Black holes and exotic compact objects

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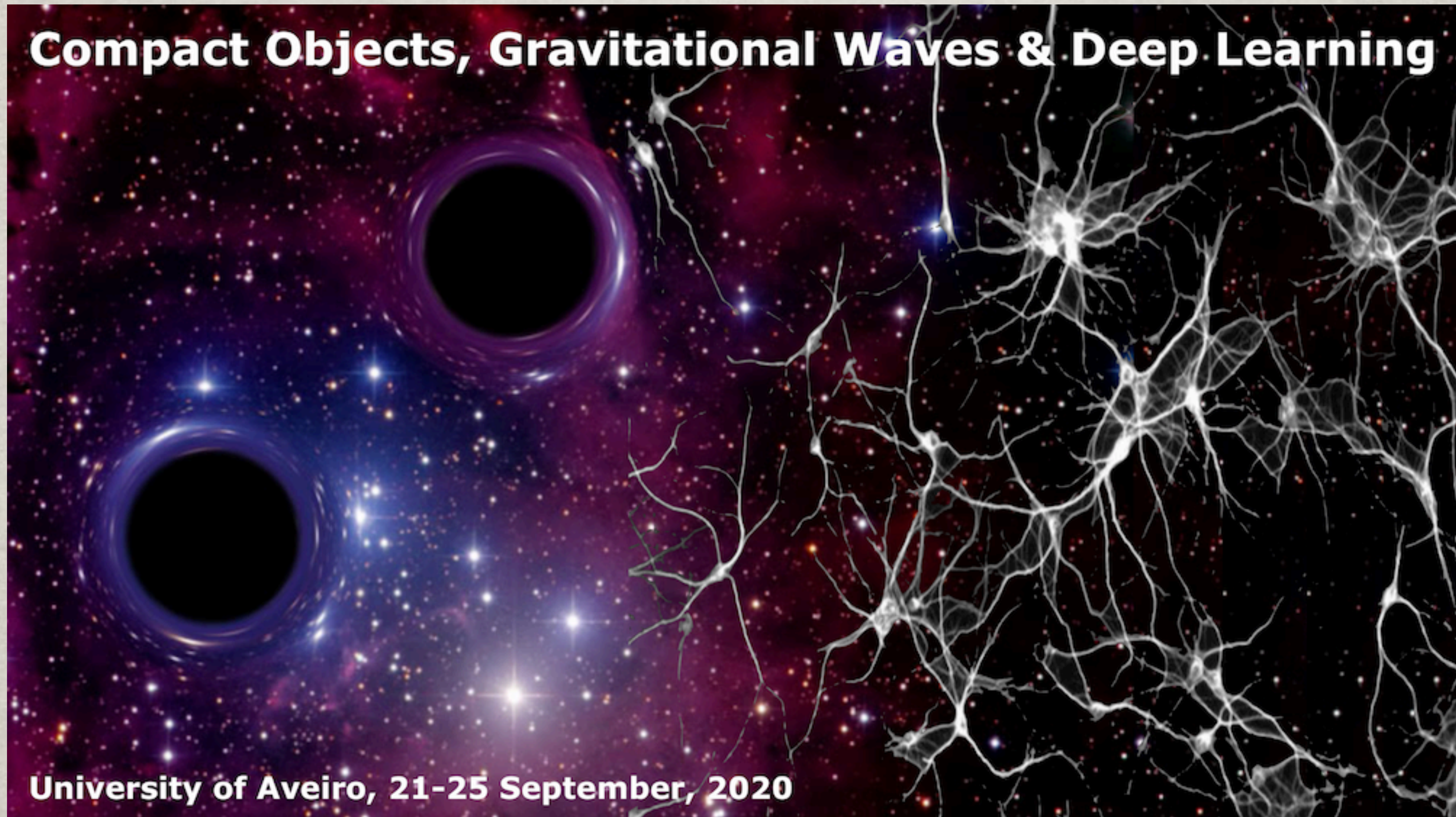


The Gravity group @ Aveiro University, Portugal

Plan of the lectures:

Lecture 1
Black holes: astrophysical evidence and a theory (brief) timeline

Lecture 2
Spherical black holes: the Schwarzschild solution

Lecture 3
Spinning black holes: the Kerr solution

Lecture 4
Exotic compact objects: the example of bosonic stars

Lecture 5
Non-Kerr black holes

Timeline

1916: **Schwarzschild's solution**

Über das Gravitationsfeld eines Massenpunktes nach der EINSTEINschen Theorie.

Von K. SCHWARZSCHILD.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

§ 1. Hr. EINSTEIN hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\left.\begin{aligned} &\delta\int ds = 0, \\ \text{wobei} \quad & \\ &ds = \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \qquad \mu,\nu = 1,2,3,4 \end{aligned}\right\} \quad (1)$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltigkeit.

Physics. — "*The field of a single centre in* EINSTEIN's *theory of gravitation, and the motion of a particle in that field.*". By J. DROSTE. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of May 27, 1916).

In two communications [1]) I explained a way for the calculation of the field of one as well as of two centres at rest, with a degree of approximation that is required to account for all observable phenomena of motion in these fields. For this I took as a starting-point the equations communicated by EINSTEIN in 1913 [2]). EINSTEIN has now succeeded in forming equations which are covariant for all possible transformations [3]), and by which the motion of the perihelion of Mercury is entirely explained [4]). The calculation of the field should henceforth be made from the new equations; we will make a beginning by calculating the field of a single centre at rest. We intend to calculate the field completely and not, as before, only the terms of the first and second order. After this, we investigate the

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$c = 1$
$G = 1$

(in the coordinates introduced by Johannes Droste, in 1916)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Analysis of singularities

1) A metric is said to be singular at some point *P* if:

either a metric coefficient diverges at *P*;
or the metric determinant vanishes at *P*.

If neither of these happen, the metric is said to be regular at P.

2) If a metric is singular at some point *P*, the singularity may be:

a coordinate singularity, if in a different coordinate system *P* is regular;
a physical singularity if there is no coordinate system in which *P* is regular.

3) Typically (but not always) the diagnosis of physical singularities is done by showing that some curvature invariant diverges at *P*;
it is then said to be a (scalar polynomial) **curvature singularity**.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Analysis of singularities

For the Schwarzschild metric, the interesting singularities are:

1) r=0, which is a physical curvature singularity,
since the Kretschmann scalar diverges

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} = \frac{48M^2}{r^6}$$

2) r=2M, which is a coordinate singularity,
since there are new coordinates that make this point regular.

Two examples of such coordinate systems are:

- ingoing Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)
- outgoing Eddington-Finkelstein (EF) coordinates (u, r, θ, ϕ)

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Ingoing Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)

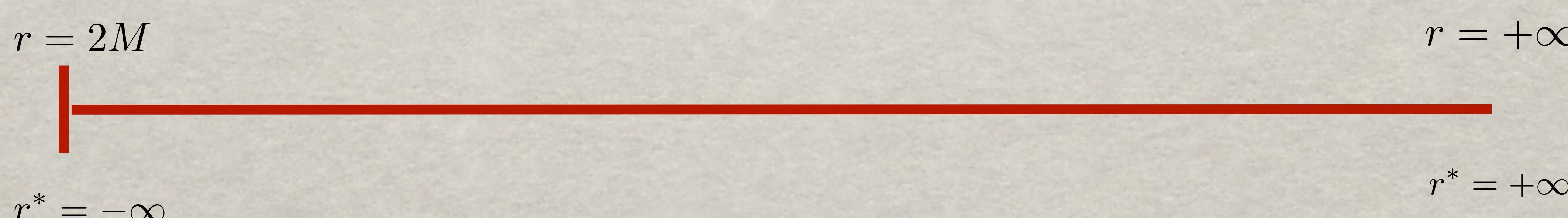
Idea: follow a radially infalling light ray:

$$ds^2 = 0 \text{ (null)} \qquad d\theta = 0 = d\phi \text{ (radial)}$$

Then:

$$dt = \pm \frac{dr}{1 - \frac{2M}{r}} \equiv \pm dr^*$$

Regge-Wheeler
radial coordinate
or
"tortoise"
radial coordinate



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Ingoing Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)

Idea: follow a radially infalling light ray (set G=1 for simplicity):

$$ds^2 = 0 \text{ (null)} \qquad d\theta = 0 = d\phi \text{ (radial)}$$

Then:

$$dt = \pm \frac{dr}{1 - \frac{2M}{r}} \equiv \pm dr^*$$

Regge-Wheeler
radial coordinate
or
"tortoise"
...ordinate

$r = 2M$ $r = +\infty$

$r^* = -\infty$ $r^* = +\infty$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Ingoing Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)

Idea: follow a radially infalling light ray (set G=1 for simplicity):

$$ds^2 = 0 \text{ (null)} \qquad d\theta = 0 = d\phi \text{ (radial)}$$

Then:

$$dt = \pm \frac{dr}{1 - \frac{2M}{r}} \equiv \pm dr^*$$

Regge-Wheeler radial coordinate or "tortoise" radial coordinate

$r = 2$

$r = +\infty$

$r^* = -\infty$

$r^* = +\infty$

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Ingoing Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)

$$dt = \pm \frac{dr}{1 - \frac{2M}{r}} \equiv \pm dr^*$$

The problematic coordinate at r=2M is actually *t*; we need to replace it. Following the ingoing, radial light rays suggest the *advanced time v*:

$$v \equiv t + r^*$$

v=constant, follows the ingoing, radial light rays. Then, changing from the Schwarzschild coordinates to the ingoing EF coordinates:

$$(t, r, \theta, \phi) \longrightarrow (v, r, \theta, \phi)$$

gives the Schwarzschild metric in the form:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Non-singular at r=2M

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

~~Ingoing~~ Eddington-Finkelstein (EF) coordinates (v, r, θ, ϕ)

Outgoing (u, r, θ, ϕ)

$$dt = \pm \frac{dr}{1 - \frac{2M}{r}} \equiv \pm dr^*$$

The problematic coordinate at r=2M is actually *t*; we need to replace it. Following the ~~ingoing~~ outgoing, radial light rays suggest the ~~*advanced time v*~~ *outgoing time u*:

$$\cancel{v \equiv t + r^*}$$
$$u \equiv t - r^*$$

~~v=constant~~ u=constant, follows the ~~ingoing~~ outgoing, radial light rays. Then, changing from the Schwarzschild coordinates to the ~~ingoing~~ outgoing EF coordinates:

$$(t, r, \theta, \phi) \longrightarrow \cancel{(v, r, \theta, \phi)}$$
$$(u, r, \theta, \phi)$$

gives the Schwarzschild metric in the form:

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\overset{du^2\ -2dudr}{\cancel{dv^2 + 2dvdr}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\boxed{\text{Non-singular at r=2M}}$$

But is there something special about the (hyper)surface r=2M?

Yes!
r=2M is a null hypersurface.
And it is also a Killing horizon.

Definition: Let $S(x^\mu)$ be a smooth function of the spacetimes coordinates x^μ and consider the family of hypersurfaces S =constant. They have normal

$$\ell = (g^{\mu\nu}\partial_\nu S)\partial_\mu \ .$$

If $\ell^2 = \ell^\mu \ell_\mu = 0$ for a particular hypersurface in the family, $\mathcal{N}$, then $\mathcal{N}$ is said to be a null surface.

Exercise 2.1

Show, using (say) ingoing EF coordinates that for the family of hypersurfaces S=r=constant in the Schwarzschild spacetime:

$$\ell^2 = 1 - \frac{2M}{r}$$

Interpretation:

In Minkowski spacetime: $ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

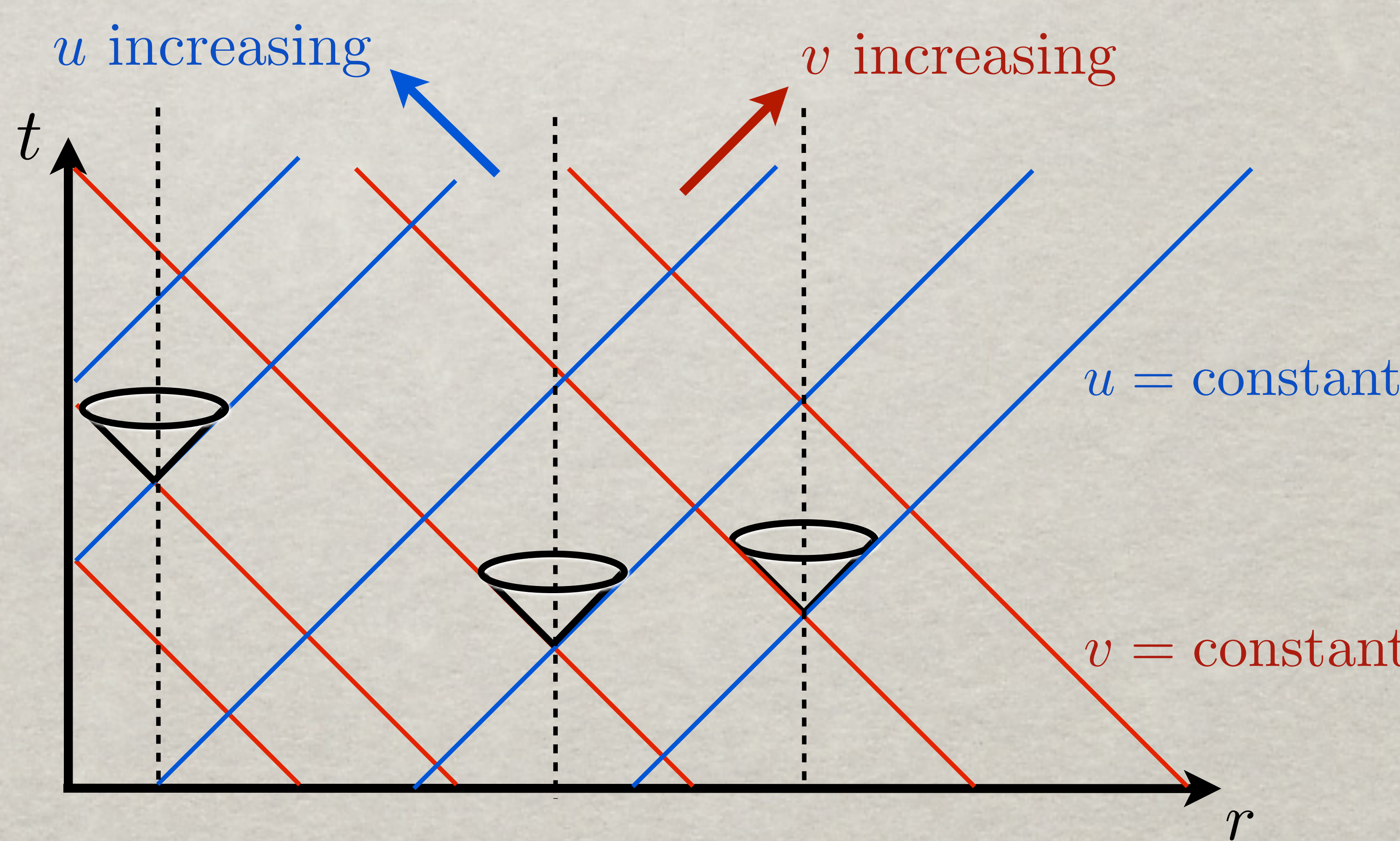
The family of hypersurfaces S=r=constant is always timelike: $\ell^2 = 1$

In a spacetime diagram:

$v = t + r$

$u = t - r$

r=constant hypersurfaces are always timelike



But in the Schwarzschild spacetime it is different:

Ingoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2dvdr = -\left[\underbrace{-ds^2}_{\substack{\geqslant 0 \\ \text{for causal} \\ \text{curves}}} - \left(1 - \frac{2M}{r}\right) dv^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Ingoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2dvdr = -\left[-ds^2 - \left(1 - \frac{2M}{r}\right) dv^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

$\geqslant 0$ for causal curves

$\geqslant 0$ for $r \leqslant 2M$

Ingoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2dvdr = -\Big[\underbrace{-ds^2}_{\substack{\geqslant 0 \\ \text{for causal} \\ \text{curves}}} - \underbrace{\left(1 - \frac{2M}{r}\right)}_{\substack{\geqslant 0 \\ \text{for } r \leqslant 2M}} dv^2 \underbrace{+ r^2(d\theta^2 + \sin^2\theta d\phi^2)}_{\geqslant 0}\Big]$$

Ingoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2dvdr = -\left[\underbrace{-ds^2}_{\geqslant 0} \underbrace{-\left(1 - \frac{2M}{r}\right) dv^2}_{\geqslant 0} + \underbrace{r^2(d\theta^2 + \sin^2\theta d\phi^2)}_{\geqslant 0}\right]$$

