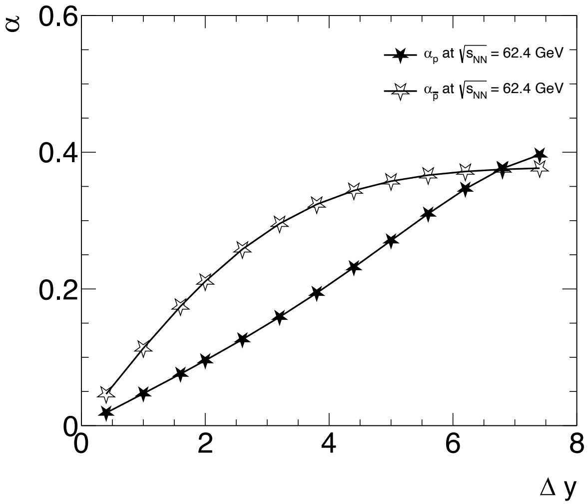
$\sqrt{s_{NN}} = 17.3$ GeV



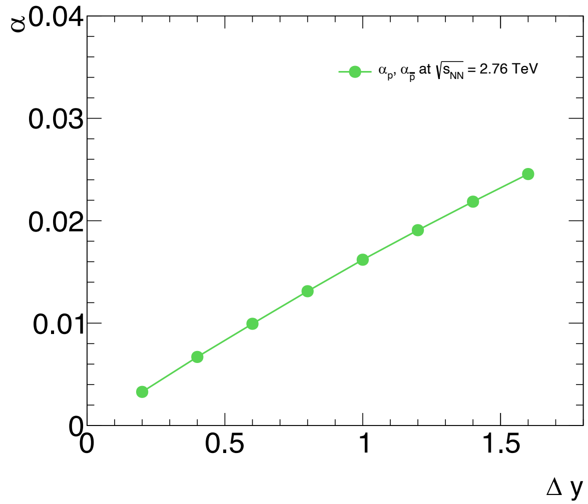
full symbols: α_p

open symbols: $\alpha_{\bar{p}}$

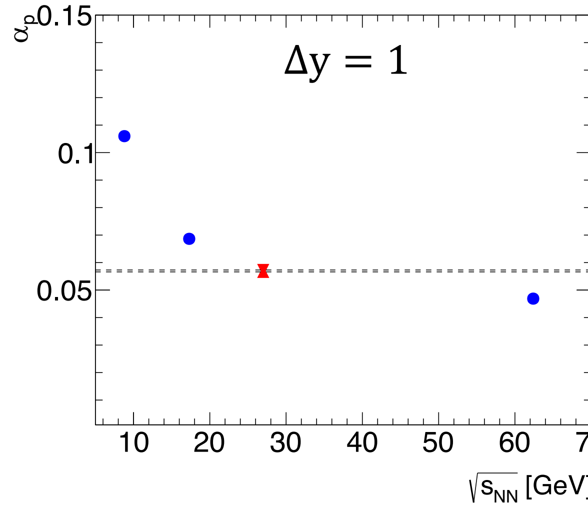
$\sqrt{s_{NN}} = 62.4$ GeV



$\sqrt{s_{NN}} = 2.76$ TeV

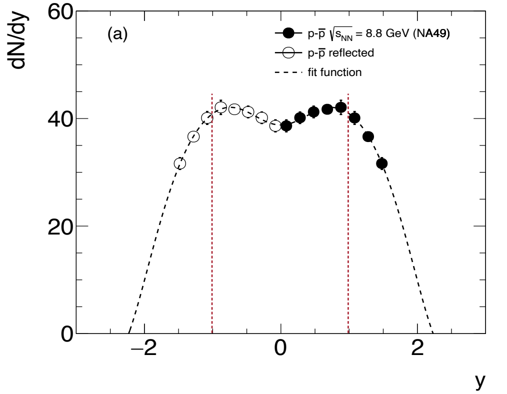


a constant cut in ***y*** introduces energy dependent acceptances

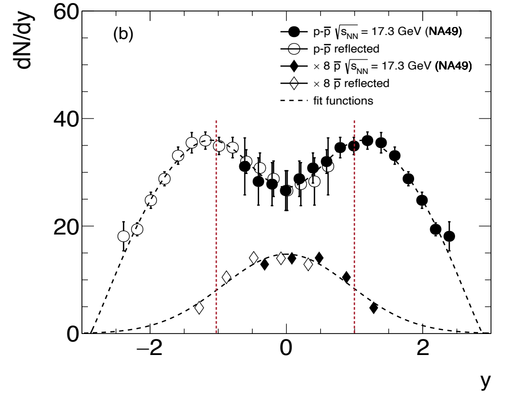


Input to event generator and analytic formulas

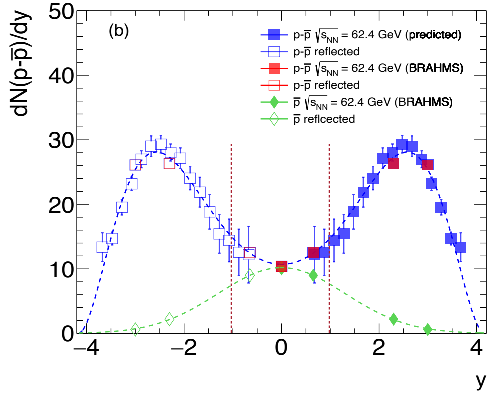
NA49 Pb-Pb, $\sqrt{s_{NN}}$=8.8 GeV



NA49 Pb-Pb, $\sqrt{s_{NN}}$=17.3 GeV



BRAHMS Au-Au, $\sqrt{s_{NN}}$=62.4 GeV



$\sqrt{s_{NN}}$ [GeV]	$\langle N_B \rangle$	$\langle N_{\bar{B}} \rangle$	$\langle N_p \rangle$	$\langle N_{\bar{p}} \rangle$	z
8.8	353	2	130	0.51	26.608
17.3	368	16	154.6	4.36	76.833
27	373 (377)	30 (34)	–	–	105.914 (113.354)
62.4	384	70	181.5	33.23	164.132

$$P_B(N_{\bar{B}}) = \frac{1}{I_B(2z)} \frac{z^B z^{2N_{\bar{B}}}}{(N_{\bar{B}} + B)!\, N_{\bar{B}}!}$$

$$N_B = N_{\bar{B}} + B$$

B = $\langle N_B \rangle - \langle N_{\bar{B}} \rangle$

$\langle N_B \rangle$, $\langle N_{\bar{B}} \rangle$ - **canonical multiplicities**

we recalculate z by solving Eq. for $\langle N_B \rangle$ or $\langle N_{\bar{B}} \rangle$

