Black holes and exotic compact objects

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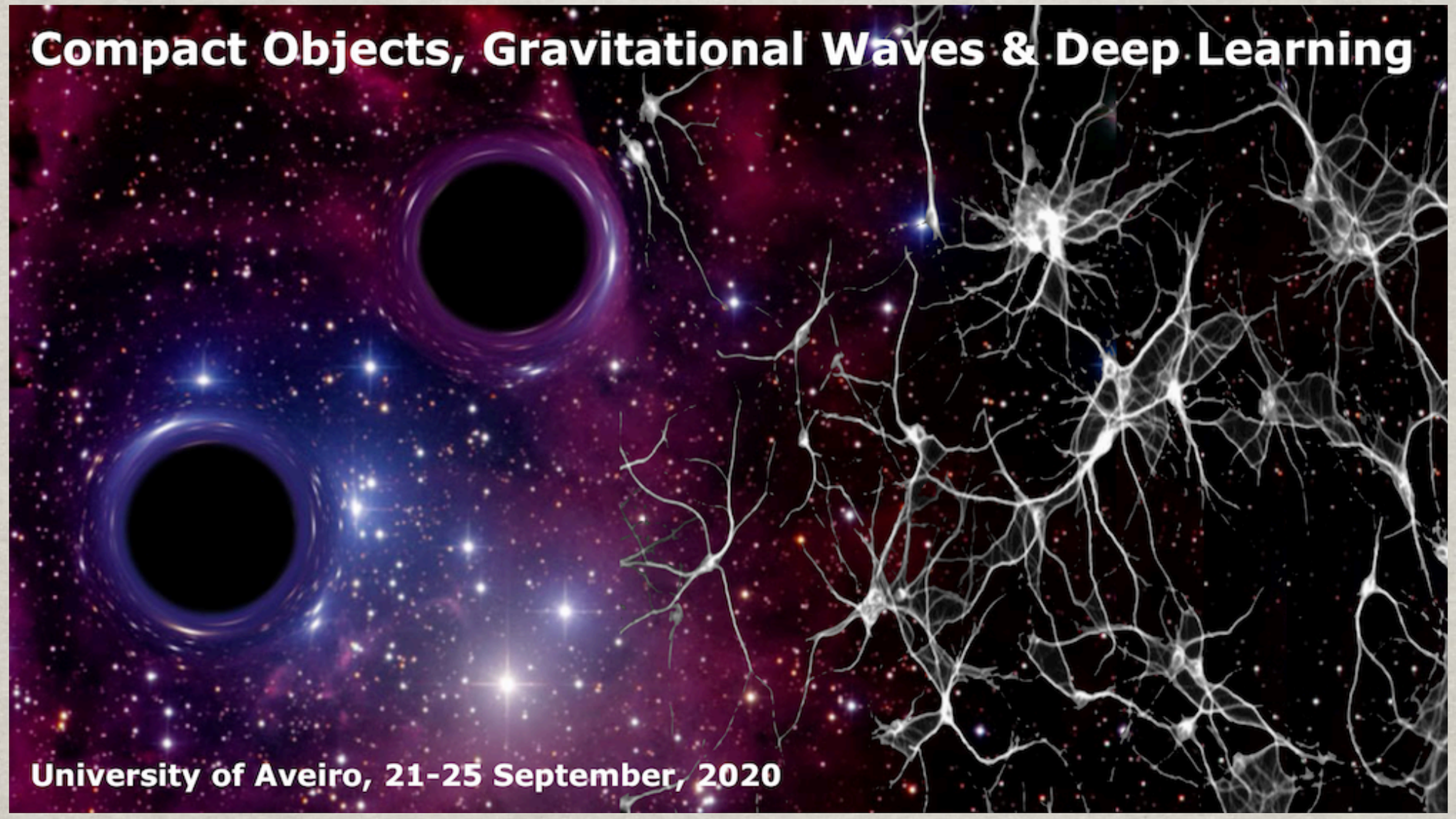

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FCT Fundação para a Ciência e a Tecnologia

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Plan of the lectures:

Lecture 1
Black holes: astrophysical evidence and a theory (brief) timeline

Lecture 2
Spherical black holes: the Schwarzschild solution

Lecture 3
Spinning black holes: the Kerr solution

Lecture 4
Exotic compact objects: the example of bosonic stars

Lecture 5
Non-Kerr black holes

The field of GW detection is
theory driven,
in the sense one needs theoretical templates to guide searches.

Efforts to produce templates in
modified gravity
and with
ECOs
are underway.

Here we shall focus on the case of bosonic stars.

Lecture 4

I) Spherical boson (scalar/vector) star solutions

II) Lensing by spherical bosonic stars

III) Rotating bosonic (scalar/vector) star solutions

IV) Lensing by rotating bosonic stars

Stationary scalar solitons in field theory

The asymptotically flat black holes we have been discussing so far have an event horizon which cloaks a curvature singularity. An interesting question in any gravitational model is whether stationary particle-like solutions exist, i.e. *gravitating solitons*: **everywhere regular configurations, without horizons**, corresponding to localized lumps of (time-independent) gravitational+matter field energy.

In field theory (without gravity) there is a long history and a large literature on finding soliton-type solutions and solitary waves, starting with the Korteweg-de-Vries equation D. J. Korteweg and G. de Vries, Philosophical Magazine 39 (1895) 422-443.

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These solutions occur for non-linear field theories and the constancy of their "shape" is interpreted as a cancellation between non-linear and dispersive effects.

There is, however, a generic argument, known as *Derrick's theorem*, against the existence of stable, time-independent solutions of finite energy in a wide class of non-linear wave equations, in three or higher (spatial) dimensions G. H. Derrick, J. Math. Phys. 5 (1964) 1252 (see also R.H. Hobart, Proc. Phys. Soc. 82 (1963)201).

Derrick's theorem (original scaling argument)

Consider the (possibly) non-linear Klein-Gordon equation in Minkowski spacetime:

$$\Box\Phi = \frac{1}{2}\frac{dV}{d\Phi}$$

This can be derived from the action (using a variational principle):

$$\mathcal{S} = \int d^4x\,[-\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)] = \int dt\,d^3\mathbf{r}\left[\left(\frac{\partial\Phi}{\partial t}\right)^2 - (\nabla\Phi)^2 - V(\Phi)\right]$$

We will prove that no stable, time-independent, localized solutions exist, for any potential energy. By "localized" we mean that the integrals:

$$I_1 \equiv \int d^3\mathbf{r}(\nabla\Phi)^2 \qquad I_2 \equiv \int d^3\mathbf{r}V(\Phi)$$

converge, when taken over the whole 3-space.

We assume the field depends only on the spatial coordinates. Then $\delta S = 0 \quad \Leftrightarrow \quad \delta E = 0$

$$E = \int d^3\mathbf{r}\left[(\nabla\Phi)^2 + V(\Phi)\right]$$

Stationary scalar solitons in field theory

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$$E = \int d^3\mathbf{r}\left[(\nabla\Phi)^2 + V(\Phi)\right]$$

Assume the existence of a localized solution of: $\delta E = 0$ Let the solution be: $\Phi(\mathbf{r})$

The solution is stable if: $\delta^2 E \geqslant 0$

Define a 1-parameter family of configurations: $\Phi_\lambda(\mathbf{r}) = \Phi(\lambda\mathbf{r})$ λ= arbitrary constant

Let:

$$E_\lambda = \int d^3\mathbf{r}\left[(\nabla\Phi_\lambda)^2 + V(\Phi_\lambda)\right] = \frac{I_1}{\lambda} + \frac{I_2}{\lambda^3}$$

It follows that:

$$\left(\frac{dE_\lambda}{d\lambda}\right)_{\lambda=1} = -I_1 - 3I_2 \qquad \left(\frac{d^2E_\lambda}{d\lambda^2}\right)_{\lambda=1} = 2I_1 + 12I_2$$

Thus:

$$\left(\frac{dE_\lambda}{d\lambda}\right)_{\lambda=1} = 0 \quad \Leftrightarrow \quad I_2 = -\frac{I_1}{3} \quad \text{and} \quad \left(\frac{d^2E_\lambda}{d\lambda^2}\right)_{\lambda=1} = -2I_1 < 0$$

Then, $\delta^2 E < 0$ for a uniform "stretching" of the hypothetical soliton and it is unstable (*q.e.∂.*).

Stationary scalar solitons in field theory

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Observe, moreover that if the potential is strictly positive (or zero if the field vanishes) **there are no solutions at all**, since

$$I_1 + 3I_2 \geqslant 0$$

and the equality only holds for the trivial solution.

Exercise 4.1

Does the argument apply in d=1,2 (spatial dims) ?

Stationary scalar solitons in field theory

Derrick observed that one way to circumvent the theorem would be allow for localized solutions that are periodic in time, rather than time independent. Various authors, starting with Rosen, considered a complex field with a harmonic time dependence, which guarantees a time-independent energy momentum tensor G. Rosen, J. Math. Phys. 9 (1968) 996:

$$\Phi(t, \mathbf{r}) = e^{-iwt}\varphi(\mathbf{r})$$

Moreover there is a global symmetry and a conserved scalar charge (typically called Q). Then, for some classes of potentials (yielding non-linear models), localized stable solutions exist, which are now known, following Coleman, as *Q-balls* S. R. Coleman, "Q Balls," Nucl. Phys. B 262 (1985) 263 [Erratum-ibid. B 269 (1986) 744]

But in the presence of gravity, no scalar non-linear interactions are required. Effectively, such non-linearities are provided by the self-gravity of the field.

Gravitating scalar solitons: boson stars

Gravitating scalar solitons: boson stars

Discussion of particle-like solutions in Einstein's gravity gained notoriety with the work of Wheeler, who searched for these configurations in electro-vacuum. He coined the term "Geon" as a contraction of "gravitational electromagnetic entity", meaning an electromagnetic or gravitational wave which is held together in a confined region by the gravitational attraction of its own field energy J. A. Wheeler, Phys. Rev. 97 (1955) 511.

Although no explicit solutions were found in electro-vacuum a Klein-Gordon Geon was found in massive scalar-vacuum by Kaup D. J. Kaup, Phys. Rev. 172 (1968) 1331. Later these gravitating solitons became known as *boson stars*.

Gravitating scalar solitons: boson stars

The model (mini-boson stars):

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi} - \frac{1}{2}g^{\alpha\beta}\left(\Phi^*_{,\alpha}\Phi_{,\beta} + \Phi^*_{,\beta}\Phi_{,\alpha}\right) - \mu^2\Phi^*\Phi\right],$$

The field equations:

$$G_{\alpha\beta} = 8\pi\left\{\Phi^*_{,\alpha}\Phi_{,\beta} + \Phi^*_{,\beta}\Phi_{,\alpha} - g_{\alpha\beta}\left[\frac{1}{2}g^{\gamma\delta}(\Phi^*_{,\gamma}\Phi_{,\delta} + \Phi^*_{,\delta}\Phi_{,\gamma}) + \mu^2\Phi^*\Phi\right]\right\}$$

$$\Box\Phi = \mu^2\Phi$$

The action is invariant under a U(1) global symmetry: $\Phi \to e^{i\alpha}\Phi$

This leads to a conserved current: $j^\alpha = -i(\Phi^*\partial^\alpha\Phi - \Phi\partial^\alpha\Phi^*)$

Integrating the temporal component of this 4-current on a timelike slice leads to a conserved charge - the *Noether charge* Q:

$$Q = \int_\Sigma j^t$$

The Noether charge counts the number of scalar particles. Notice that this is conserved in the sense of a local continuity equation; **there is no associated Gauss law**!

Gravitating scalar solitons: boson stars

Spherically symmetric solutions ansatz (**three** unknown functions):

$$ds^2 = -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ , \quad N(r) \equiv 1 - \frac{2m(r)}{r} \ , \quad \Phi = \phi(r)e^{-iwt}$$

The time dependence cancels at the level of the energy momentum tensor, being therefore compatible with a stationary metric. Thus k = ∂/∂t is a Killing vector field, but it does not preserve the scalar field - the metric and the matter field **do not share the same symmetries**.

The above ansatz makes the Einstein equations simpler as compared to other choices (such as isotropic coordinates). The two "essential" Einstein equations read:

$$m' = 4\pi r^2 \left(N\phi'^2 + \mu^2\phi^2 + \frac{w^2\phi^2}{N\sigma^2} \right) \ , \quad \sigma' = 8\pi\sigma r \left(\phi'^2 + \frac{w^2\phi^2}{N^2\sigma^2} \right)$$

(one further constraint equation is found, but which is a differential consequence of these).

The Klein-Gordon equation gives (thus completing **three** equations):

$$\phi'' + \frac{2\phi'}{r} + \frac{N'\phi'}{N} + \frac{\sigma'\phi'}{\sigma} - \frac{\mu^2\phi}{N} + \frac{w^2\phi}{N^2\sigma^2} = 0$$

Exercise 4.2

Obtain these equations.

Gravitating scalar solitons: boson stars

Details ? Talk to Alexandre Pombo

These equations are solved numerically subject to the following boundary conditions:

$$m(0) = 0, \quad \sigma(0) = s_0, \quad \phi(0) = b \ , \qquad\qquad m(\infty) = M, \quad \sigma(\infty) = 1, \quad \phi(\infty) = 0$$

To start integrating from the origin, one finds a power series solution of the field equations near the origin:

$$m(r) = \frac{4\pi(\mu^2 s_0^2 + w^2)}{3s_0} b^2 r^3 + \mathcal{O}(r^5) \ , \quad \sigma(r) = s_0 + \frac{4\pi w^2 b^2}{s_0} r^2 + \mathcal{O}(r^4) \ , \quad \phi(r) = b + \frac{1}{6}\left(\mu^2 - \frac{w^2}{s_0^2}\right) b r^2 + \mathcal{O}(r^4)$$

Then, one integrates the coupled ODEs by a standard Runge-Kutta solver, with input parameters b, w, s_0 . The solution has 2-parameters (w,n); don't define the solution uniquely.

A shooting strategy is implemented to assure that (at the level of machine precision), the scalar field value vanishes at some large r. Then, one extracts the ADM mass M from the value of m.

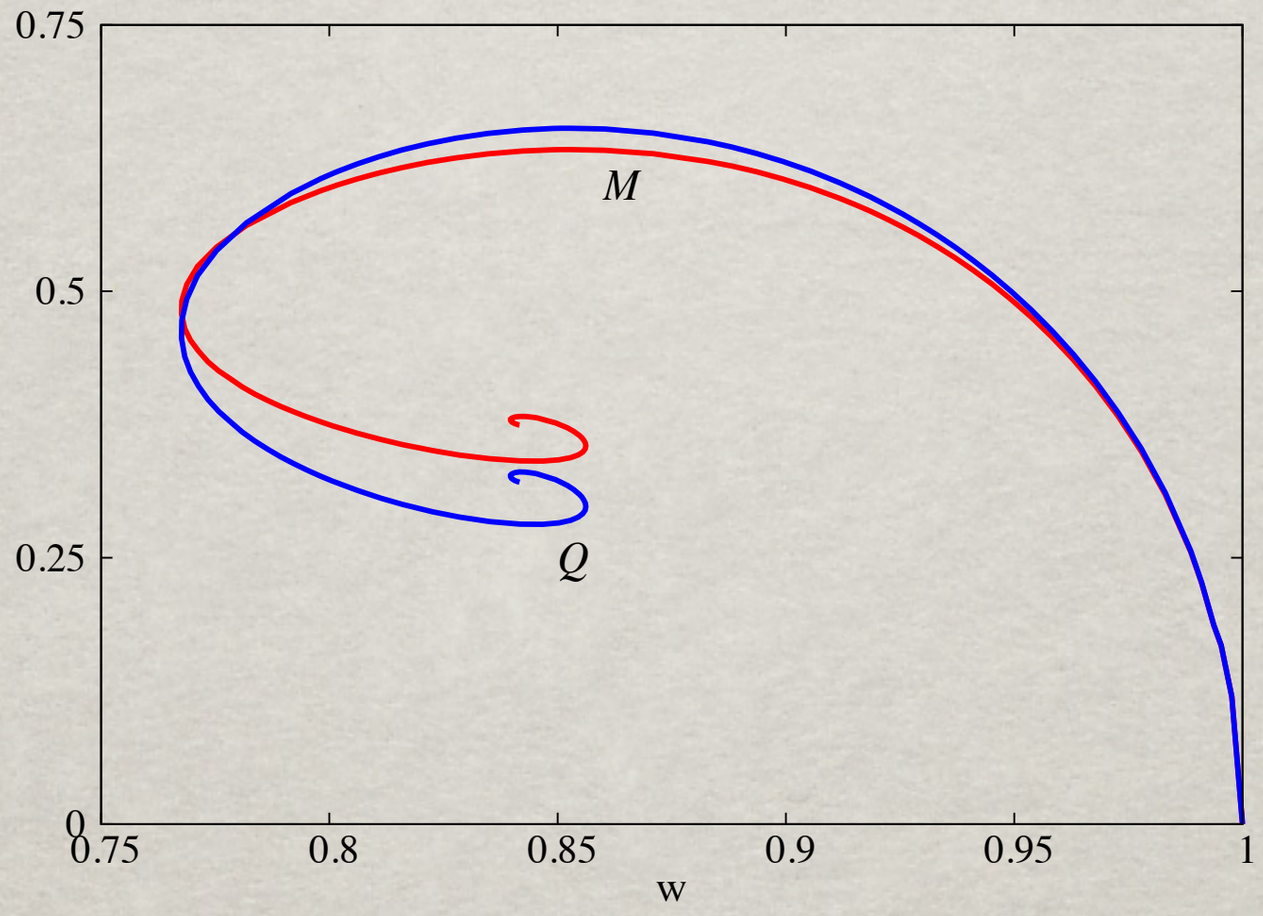
The shooting method is implemented in terms of the scalar field value at r=0, for fixed w, s_0 =1. This is actually a 1-parameter shooting, with shooting parameter b. At some large value of r, $\phi(r_{\max}) = 0$, to machine precision, but $\sigma(r_{\max}) = s_{\text{fin}} \neq 1$. But there is a scaling symmetry of the field equations ($\sigma \to \sigma/\lambda, \quad w \to w/\lambda$) that maps the solution to another w and the $\sigma(r_{\max}) = 1$.

One finds various (or no) solutions, corresponding to different ADM masses M, for the same number of nodes and for a given frequency.

Solutions **without**/**with** nodes are the **fundamental family**/**excited** states.

Gravitating scalar solitons: boson stars

ADM mass M (and Noether charge Q) vs. frequency w diagram:



In units of μ

$M\mu \to M \ , \quad w/\mu \to w$

- Solutions only exist for a range of frequencies: $\frac{w_{\rm min}}{\mu} < \frac{w}{\mu} < 1$ $\qquad w_{\rm min} \simeq 0.767\mu$

- There is a range of frequencies for which more than one solution exists. This defines the first, second, third, etc, **branches**.

- There is a maximum value for the ADM mass: $M_{\rm ADM}^{\rm max} \simeq \alpha_{\rm BS} \frac{M_{\rm Pl}^2}{\mu} \simeq \alpha_{\rm BS}\, 10^{-19} M_\odot \left(\frac{\rm GeV}{\mu}\right)$

  $\alpha_{\rm BS} = 0.633$

Gravitating scalar solitons: boson stars

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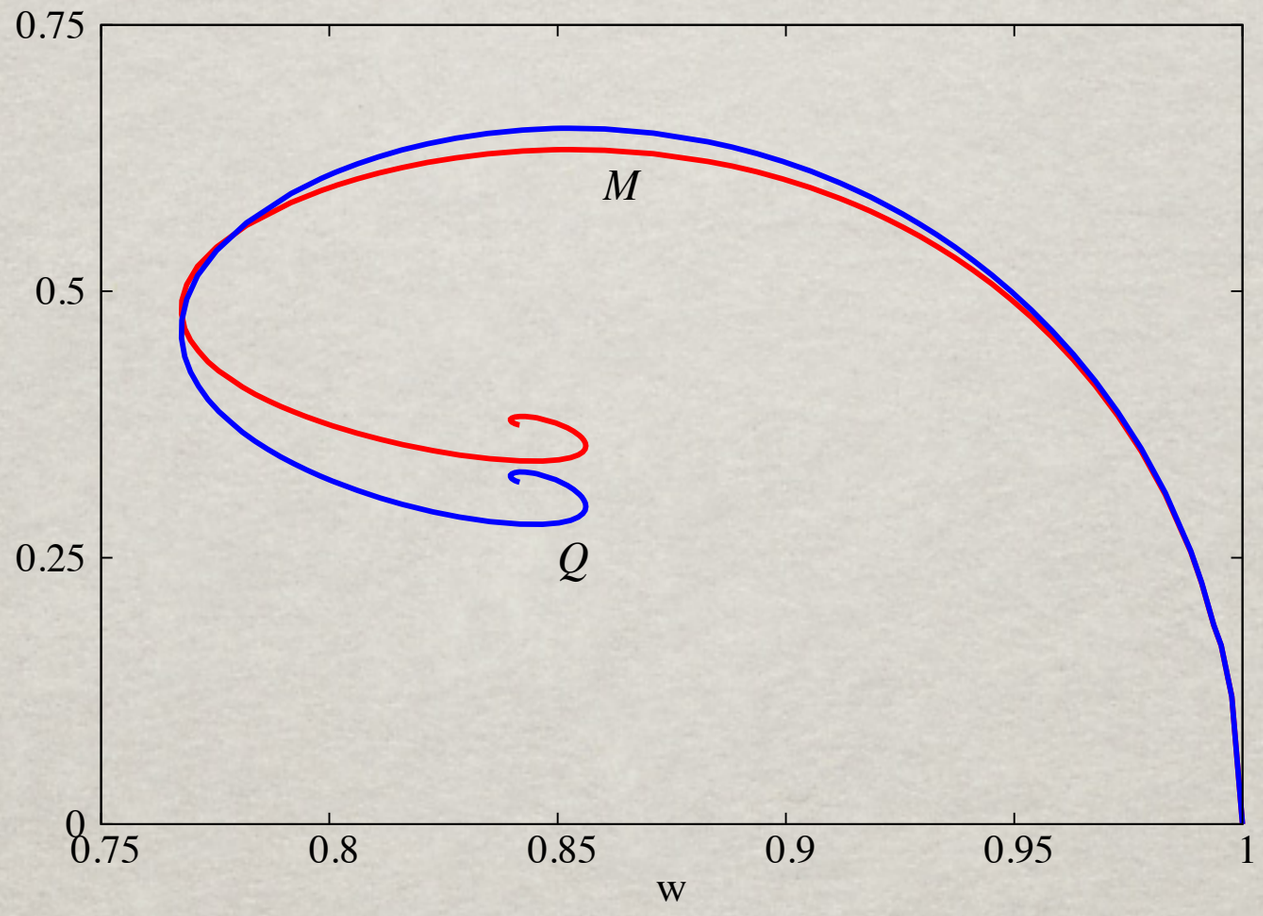