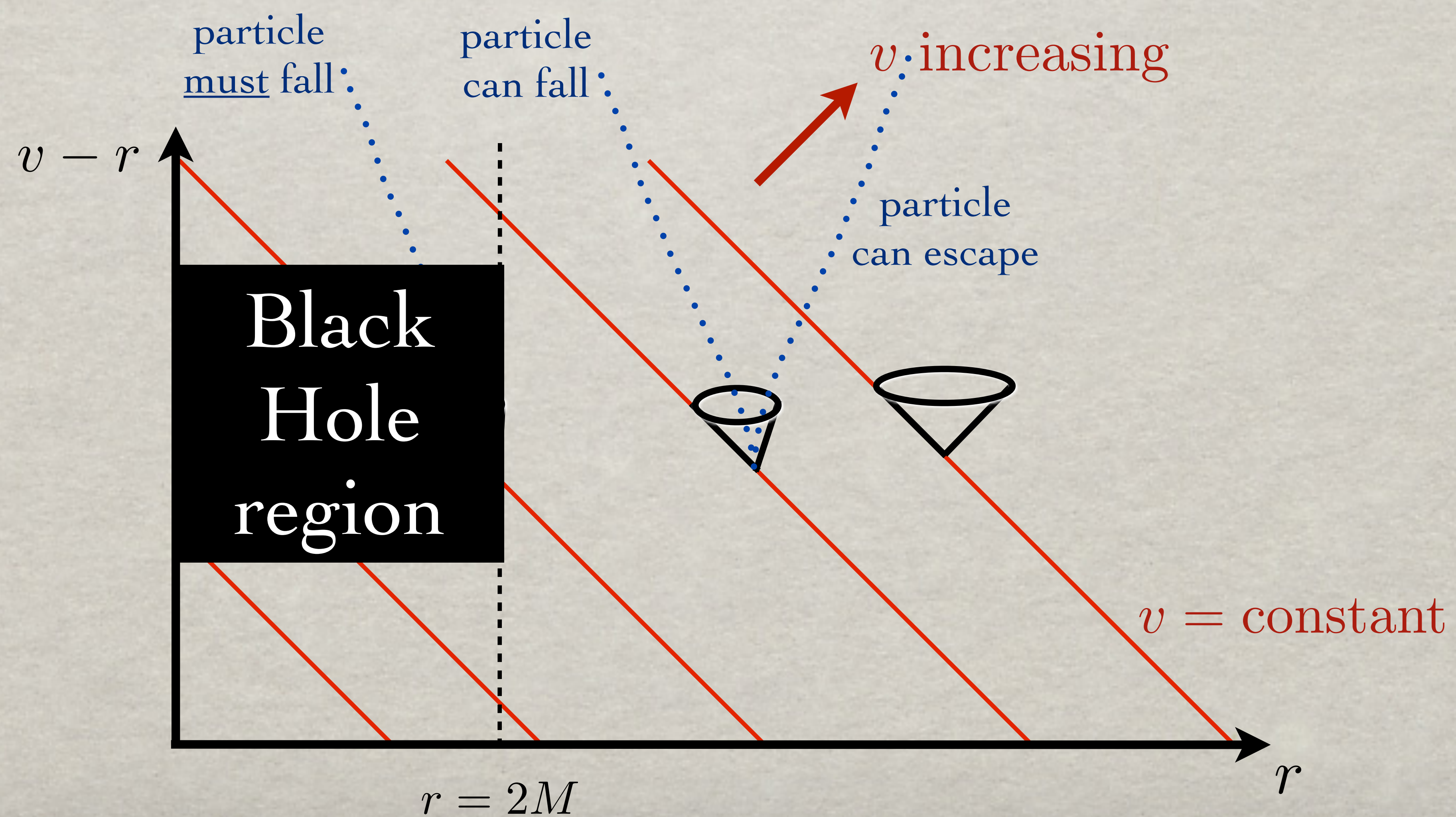
$\geqslant 0$ for causal curves

$\geqslant 0$ for $r \leqslant 2M$

$\geqslant 0$

Thus if $dv > 0$ (future directed) then $dr < 0$ (infalling)

Easily visualised in Finkelstein diagram



Outgoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) du^2 - 2dudr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2dudr = \left[\underbrace{-ds^2}_{\geqslant 0 \text{ for causal curves}} - \left(1 - \frac{2M}{r}\right) du^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Outgoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right) du^2 - 2du dr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Or, equivalently:

$$2du dr = \left[\underbrace{-ds^2}_{\substack{\geqslant 0 \\ \text{for causal} \\ \text{curves}}} \underbrace{- \left(1 - \frac{2M}{r}\right) du^2}_{\substack{\geqslant 0 \\ \text{for } r \leqslant 2M}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

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$$ds^2 = -\left(1 - \frac{2M}{r}\right) du^2 - 2du dr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

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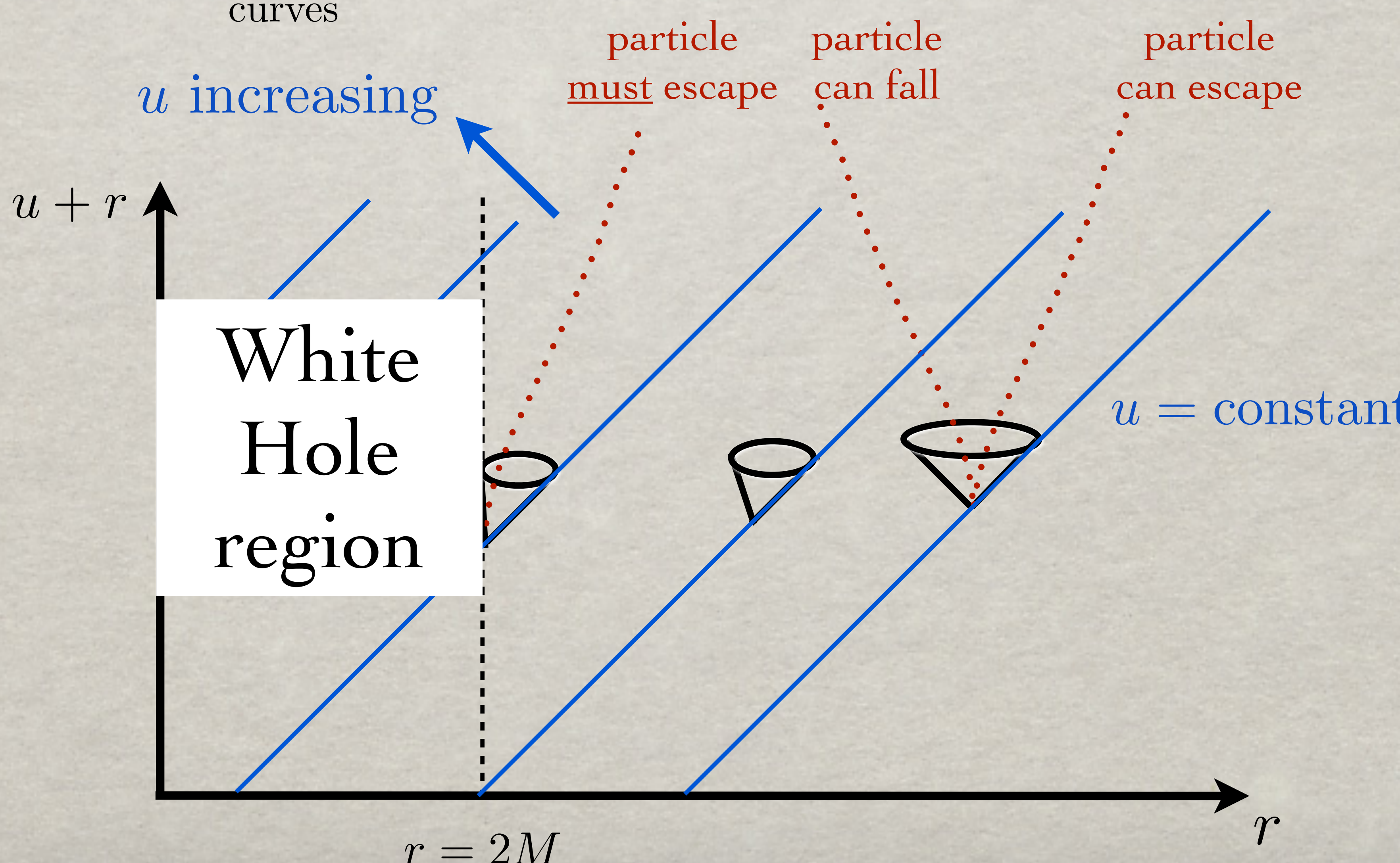
Thus if $du > 0$ (future directed) then $dr > 0$ (escaping)



Easily visualised in Finkelstein diagram

Thus, the Schwarzschild spacetime has a richer structure, (more regions) that one would initially have guessed!

The global structure of a spacetime is determined by its maximal analytic extension:

Definition: The maximal analytic extension of a spacetime is a coordinate system (or set of coordinate systems) in which all geodesics that to not terminate on a physical (irremovable) singularity, can be extended to arbitrary values of their affine parameters.

For the Schwarzschild spacetime the maximal analytic extension is given by the Kruskal-Szekers coordinates.

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Exercise 2.2

Introduce the Kruskal-Szekers (KS) coordinates from the Schwarzschild coordinates as:

$$(t, r, \theta, \phi) \to (U, V, \theta, \phi)$$

$$U = -e^{-\frac{u}{4M}} \qquad V = e^{\frac{v}{4M}}$$

where

$$v = t + r^* \qquad u = t - r^* \qquad dr^* = \frac{dr}{1 - \frac{2M}{r}}$$

Show that the Schwarzschild metric becomes:

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dU dV + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

where r is implicitly defined in terms of U,V as:

$$UV = -\left(\frac{r-2M}{2M}\right) e^{\frac{r}{2M}}$$

Initially, the KS coordinates are only defined for U<0 and V>0:

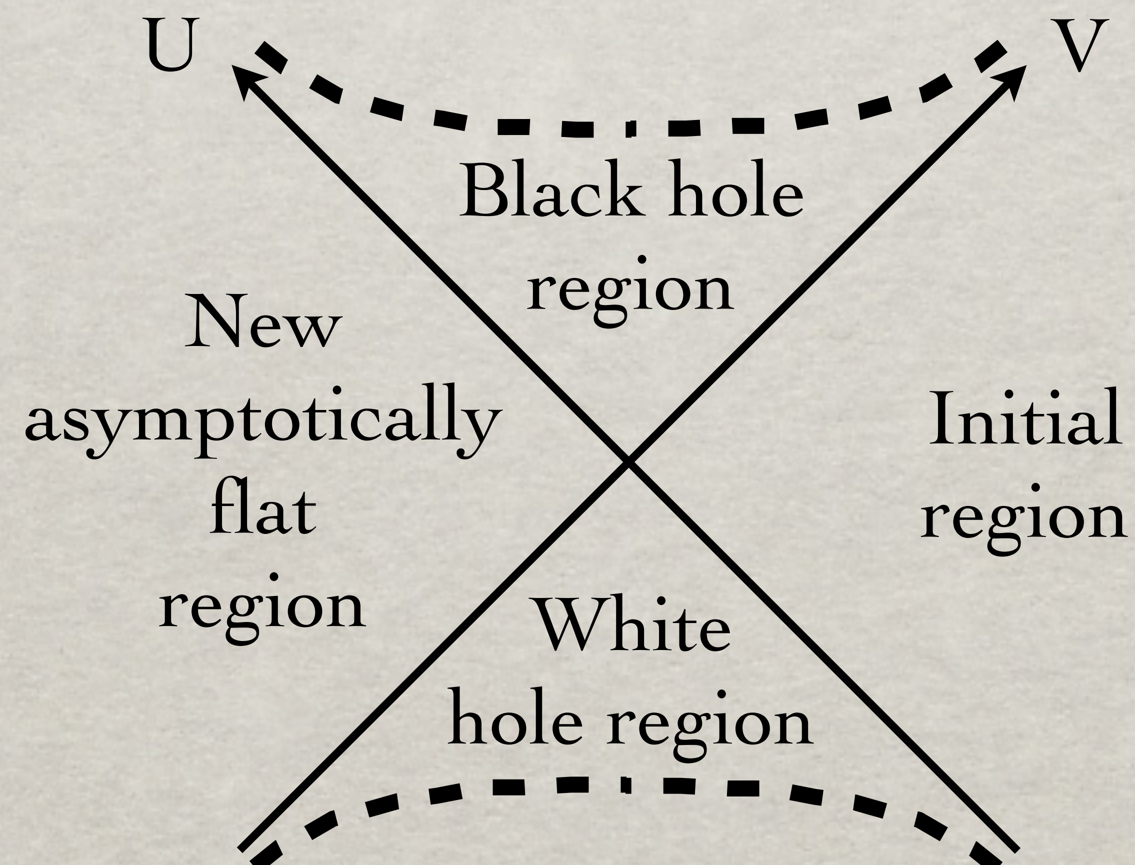
$$U = -e^{-\frac{u}{4M}}$$

$$V = e^{\frac{v}{4M}}$$

But in KS coordinates we can extend beyond U=0 and V=0 into arbitrary values; the metric is only singular at r=0 i.e. UV=1

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dU dV + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$UV = -\left(\frac{r-2M}{2M}\right) e^{\frac{r}{2M}}$$



Finkelstein diagram for the eternal Schwarzschild black hole

An even nicer way to visualize the black hole spacetime, and its causal structure is by means of a <u>conformal compactification</u>.

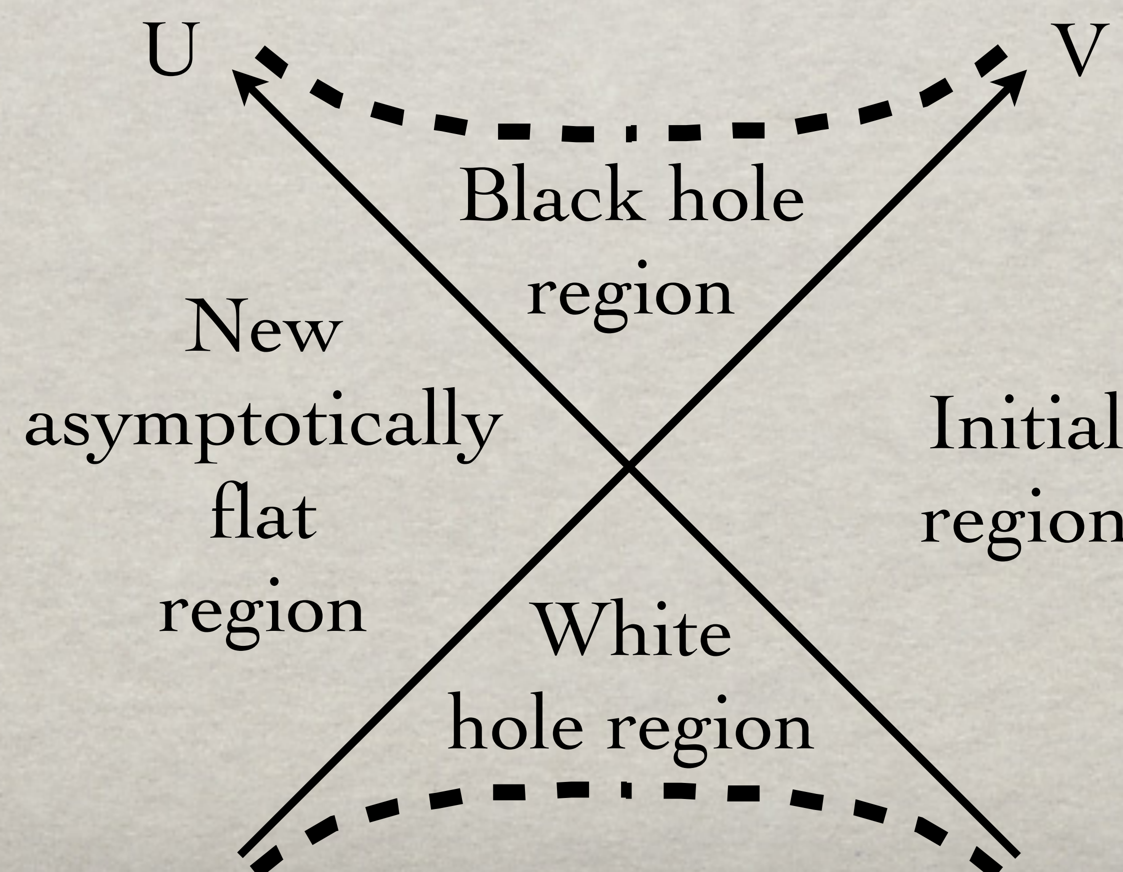
Definition: A conformal compactification is a conformal transformation of the metric, thus preserving the light cone structure,

$$ds^2 \longrightarrow d\tilde{s}^2 = \Lambda(t, \vec{x})^2 ds^2 \ ,$$

such that all points at infinity in the original metric are at finite affine parameter in the new metric. Thus, we choose $\Lambda(t, \vec{x})$ such that

$$\Lambda(t, \vec{x}) \to 0 \quad \text{as} \quad |\vec{x}| \to 0 \quad \text{and/or} \quad |t| \to 0 \ .$$