Obtained results

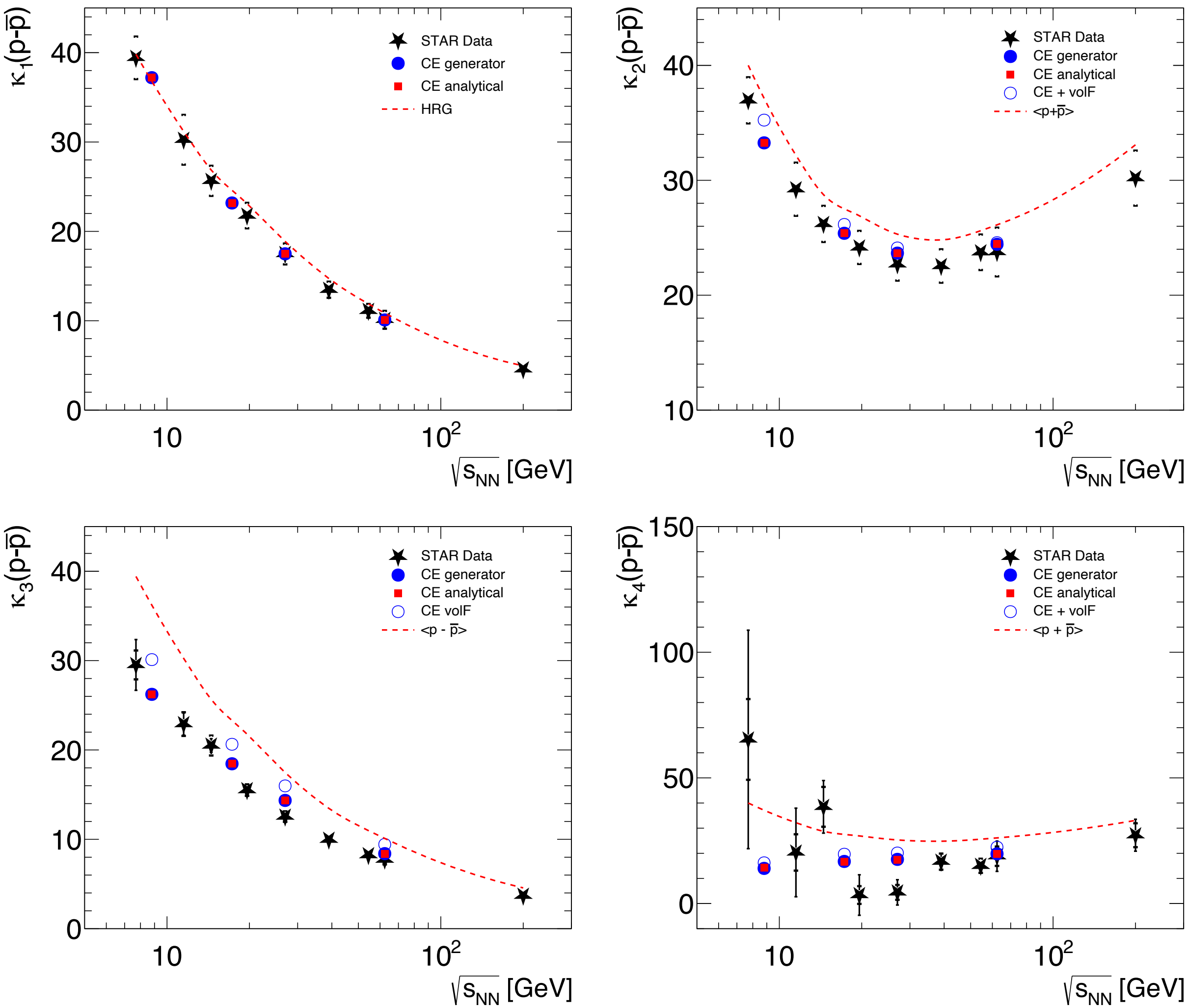
PURE CUMULANTS VS. STAR DATA

CUMULNT RATIOS VS. STAR DATA

REMARK ON FREEZE-OUT PARAMETERS

PYTHON SOFTWARE PACKAGE

Pure cumulants vs. STAR data



κ_1 values from STAR are used as input to our calculations, presented for consistency only

remarkable agreement between analytical/generated values and the STAR data.

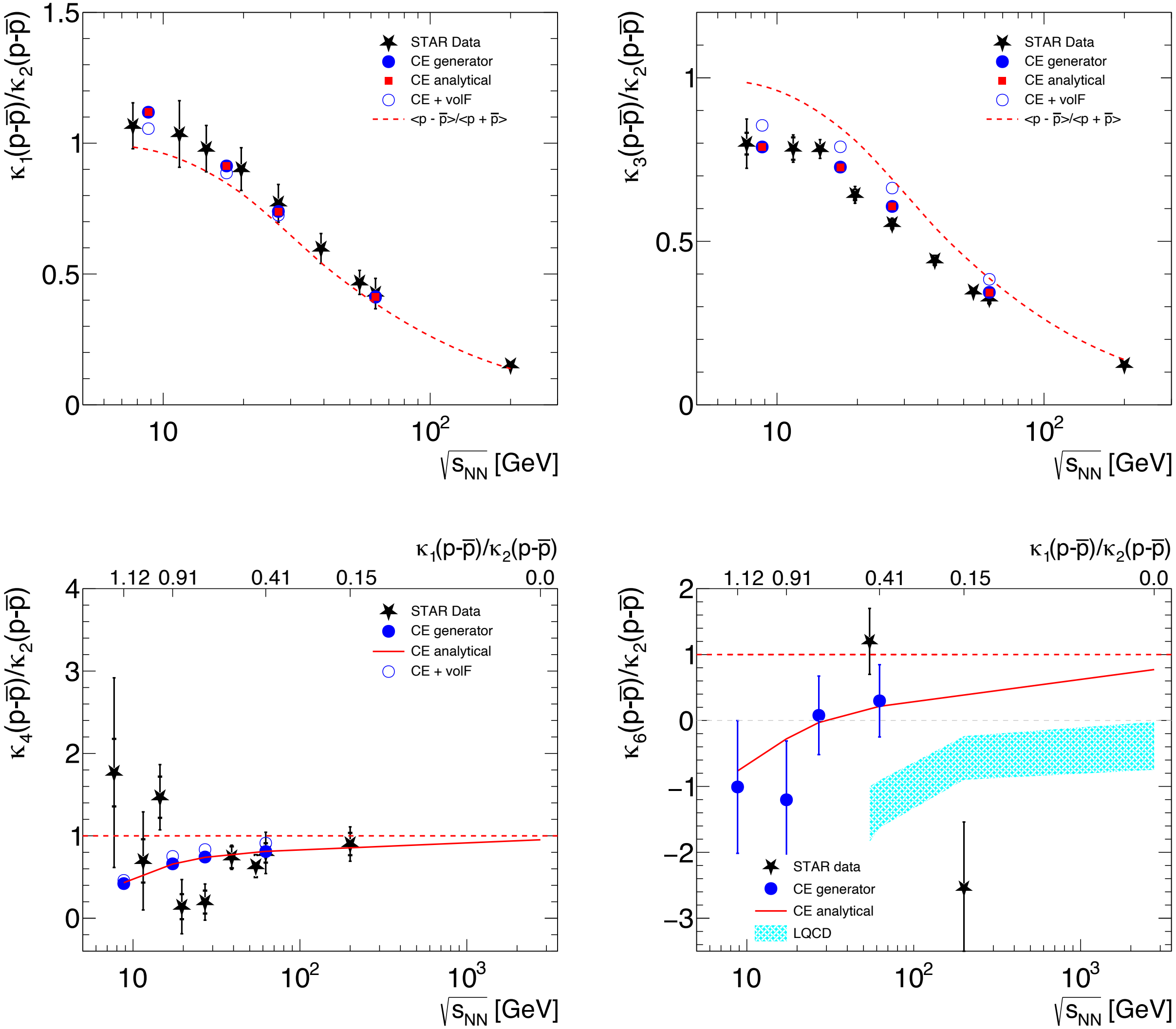
$\kappa_2, \kappa_3, \kappa_4$ values are suppressed compared to the GCE baseline