In units of μ

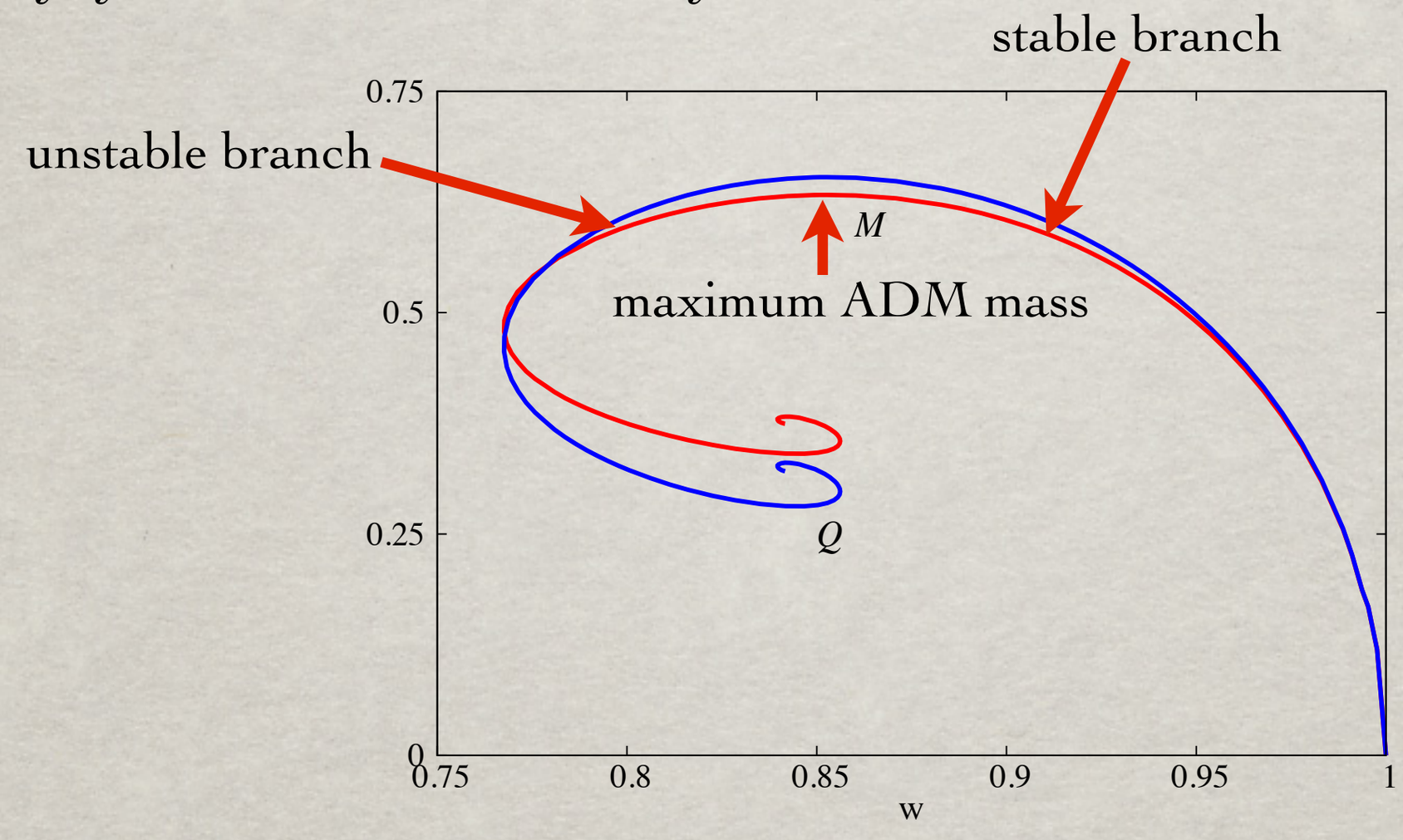
$M\mu \to M \ , \quad w/\mu \to w$

- This spiral corresponds to nodeless solutions. These are regarded as fundamental modes. Excited solutions also exist.

- Along the spiral, M becomes larger than Q. This signals instability, since there is **excess**, rather than **binding**, energy.

Gravitating scalar solitons: boson stars

Spherically symmetric solutions: Stability



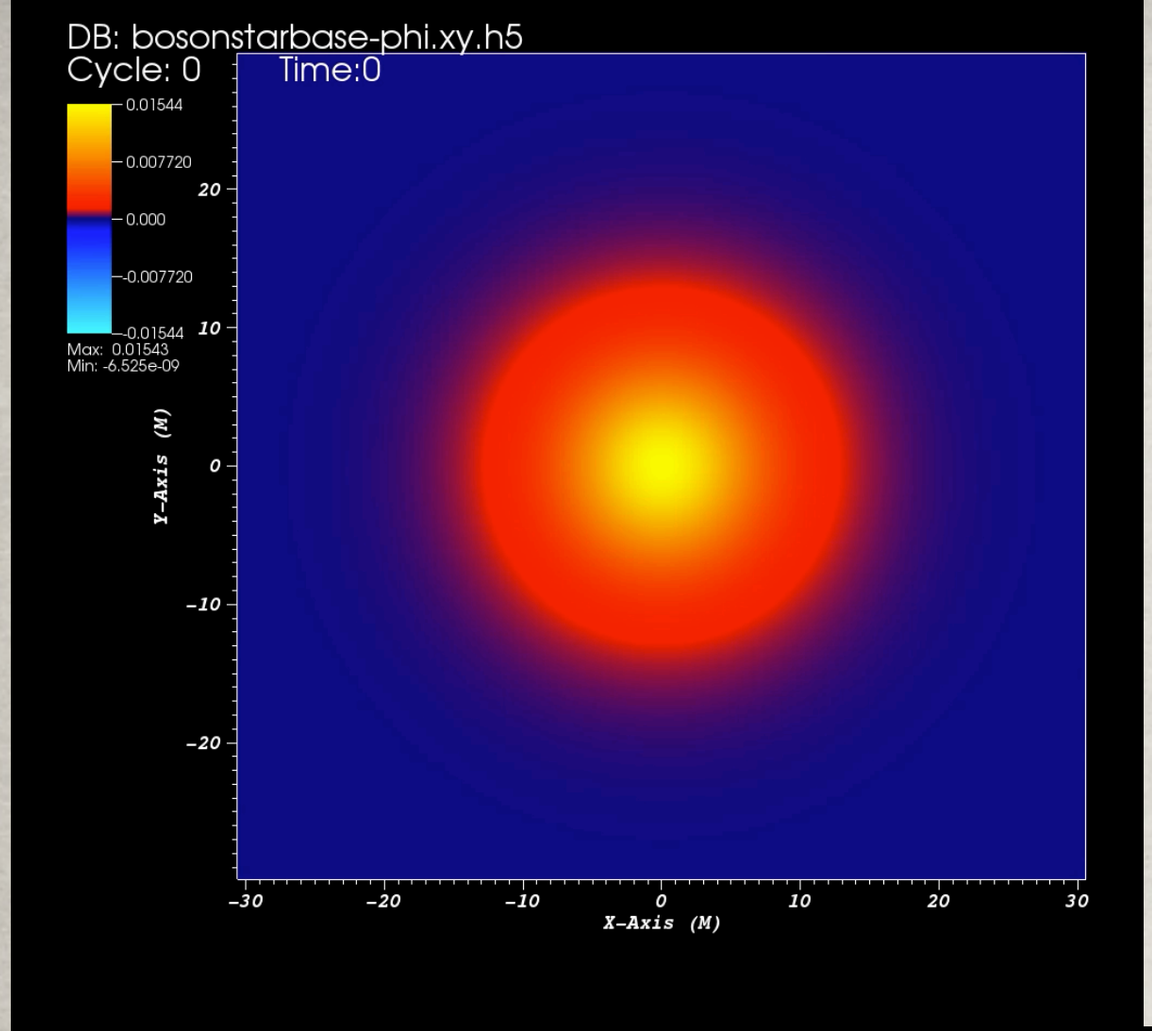
In units of μ

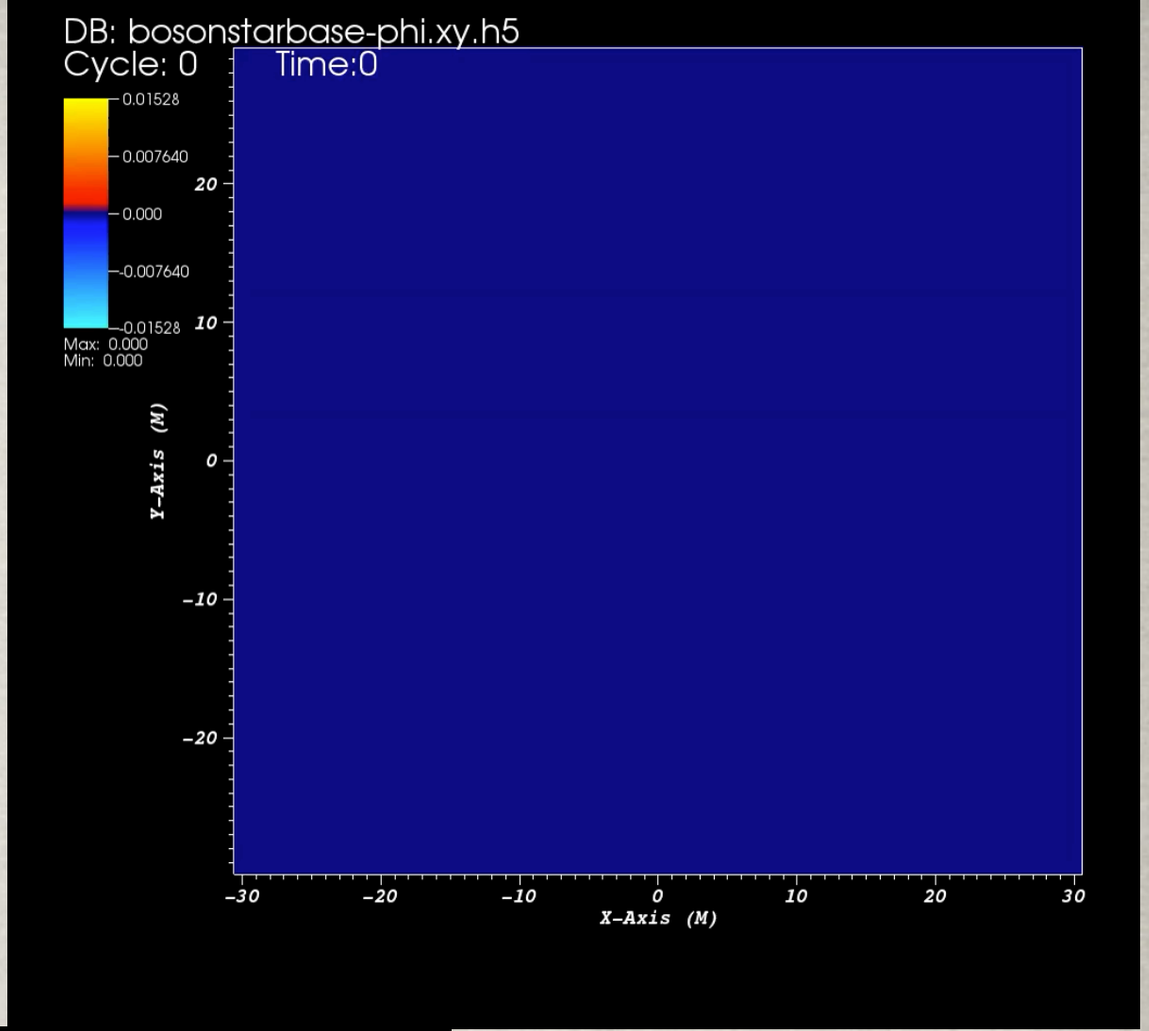
$M\mu \to M \ , \quad w/\mu \to w$

Studying linearized radial perturbations of the coupled metric-scalar field system shows that an unstable mode arises precisely at the maximum of the ADM mass M. Gleiser and R. Watkins, Nucl. Phys. B319 (1989) 733; T. D. Lee and Y. Pang, Nucl. Phys. B315, 477 (1989).

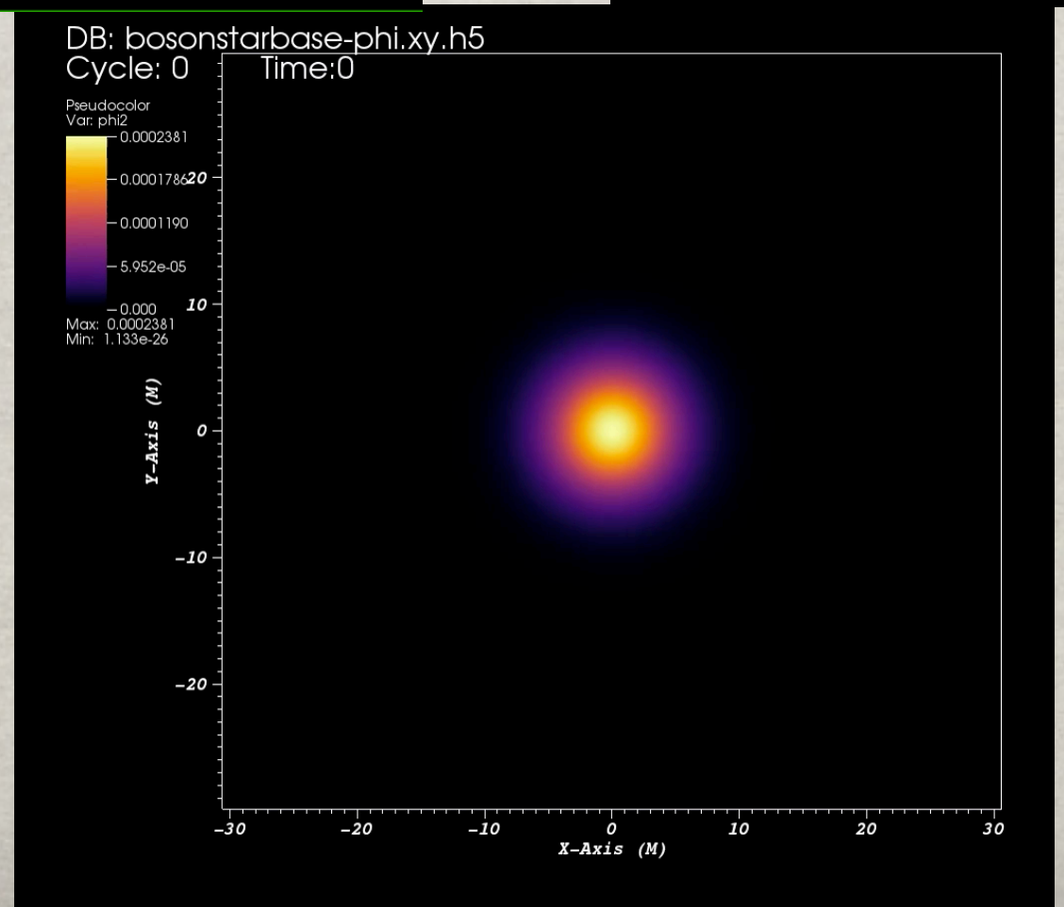
Unstable BSs can migrate, decay into a Schwarzschild black hole or disperse entirely Seidel and Suen, PRD 42 (1990) 384; Guzman, PRD 70 (2004) 044033; Hawley and Choptuik, PRD 62 (2000)104024

No-hairy-BHs in this case, cf. Pena and D. Sudarsky, Class. Quant. Grav. 14 (1997) 3131





$\mathcal{R}(\Phi)$



$\mathcal{I}(\Phi)$

w=0.95
first branch

Simulations
Zilhão (2017)

$|\Phi|^2$

Scalar Boson Stars can form dynamically:

Seidel and D. Suen, Phys. Rev. Lett. 72 (1994) 2516

Initial data: a coherent complex scalar field: $\Phi = 0.0025 e^{-r^2/90^2}$, $\dot{\Phi} = 0.9iw$

Evolution:
time in units of $1/\mu$

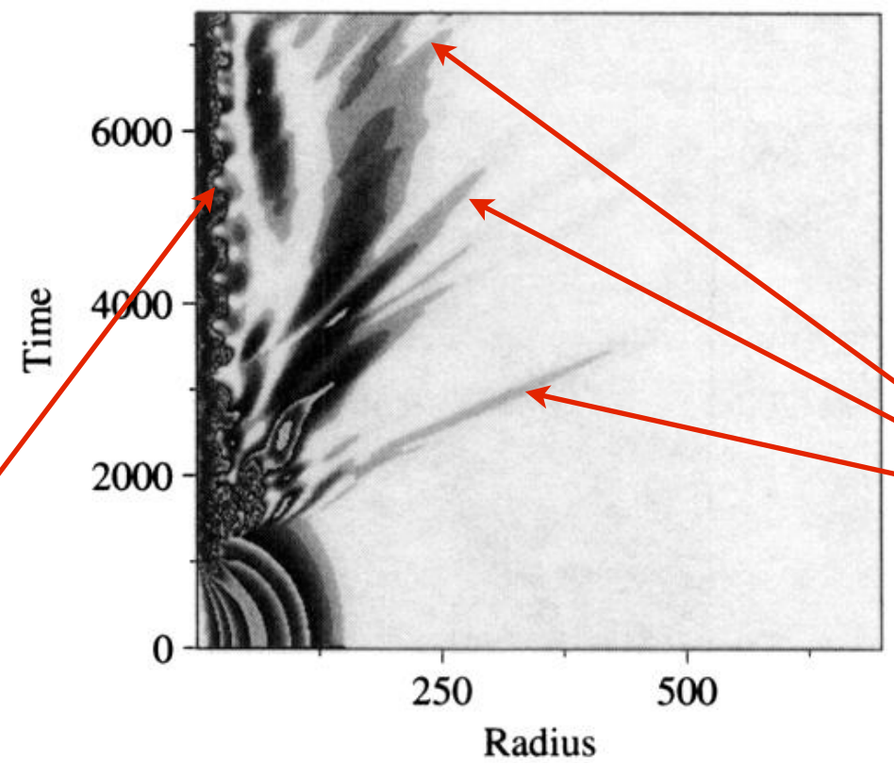


FIG. 1. The evolution of $r^2\rho$ for a massive, self-gravitating complex scalar field is shown. Because of the self-gravitation, the field collapses quickly and a perturbed boson star is formed at the center. The star oscillates and begins to settle down as scalar material is radiated through the gravitational cooling process discussed in the text.

Gravitational cooling:
ejected scalar field "radiation" carries excess kinetic energy
(analogous to violent relaxation in collisionless stellar systems)

Final state:
(perturbed) boson star with mass

$$\sim 0.56 \frac{M_{Pl}^2}{\mu}$$

$$M_i \sim 0.644 \frac{M_{Pl}^2}{\mu} > 0.633 \frac{M_{Pl}^2}{\mu}$$

(maximal mass for boson stars in this model; without cooling a BH would form)

$$\frac{M_i - M_f}{M_i} \sim 0.13$$

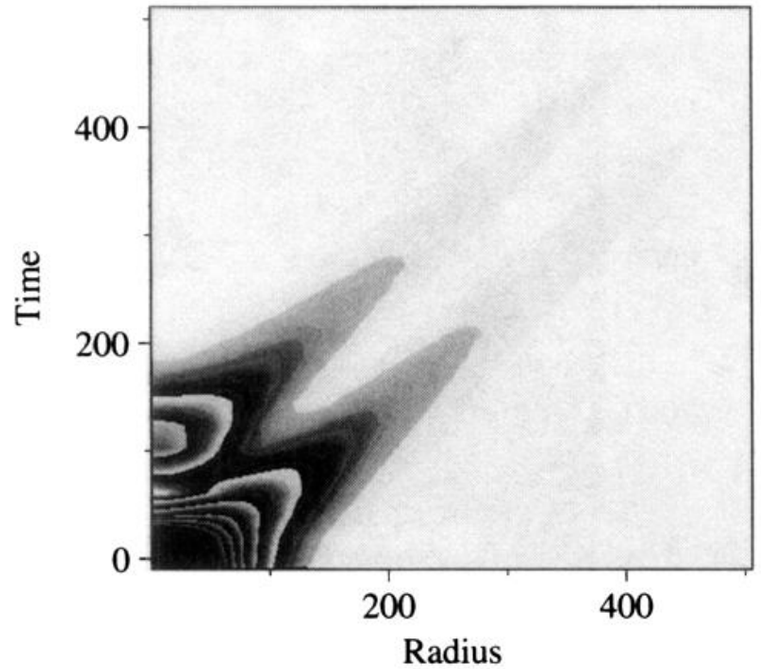


FIG. 2. To contrast the formation process for a massive field, the evolution of the energy density ρ for a massless, self-gravitating complex scalar field is shown. No boson star can form for a massless field.

For **massless** field there is complete dispersion: collapse, rebounces and complete dispersion to infinity

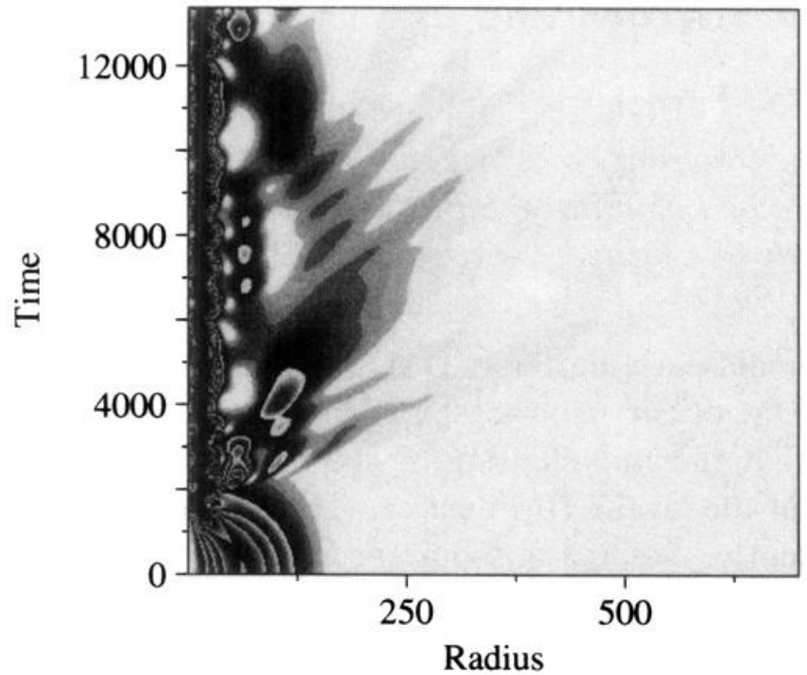


FIG. 4. The evolution of $r^2\rho$ for a massive, self-gravitating real scalar field is shown. The evolution is very similar to that shown in Fig. 1 for a complex configuration with the same initial profile for ϕ.