For Schwarzschild:



First introduce compact coordinates:

$$U = \tan \tilde{U}$$

$$V = \tan \tilde{V}$$

$$-\infty < U, V < +\infty$$

$$-\frac{\pi}{2} < \tilde{U}, \tilde{V} < \frac{\pi}{2}$$

$$ds^2 \longrightarrow d\tilde{s}^2 = \Lambda(t,\vec{x})^2 ds^2 \ ,$$

such that all points at infinity in the original metric are at finite affine parameter in the new metric. Thus, we choose $\Lambda(t,\vec{x})$ such that

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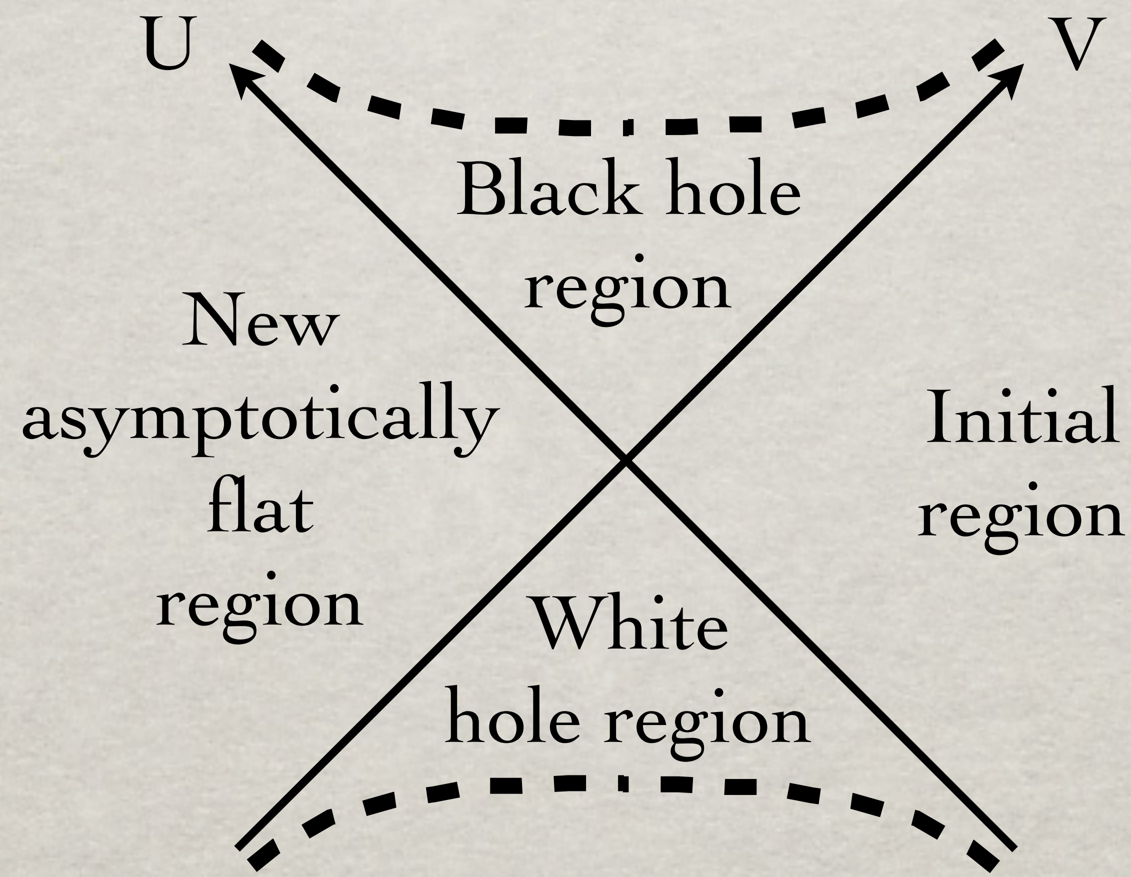

First introduce compact coordinates:

$$U = \tan \tilde{U}$$

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$$-\infty < U, V < +\infty$$

$$-\frac{\pi}{2} < \tilde{U}, \tilde{V} < \frac{\pi}{2}$$

The metric becomes:

$$ds^2 = \frac{1}{\cos^2 \tilde{U} \cos^2 \tilde{V}} \left[-\frac{32M^3}{r} e^{-\frac{r}{2M}} d\tilde{U} d\tilde{V} + r^2 \cos^2 \tilde{U} \cos^2 \tilde{V} (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

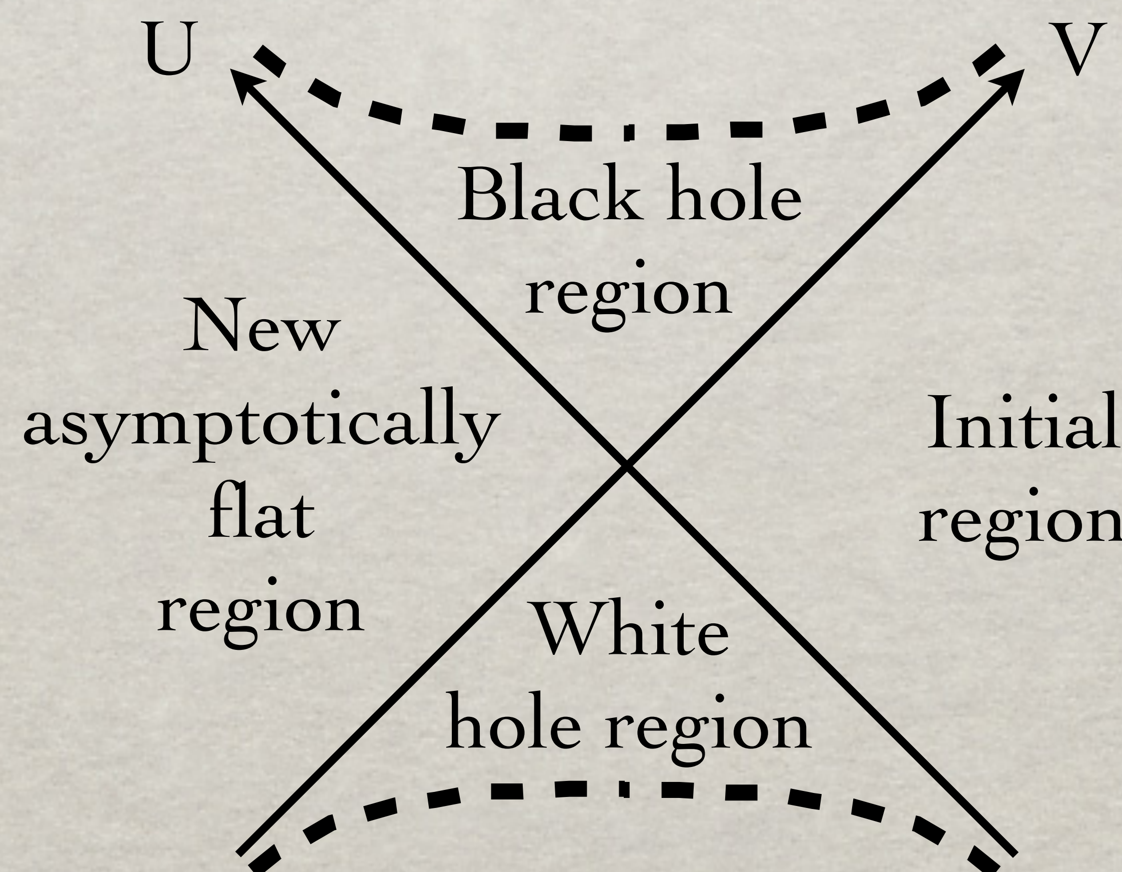
And we define:

$$d\tilde{s}^2 = \underbrace{\cos^2 \tilde{U} \cos^2 \tilde{V}}_{\Lambda(t,\vec{x})^2} ds^2$$

Infinity is:

$$\Lambda(t,\vec{x}) = 0 \ \Leftrightarrow \ \tilde{U} = \pm\frac{\pi}{2} \ \text{or} \ \tilde{V} = \pm\frac{\pi}{2}$$

We can now draw the spacetime diagram for the conformal compactification of the Schwarzschild spacetime called the Carter-Penrose diagram.
First change to the compact coordinates:



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We can now draw the spacetime diagram for the conformal compactification of the Schwarzschild spacetime called the Carter-Penrose diagram.
First change to the compact coordinates:

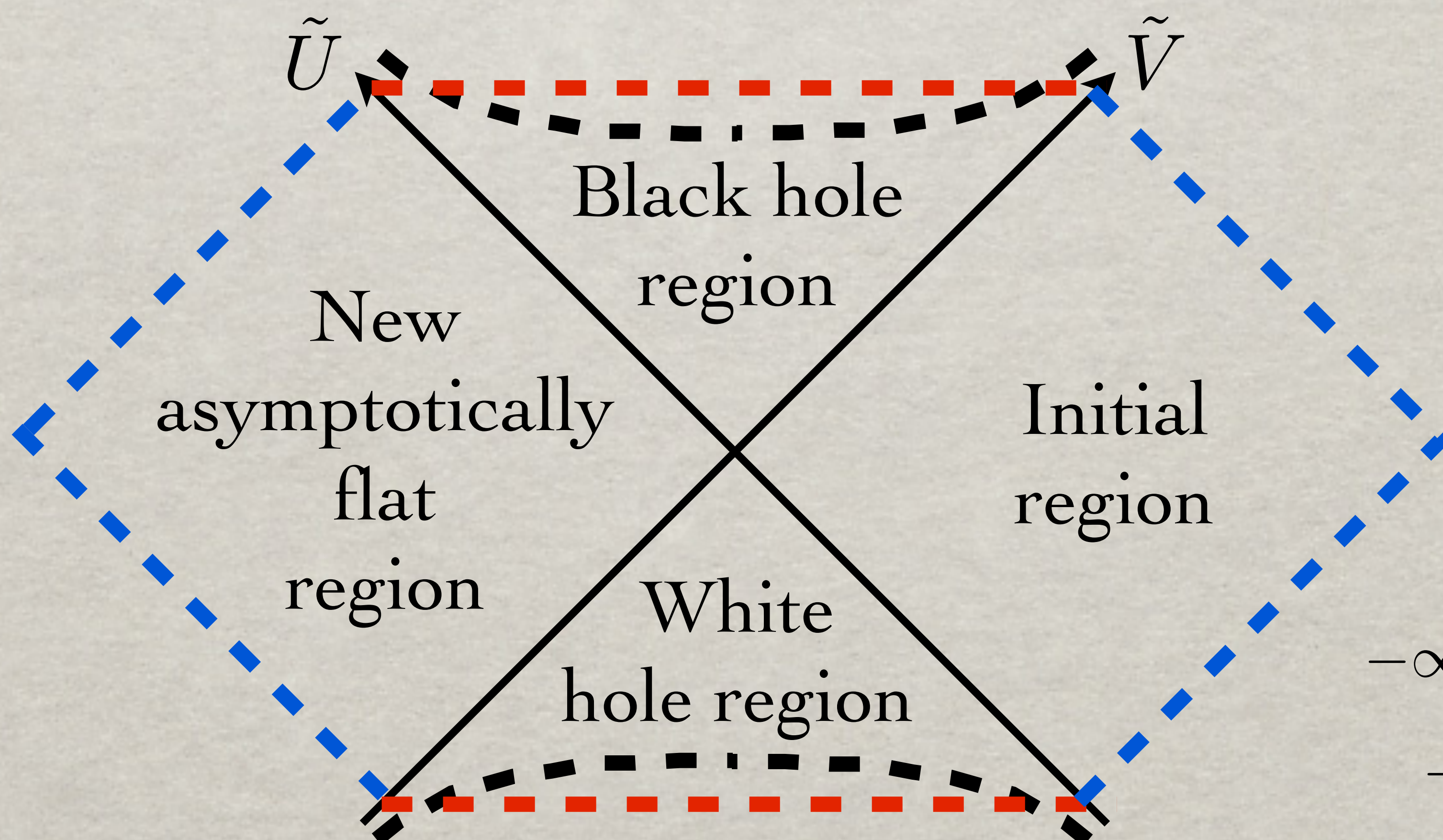
Next consider the spacetime boundaries:

Singularity:

$$UV = 1 \;\Leftrightarrow\; \tan\tilde{U}\tan\tilde{V} = 1 \Leftrightarrow \begin{array}{l} \tilde{U} = \frac{\pi}{2} - \tilde{V} \\ \tilde{U} = -\frac{\pi}{2} - \tilde{V} \end{array}$$

Infinity:

$$\Lambda(t, \vec{x}) = 0 \;\Leftrightarrow\; \tilde{U} = \pm\frac{\pi}{2} \text{ or } \tilde{V} = \pm\frac{\pi}{2}$$



First introduce compact coordinates:

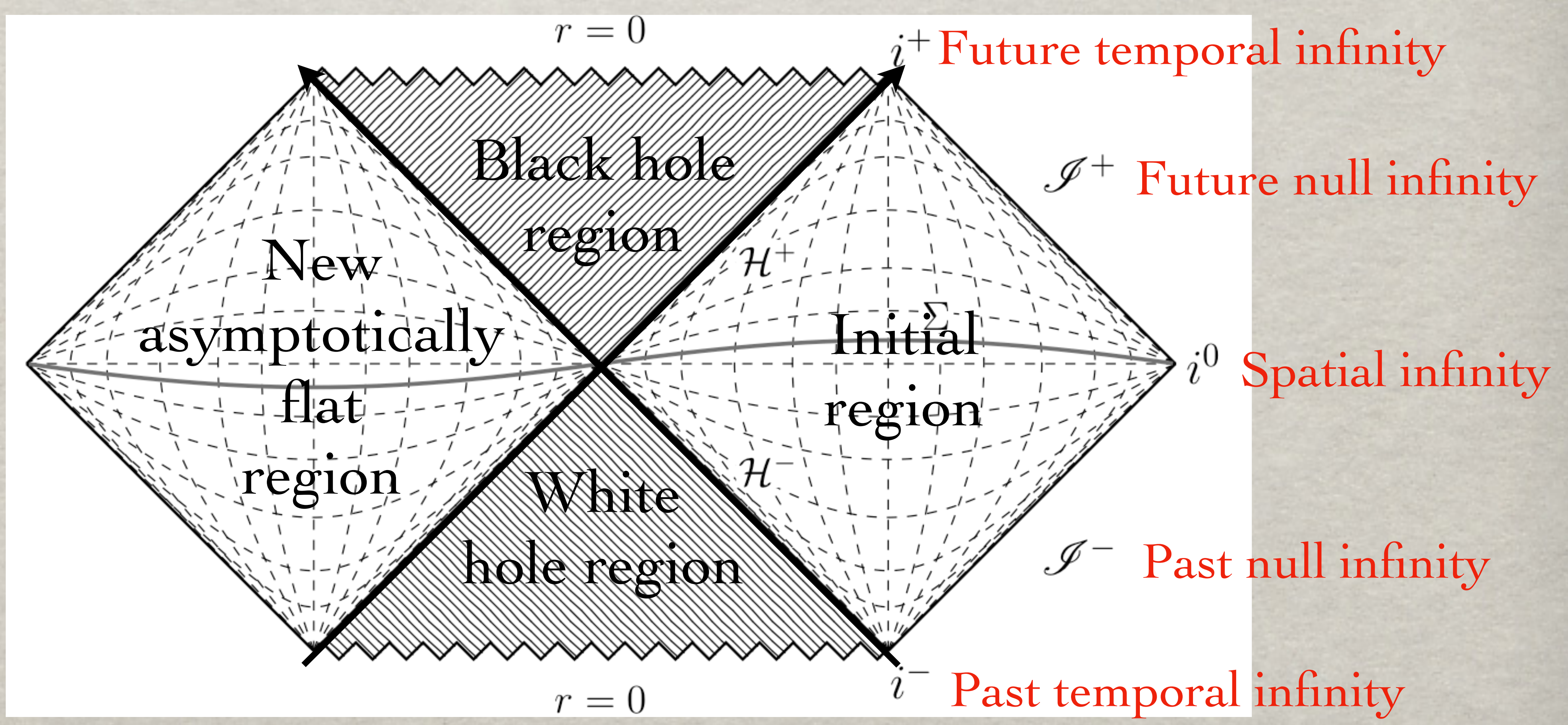
$$U = \tan\tilde{U}$$

$$V = \tan\tilde{V}$$

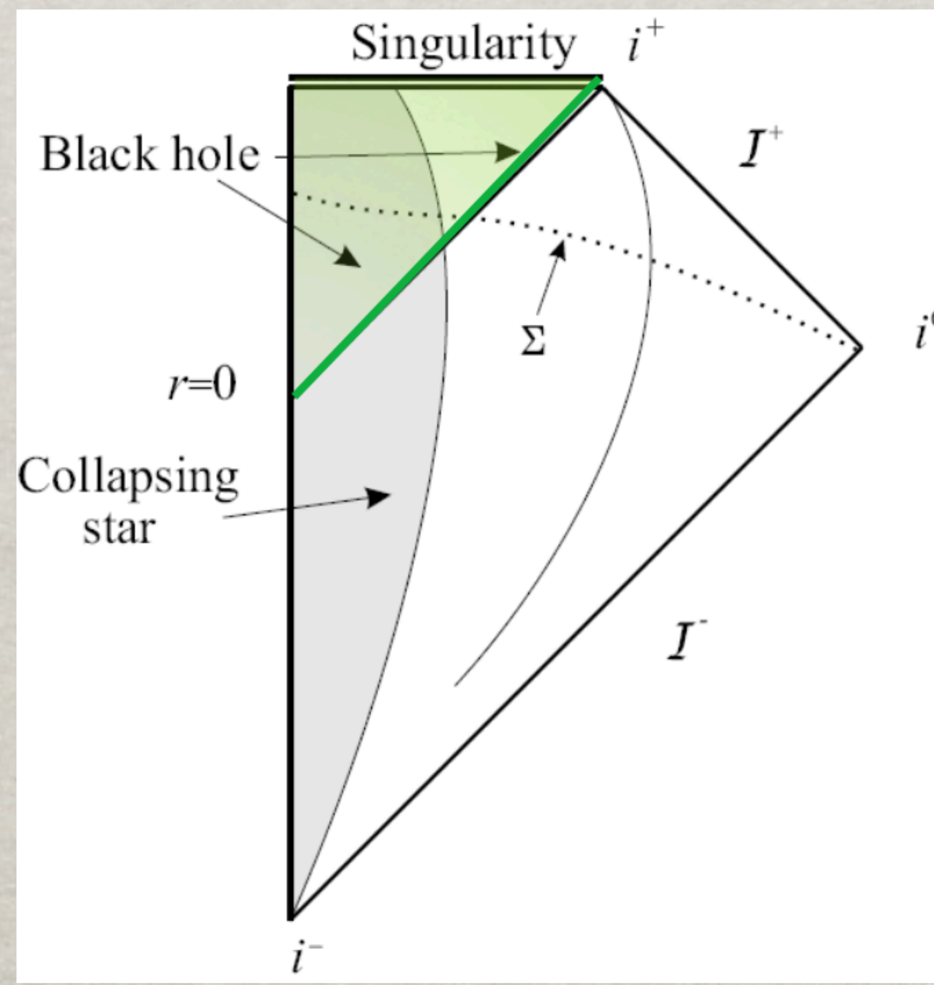
$$-\infty < U, V < +\infty$$

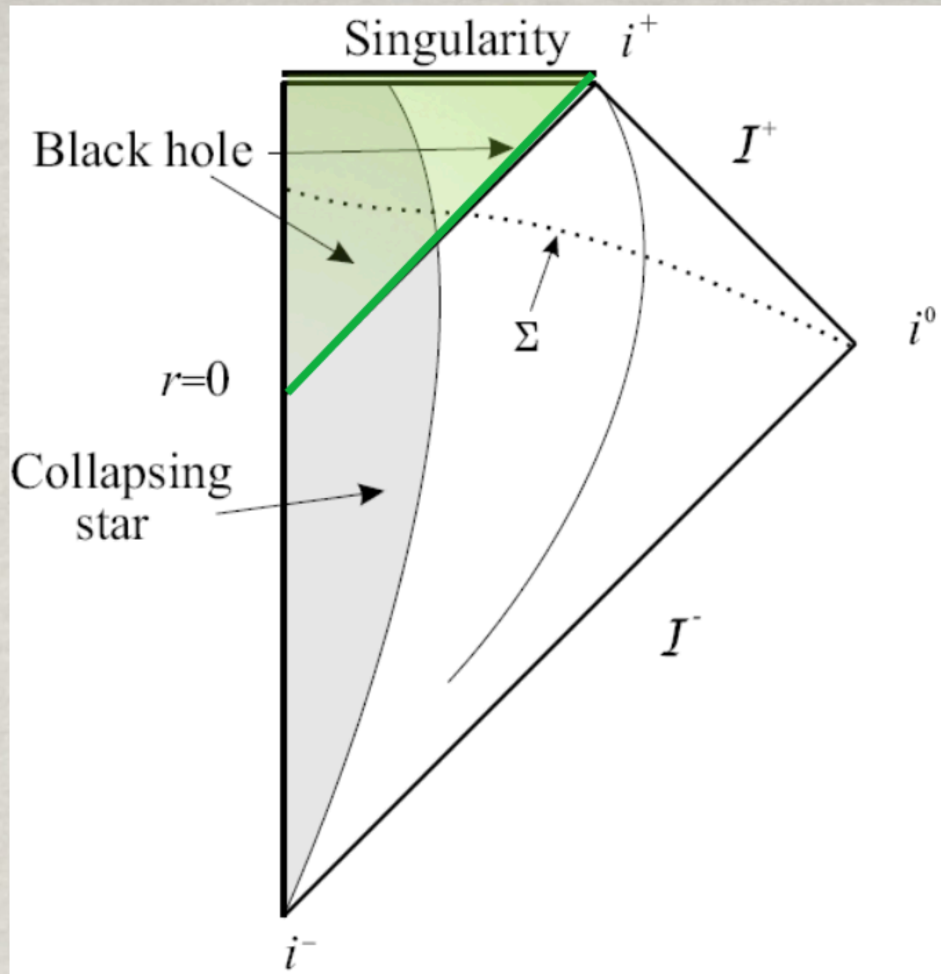
$$-\frac{\pi}{2} < \tilde{U}, \tilde{V} < \frac{\pi}{2}$$

This is the Carter-Penrose diagram
for the eternal Schwarzschild spacetime.



For an astrophysically realistic black hole, on the other hand, only the initial region and the black hole region are expected to be present.

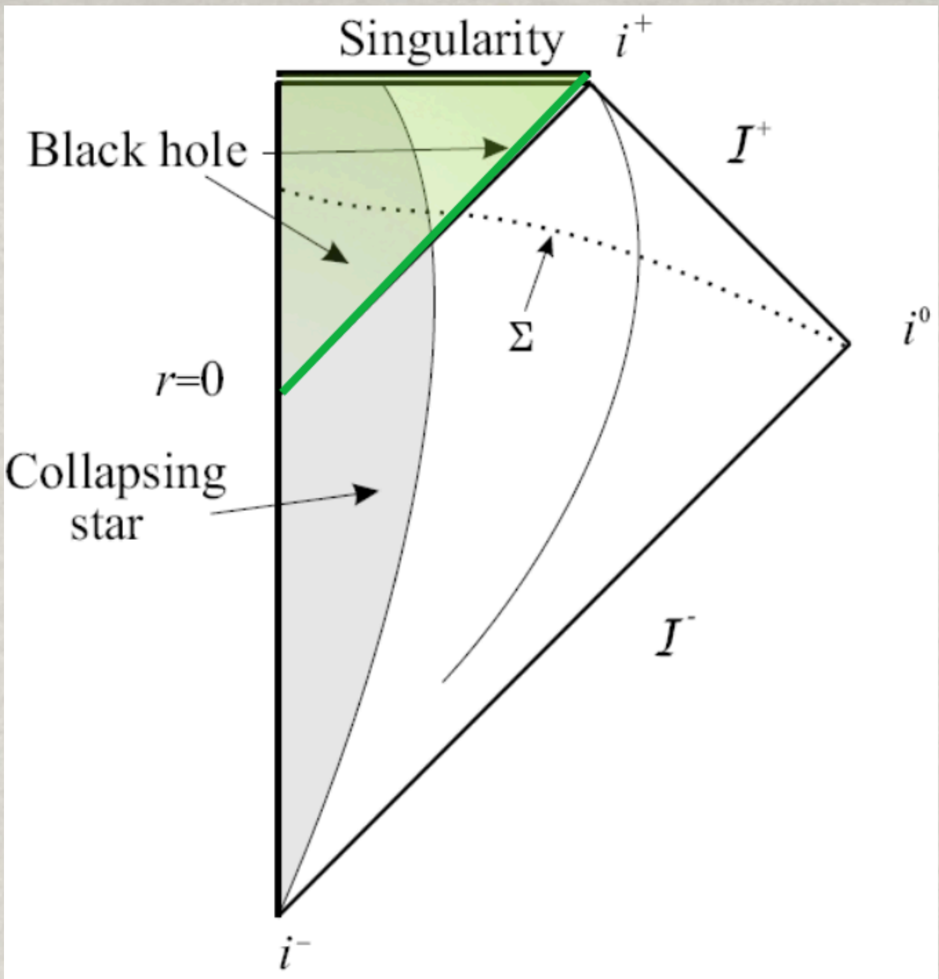




The surface r=2M in Schwarzschild is the black hole <u>event horizon</u>.

Definition: The event horizon of an asymptotically flat spacetime is the boundary of the causal past of future null infinity.

This definition is teleological (i.e. global); one needs to know the whole spacetime history to determine where the event horizon is.



In a stationary spacetime, however, the event horizon coincides with a local concept: a <u>Killing horizon</u>.

Definition: A null hypersurface, $\mathcal{N}$, is a Killing horizon of a Killing vector field, ξ, if, on $\mathcal{N}$, ξ is normal to $\mathcal{N}$, *i.e.*, $\xi^2 = 0$.

Hawking's Rigidity Theorem (1972): The event horizon of a stationary, regular, asymptotically flat spacetime is a Killing horizon.

For Schwarzschild, using regular coordinates on the horizon, say, ingoing EF coordinates:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Then, clearly, a Killing vector field is: $\xi = \dfrac{\partial}{\partial v}$

Its norm is: $\xi^2 = \xi^\mu \xi^\nu g_{\mu\nu} = g_{vv} = -\left(1 - \dfrac{2M}{r}\right)$

Thus, the norm vanishes on the null surface r=2M.
Consequently, this surface is a Killing horizon.
Consequently it is the spacetime event horizon.

Bottom line:
The Schwarzschild spacetime represents a black hole.
The hypersurface r=2M is the event horizon.

Probing the Schwarzschild black hole
with
light

Schwarzschild space-time:
BH with mass M (G=1=c)

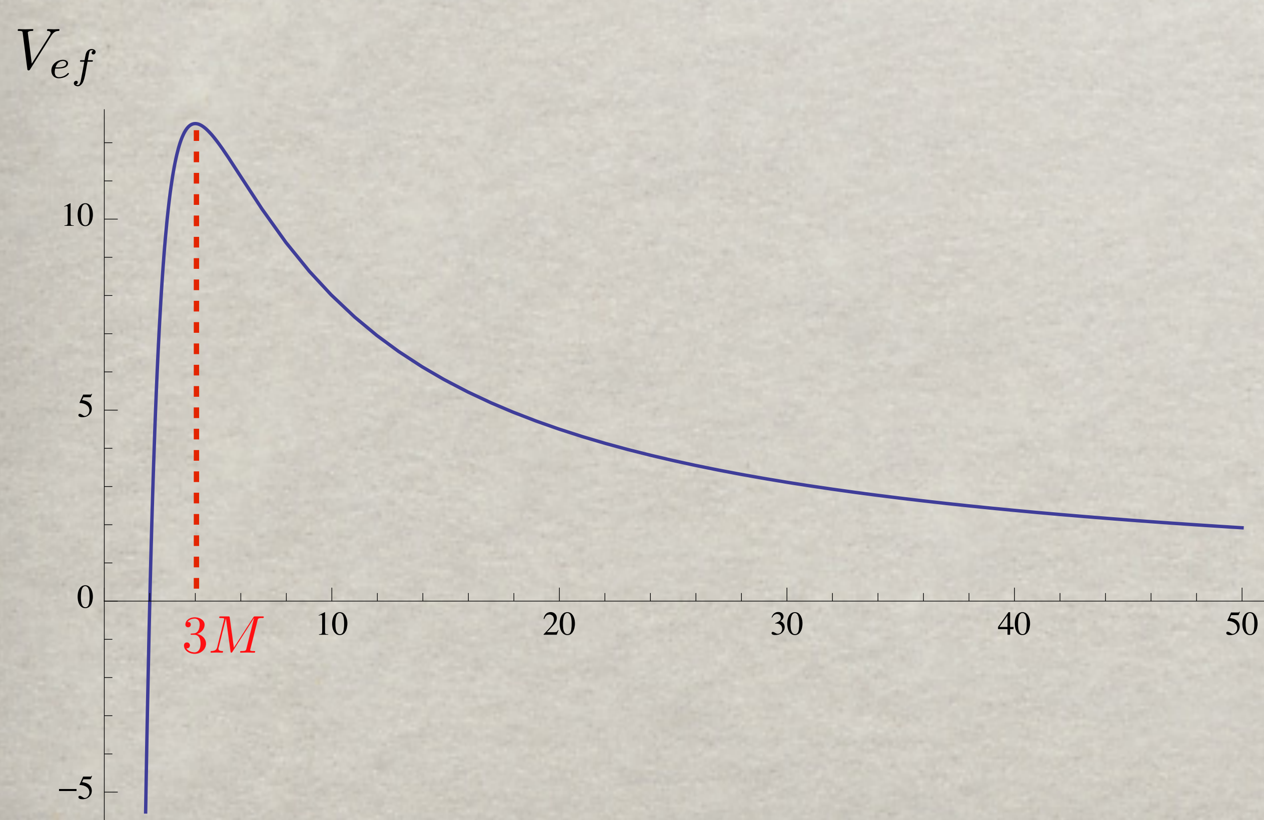
$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega_2$$

Exercise 2.3

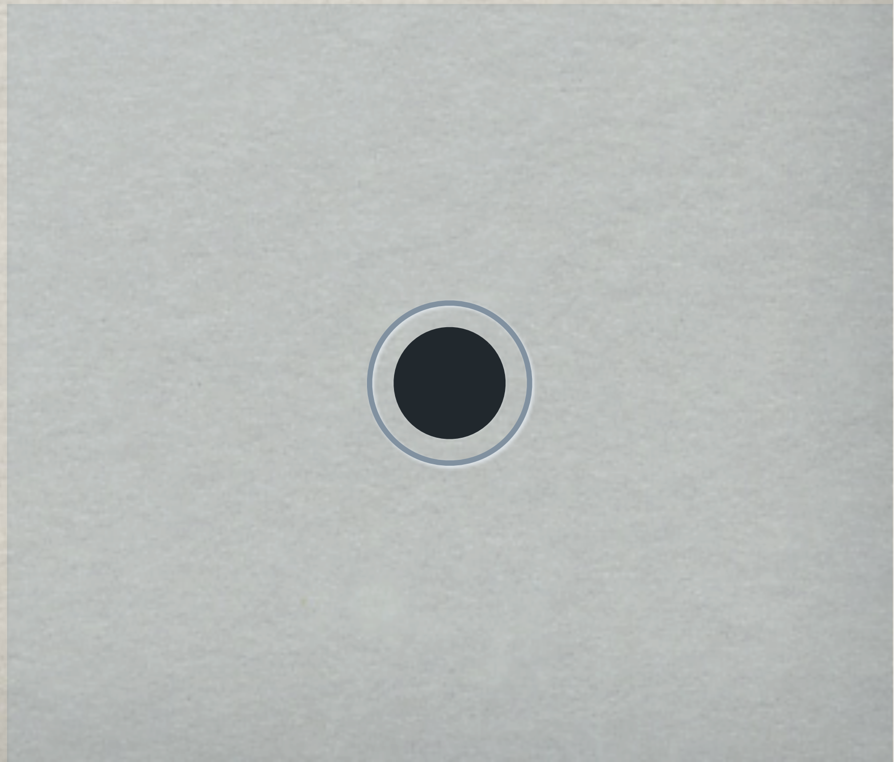
Null geodesics obey:

Obtain this equation!