the amount of suppression in data is consistent with canonical effects

the data points for κ_4 fluctuate around the canonical baseline

STAR data: B. Mohanty, Nu Xu, this workshop

Cumulant ratios vs. STAR data



as already observed for pure cumulants, remarkable agreement between calculations and the STAR data is obvious

artefact of fixed acceptance in rapidity:

for higher energies the ratios approach the HRG baseline

significant reduction of κ_6/κ_2 going from positive values at LHC to negative values at lower energies

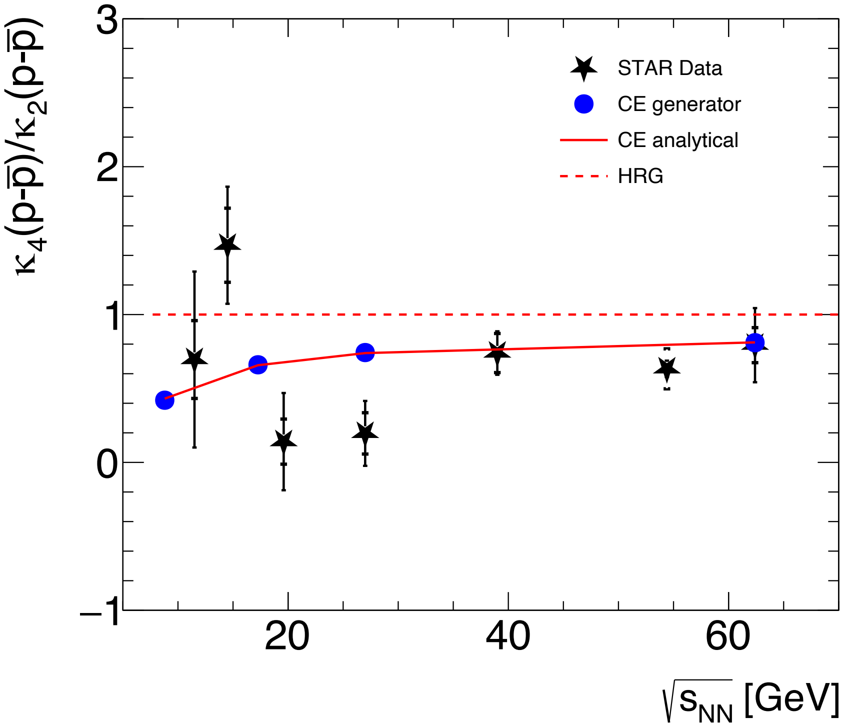
LQCD results for κ_6/κ_2 are negative for all energies

STAR data: B. Mohanty, Nu Xu, this workshop

LQCD: A. Bazavov et al., Phys.Rev.D 101 (2020) 7, 074502

Hypothesis test for κ_4/κ_2

$\sqrt{s_{NN}} = 8.8 - 62.4$ GeV



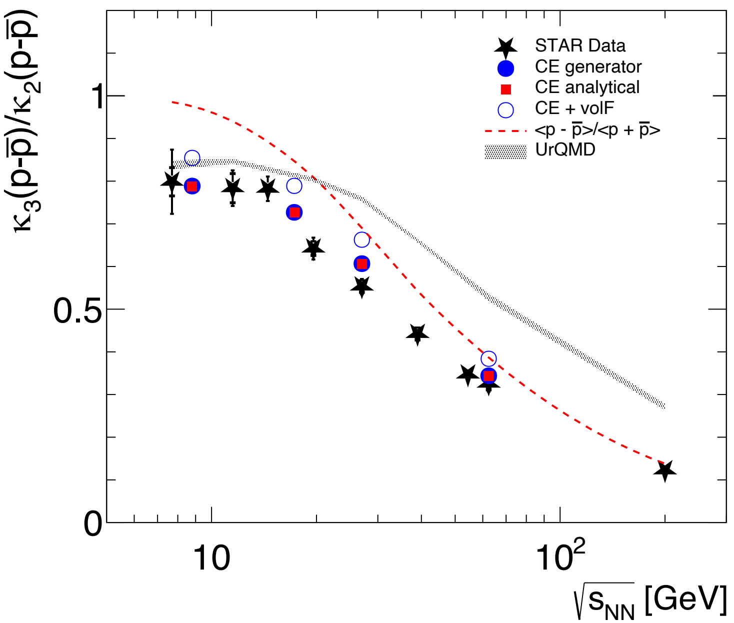
Kolmogorov-Smirnov test

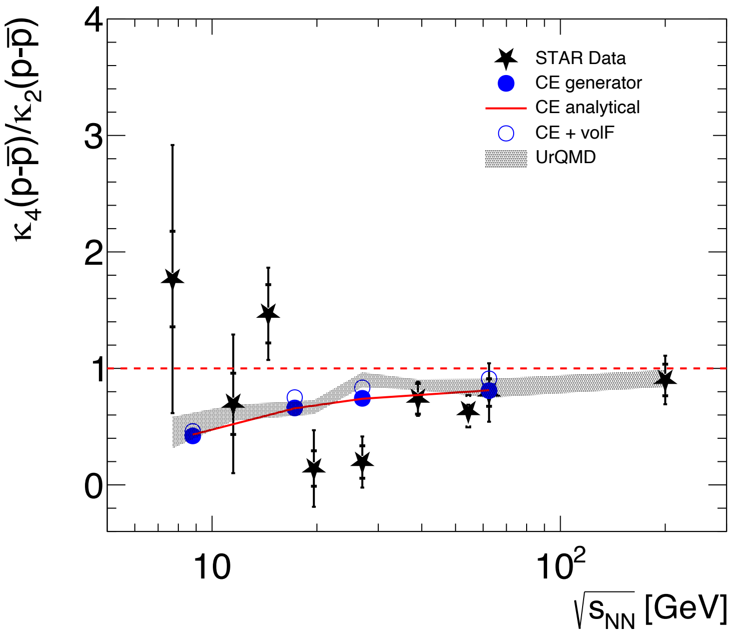
- null-hypothesis
  the data and CE baseline are consistent,
  rejected when p-value is < 0.1
- obtained p-value: > 0.3

the observed deviations between the STAR data and the canonical baseline are not statistically significant

STAR data: B. Mohanty, Nu Xu, this workshop

Note on comparison to UrQMD





conflicting behavior in UrQMD

κ_3/κ_2 : **above 20 GeV the UrQMD results are significantly above the STAR data and the HRG baseline**