Similar results for a **real** (rather than complex) massive scalar field:
an oscillaton (rather than boson star) forms

Result holds for generic initial data;
main restriction spherical symmetry

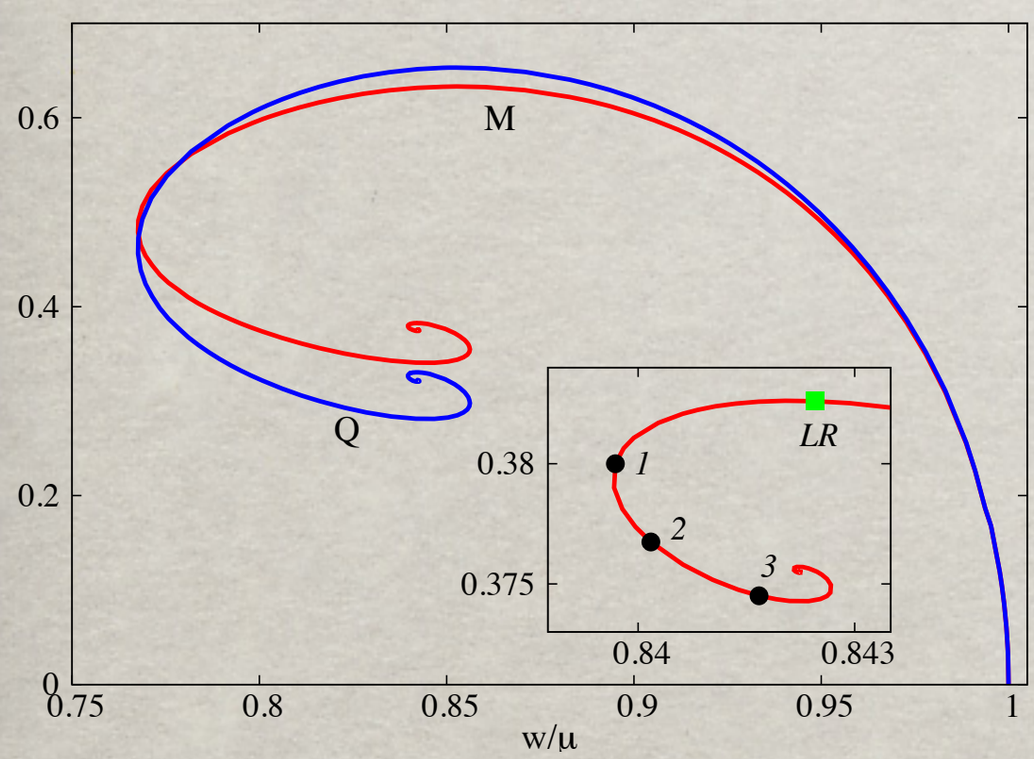
The vector cousin: spherical Proca stars

Brito, Cardoso, Herdeiro and Radu, Phys. Lett. B 752 (2016) 291

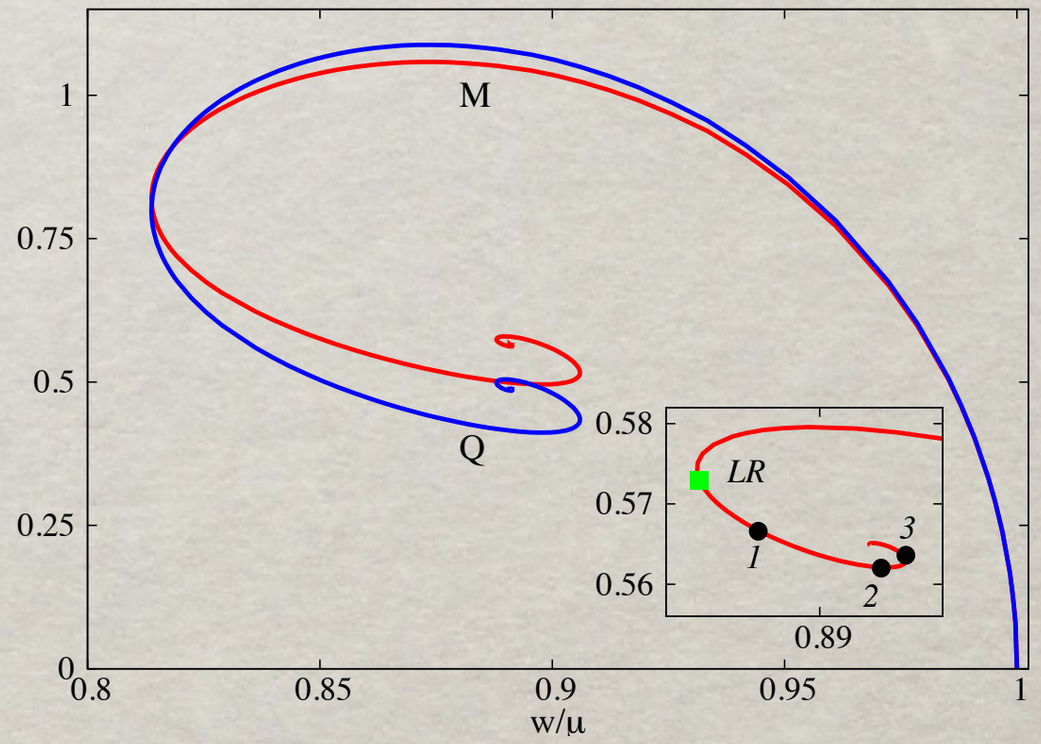
$$\mathcal{S} = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi G}R - \frac{1}{4}\mathcal{F}_{\alpha\beta}\bar{\mathcal{F}}^{\alpha\beta} - \frac{1}{2}\mu^2\mathcal{A}_\alpha\bar{\mathcal{A}}^\alpha\right) .$$

A similar construction holds yielding spherical solitonic objects: spherical Proca stars

scalar



vector/Proca

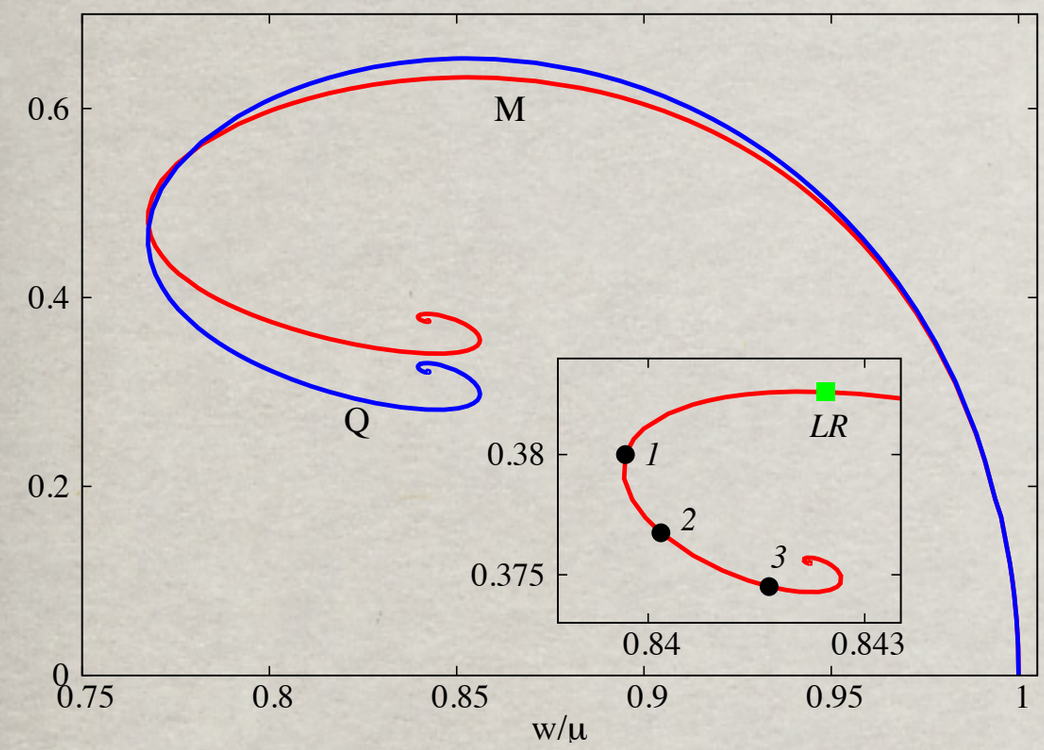


Very similar domain of existence;
Energetic instability around minimal frequency;
Similar structure of fundamental family and excited states;
but in Proca case $\mathcal{A}_0$ has at least one node.

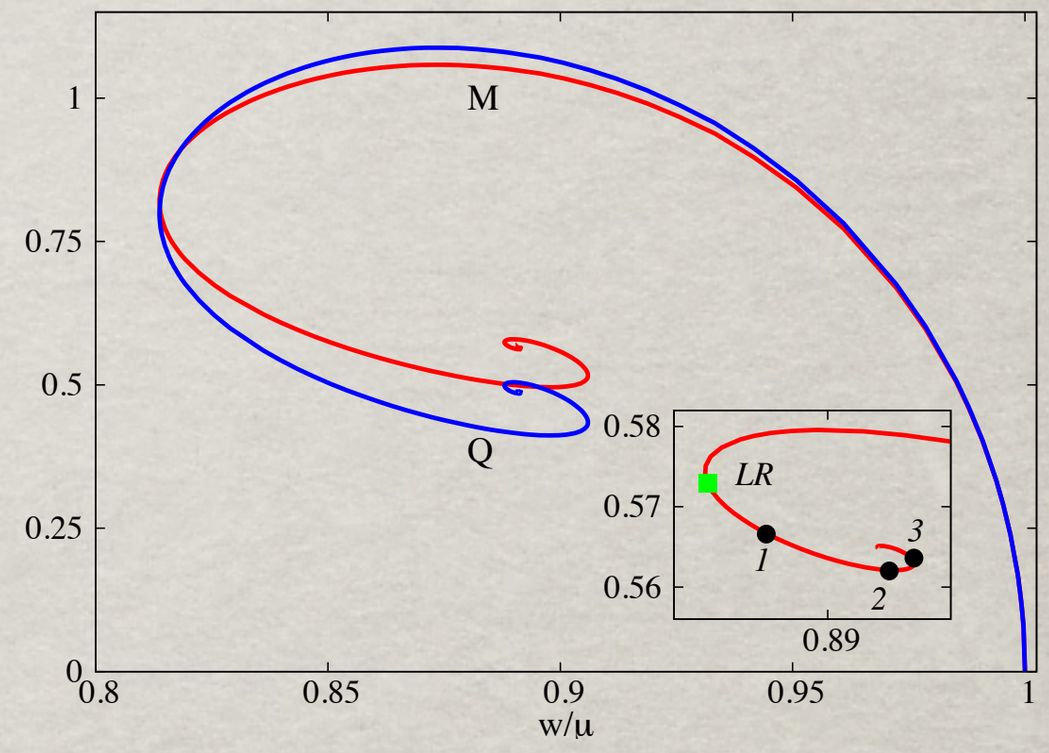
Ultra-compact spherical Proca stars

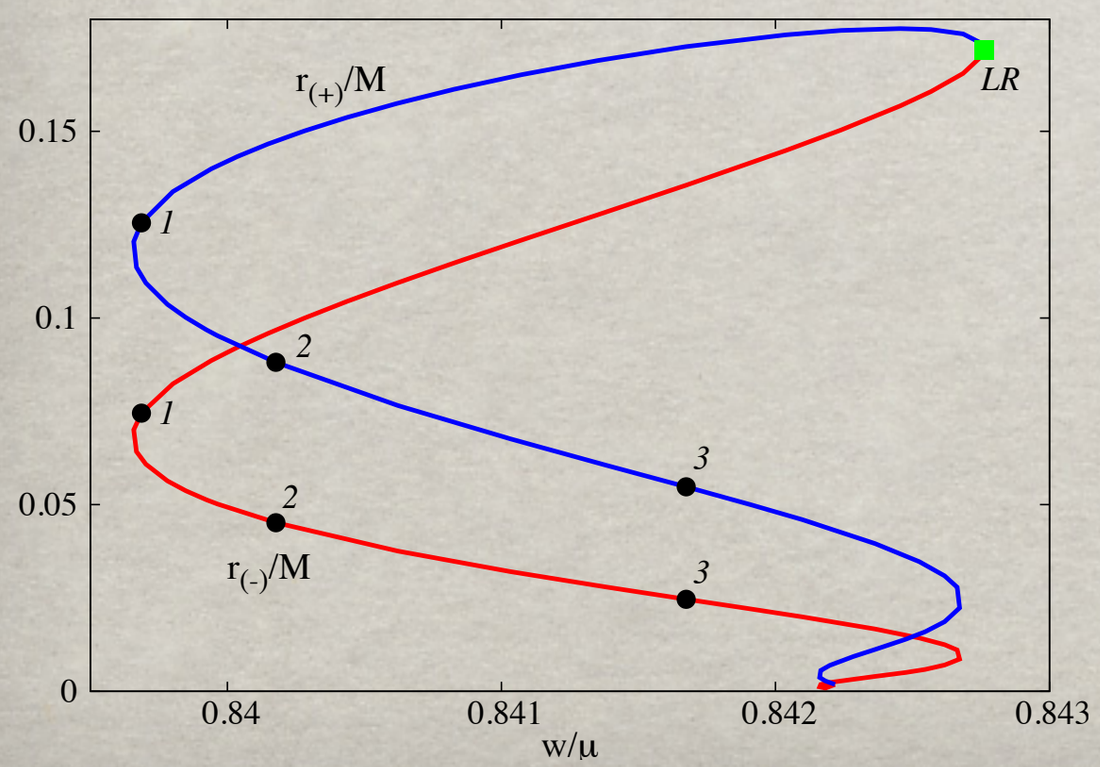
Cunha, Font, Herdeiro, Radu, Sanchis-Gual, Zilhão, PRD 96 (2017) 104040

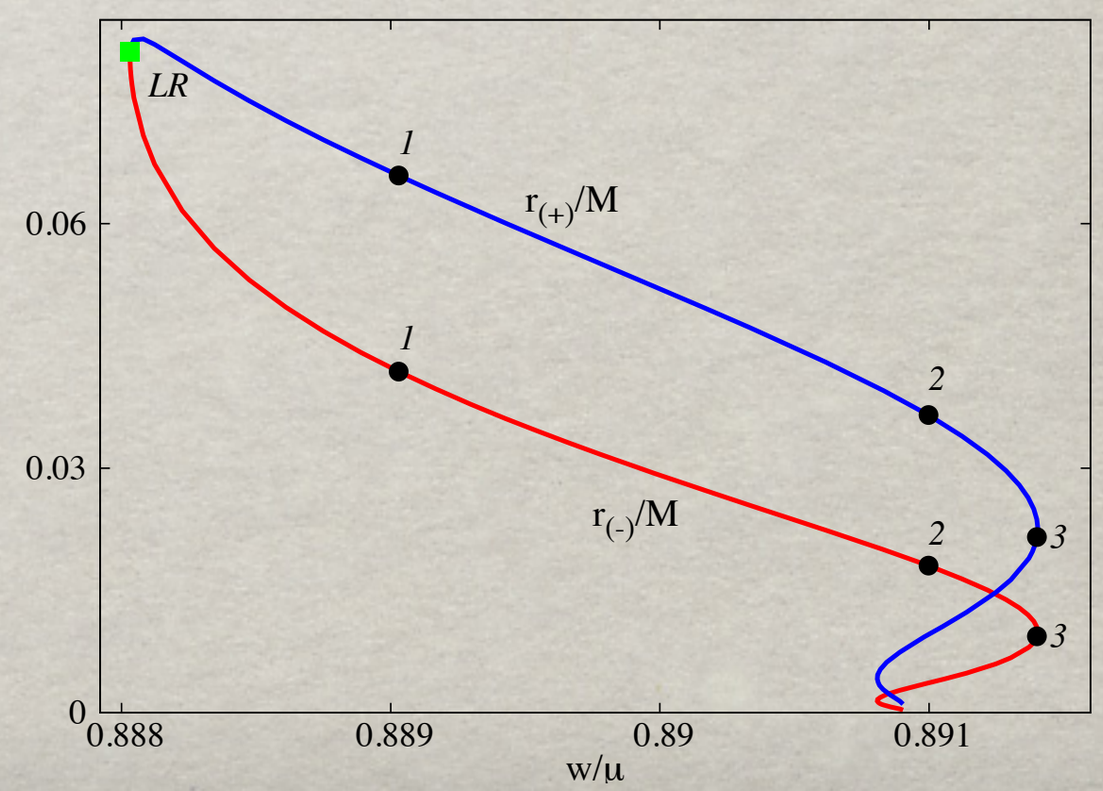
scalar



vector/Proca







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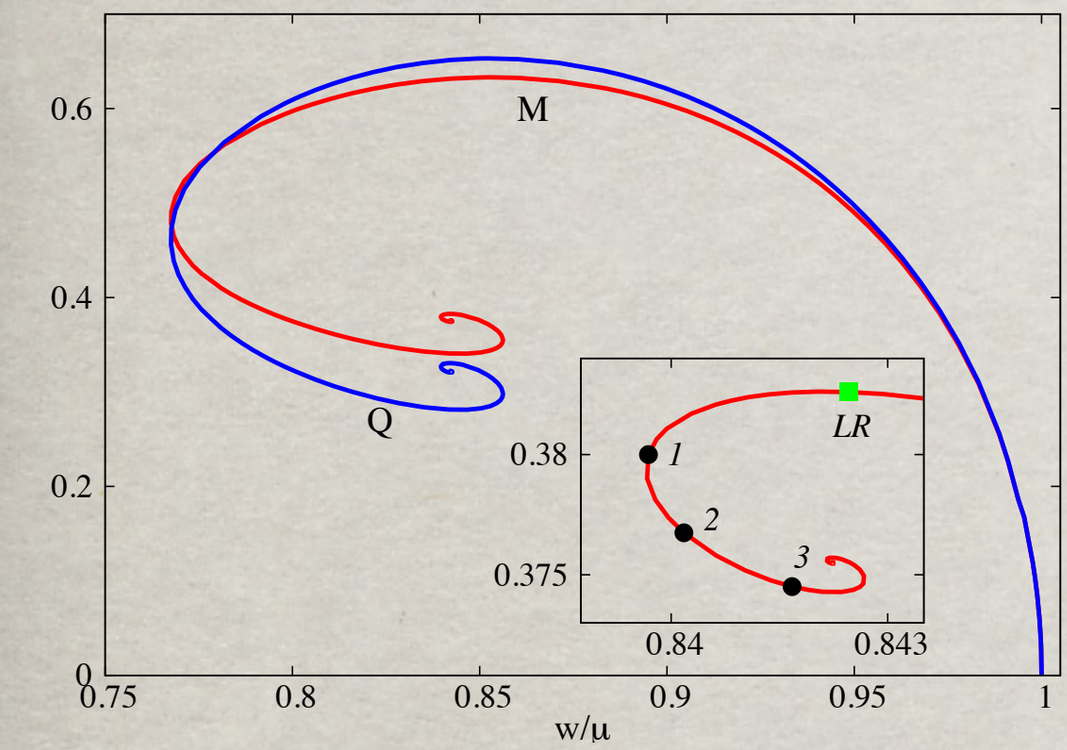
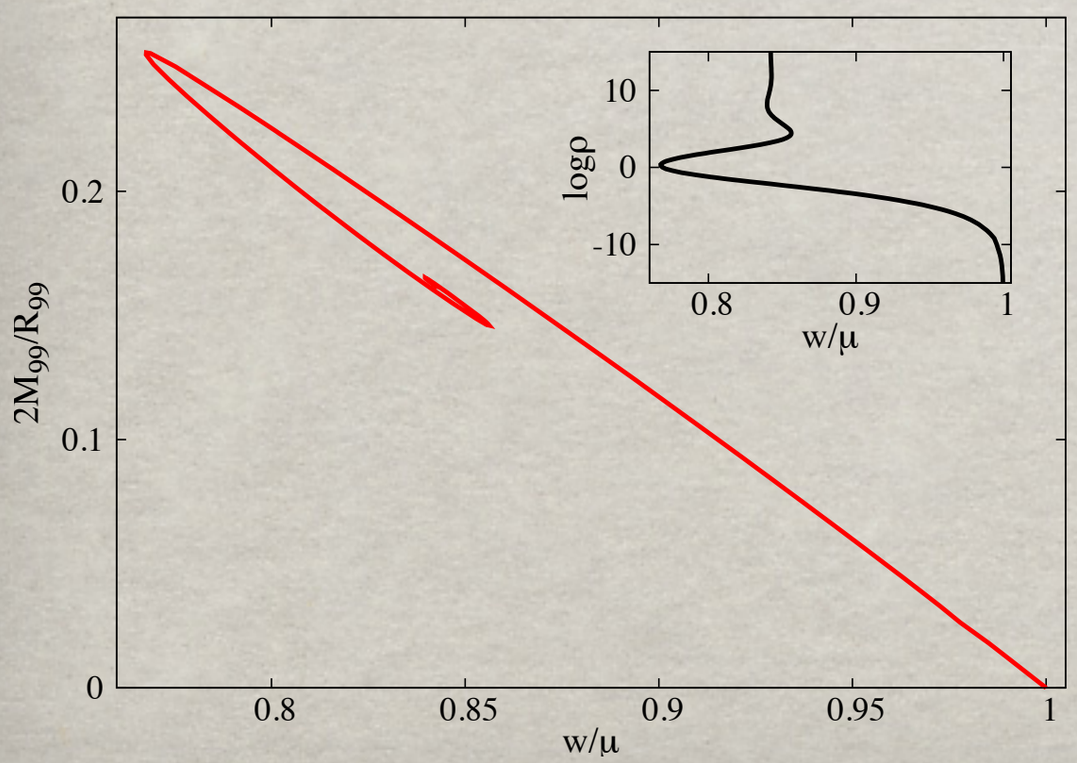
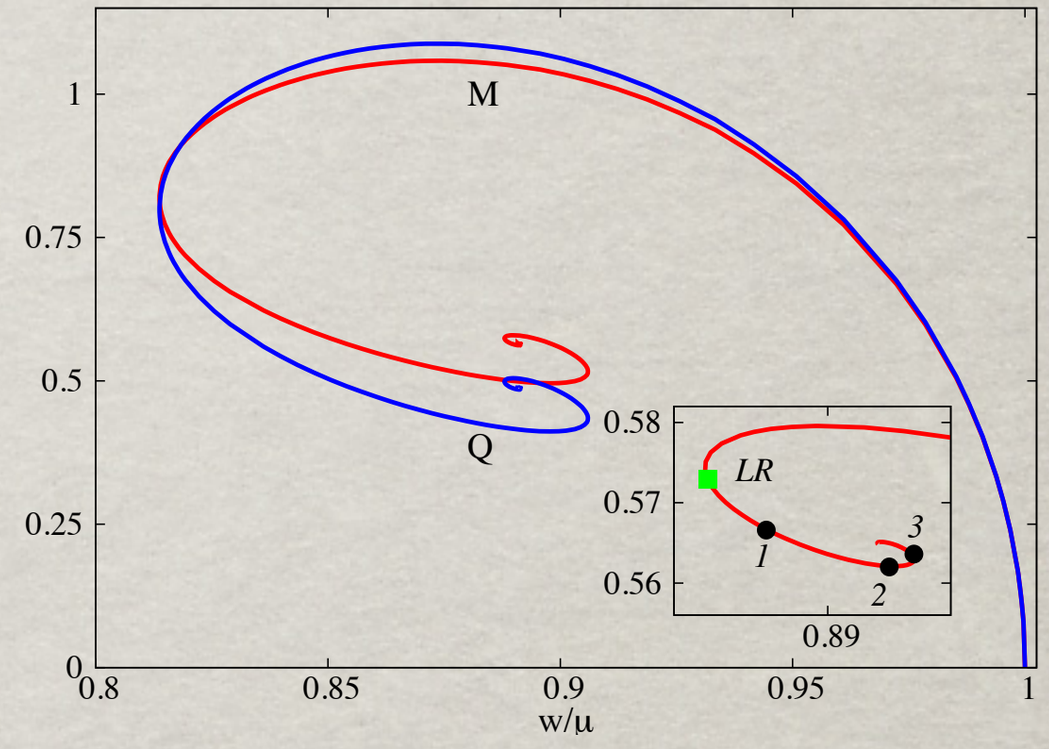
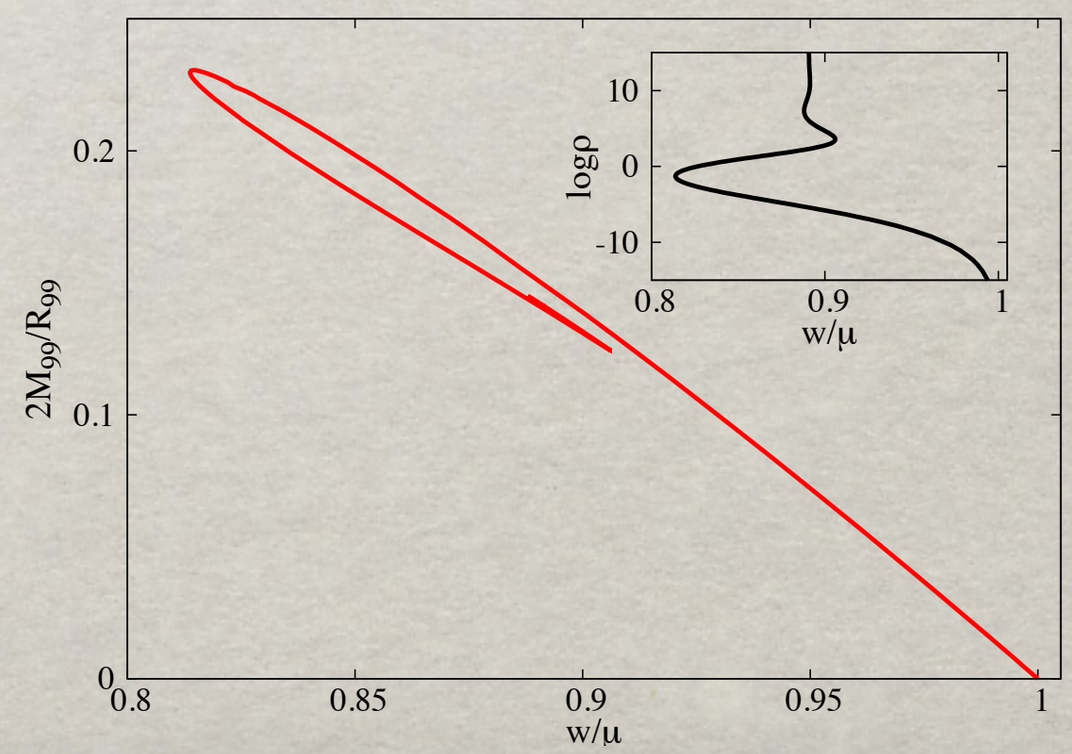
scalar