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 - \boxed{\left(1-\frac{2M}{r}\right)\frac{j^2}{r^2}} \quad V_{ef}$$



Real space motion



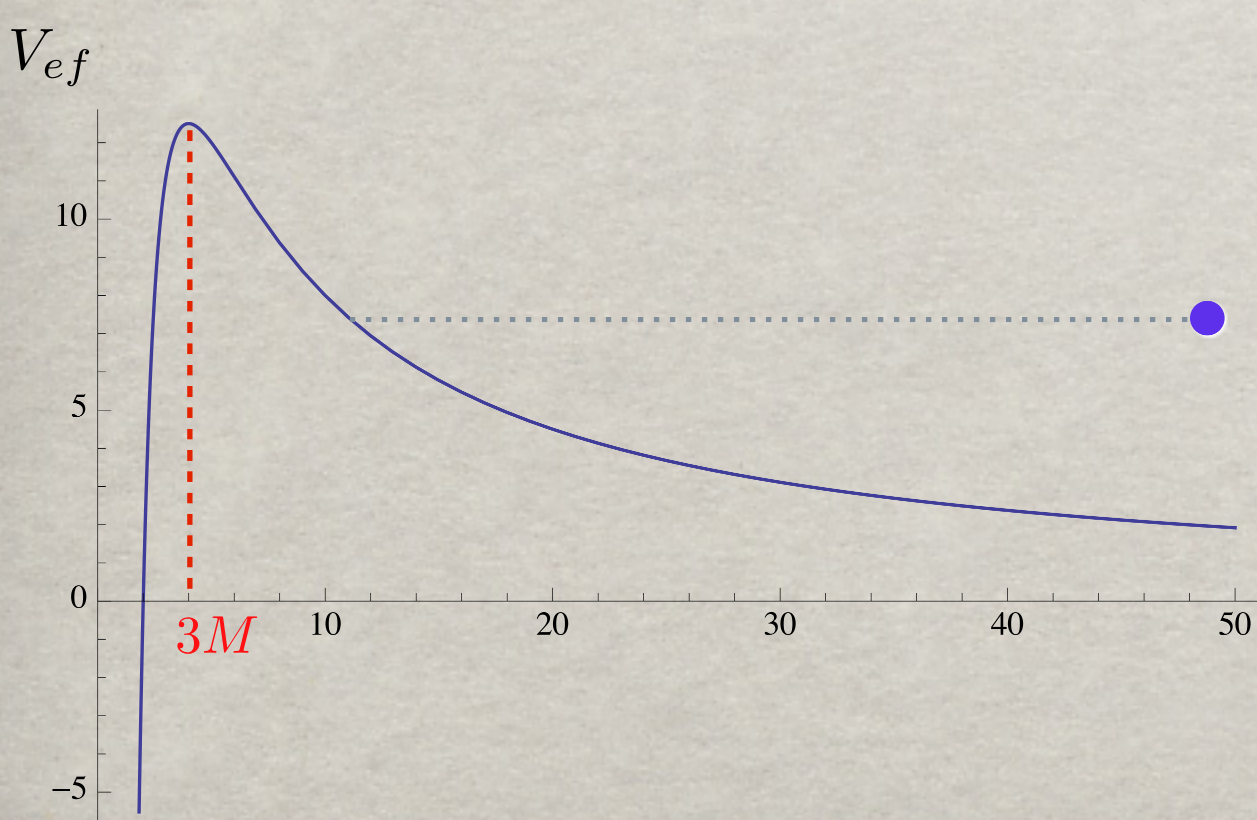
Schwarzschild space-time:

BH with mass M

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega_2$$

Null geodesics obey:

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 - \boxed{\left(1 - \frac{2M}{r}\right)\frac{j^2}{r^2}}\ V_{ef}$$



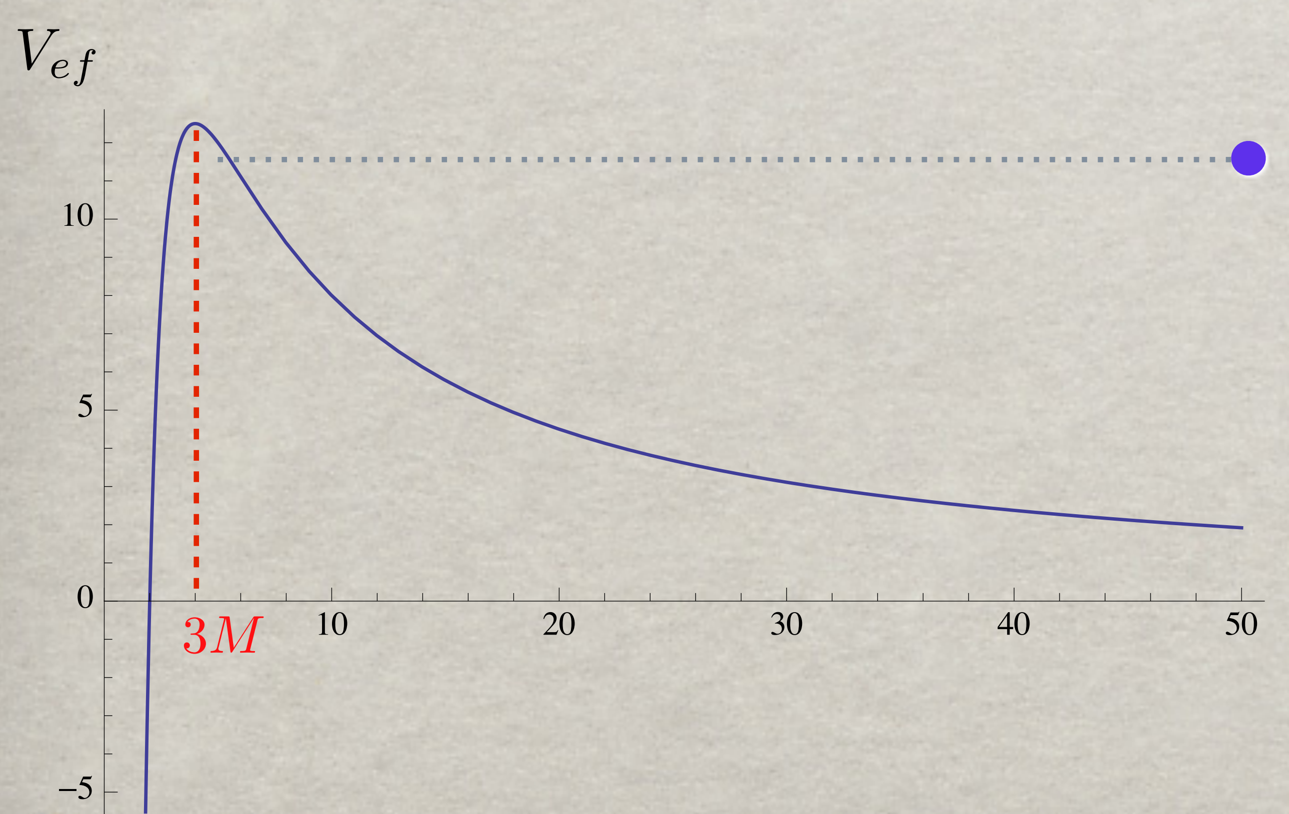
Real space motion

Schwarzschild space-time:
BH with mass M

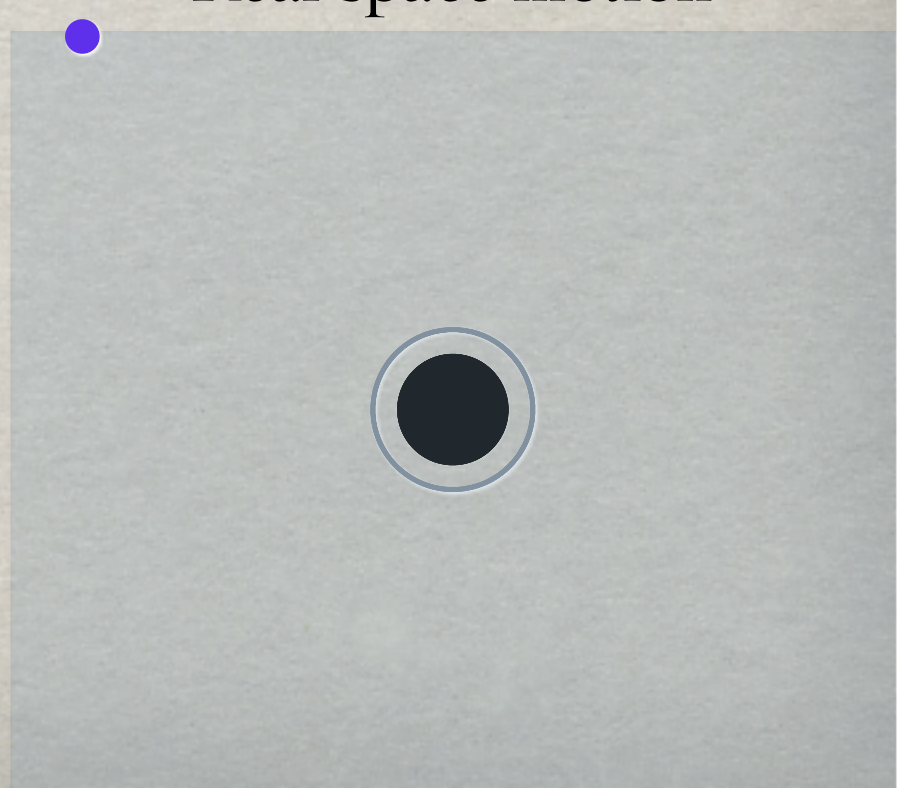
$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega_2$$

Null geodesics obey:

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 - \underbrace{\boxed{\left(1 - \frac{2M}{r}\right)\frac{j^2}{r^2}}}_{V_{ef}}$$



Real space motion



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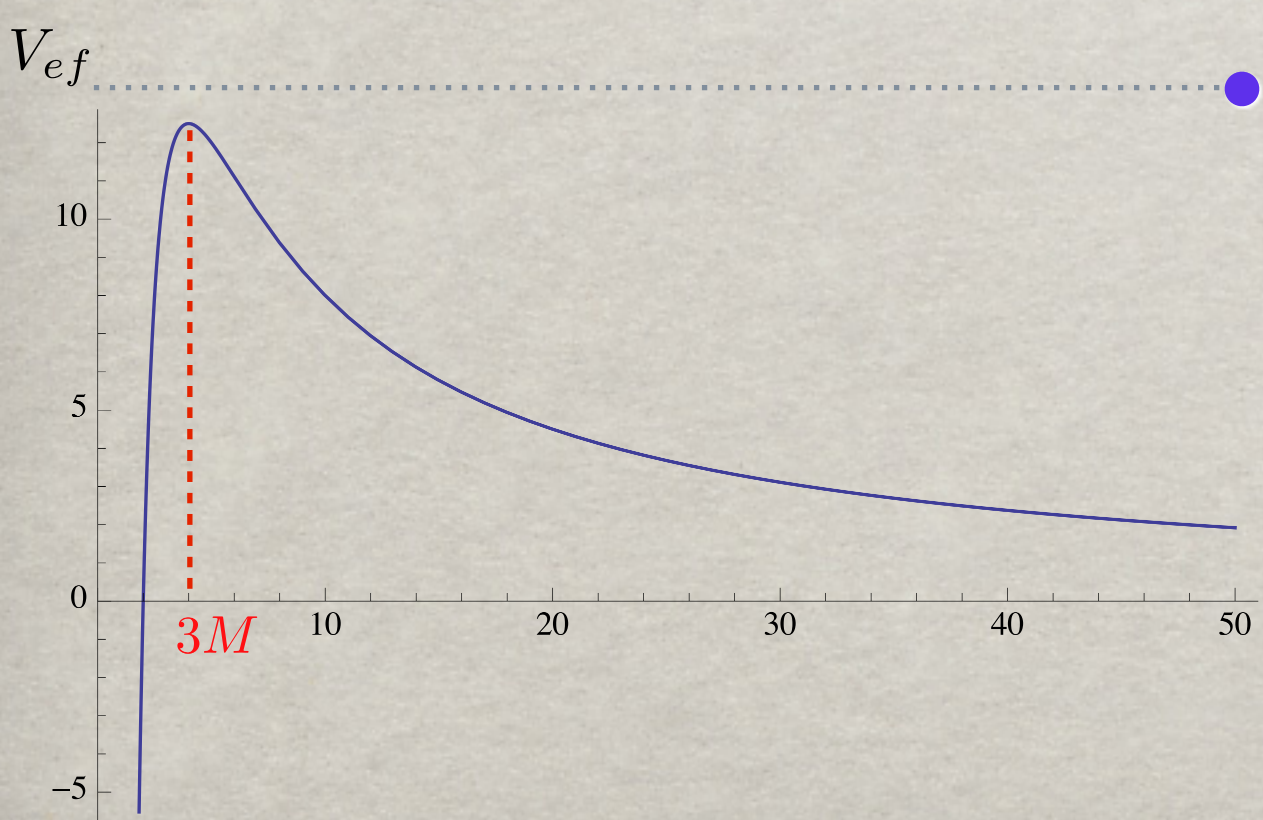
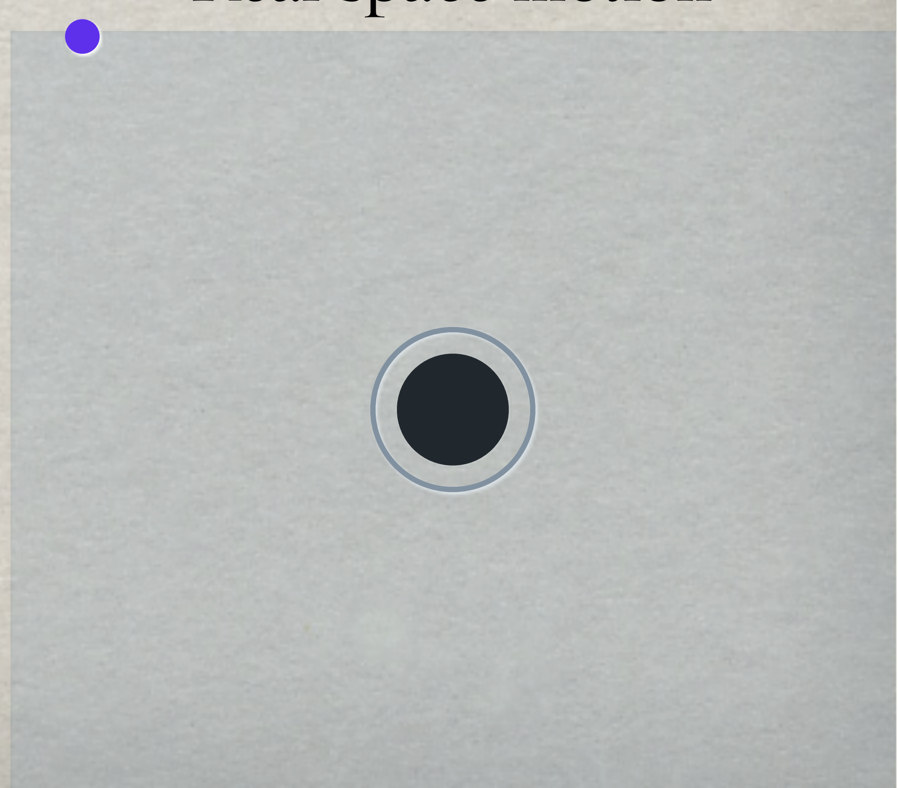


Real space motion



Lesson:

The photon escapes if it has an impact parameter greater than a **critical value**.

What is this value?