κ_4/κ_2 : **the UrQMD results are in agreement with the canonical suppression**

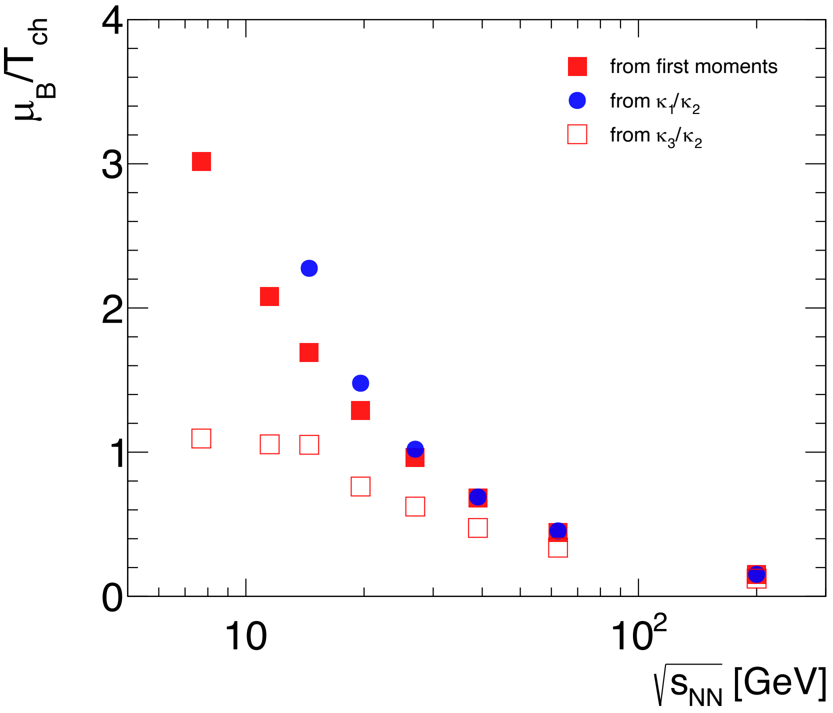
STAR data, UrQMD: B. Mohanty, Nu Xu, this Workshop

Remarks on freeze-out parameters

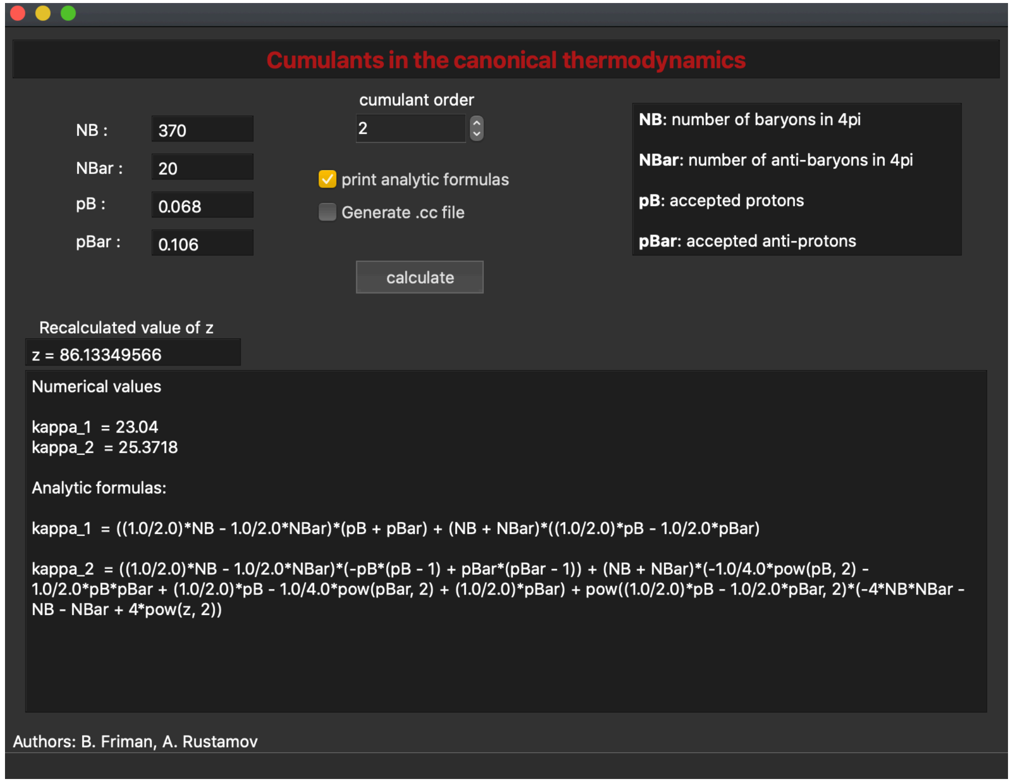
freeze-out parameters from higher cumulants using **G**rand **C**anonical **E**nsemble formulation

$$\frac{\mu_B}{T_{ch}} = \text{atanh}\left(\frac{\kappa_1}{\kappa_2}\right) = \text{atanh}\left(\frac{\kappa_3}{\kappa_2}\right)$$

if canonical effects alter cumulant ratios the extracted freeze-out parameters will give spurious results



A dedicated Python package



a Python package for calculating both analytic formulas and numerical values for net-baryon cumulants of any order in the finite acceptance is available for download