vector/Proca





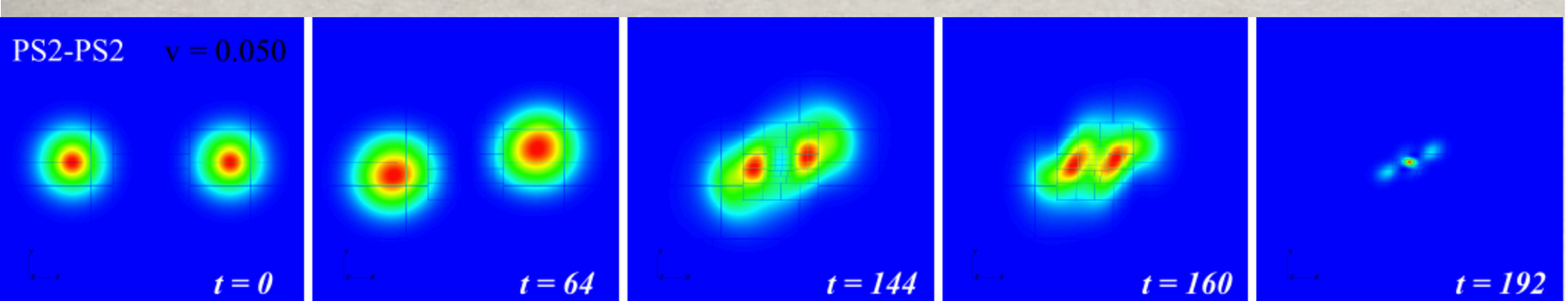
Dynamics of spherical Proca stars

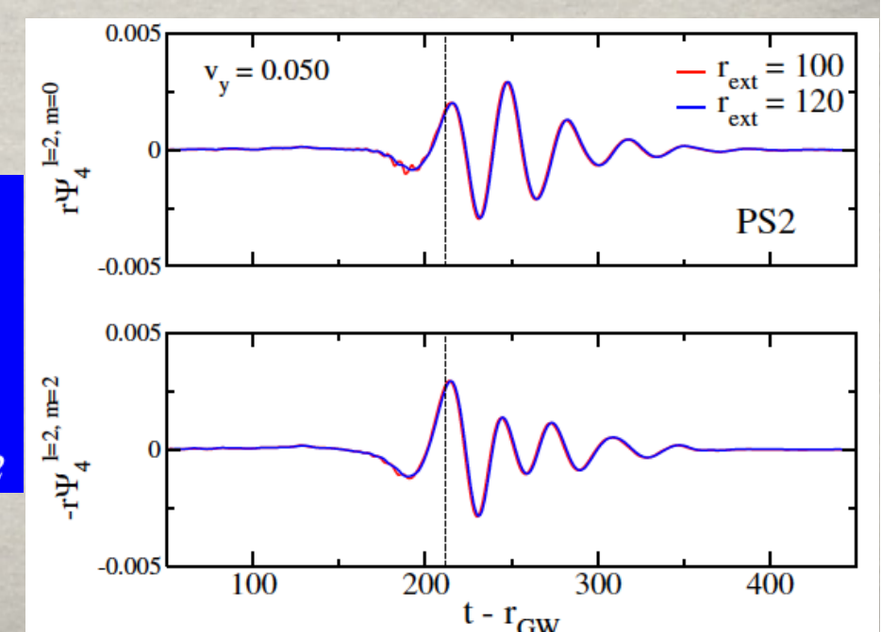
1) As in the scalar case, vector boson stars are perturbatively stable up to the maximal mass; then they share the same three possible fates: migration, collapse or dispersion
Brito, Cardoso, Herdeiro and Radu, Phys. Lett. B 752 (2016) 291

2) As in the scalar case, ultracompact vector boson stars are unstable against collapse
Cunha, Font, Herdeiro, Radu, Sanchis-Gual, Zilhão, PRD 96 (2017) 104040

3) As in the scalar case, vector boson stars can form dynamically via gravitational cooling
Di Giovanni, Sanchis-Gual, Herdeiro and Font, PRD 98 (2018) 064044

4) As in the scalar case, one can study binaries of spherical Proca stars and their gravitational wave emission
Sanchis-Gual, Herdeiro, Font, Radu and Di Giovanni, Phys. Rev. D 99 (2019) 024017





Stable model; apparent horizon forms at t~200

Lecture 4

I) Spherical boson (scalar/vector) star solutions

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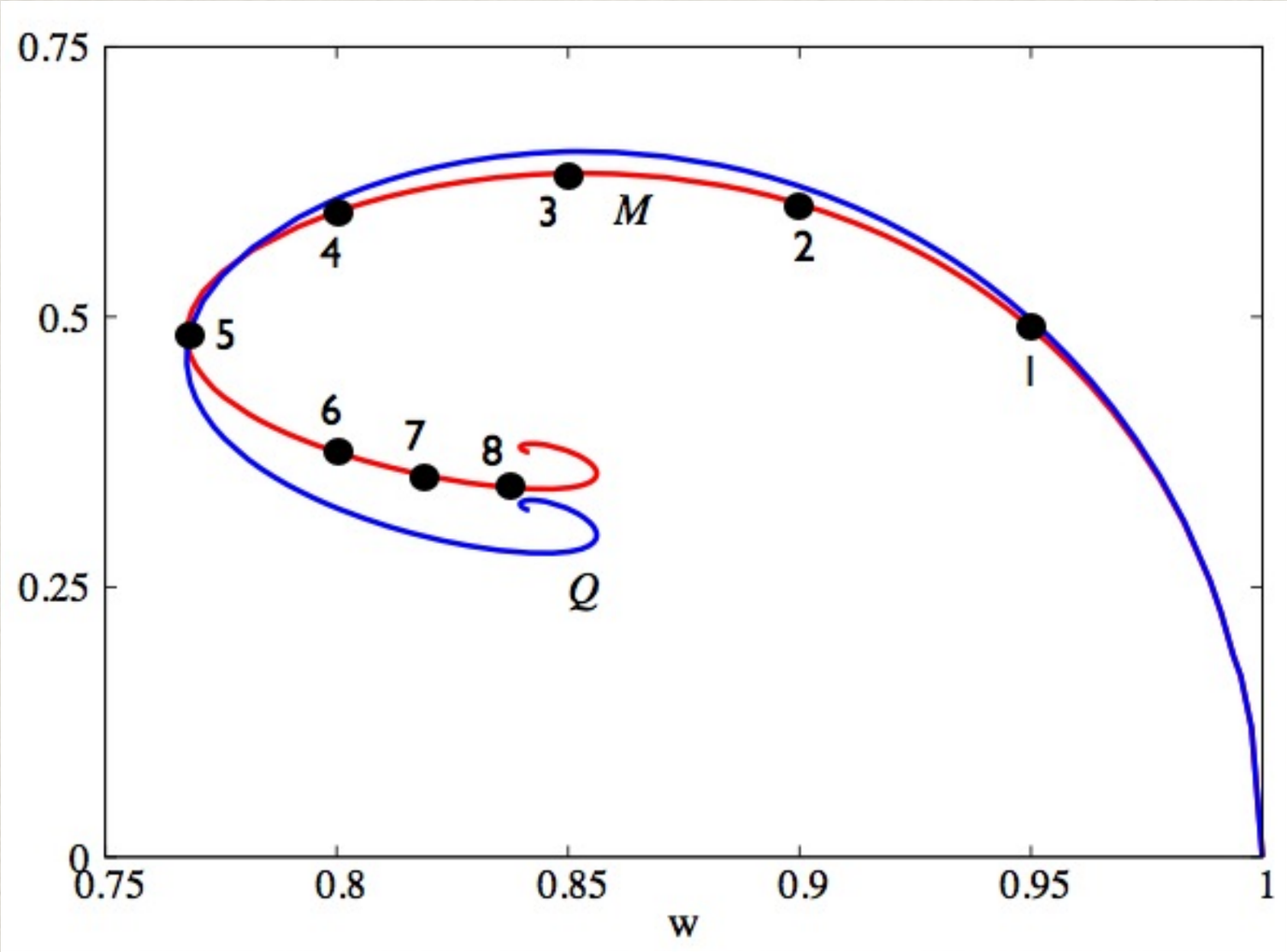
Lensing by spherically symmetric scalar boson stars.
We assume only minimal interaction between light and the scalar field.



We take

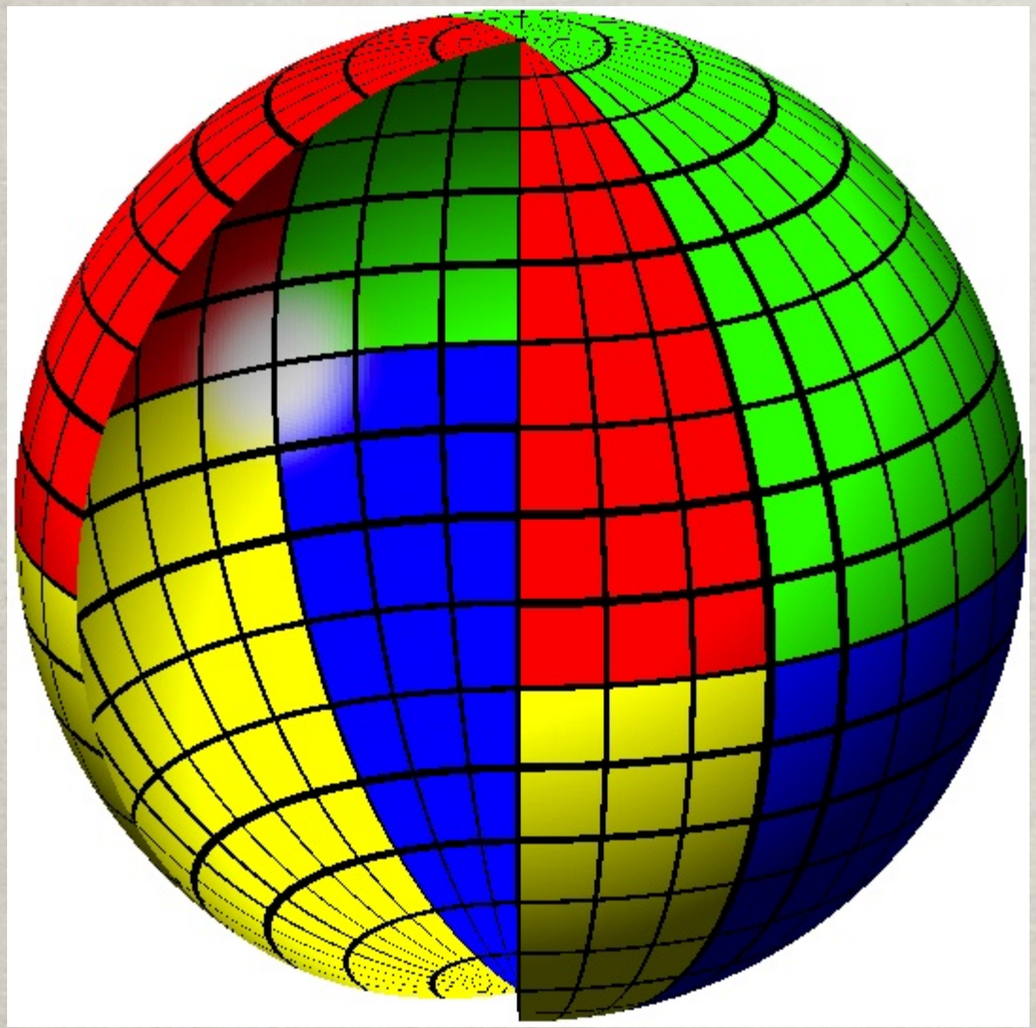
$$\mu = constante$$

so the solutions have varying mass M

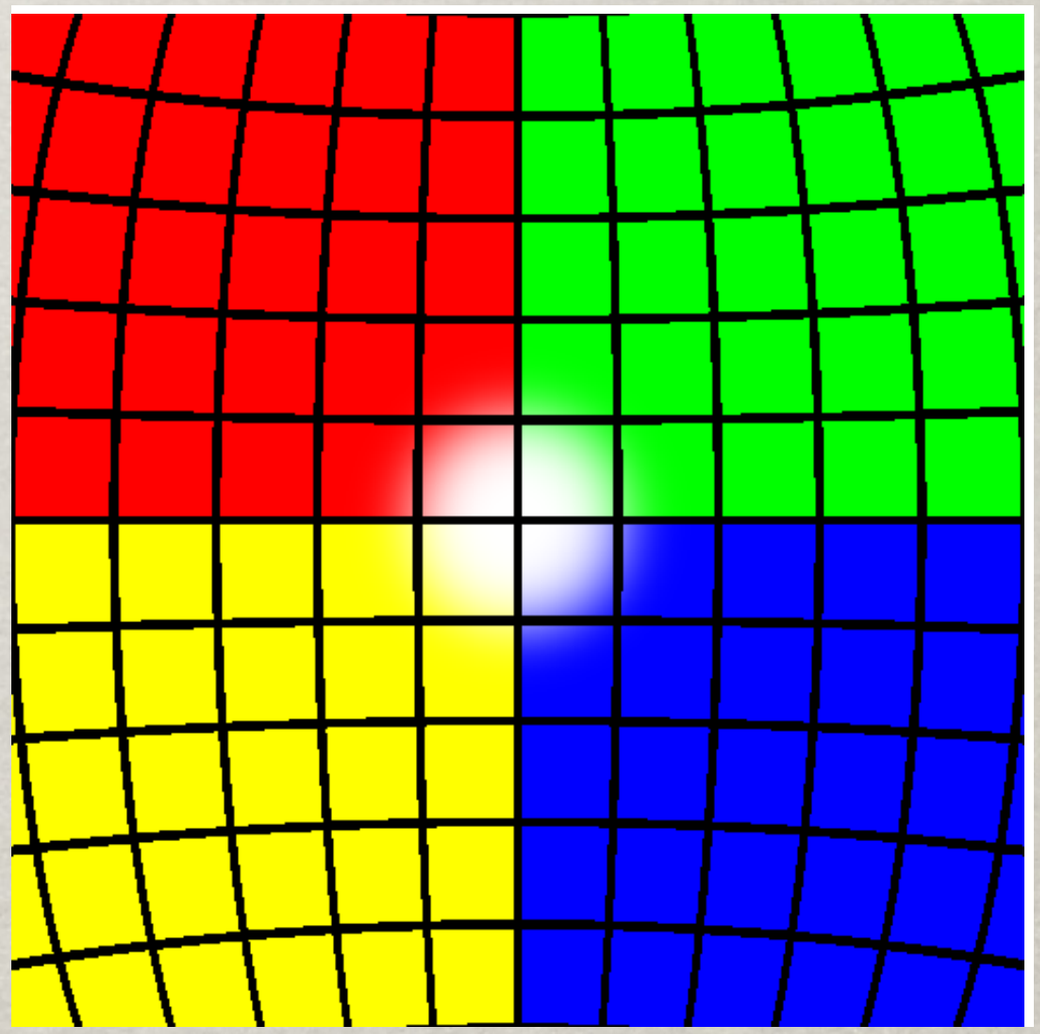
We have performed ray tracing to compute lensing and shadows.

Cunha, Herdeiro, Radu, Runarsson, PRL 115 (2015) 211102

Fix the observer's distance.