From the maximum of the potential, r=3M; then, from the radial equation:

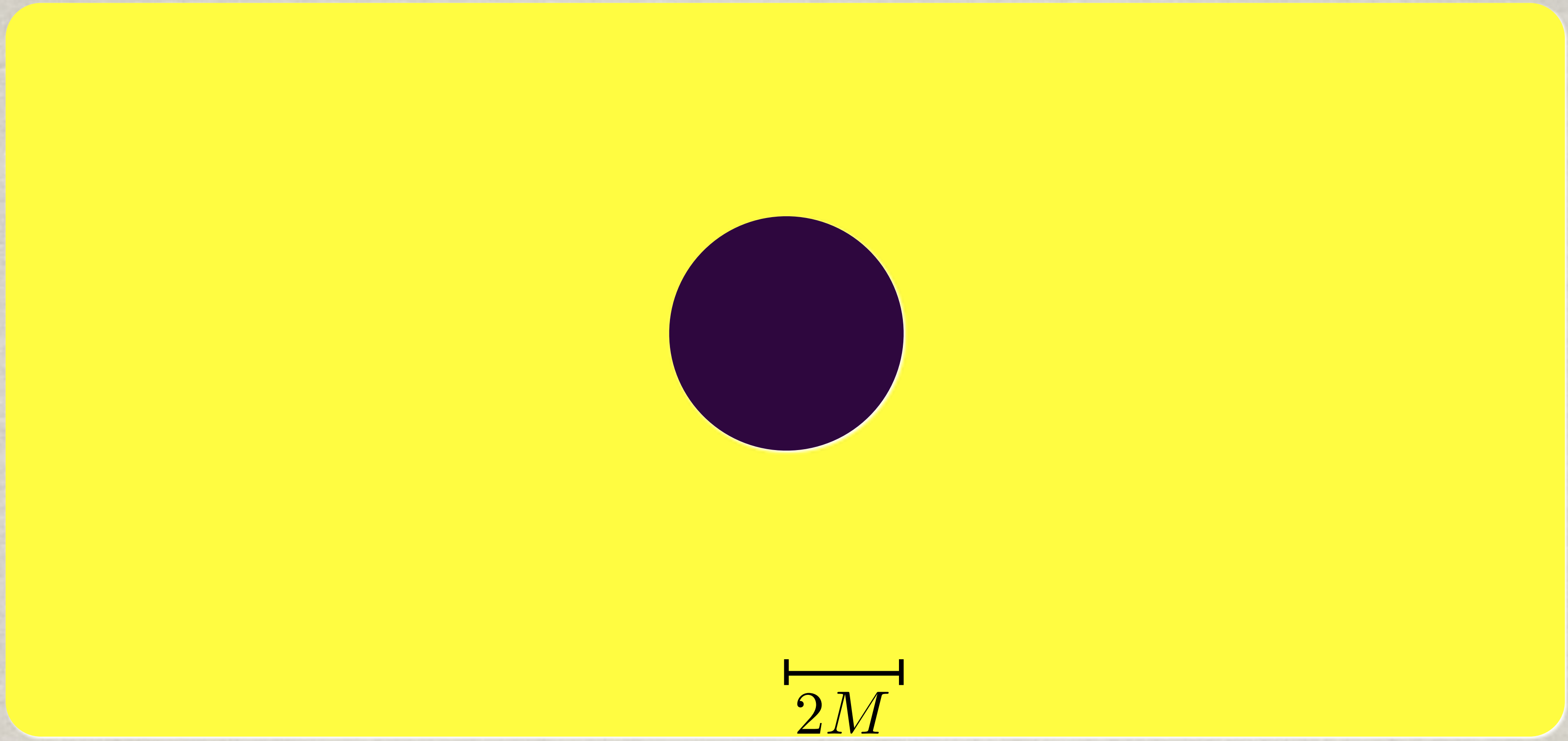
$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 - \left(1 - \frac{2M}{r}\right)\frac{j^2}{r^2}$$

the circular null orbit occurs for impact parameter

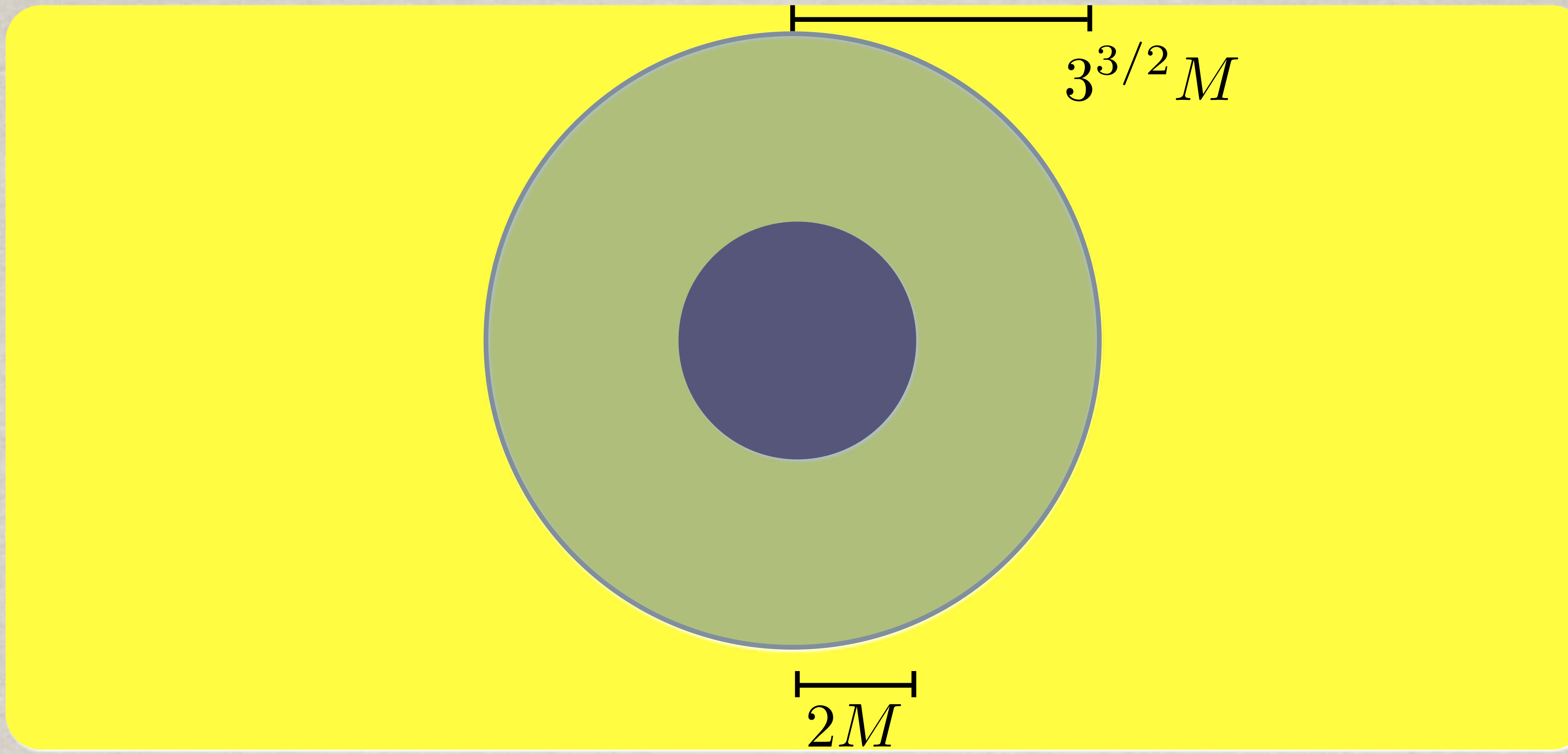
$$d \equiv \frac{j}{E} = 3^{3/2} M$$

This is the critical value.

Consider a “bright” homogeneous background with angular size much larger than the BH



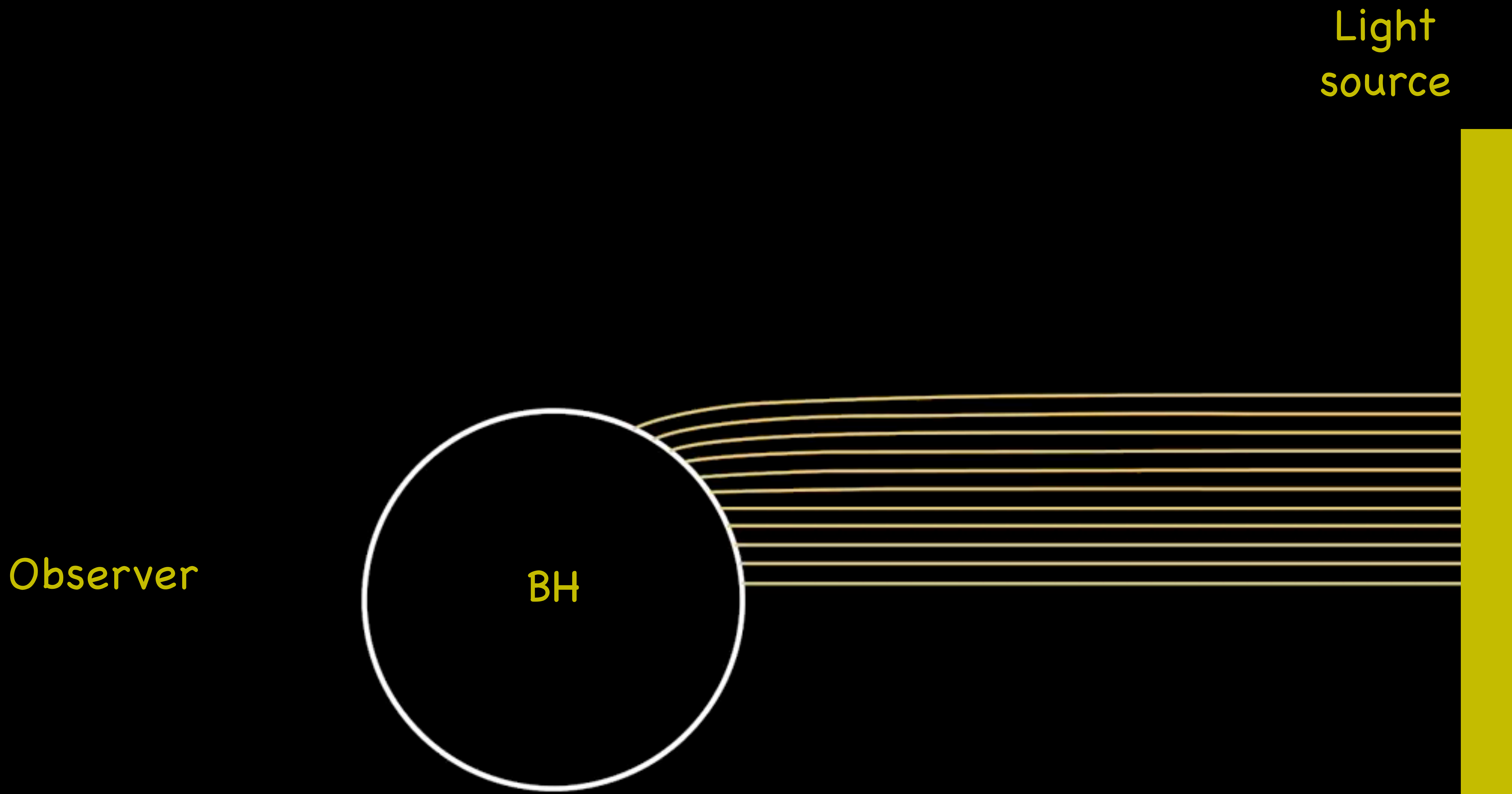
Consider a “bright” homogeneous background with angular size much larger than the BH



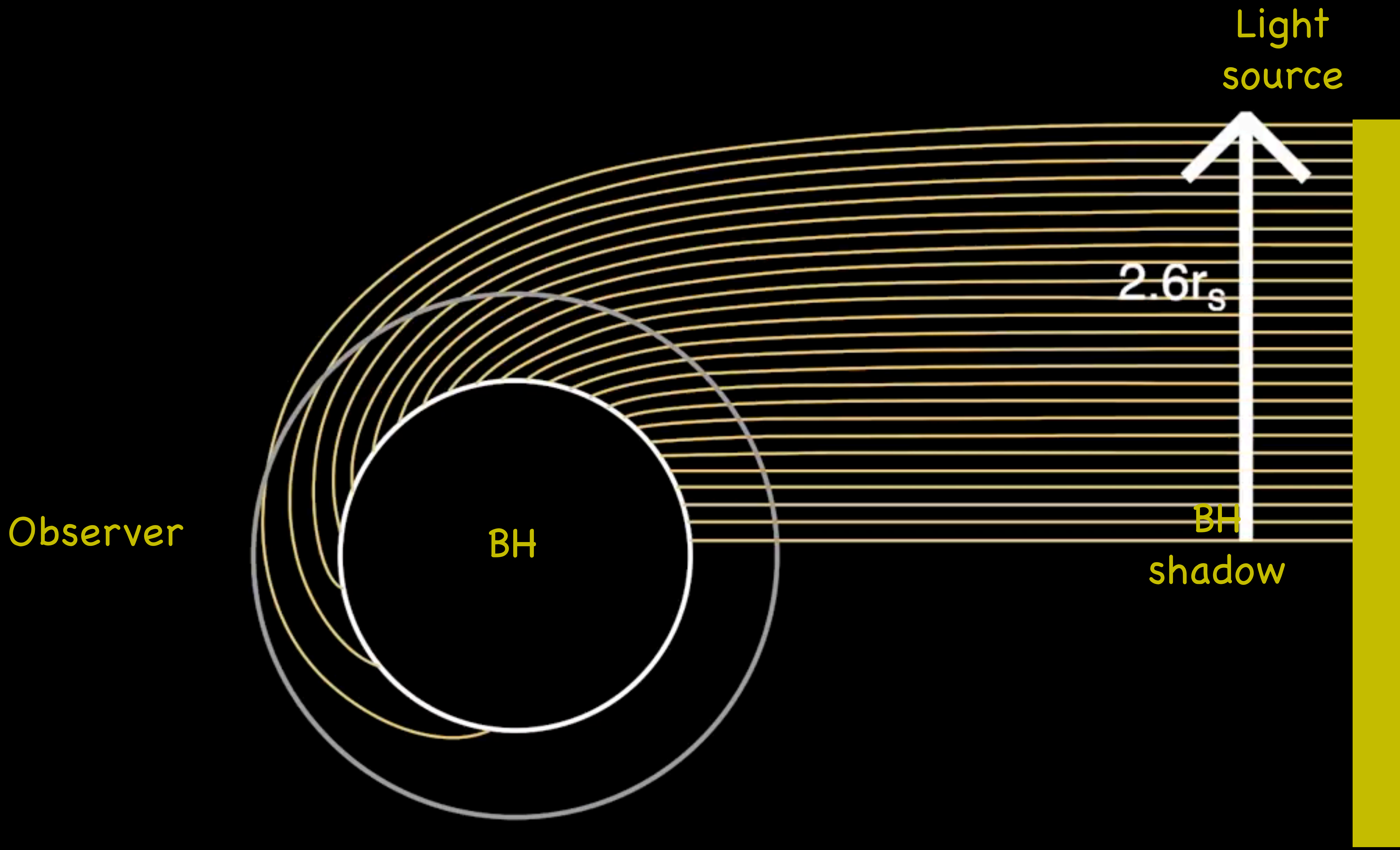
As seen by the distant observer the BH will cast a **shadow** in the middle of the large bright source, larger than the horizon scale

The rim of the BH **shadow** corresponds to the critical impact parameter:

$$d \equiv \frac{j}{E} = 3^{3/2}M$$



Light
source
Observer
BH



Light
source
2.6r_s
Observer
BH
BH
shadow

The importance of the light ring

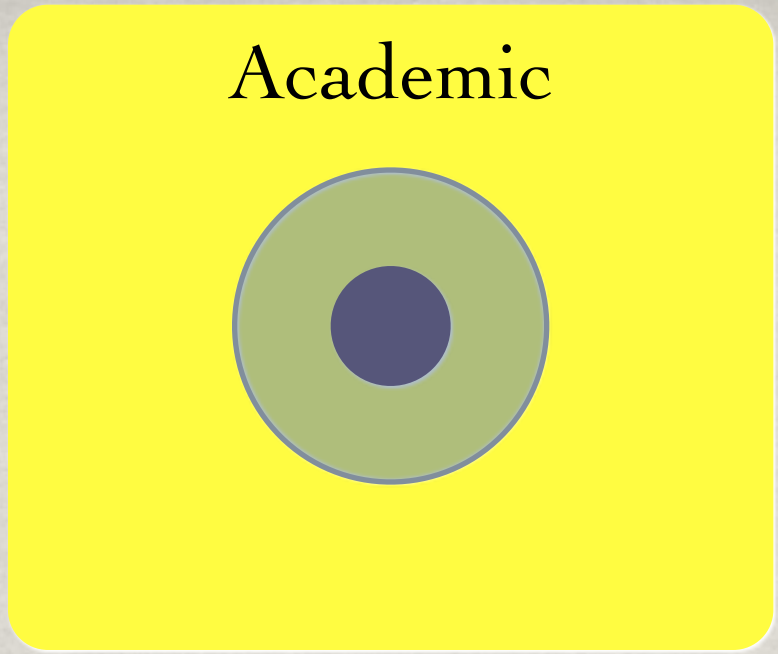
The stereotypical
light ring
arises in the Schwarzschild black hole solution
of vacuum general relativity

Why is it relevant ?