git clone https://github.com/e-by-e/Cumulants-CE.git

global vs. local conservations

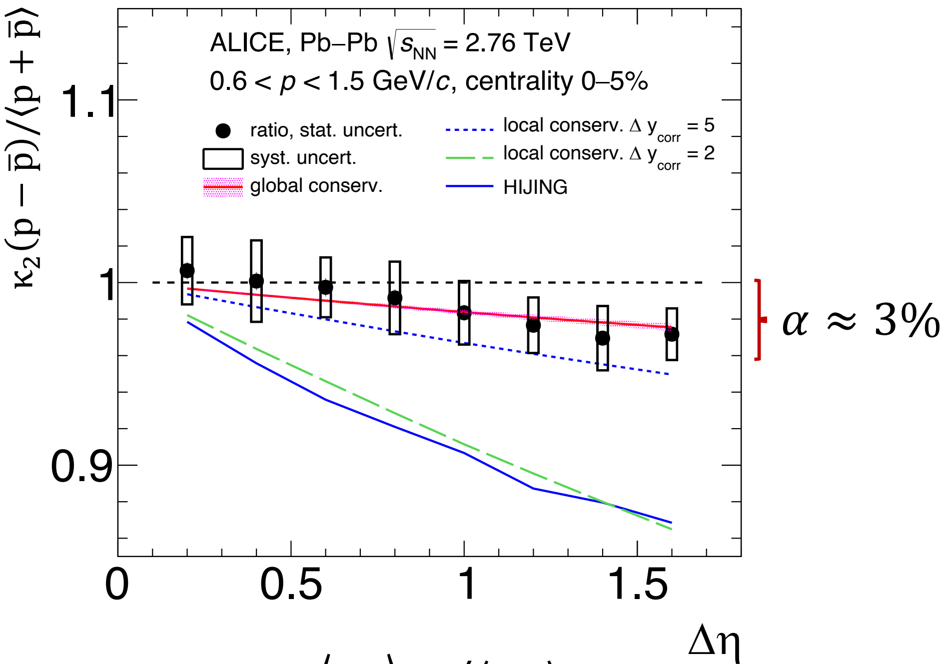
multi-particle correlations

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032

B. Ling and M. A. Stephanov, Phys. Rev. C 93, 034915 (2016).

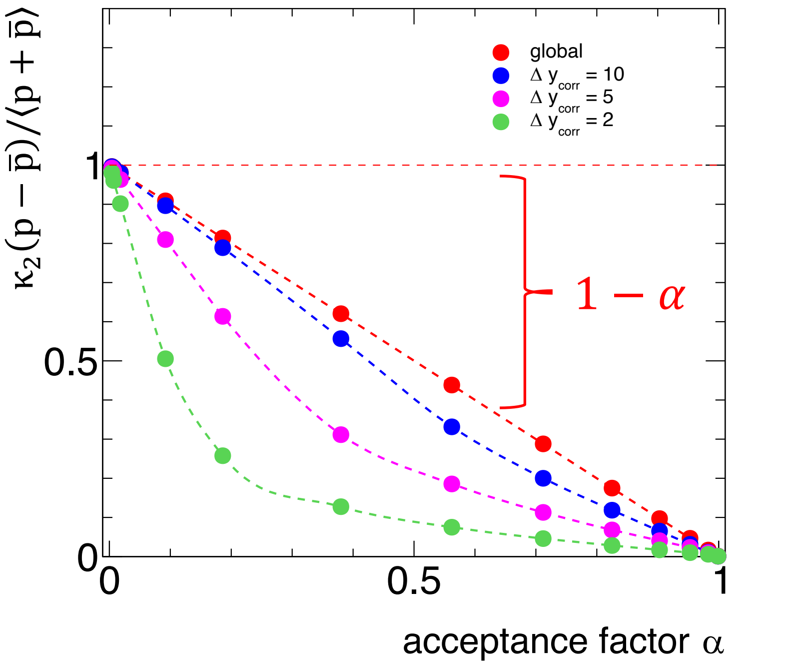
A. Bzdak, V. Koch, and N. Strodthoff, Phys. Rev. C 95, 054906 (2017)

Global vs. local correlations



$$\alpha = \langle n_p \rangle_{\Delta\eta} / \langle n_B \rangle_{4\pi}$$

ALICE: Phys. Lett. B 807 (2020) 135564



P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032

- The data are best described by global baryon number conservation: $\approx 1-\alpha$
- HIJING corresponds to $\Delta y_{corr} = 2$, not consistent with the data

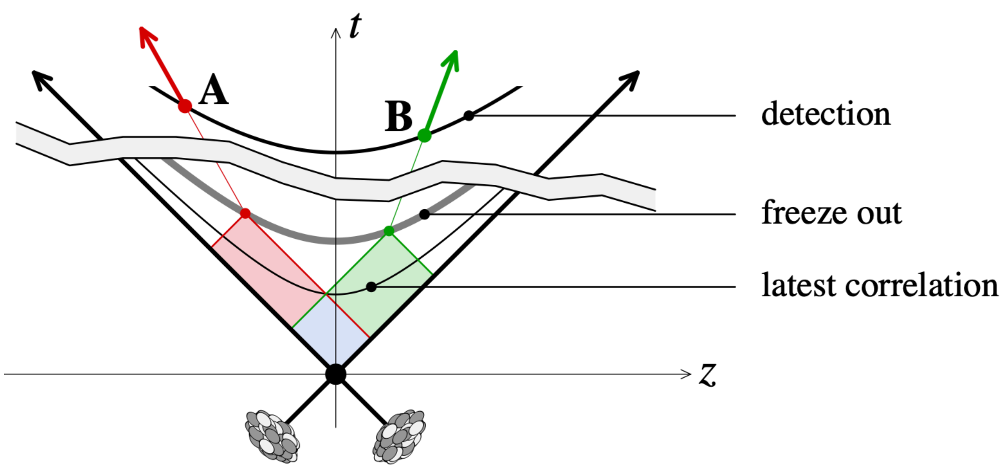
baryon production in string models is not consistent with data

the ALICE data indicate long range correlations

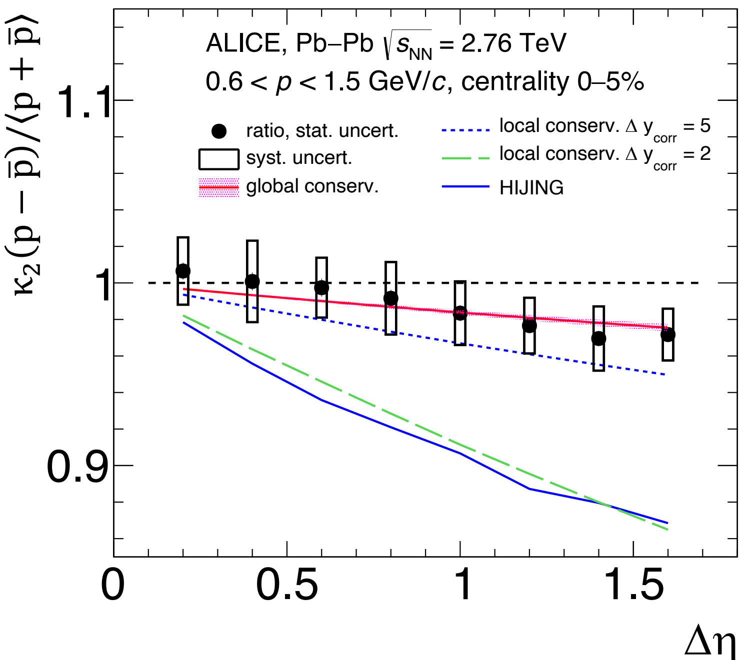
cf. also J. Adolfsson et al., 2003.10997 [hep-ph]

Probing the early collision times

A. Dumitru, F. Gelis, L. McLerran, R.Venugopalan , Phys. A810 (2008) 91–108



$$\tau_i \leq \tau_f e^{-|y_A - y_B|/2}$$

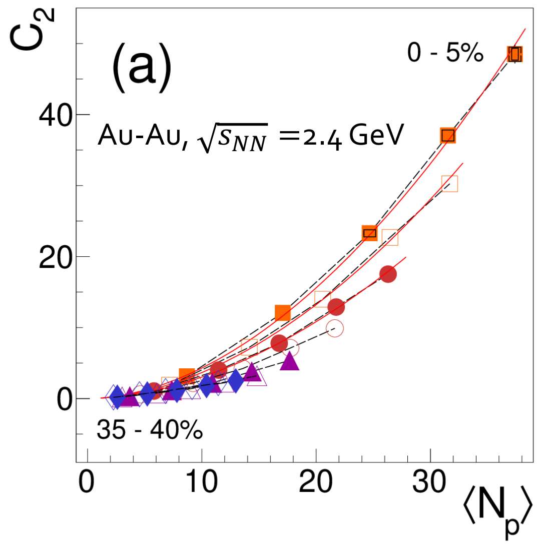


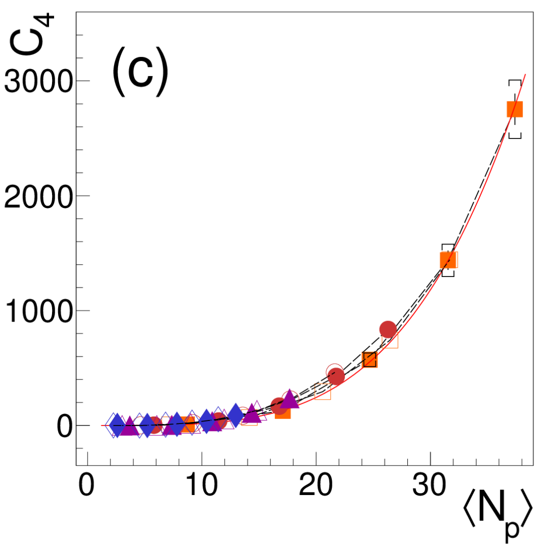
ALICE: Phys. Lett. B 807 (2020) 135564

long range rapidity correlations can only be created at early times; shortly after the collision …