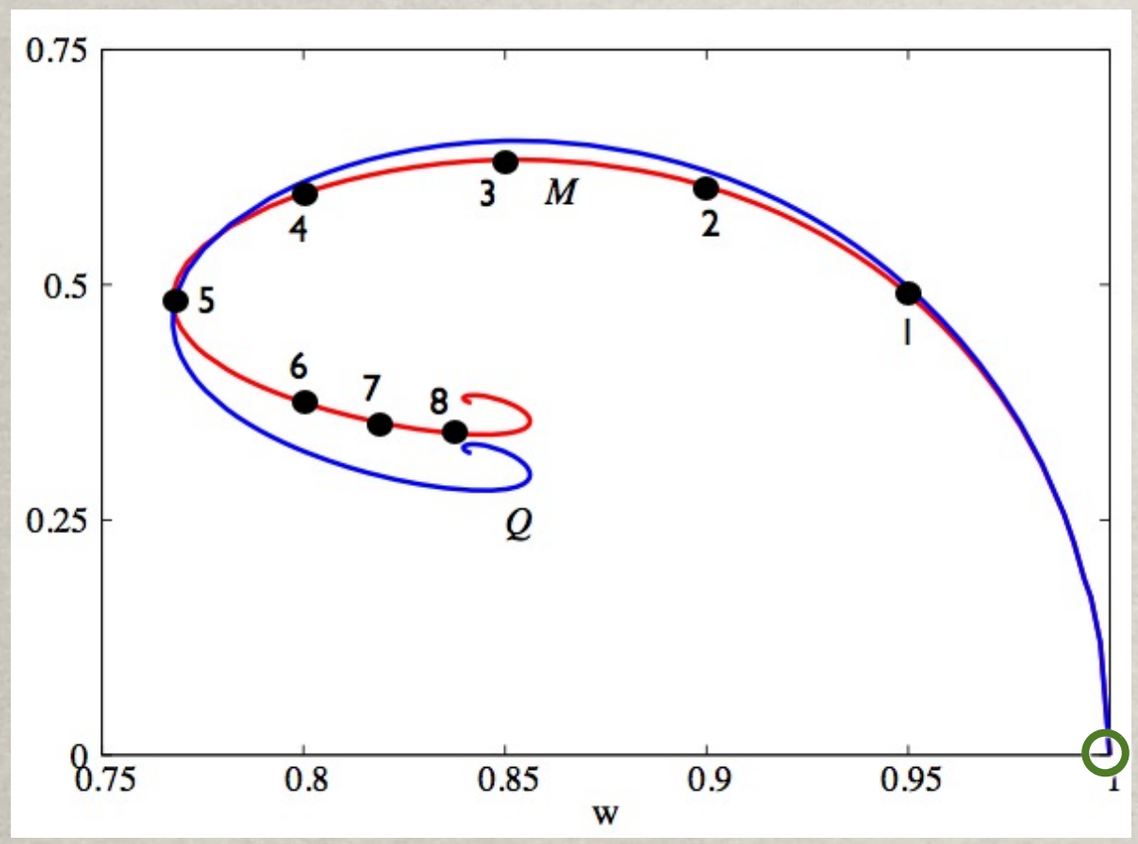
The full celestial sphere

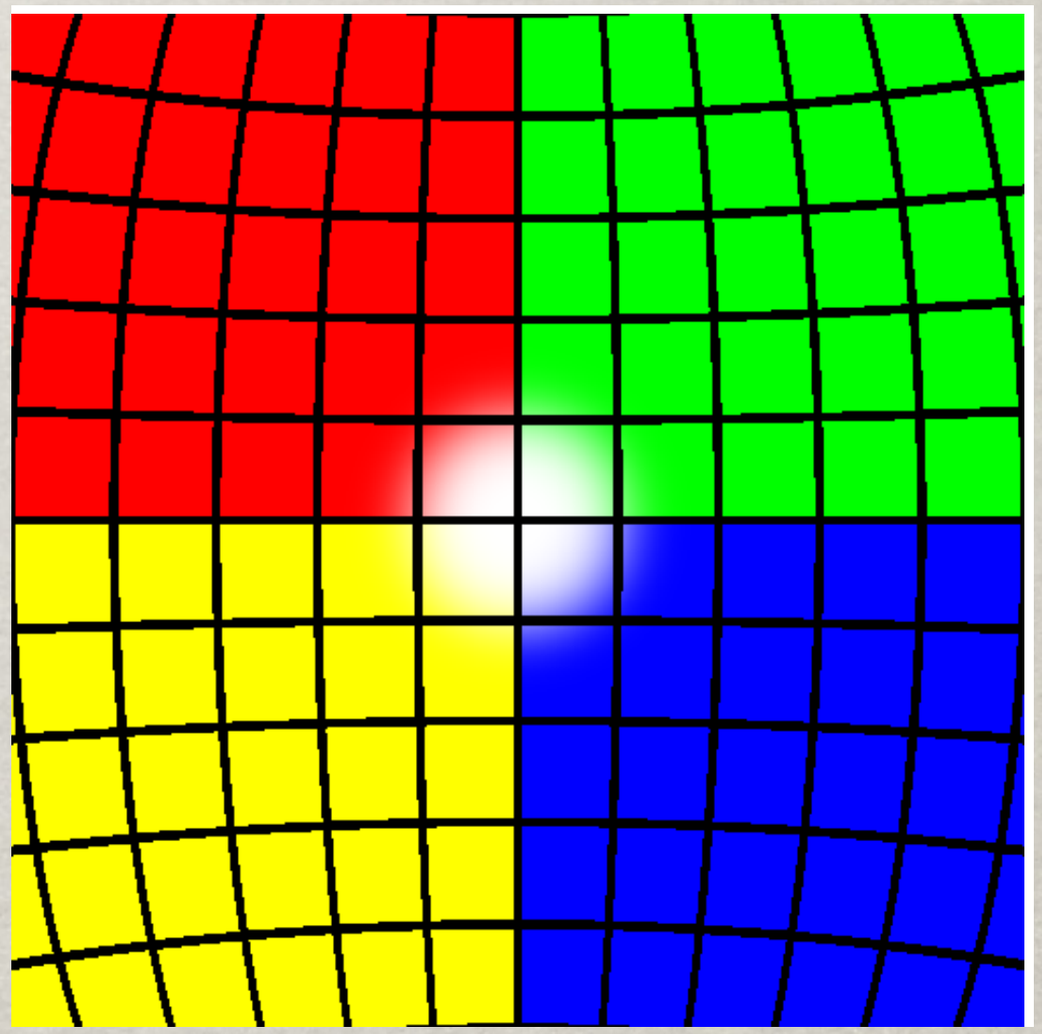


The "camera" opening angle

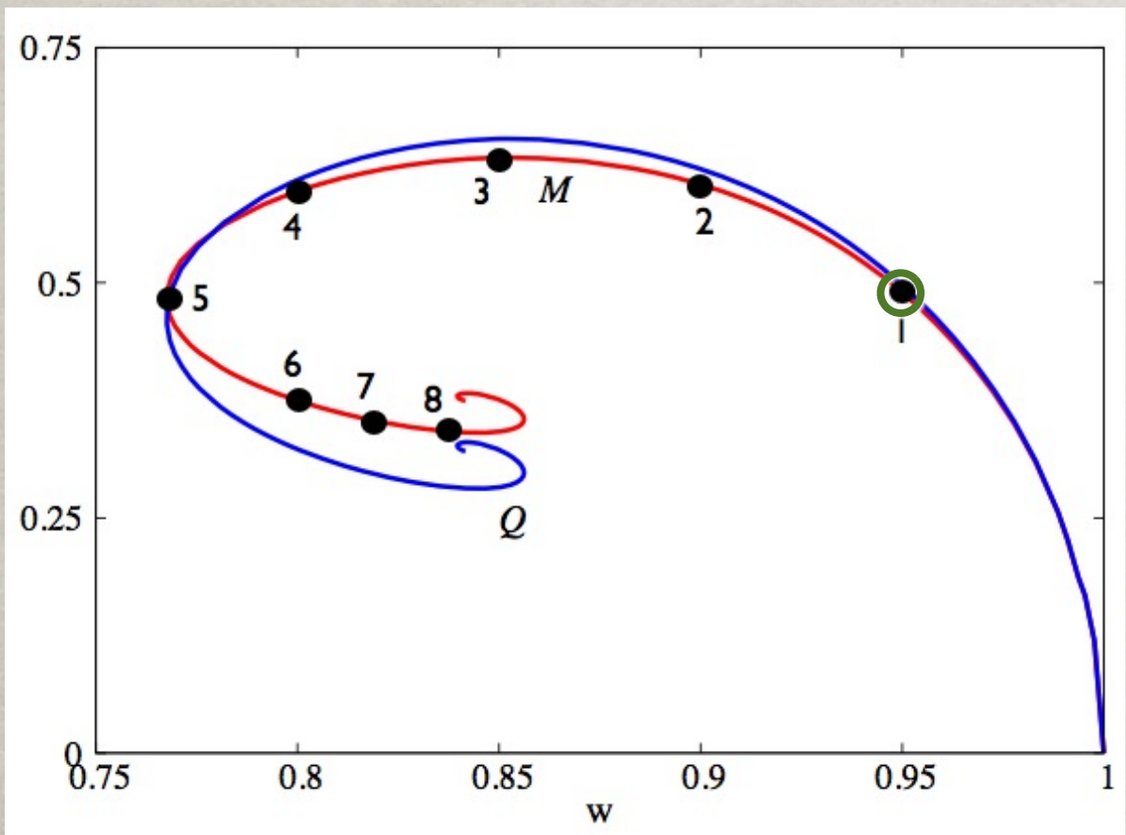
Following A. Bohn et al. CQG32(2015)065002

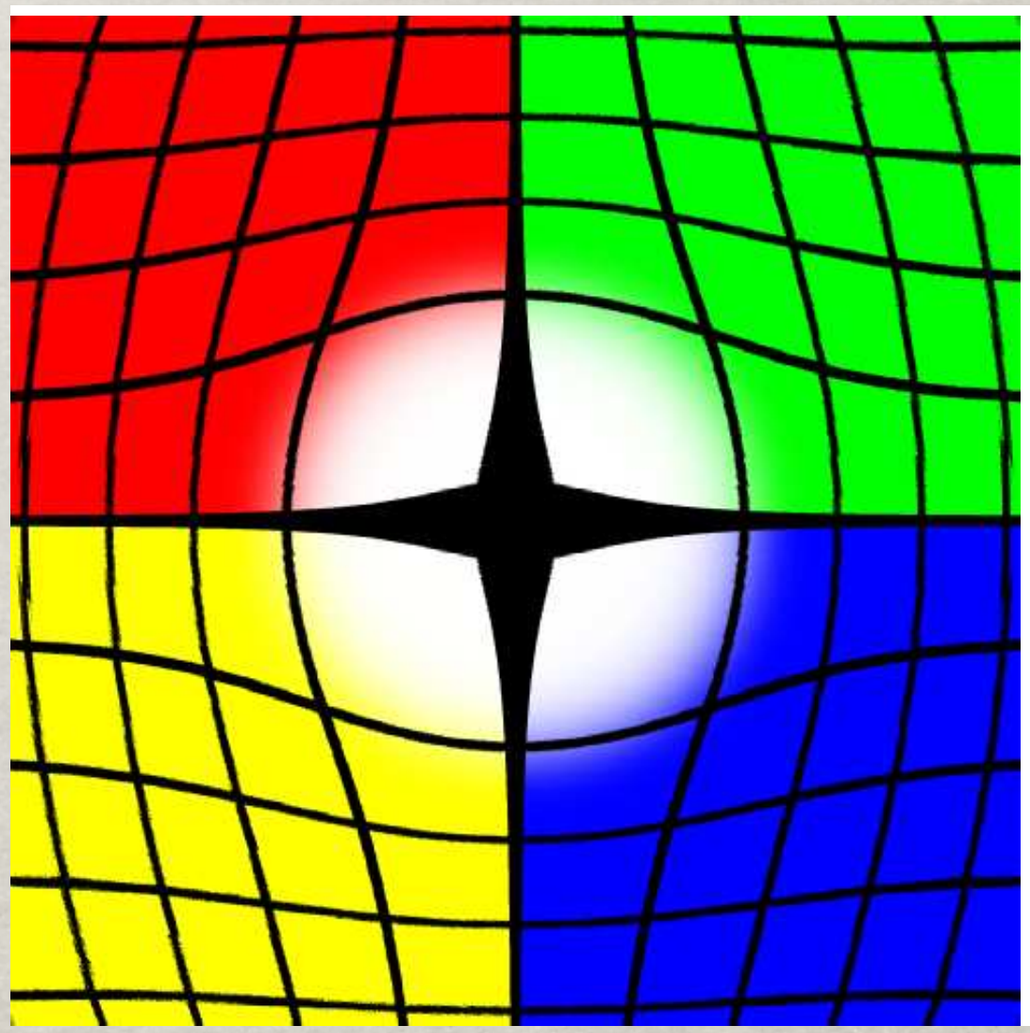
Configuration 0 (no boson star)





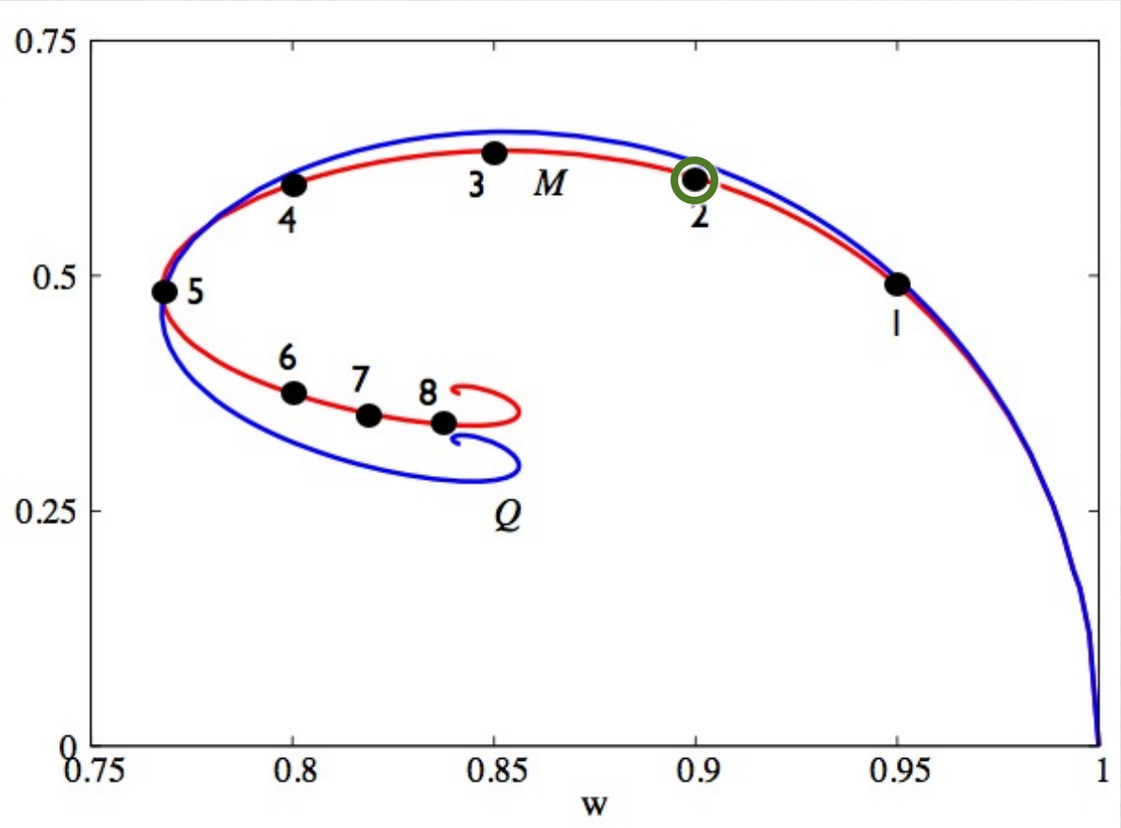
Configuration 1, first branch, w=0.95





Lensing is visible; but no multiplication of images

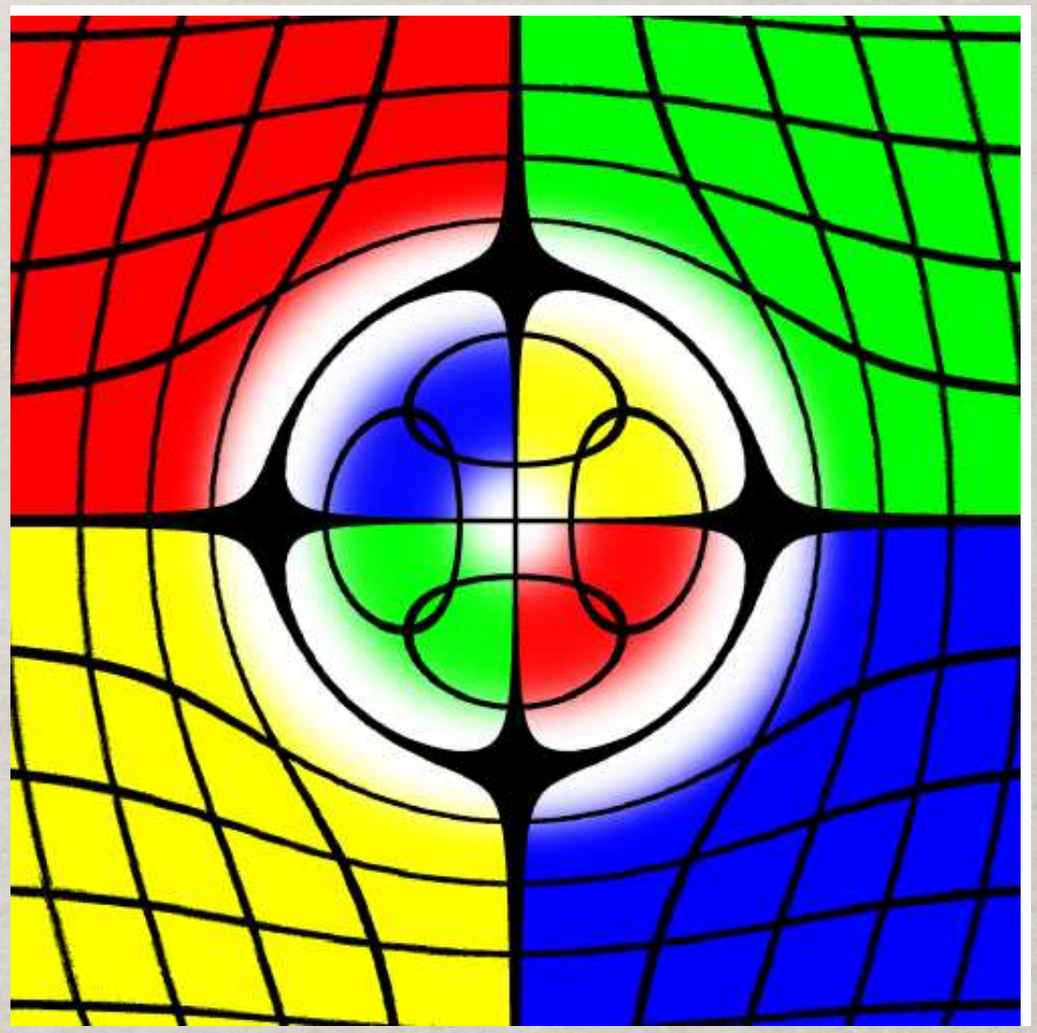
Configuration 2, first branch, w=0.9



Einstein ring appears at:

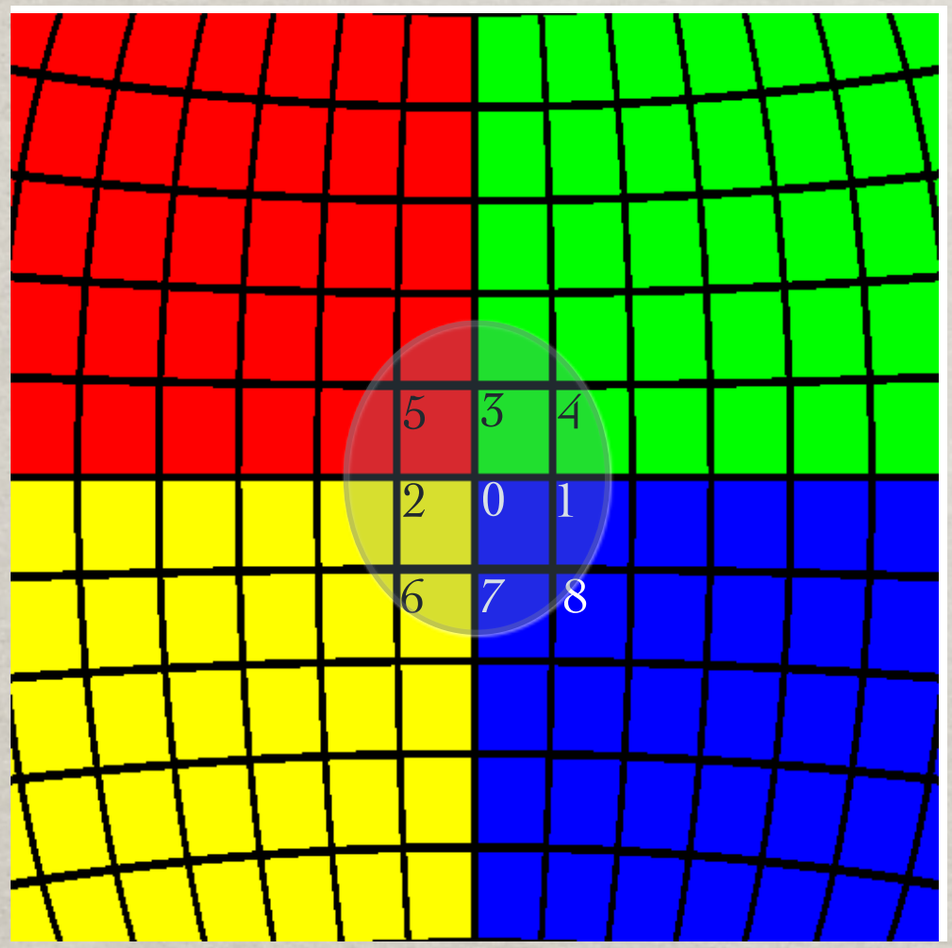
$$w_{ER1}^{(b1)} \simeq 0.94$$

Einstein rings (plus central point):
F F

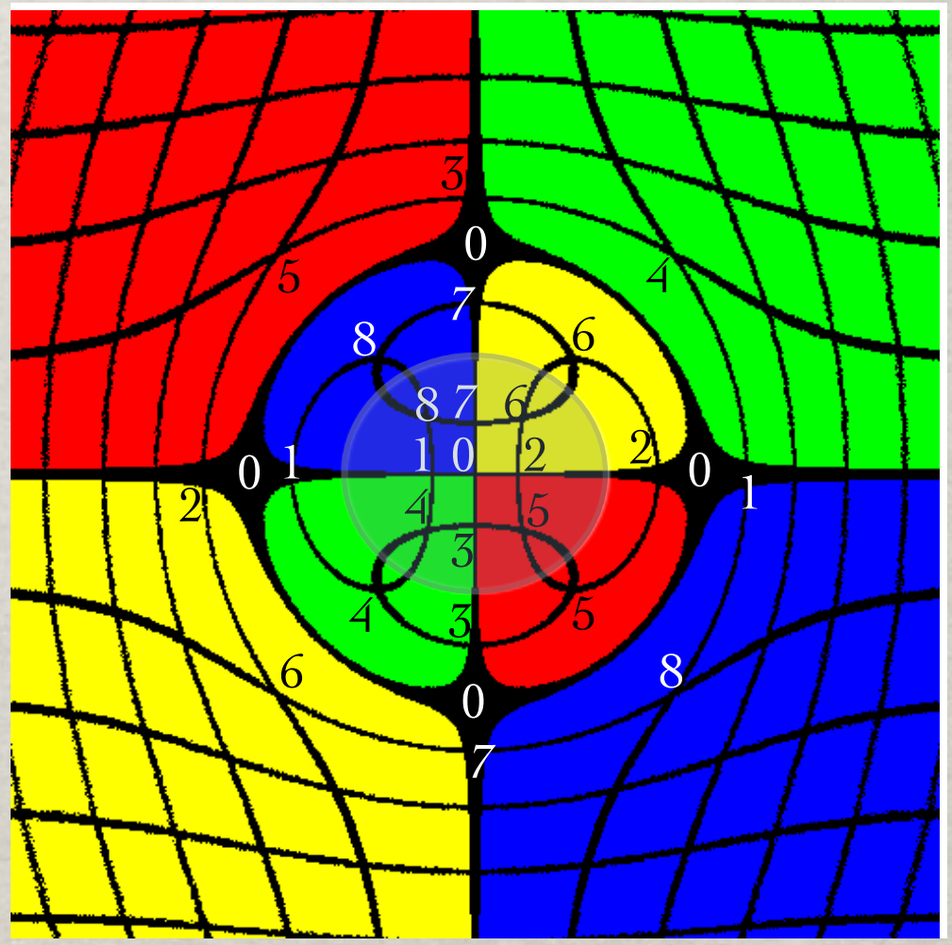


An Einstein ring arises, corresponding to the lensing of the front point (F); two further images for a part of the celestial sphere