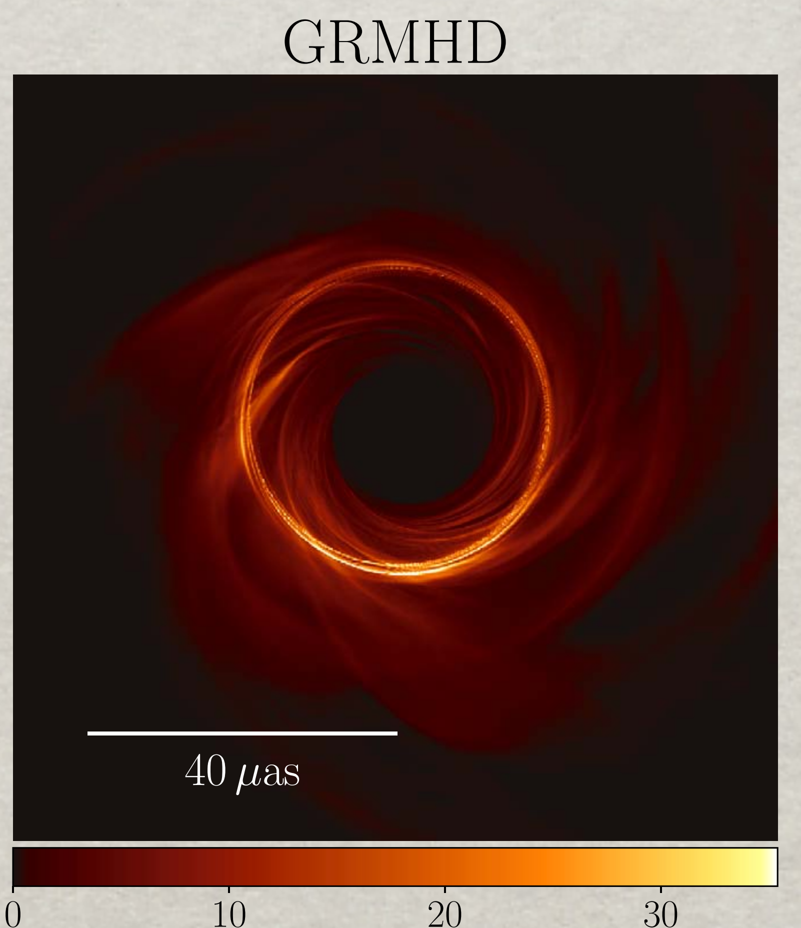
1) The light ring determines the “shadow” of the black hole.

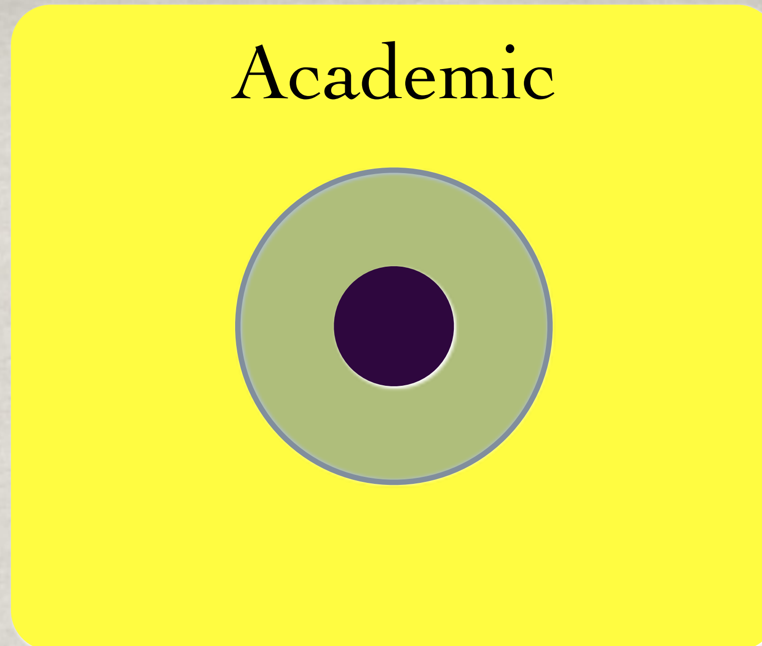
In a more astrophysical scenario:

background light is replaced by synchrotron radiation from accretion disk



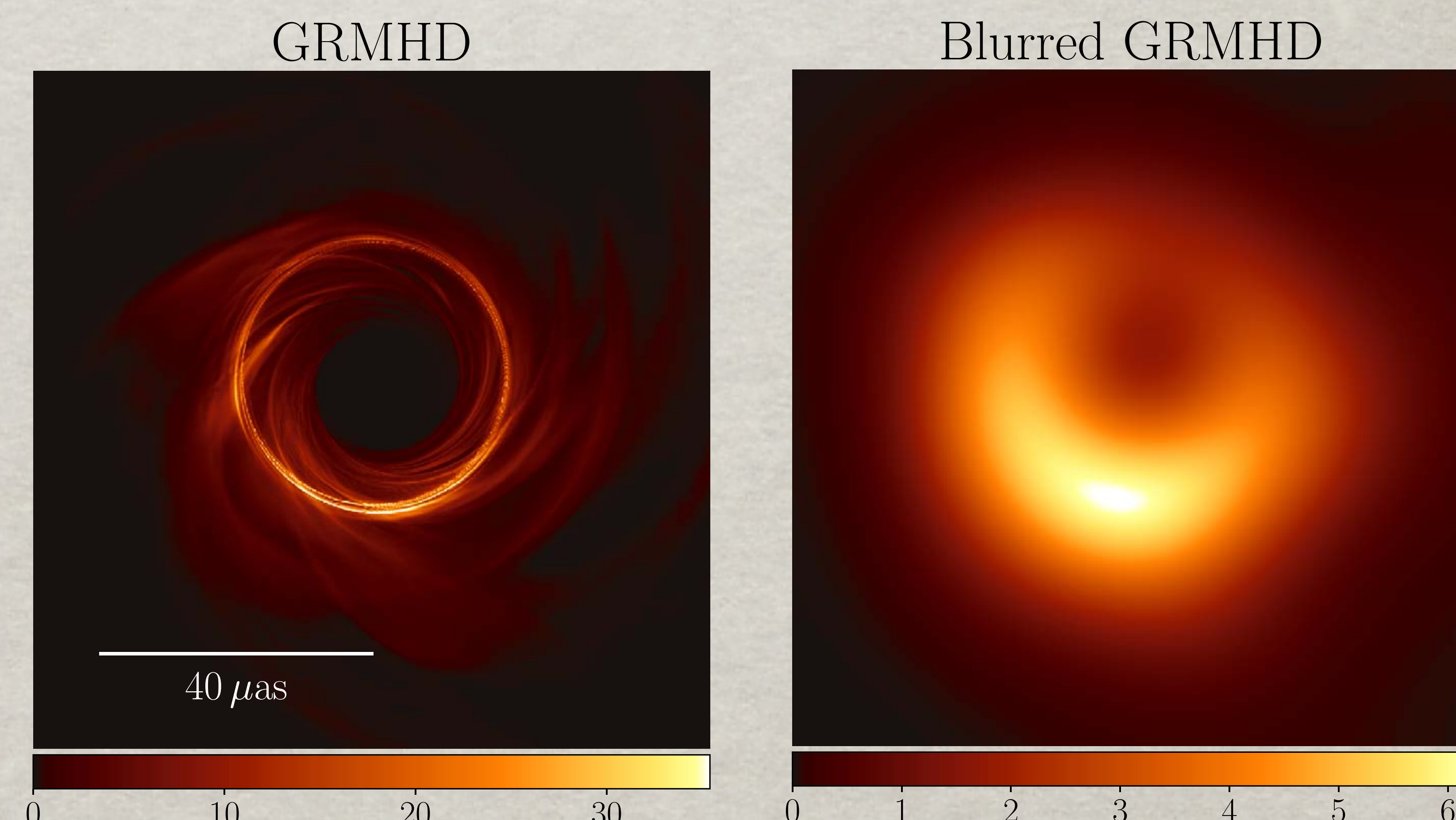
Synthetic images are generated by General Relativistic Magneto-Hydrodynamics (GRMHD) simulations using the Kerr metric



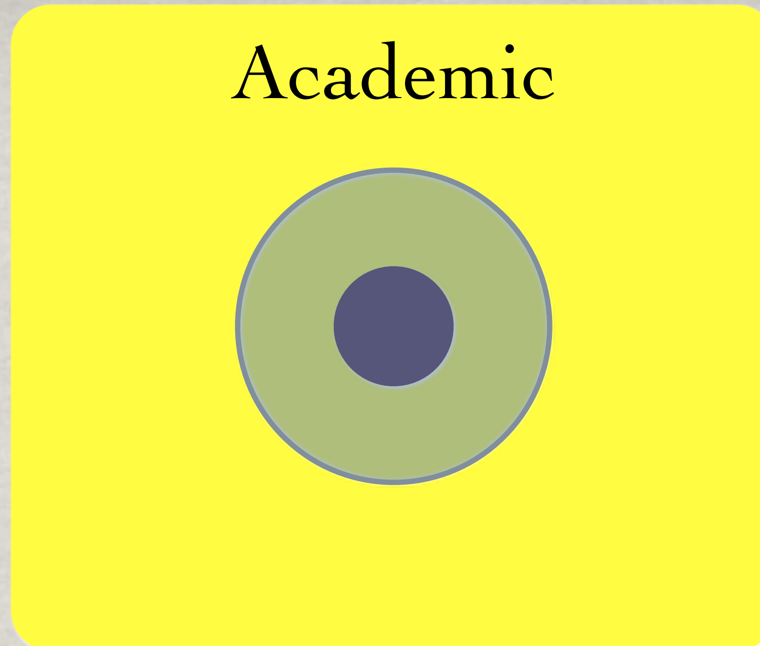


A Gaussian Blurring filter
is applied to a synthetic image
to reproduce
real EHT observations

Synthetic images are generated
by General Relativistic
Magneto-Hydrodynamics
(GRMHD)
simulations
using the Kerr metric



The synthetic blurred image
is similar do real data,
consistent with a Kerr black hole



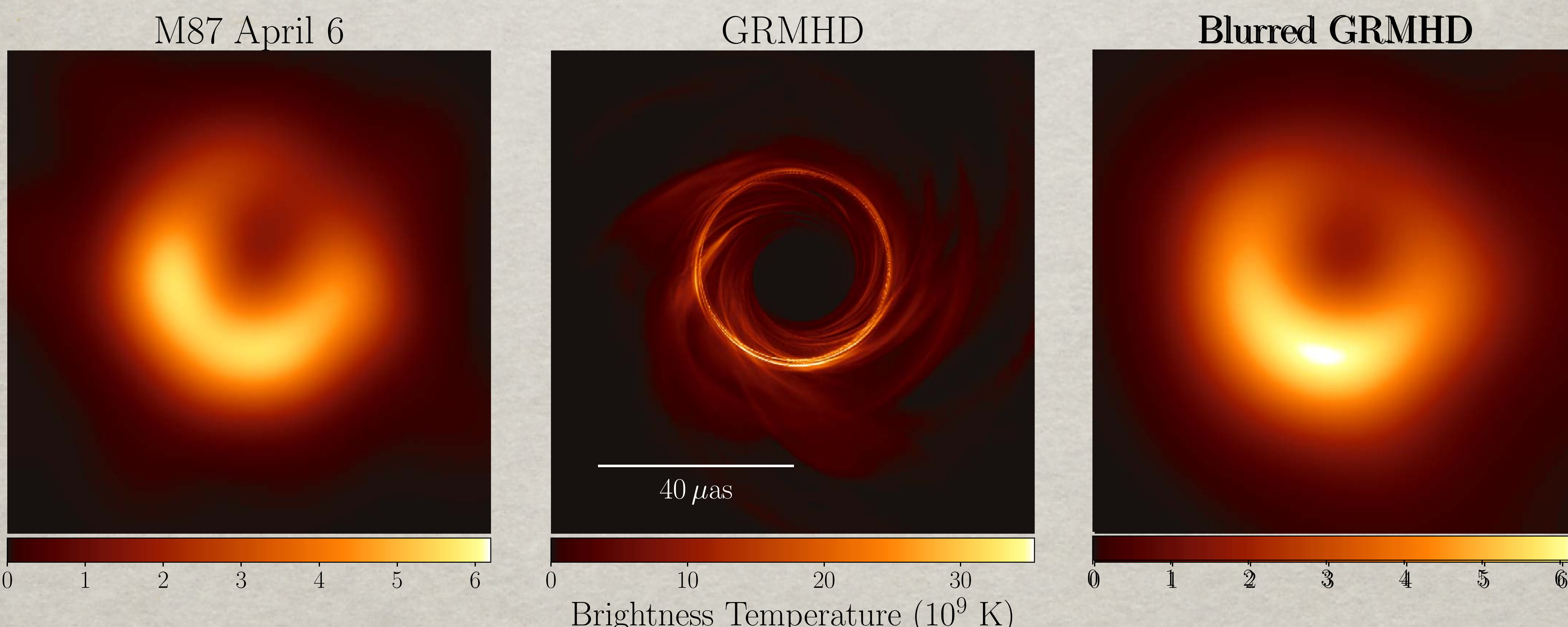


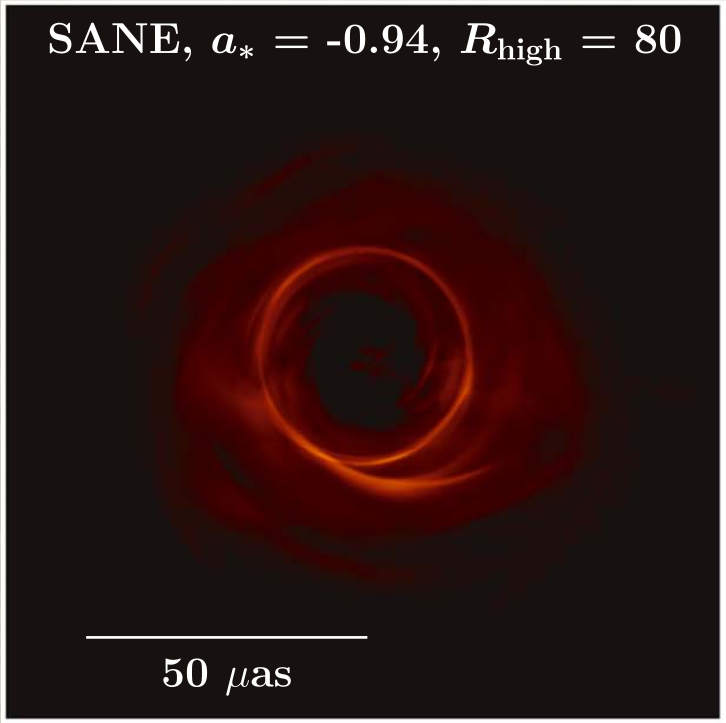
Figure 1. Left panel: an EHT2017 image of M87 from Paper IV of this series (see their Figure 15). Middle panel: a simulated image based on a GRMHD model. Right panel: the model image convolved with a 20 μas FWHM Gaussian beam. Although the most evident features of the model and data are similar, fine features in the model are not resolved by EHT.

ApJ Lett. 875 (2019) L5

...we do not simply assume that the measured emission diameter is that of the photon ring itself. We instead directly calibrate to the emission diameter found in model images from GRMHD simulations. The structure and extent of the emission preferentially from outside the photon ring leads to a < 10% offset between the measured emission diameter in the model images and the size of the photon ring.