A. Dumitru, F. Gelis, L. McLerran, R.Venugopalan , Phys. A810 (2008) 91–108

Multi-particle correlations at HADES





$\kappa_2 = \kappa_1 + C_2$ $\kappa_3 = \kappa_1 + 3C_2 + C_3$ $\kappa_4 = \kappa_1 + 7C_2 + 6C_3 + C_4$

B. Ling and M. A. Stephanov, Phys. Rev. C 93, 034915 (2016).

A. Bzdak, V. Koch, and N. Strodthoff, Phys. Rev. C 95, 054906 (2017)

long range correlations $(\Delta y_{corr} \gg \Delta y)$: $C_n \sim \langle N_p \rangle^n$

short range correlations $(\Delta y_{corr} \ll \Delta y)$: $C_n \sim \langle N_p \rangle$

Analyzed rapidity intervals:

- $\Delta y = 0.2, 0.4, 0.6, 0.8, 1$
- $\langle N_p \rangle$ - mean number of protons in selected Δy
- fit function: $C_0 \langle N_p \rangle^{\alpha}$

HADES: PRC 102 (2020) 024914

centrality	$\alpha[C_2]$	$\alpha[C_3]$	$\alpha[C_4]$
0-5%	1.86±0.04	2.84±0.05	3.89±0.14

$\alpha \approx n \rightarrow \Delta y_{corr} \geq 1$

NOTE: canonical effects are not taken into account

the data indicates strong multi-particle correlations

The high μ_B corner of the phase diagram

Near future at HADES

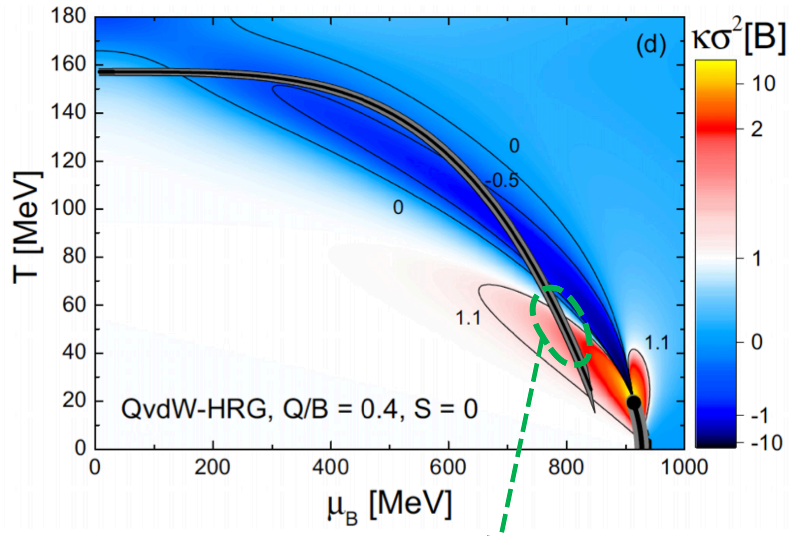
Au-Au collisions, with projectile kinetic energies of 0.8*A*, 0.6*A*, 0.4*A*, 0.2*A* GeV

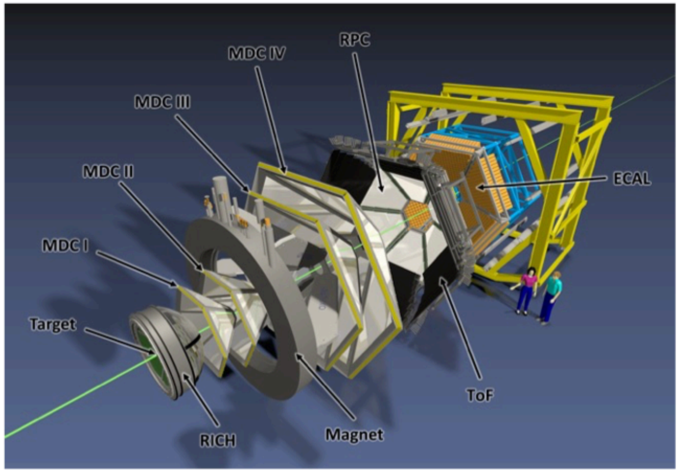
$\sim 10^9$ events for each energy

the proposal is submitted

- systematic study of fluctuations and correlation functions and the high values of μ_B
- probing the artefacts of nuclear liquid-gas phase transition
- studying contributions of stopped protons to multi-particle correlations
- ...

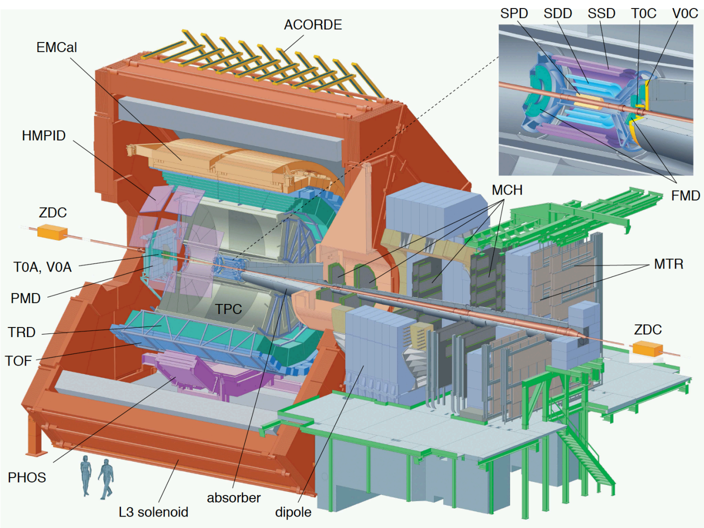
R. Poberezhnyuk, V. Vovchenko, A. Motornenko, M.I. Gorenstein, H. Stoecker, PRC 100 (2019) no.5, 054904