Configuration 2, first branch, w=0.9

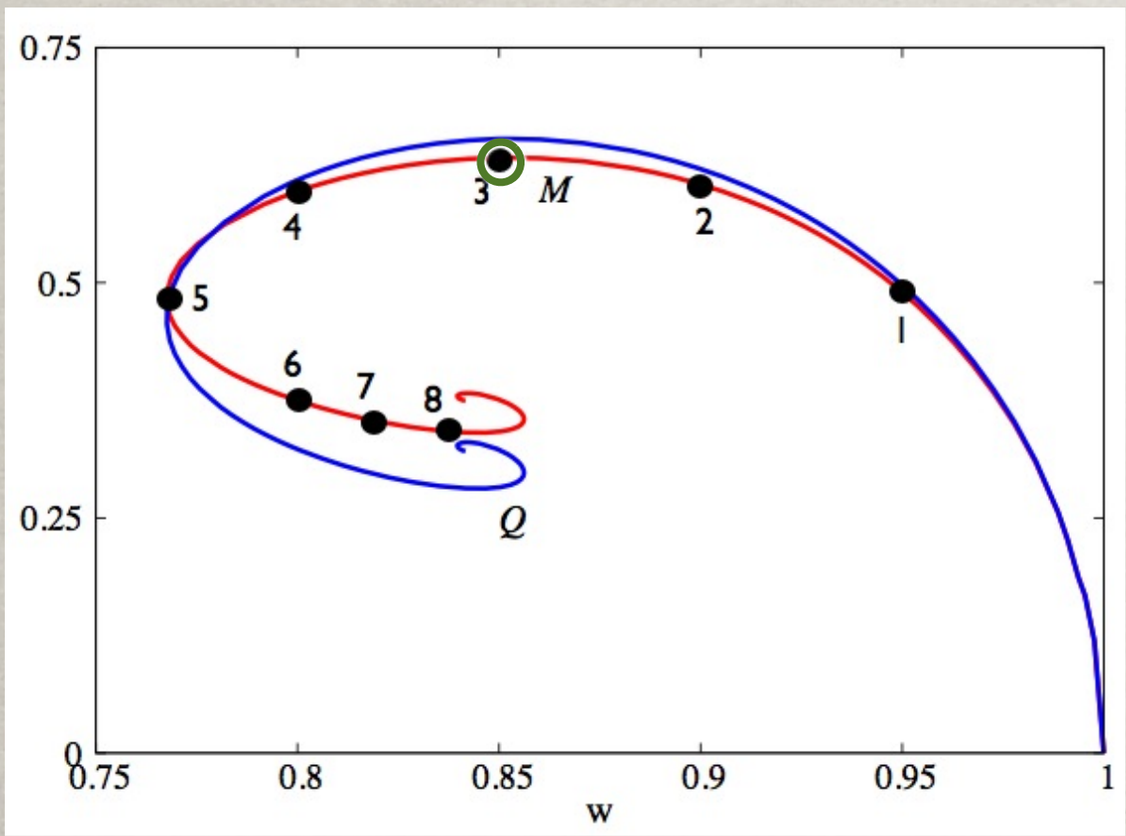


Region with two extra images

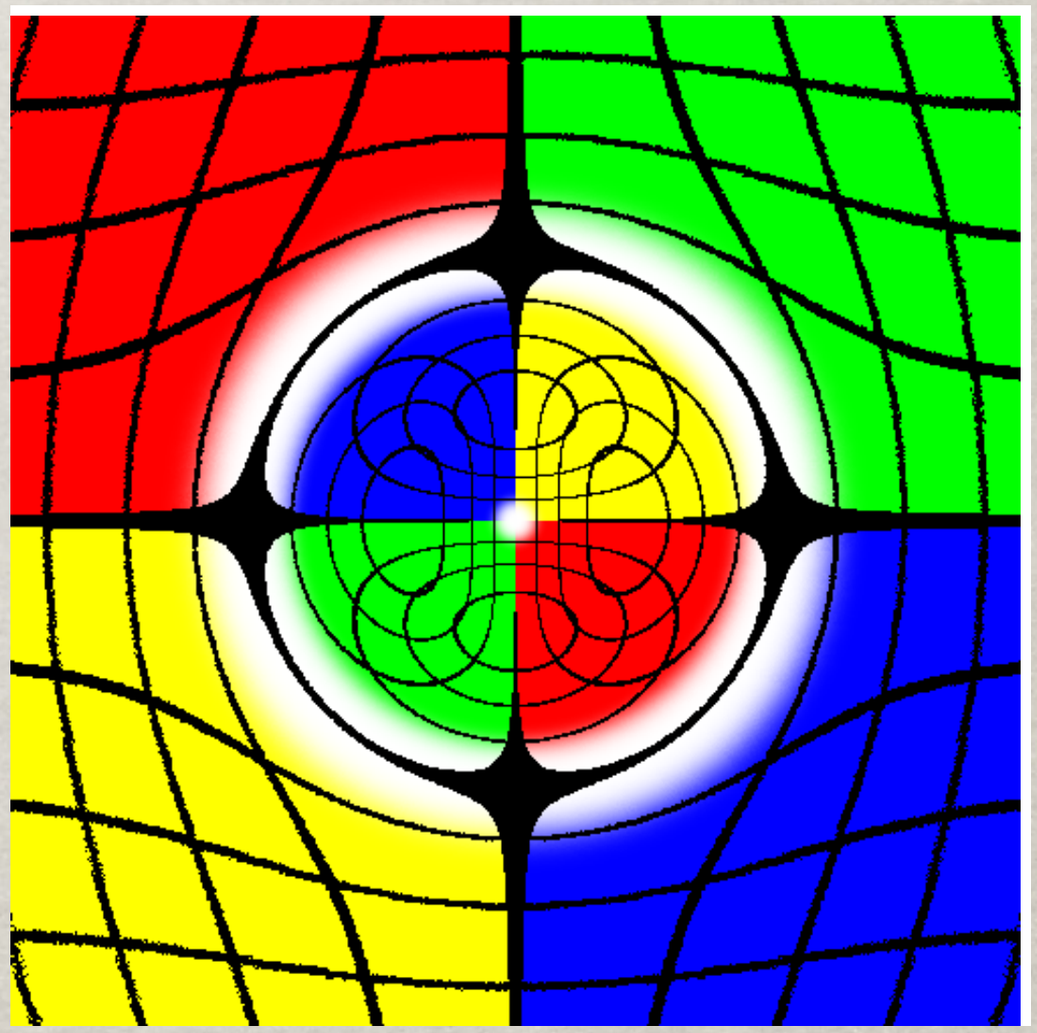


Lensing yields triple image

Configuration 3, first branch, w=0.85

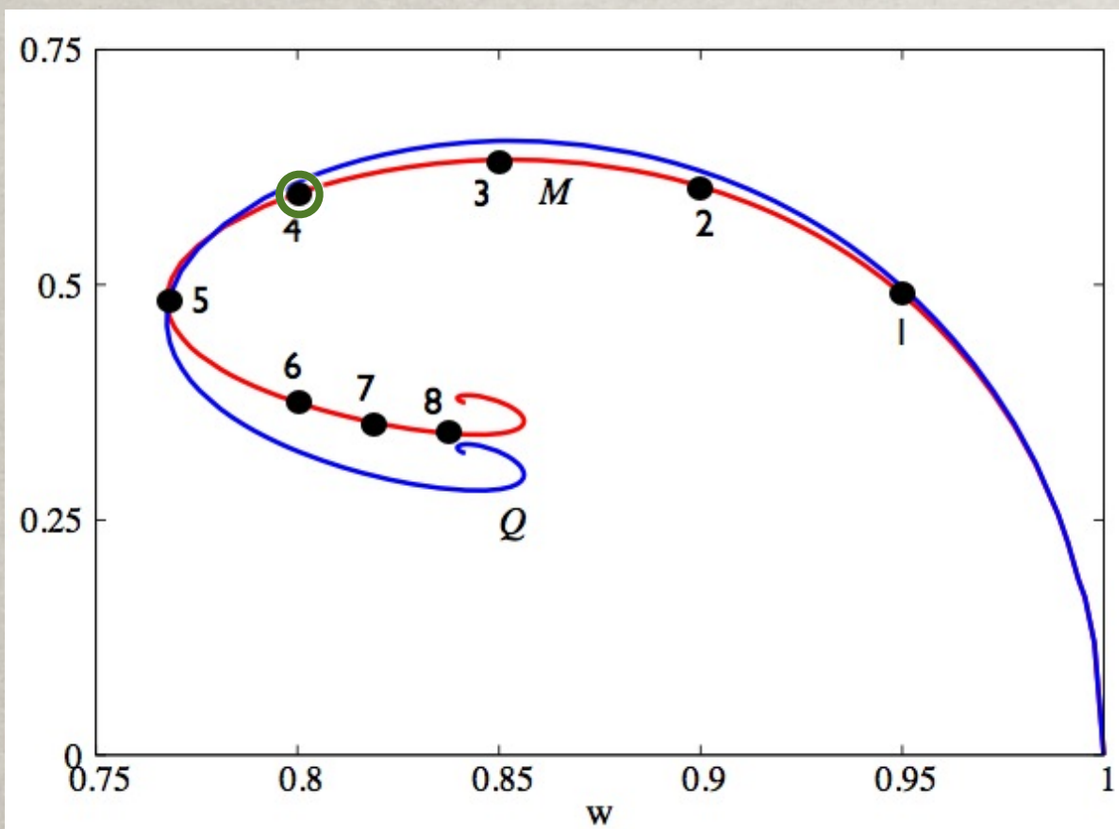


Einstein rings (plus central point):
F F

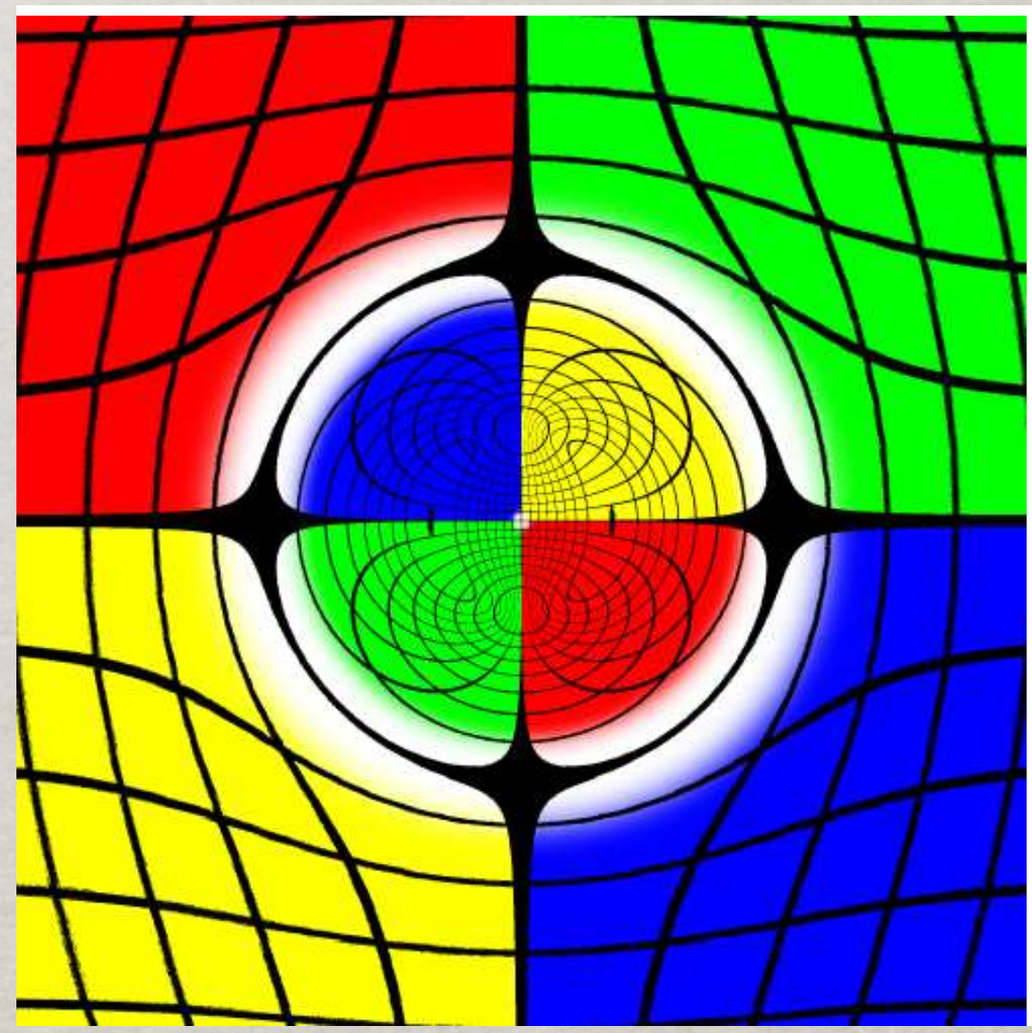


Central circle gets to maximal size in agreement with ADM mass

Configuration 4, first branch, w=0.8

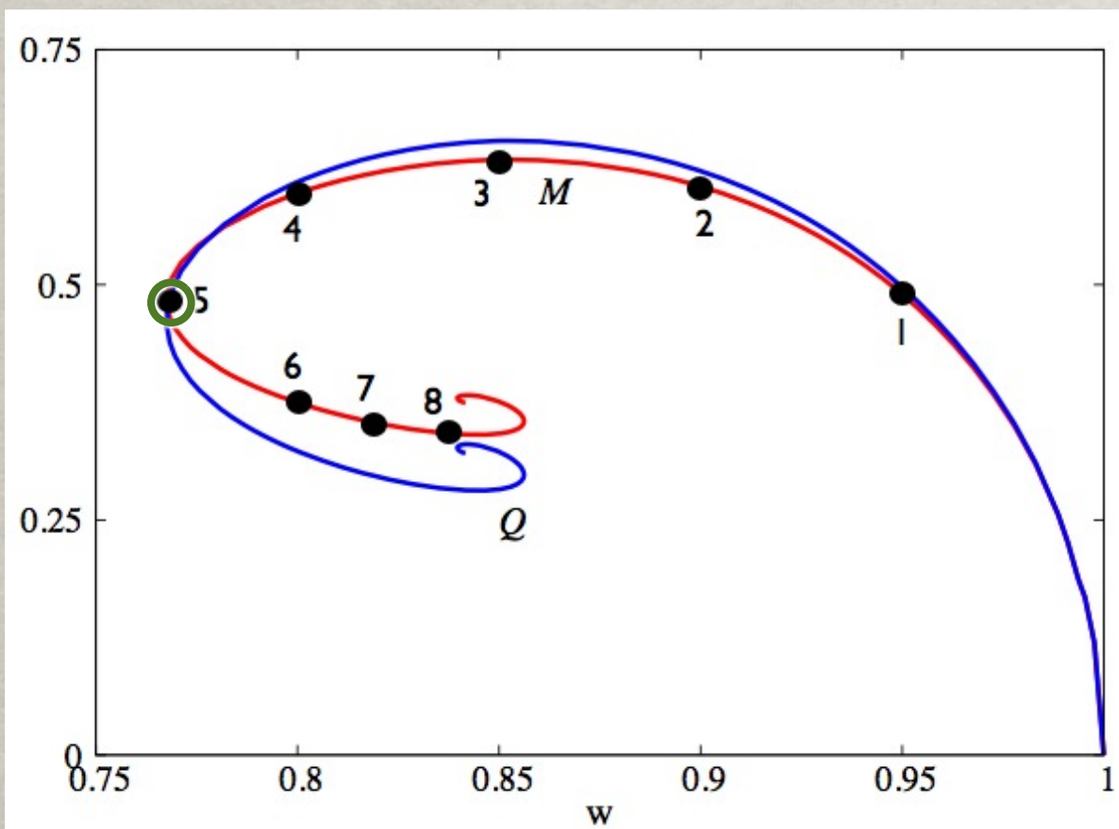


Einstein rings (plus central point):
F F

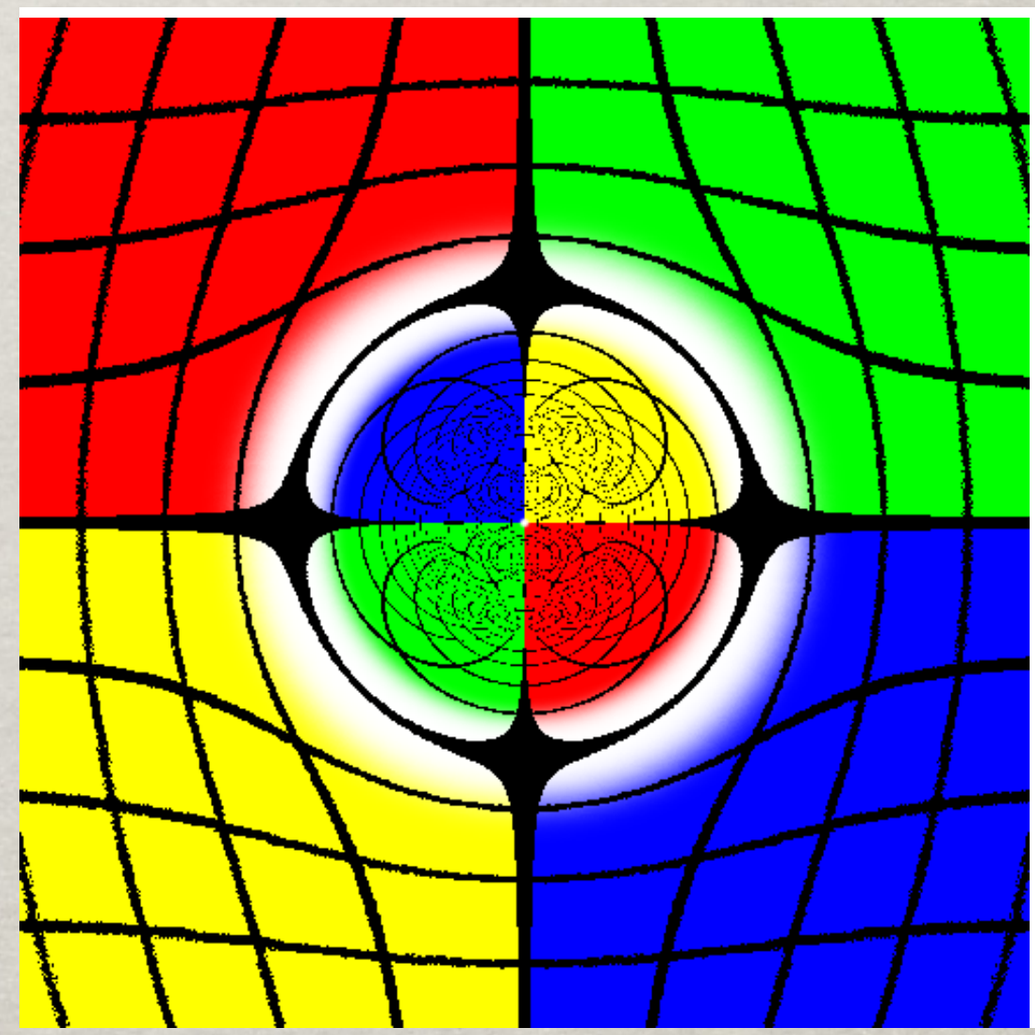


Larger region is “duplicated”

Configuration 5, first branch, w=0.7677

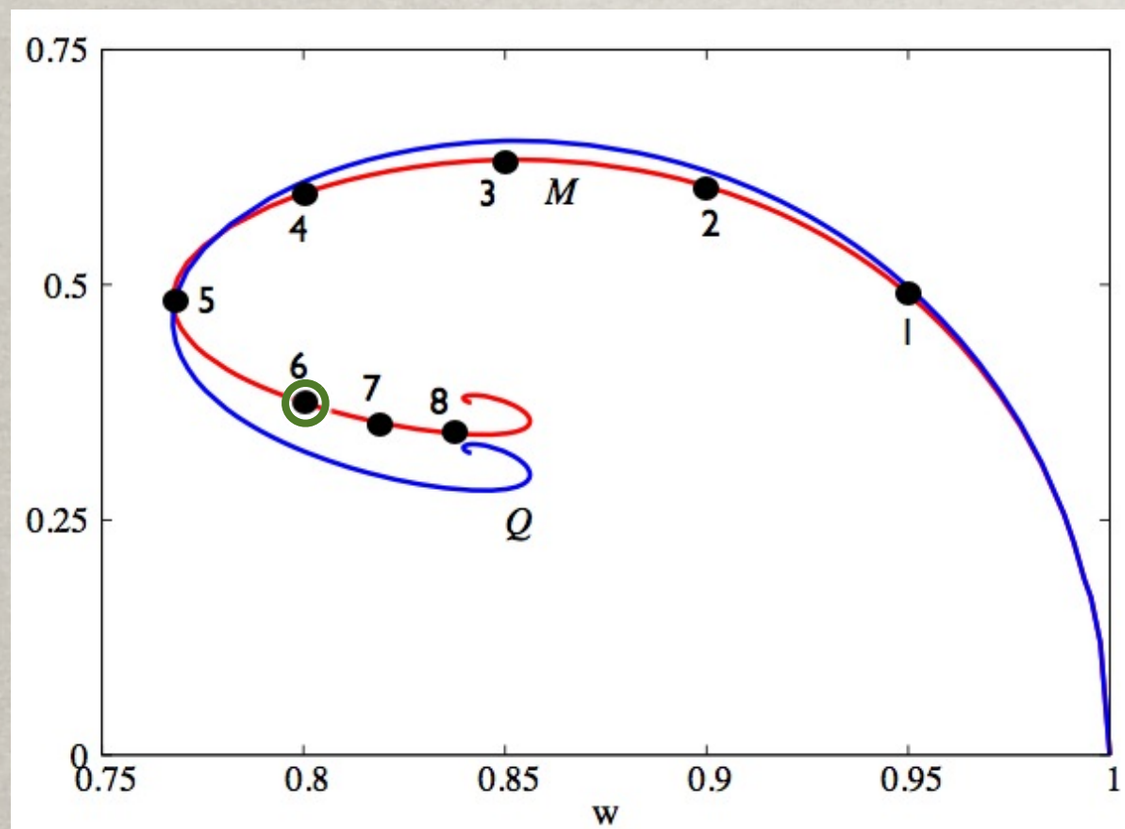


Einstein rings (plus central point):
F F

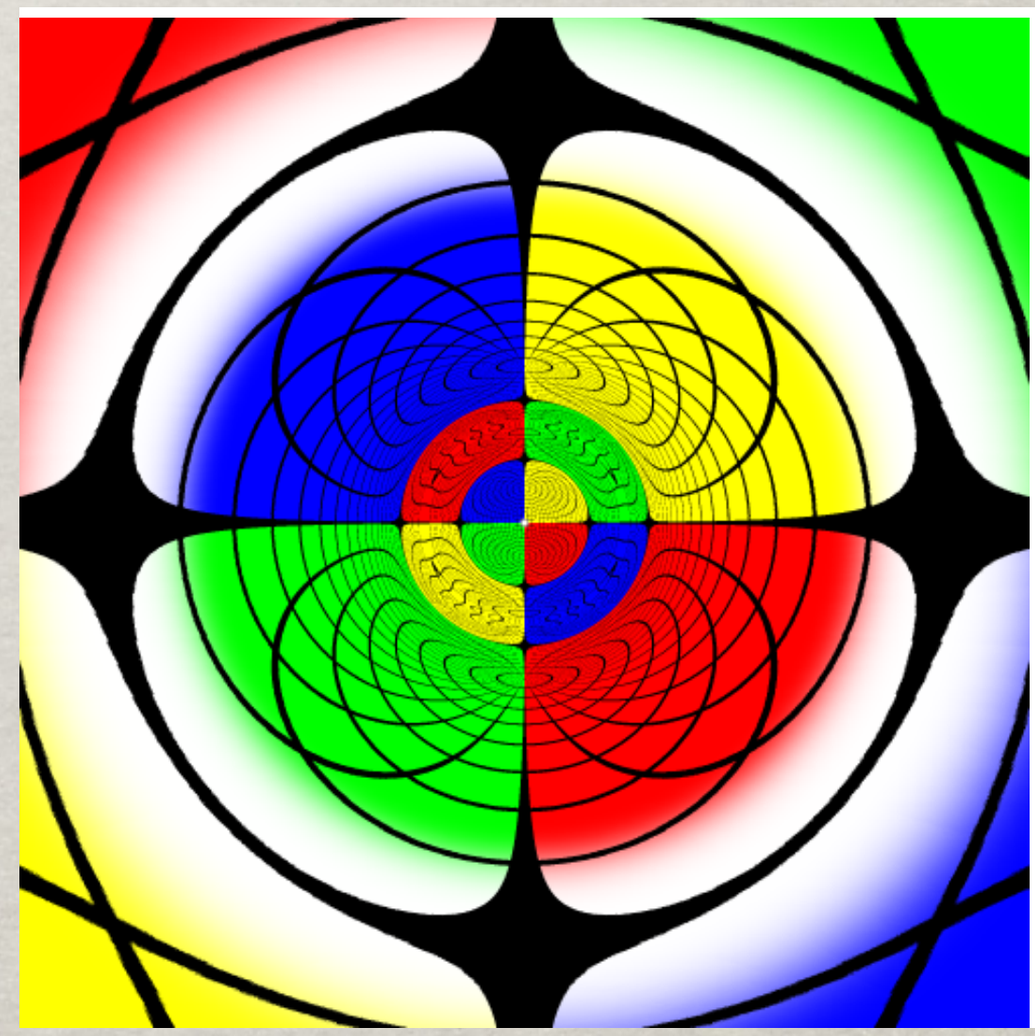


Configuration 6, second branch, w=0.8 (zoomed)