ApJ Lett. 875 (2019) L6

GRMHD models



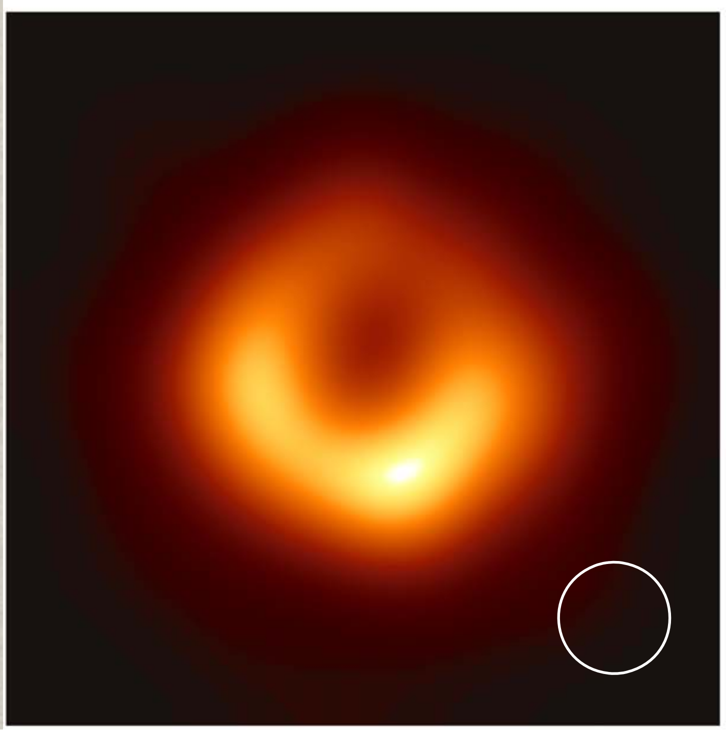


Table 1
Parameters of M87*

Parameter	Estimate
Ring diameter [a] d	42 ± 3 μas
Ring width [a]	<20 μas
Crescent contrast [b]	$>10:1$
Axial ratio [a]	$<4:3$
Orientation PA	150°–200° east of north
$\theta_{\rm g} = GM/Dc^2$ [c]	3.8 ± 0.4 μas
$\alpha = d/\theta_{\rm g}$ [d]	$11^{+0.5}_{-0.3}$
M[c]	$(6.5 \pm 0.7) \times 10^9\ M_\odot$
Parameter	**Prior Estimate**
D [e]	(16.8 ± 0.8) Mpc
M(stars) [e]	$6.2^{+1.1}_{-0.6} \times 10^9\ M_\odot$
M(gas) [e]	$3.5^{+0.9}_{-0.3} \times 10^9\ M_\odot$

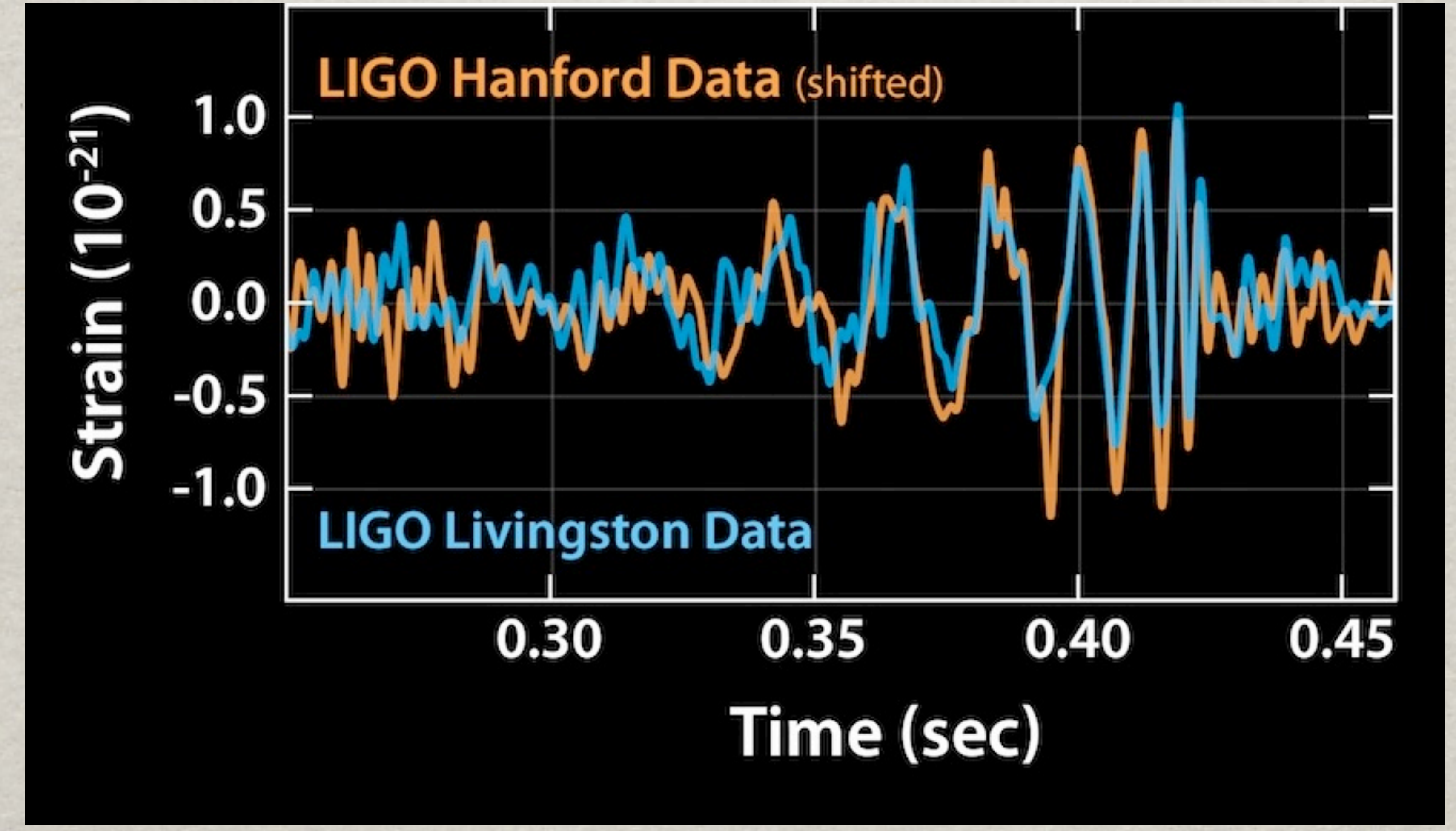
Notes.
[a] Derived from the image domain.
[b] Derived from crescent model fitting.
[c] The mass and systematic errors are averages of the three methods (geometric models, GRMHD models, and image domain ring extraction).
[d] The exact value depends on the method used to extract d, which is reflected in the range given.
[e] Rederived from likelihood distributions (Paper VI).

Schwarzschild based estimate of the light ring sky angle:

$$2 \times \sqrt{27} \times 3.8\ \mu\text{as} \simeq 39.5\ \mu\text{as}$$

2) The light ring determines the
initial “ringdown” of a perturbed black hole

Gravitational signals describe an inspiral, merger and ringdown



GW150914

Abbot et al., PRL 116 (2016) 061102

The ringdown is associated to the light ring:

THE ASTROPHYSICAL JOURNAL, **172**:L95–L96, 1972 March 15

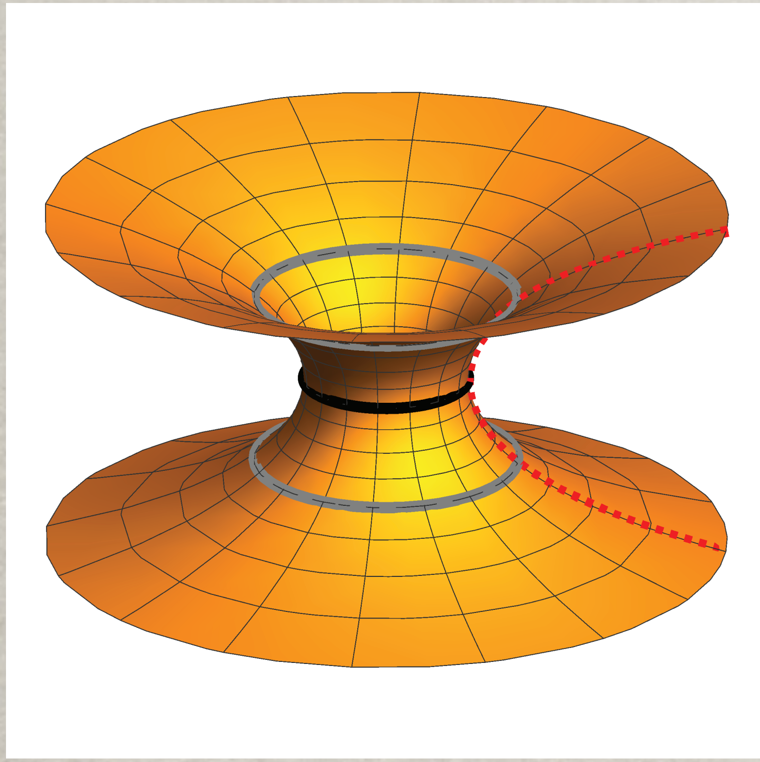
COMMENTS ON THE "VIBRATIONS" OF A BLACK HOLE

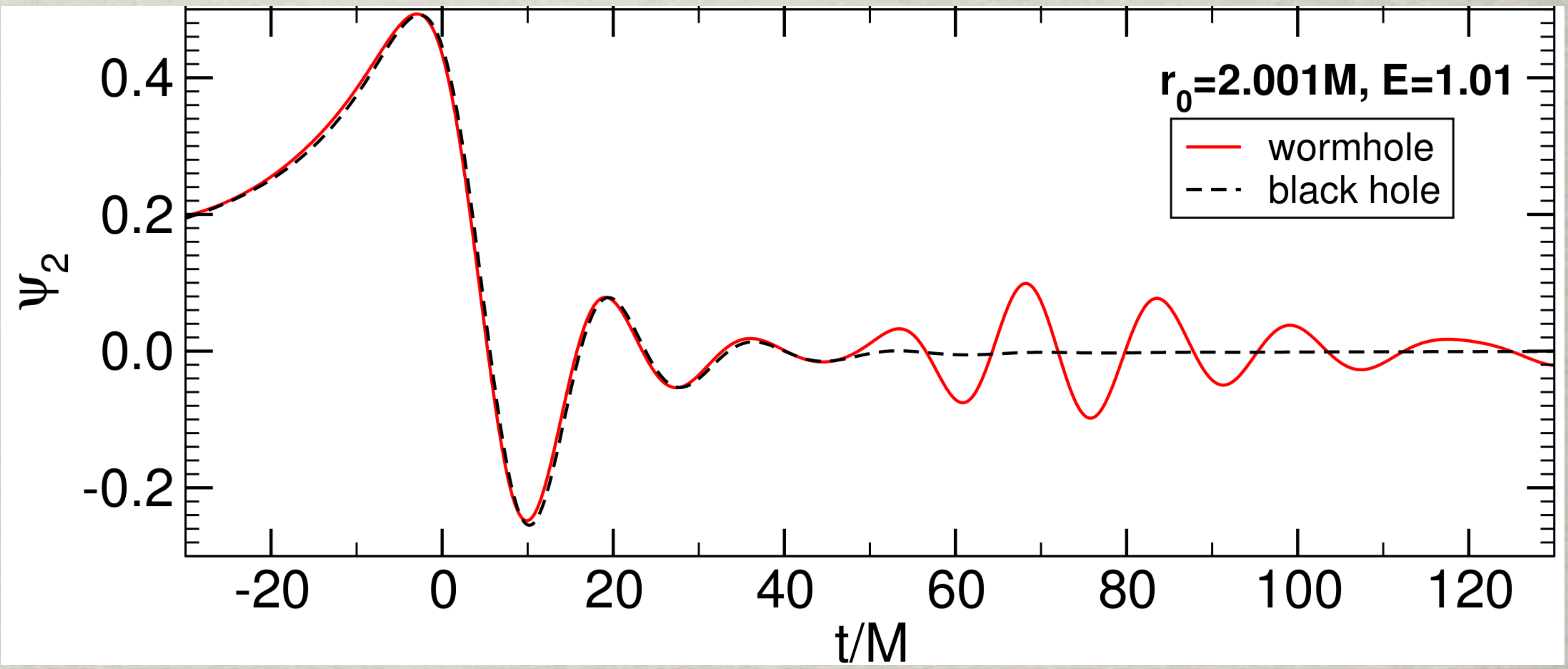
C. J. GOEBEL
University of Wisconsin, Physics Department, Madison
Received 1972 January 4; revised 1972 January 27

ABSTRACT

It is shown that the "vibrations of a black hole" of Press are gravitational waves in spiral orbits close to the well-known unstable circular orbit at $r = 3M$. The corresponding "vibrations" of a spinning black hole are discussed. It is emphasized that these "vibrations" provide, not a source, but only a temporary storage, of high-frequency gravitational radiation.

So, a hypothetical horizonless exotic compact object (ECO) with a similar light ring could vibrate similarly, initially...





Cardoso, Franzin, Pani, PRL 117 (2016) 089902

Thus,

if there are black hole mimickers with similar light rings
to black holes but no "event horizon"
they could mimic the black hole phenomenology.

Additionally, ECOs **with or without** light rings have been proposed
motivated by the singularity problem, the dark matter mystery,…

Can there be such speculative exotic compact objects (ECOs)?

Black holes have a horizon and (abiding energy conditions) a curvature singularity: issues

To avoid this difficulties, models of horizonless ECOs (black hole mimickers) have been considered:

a) "geons", realized by Boson stars (Schunck, Mielke, CQG 20 (2003) R301) and Proca stars (Brito, Cardoso, CH, Radu, PLB752 (2016) 291); can form dynamically (Seidel, Suen, PRL 72 (1994) 2516); Perturbatively stable Gleiser and Watkins, NPB 319 (1989) 733; Lee and Pang, NPB 315 (1989) 477 ; Can be studied dynamically in binaries (Liebling and Palenzuela LRR 20 (2017) 5)

b) wormholes

c) gravastars (Mazur and Mottola, gr-qc/0109035)

d) fuzzballs (Mathur, Fortsch. Phys. 53 (2005) 793)

e) anisotropic stars, ...

- There are viability issues. But to be able to mimic (up to a fine structure) the GW ringdown the object must be ultracompact, i.e. have a light ring;

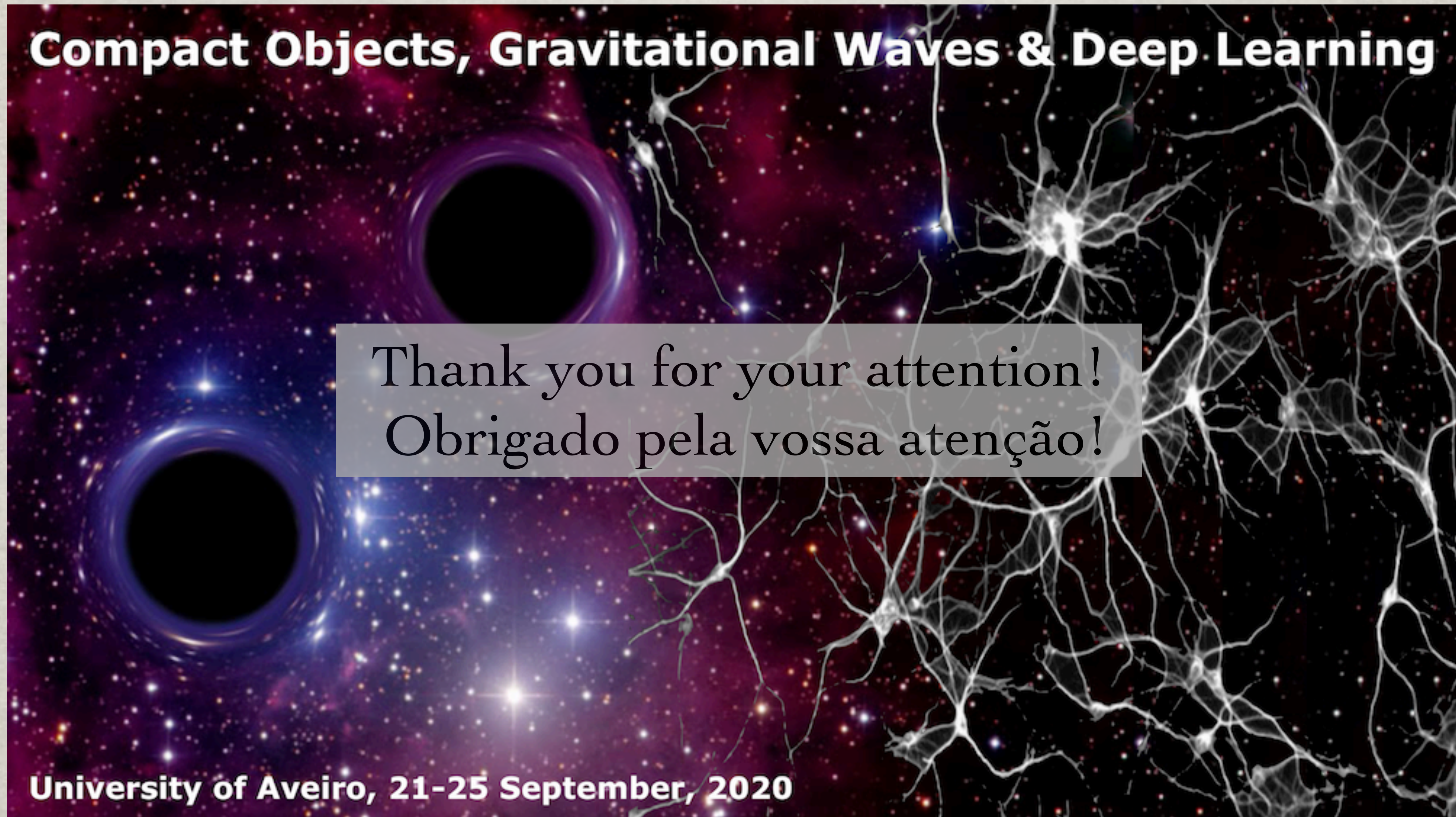
- Possible issue: for ultracompact ECOs resulting from a smooth, incomplete gravitational collapse, light rings come in pairs and one is stable Cunha, Berti, CH, PRL 119 (2017) 251102

In lecture 4 we will turn our attention to
a class of ECOs:
bosonic stars

(see also Nico's lectures and Juan's seminar on wednesday!)

Black holes and exotic compact objects

C. Herdeiro
Departamento de Matemática and CIDMA, Universidade de Aveiro, Portugal



Thank you for your attention!
Obrigado pela vossa atenção!