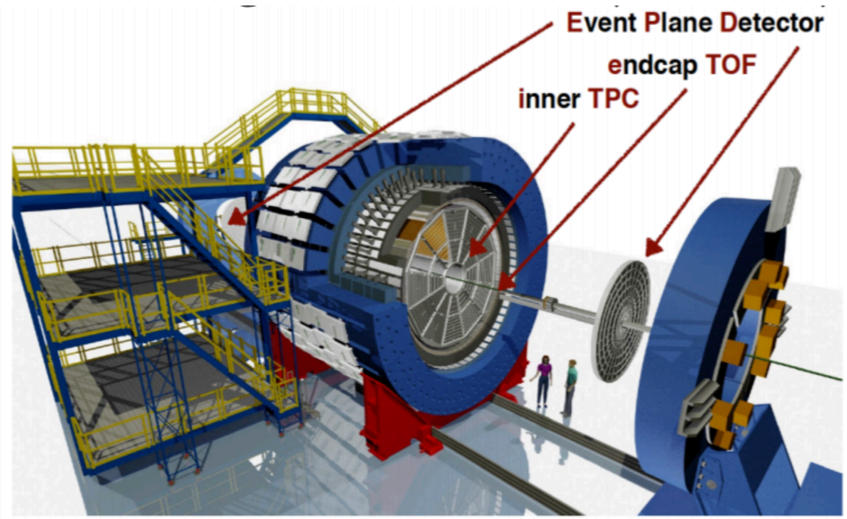


Near Future Experiments

- **ALICE upgrade**
  - new ITS: better vertexing
  - faster TPC: MWPC → GEMs
  - record minimum-bias Pb-Pb data at 50 kHz (currently < 1 kHz)
    - order of magnitude more events
    - measuring κ_6, may be beyond



- **STAR upgrade, BES - II**
  - iTPC: $|\eta| < 1.5$
    - better dE/dx resolution
    - lower momentum acceptance
  - EPD: $2.1 < |\eta| < 5.1$
    - centrality determination
    - ~ factor 20 more statistics



Zhangbu Xu: QM19

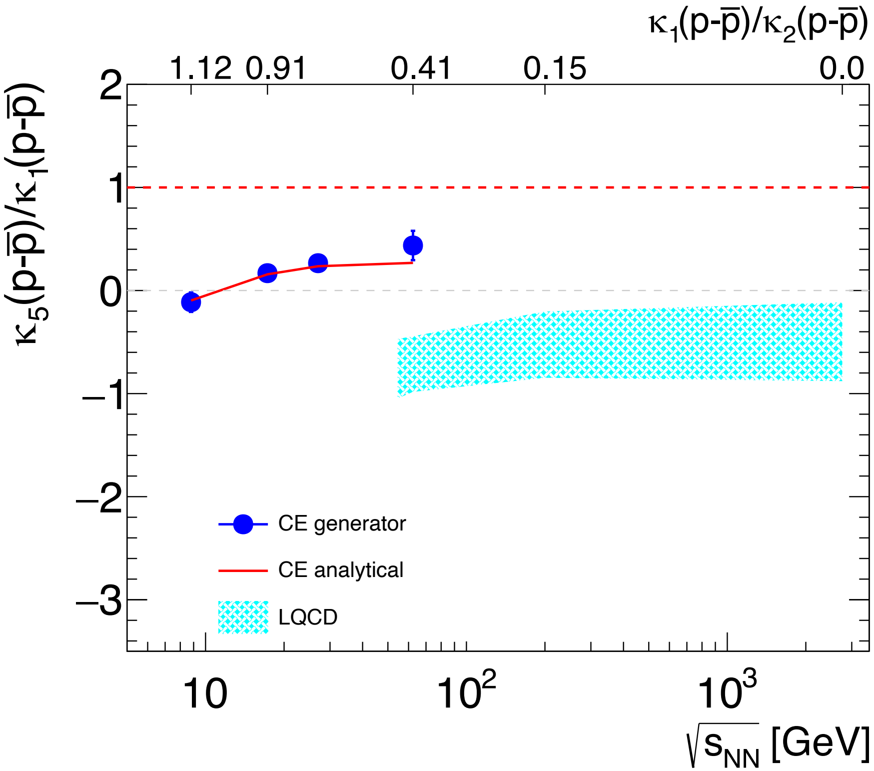
Summary

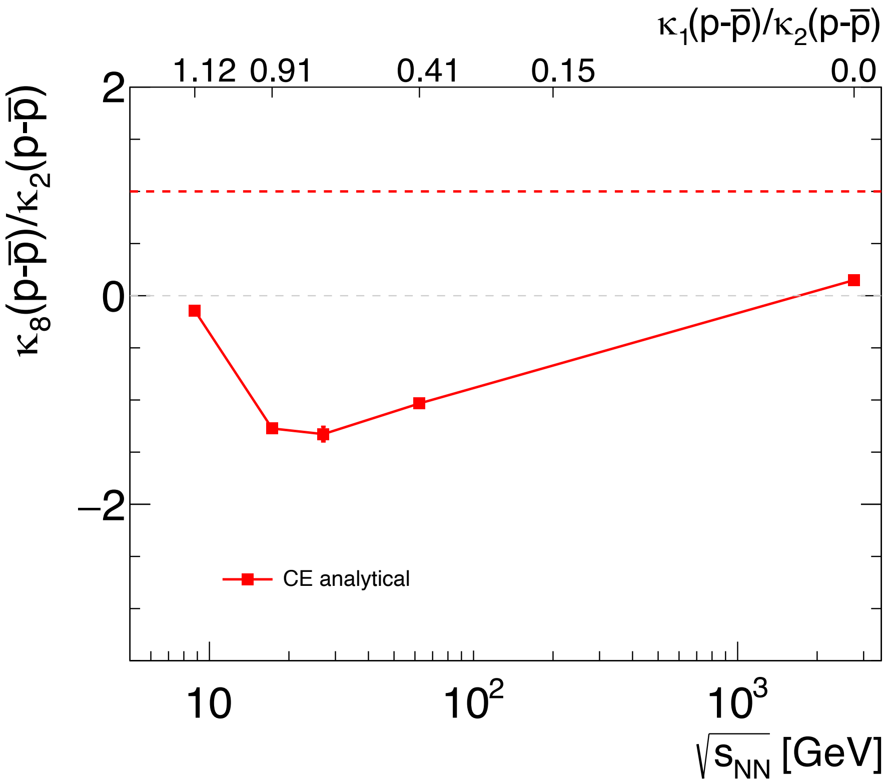
- The non-critical baseline for net-baryon cumulants is developed
- Overall the experimental results from **STAR** and **ALICE** follow the non-critical baseline predictions
- Contributions due to local baryon number conservation at LHC energies are negligible
  - The **ALICE** data strongly indicate long range correlations, implying sensitivity to early stages of collisions
- The data from **HADES** indicate strong multi-particle correlations
  - For firm conclusions canonical effects are to be accounted for
  - Proposed experiments in **HADES** will shed light on the nature of multi-particle correlations
- Near future experiments at **ALICE, HADES, STAR** will allow for high precision measurements of cumulants beyond fourth order

Thank you for your attention!