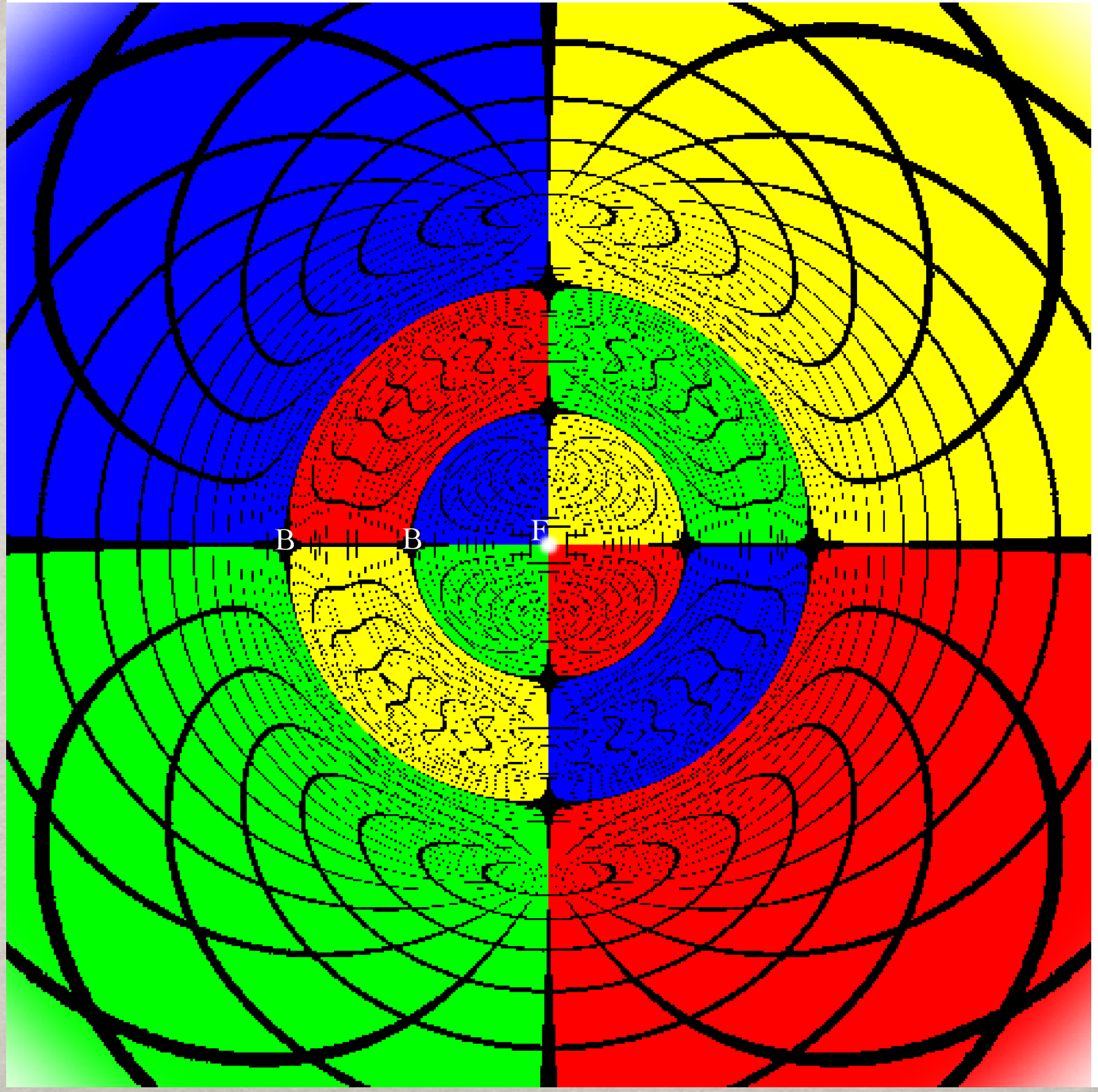
Full celestial sphere is "duplicated" just after the backbending; then, a new pair of Einstein rings appears



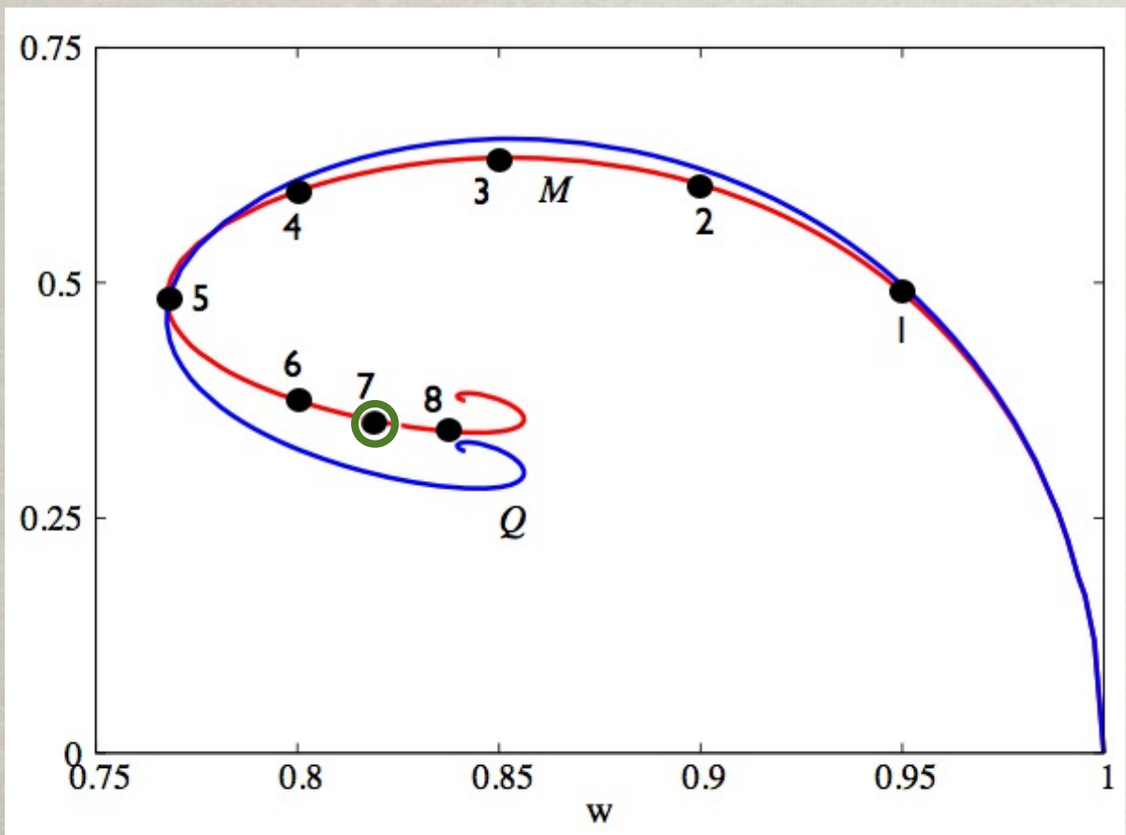
Einstein rings (plus central point):
F B B F



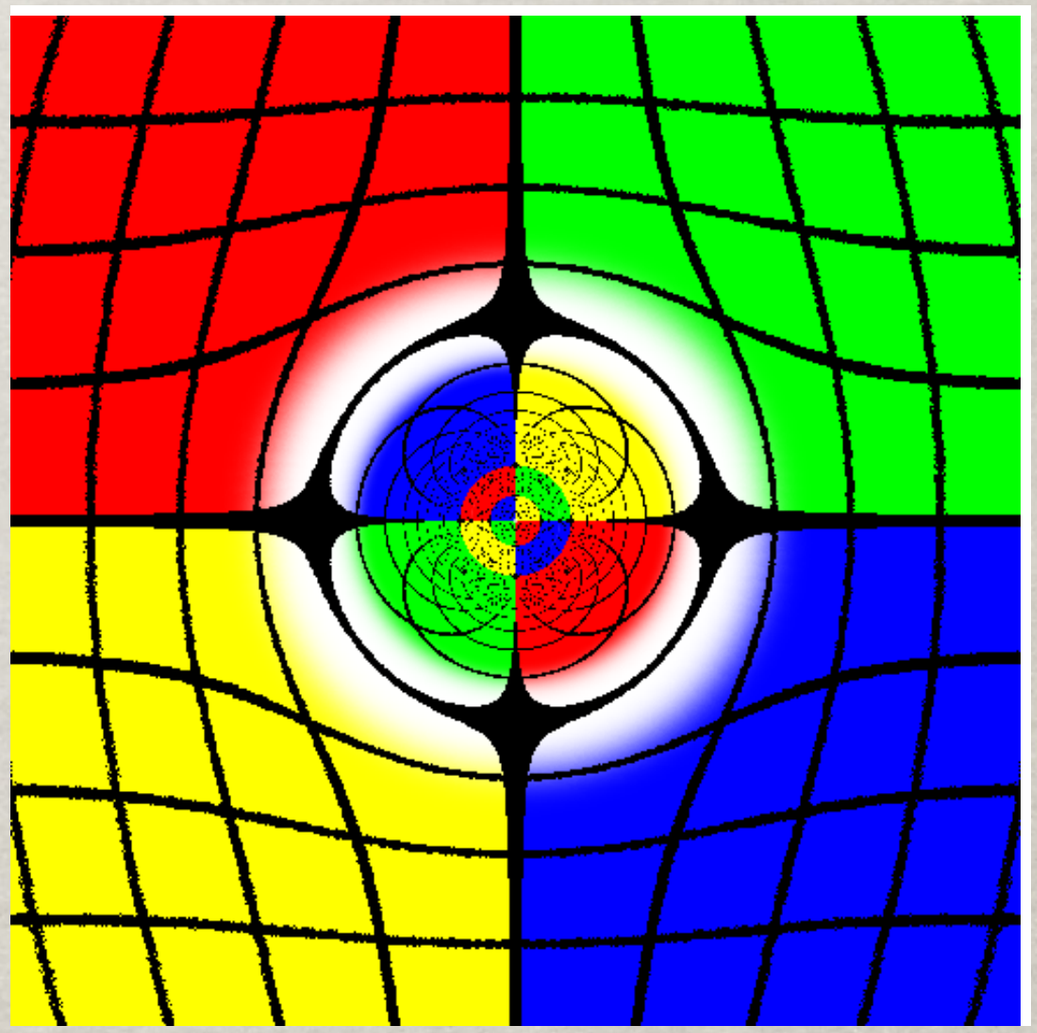
Appearance of two inner Einstein rings means two extra images of the full celestial sphere and four extra images of a part of the celestial sphere. The two new Einstein rings are images of the point behind the observer (B).


B
B
F

Configuration 7, second branch, w=0.82

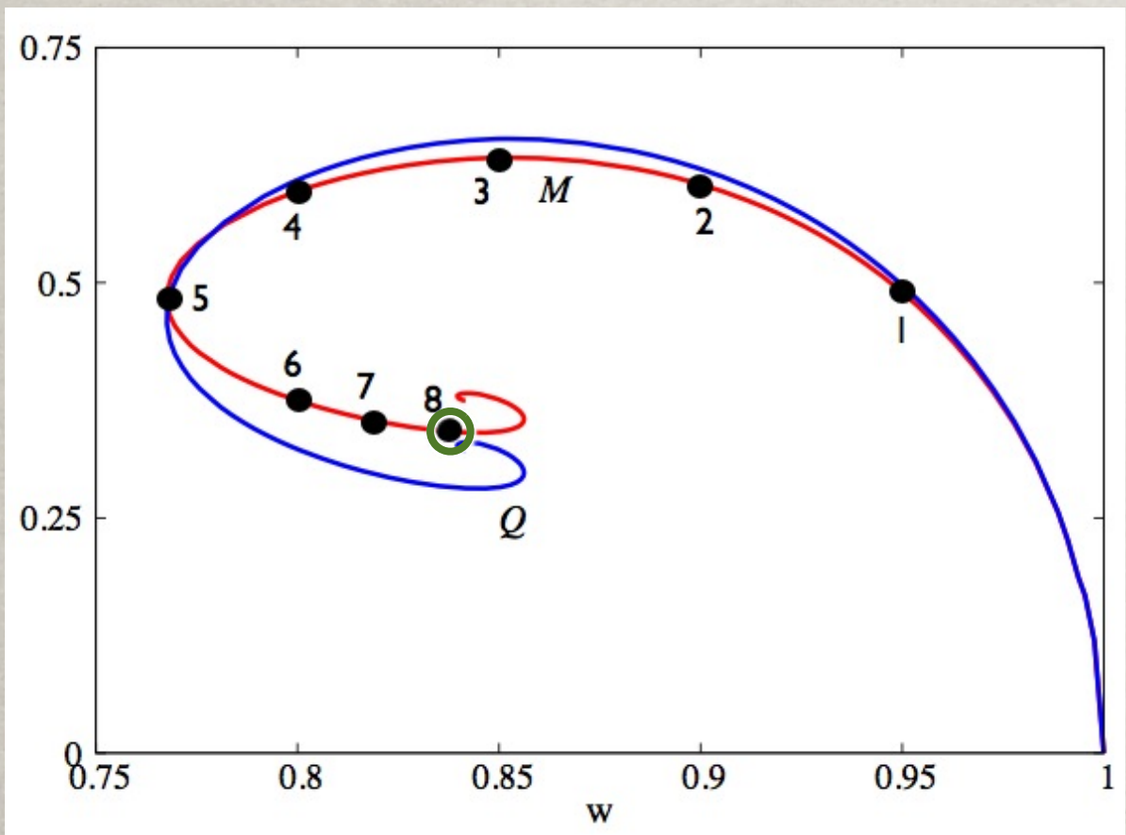


Einstein rings (plus central point):
F B B F

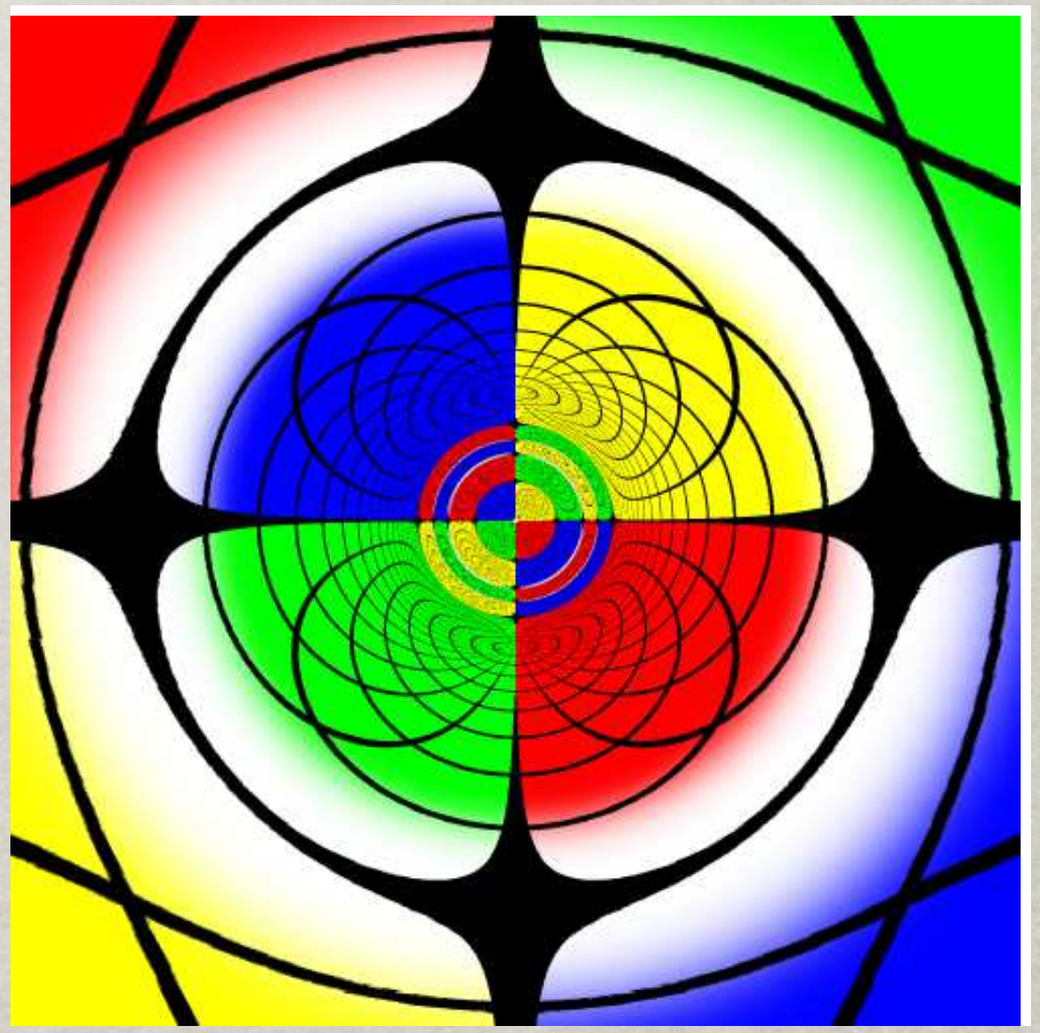


Configuration 8, second branch, w=0.84 (zoomed)

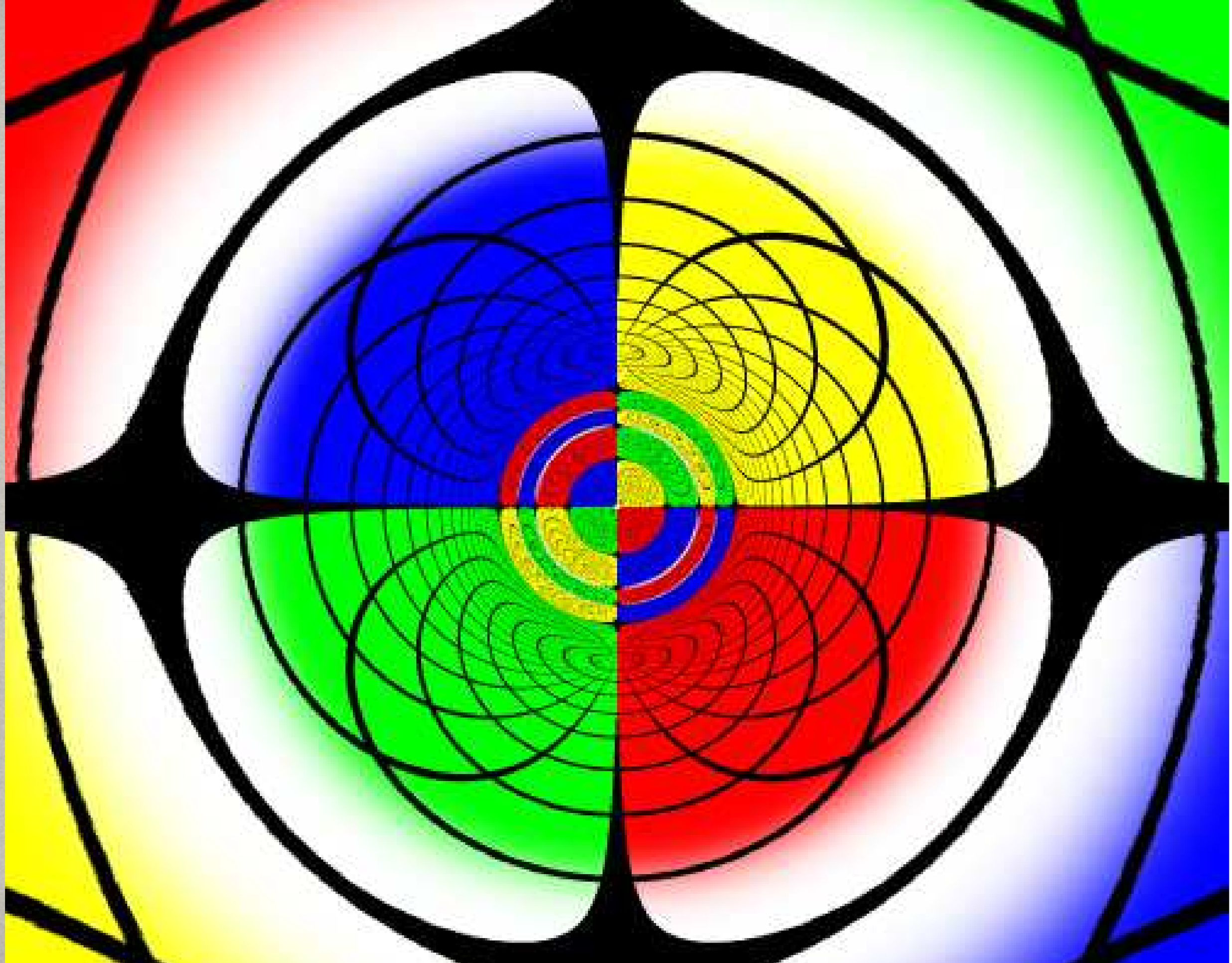
In between the latter pair of Einstein rings, new pairs of Einstein rings appear.



Einstein rings (plus central point):
F B F F B F



Two new Einstein rings appear, implying four extra images of the full celestial sphere and six extra images of a part of the celestial sphere.