BACKUP SLIDES

5^{th} and 8^{th} order cumulants





P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, arXiv:2007.02463

Rapidity distributions

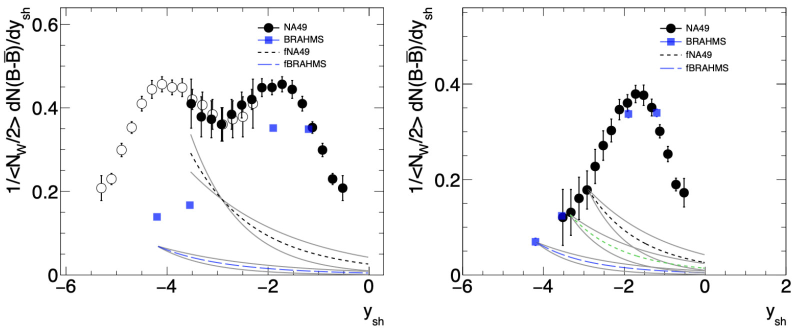


Figure 3: Left panel: Normalized net-baryon rapidity densities as measured by the NA49 [64] collaboration (black solid circles) and their reflected values (black open circles) in central Pb–Pb collisions at beam momentum of 158A GeV/c ($\sqrt{s_{NN}}$ = 17.3 GeV), plotted in the beam rapidity frame. Here, the shifted rapidity is defined as $y_{sh} = y - y_b$ with y_b the beam rapidity in the nucleon-nucleon center-of-mass frame. The blue solid squares represent the corresponding BRAHMS data [66] at $\sqrt{s_{NN}}$ = 62.4 GeV. The black dashed and blue long dashed lines, calculated with Eq. 23, represent the contributions from the 'target' regions for NA49 and BRAHMS data, respectively. Right panel: Normalized net baryon rapidity densities after subtracting the corresponding target contributions. In addition the 'target' contribution for $\sqrt{s_{NN}}$ = 27 GeV is shown as green dashed line.

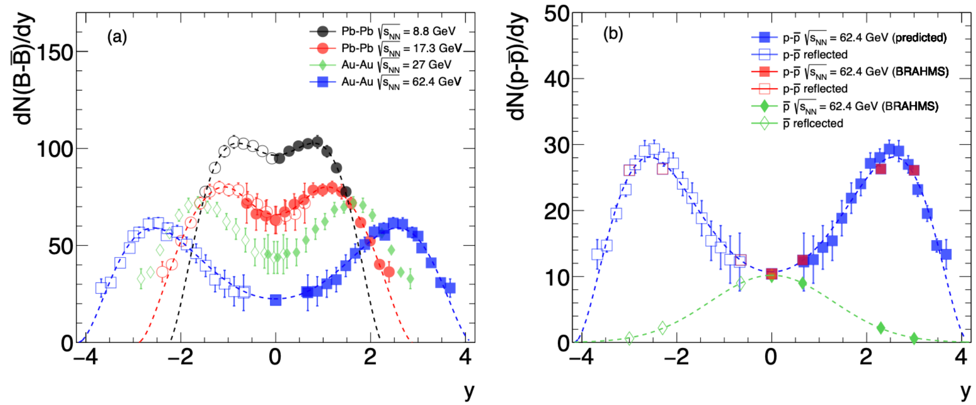


Figure 4: (a): Rapidity distributions of net-baryons at $\sqrt{s_{NN}}$ = 8.8 and 17.3 GeV (measured distributions from NA49) and 27 and 62.4 GeV (constructed using the limiting fragmentation concept described in the text). (b): Constructed (blue symbols) and BRAHMS measured (red symbols) rapidity distributions of net-protons at $\sqrt{s_{NN}}$ = 62.4 GeV.

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, arXiv:2007.02463

Energy dependece of $\alpha_{\bar{p}}$ and α_p

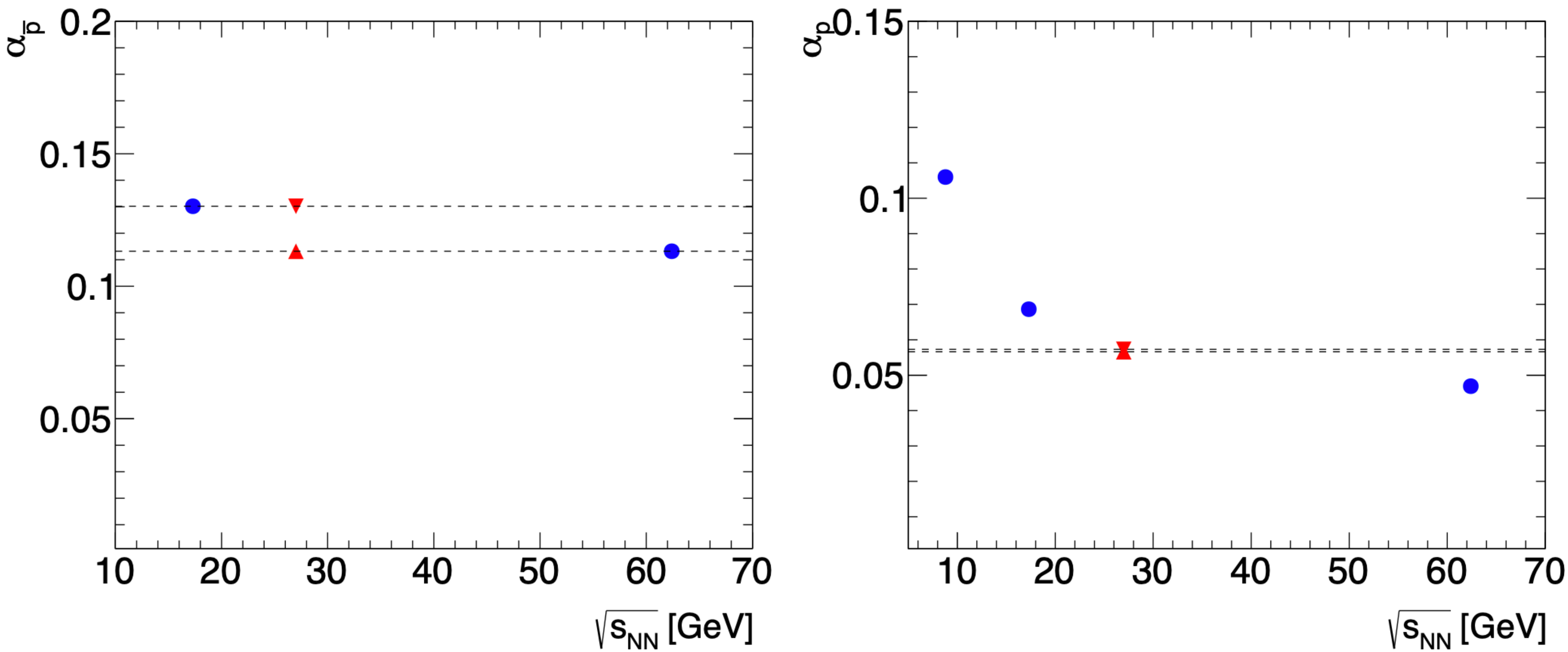


Figure 6: Collision energy dependence of $\alpha_{\bar{p}}$ (left panel) and α_p (right panel) for $\Delta y = 1$. The red triangles indicate the estimated upper a lower bounds for $\sqrt{s_{NN}} = 27$ GeV.

P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel, arXiv:2007.02463