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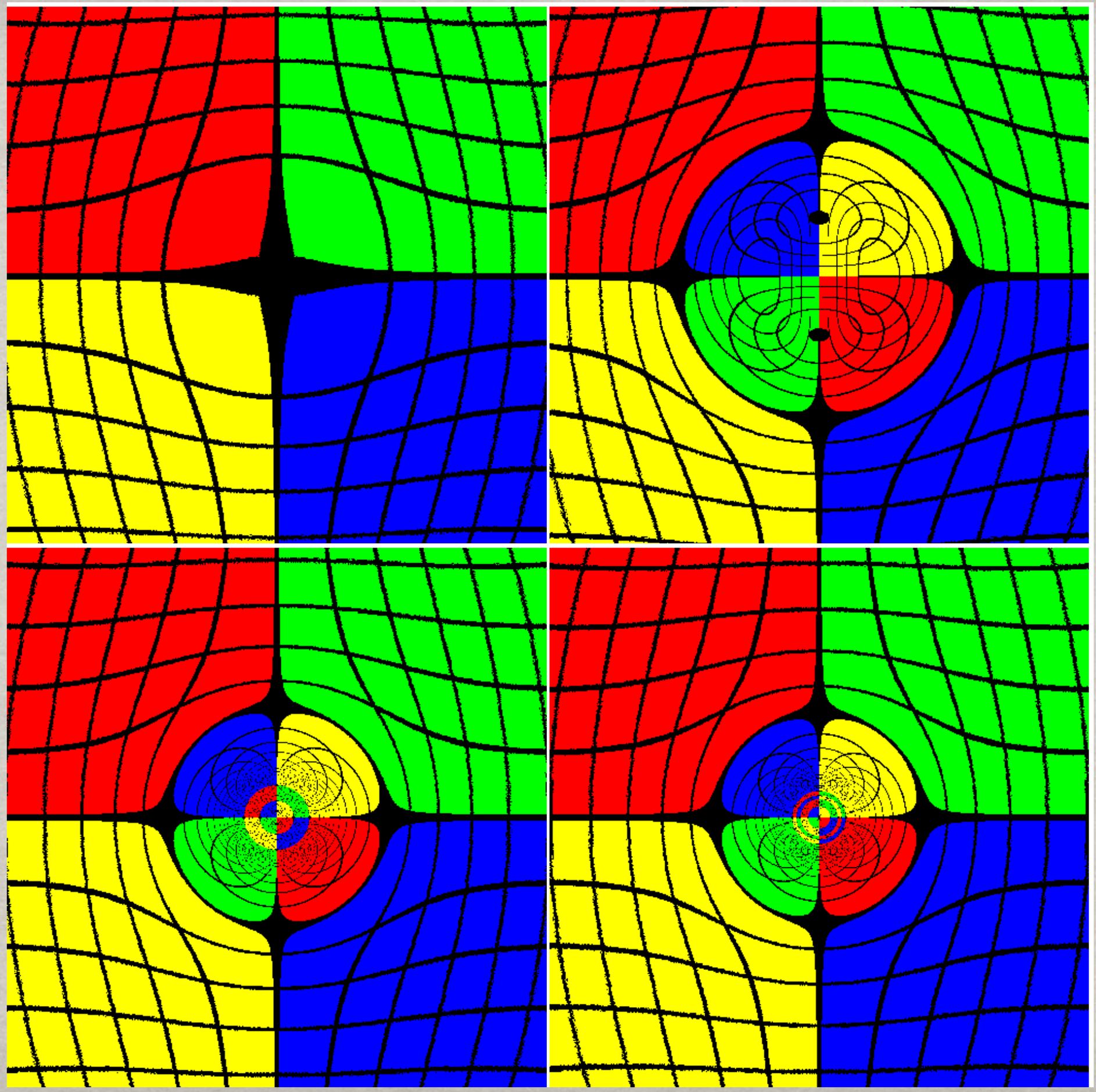
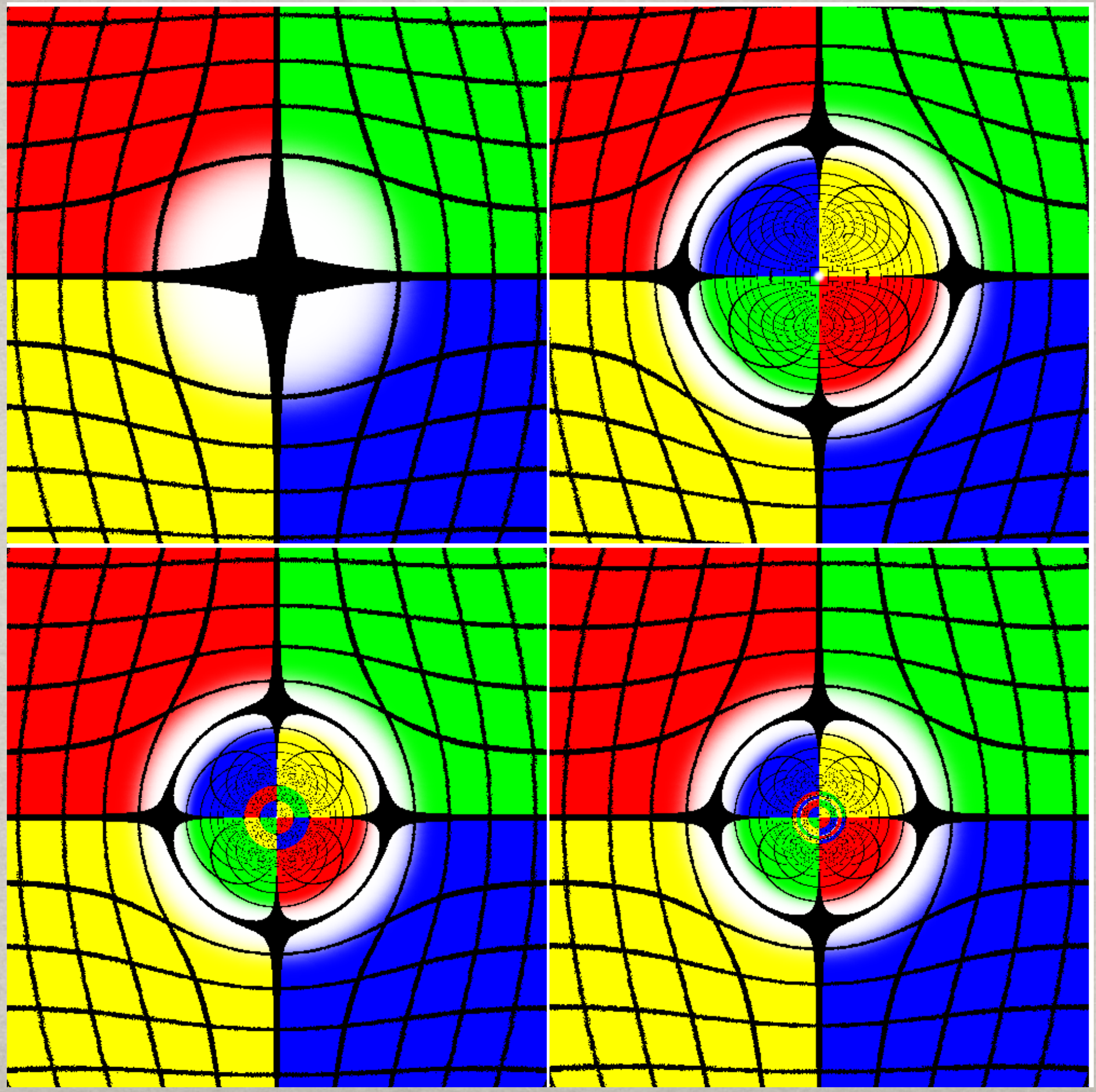
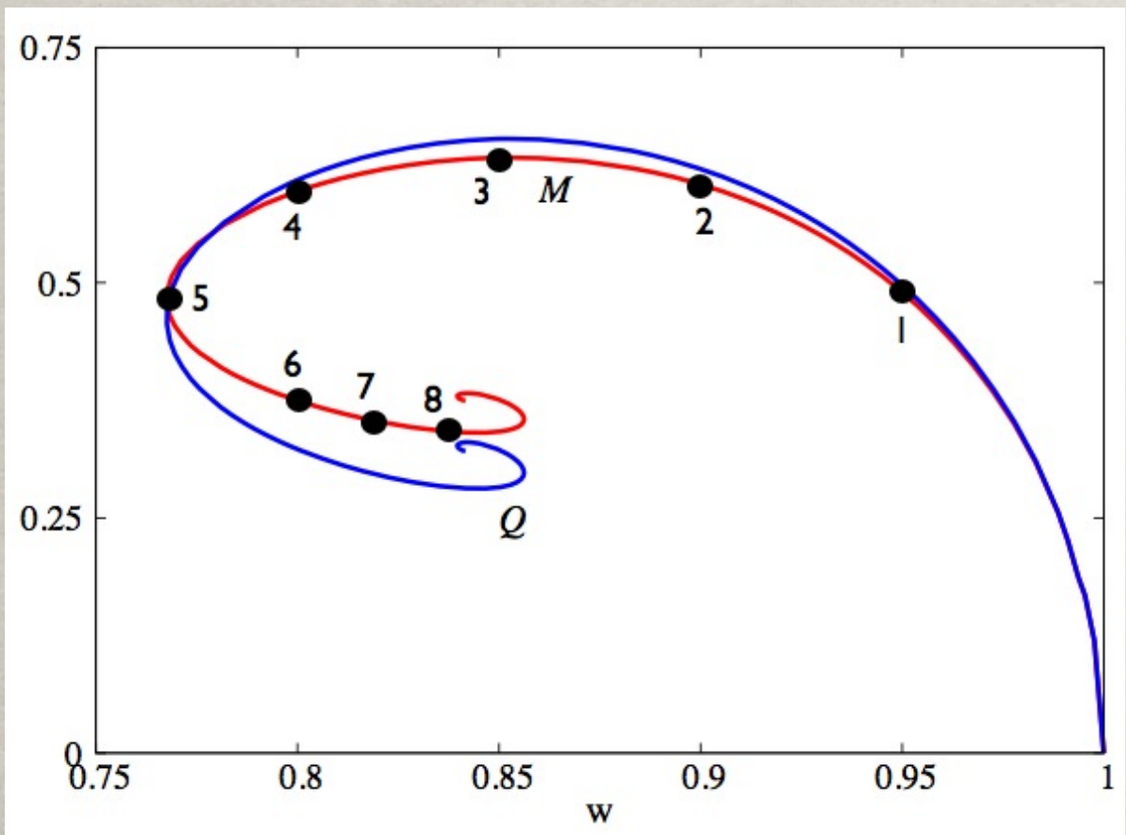
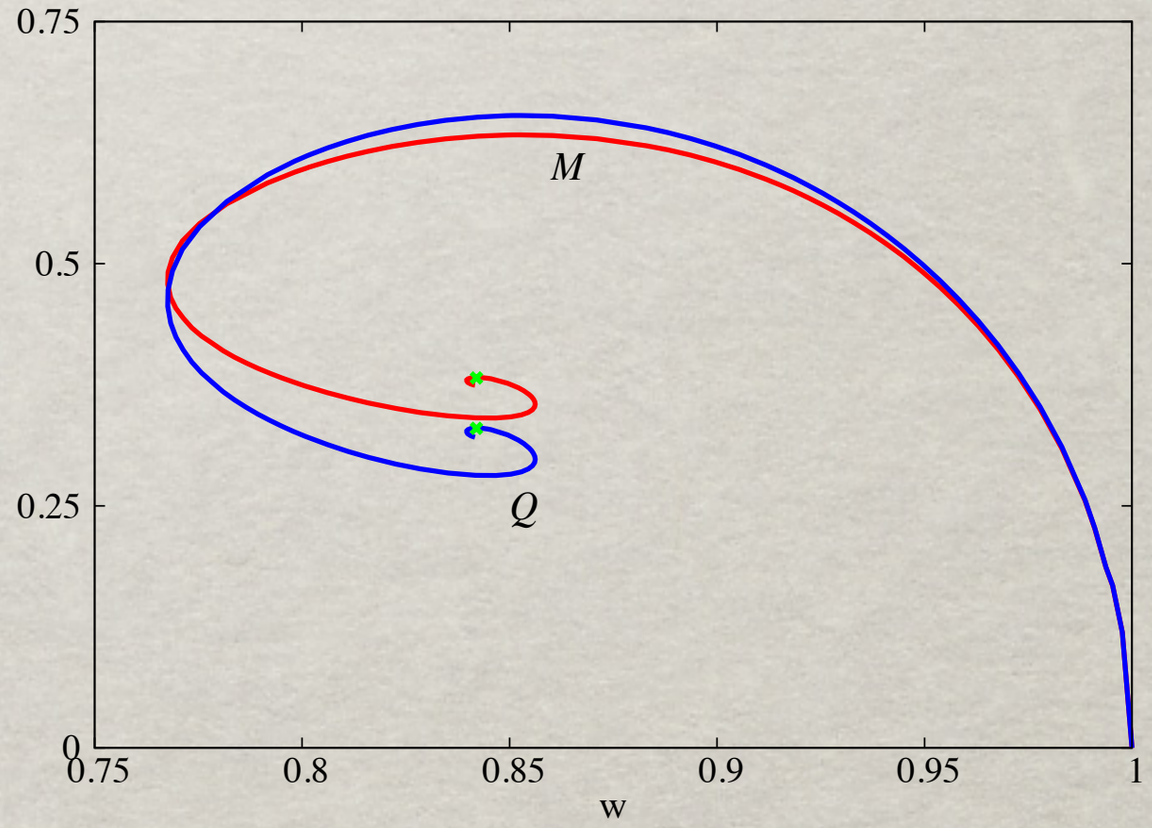




Appearance of light ring - third branch

w=0.842, M=0.382, Q=0.330, Phi(0)=1.1375

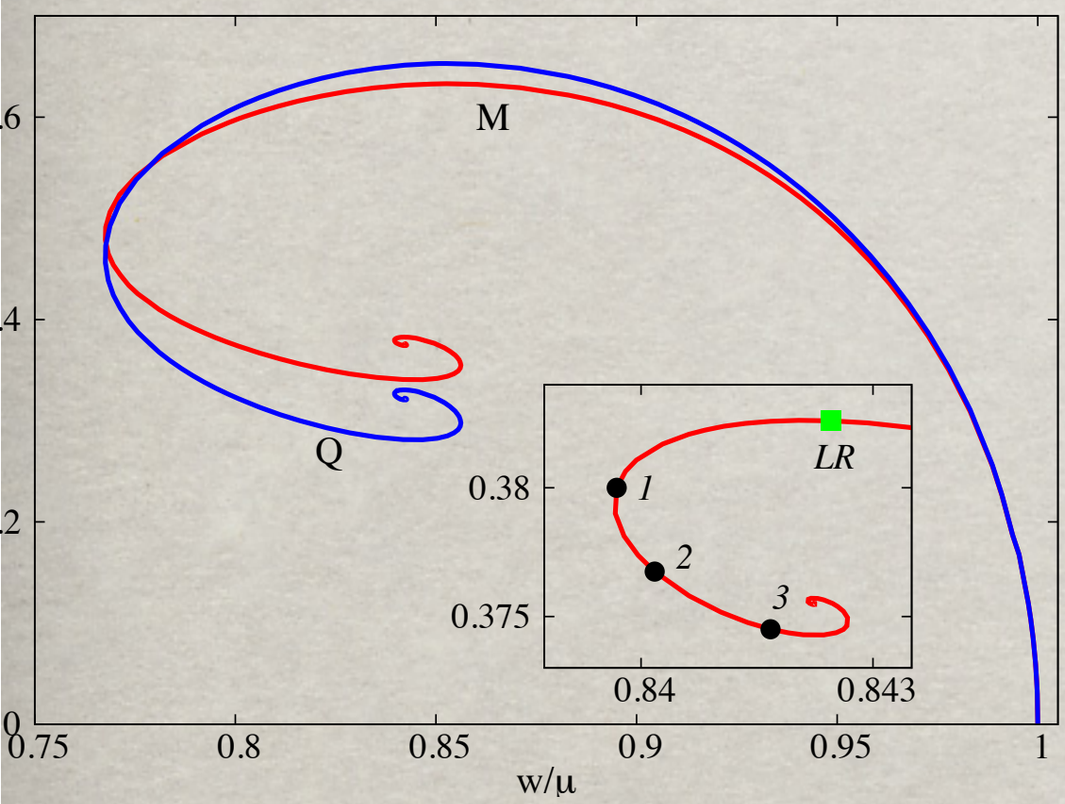


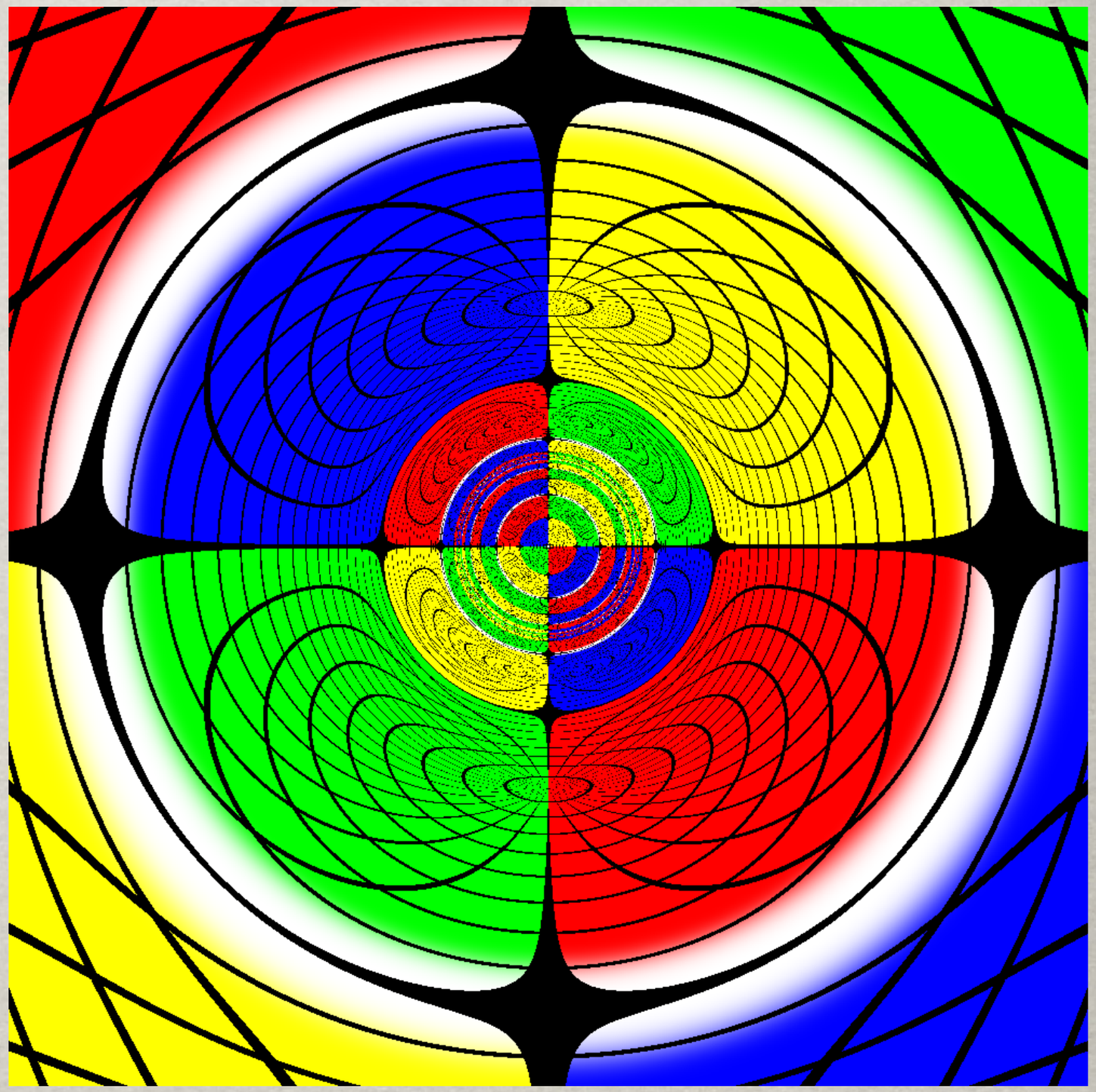


Lensing of a boson star beyond the light ring?
A self-similar structure?

Analysis of ultracompact spherical boson stars

Cunha, Font, Herdeiro, Radu, Sanchis-Gual, Zilhão, PRD 96 (2017) 104040

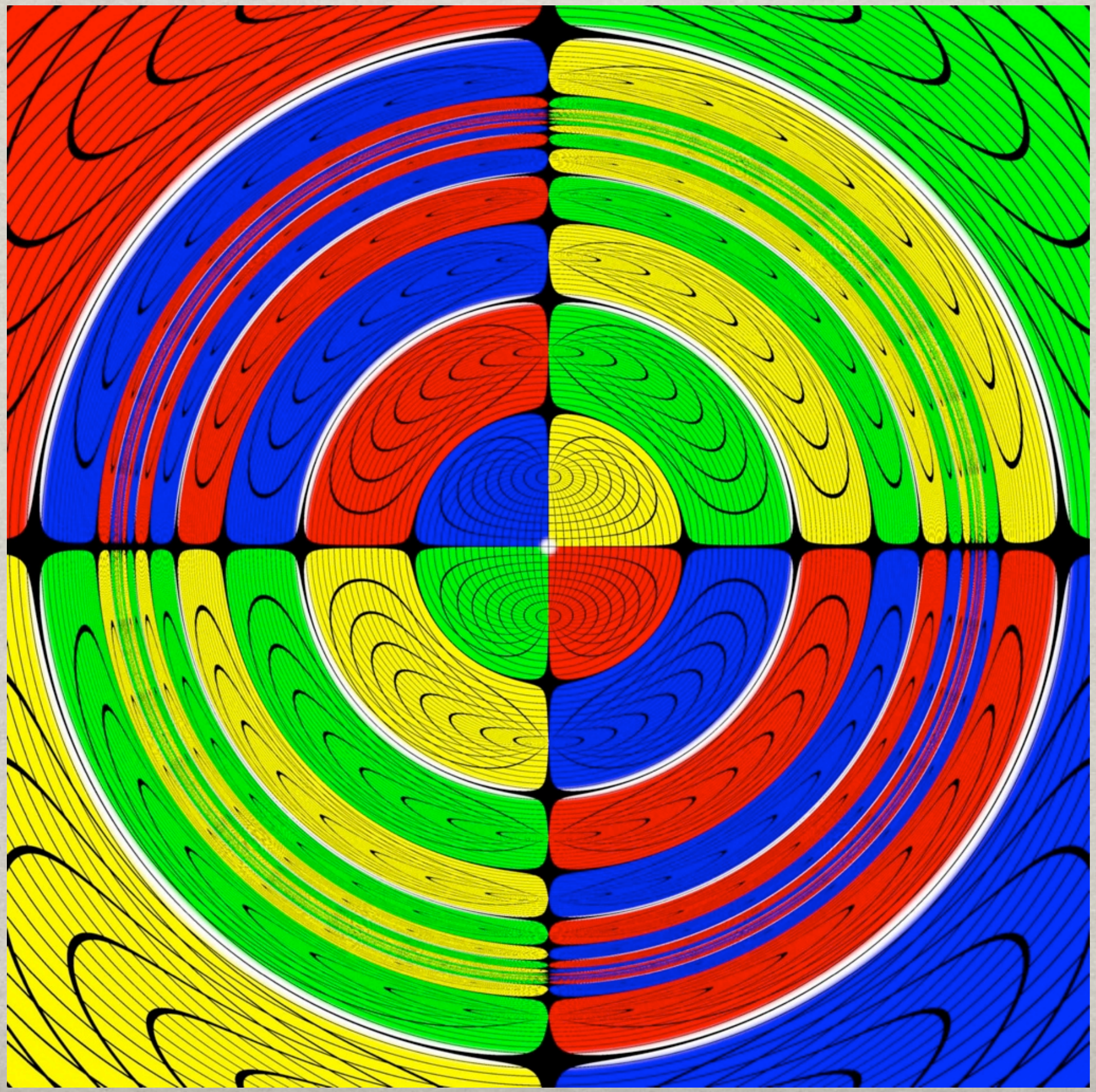




Configuration 2

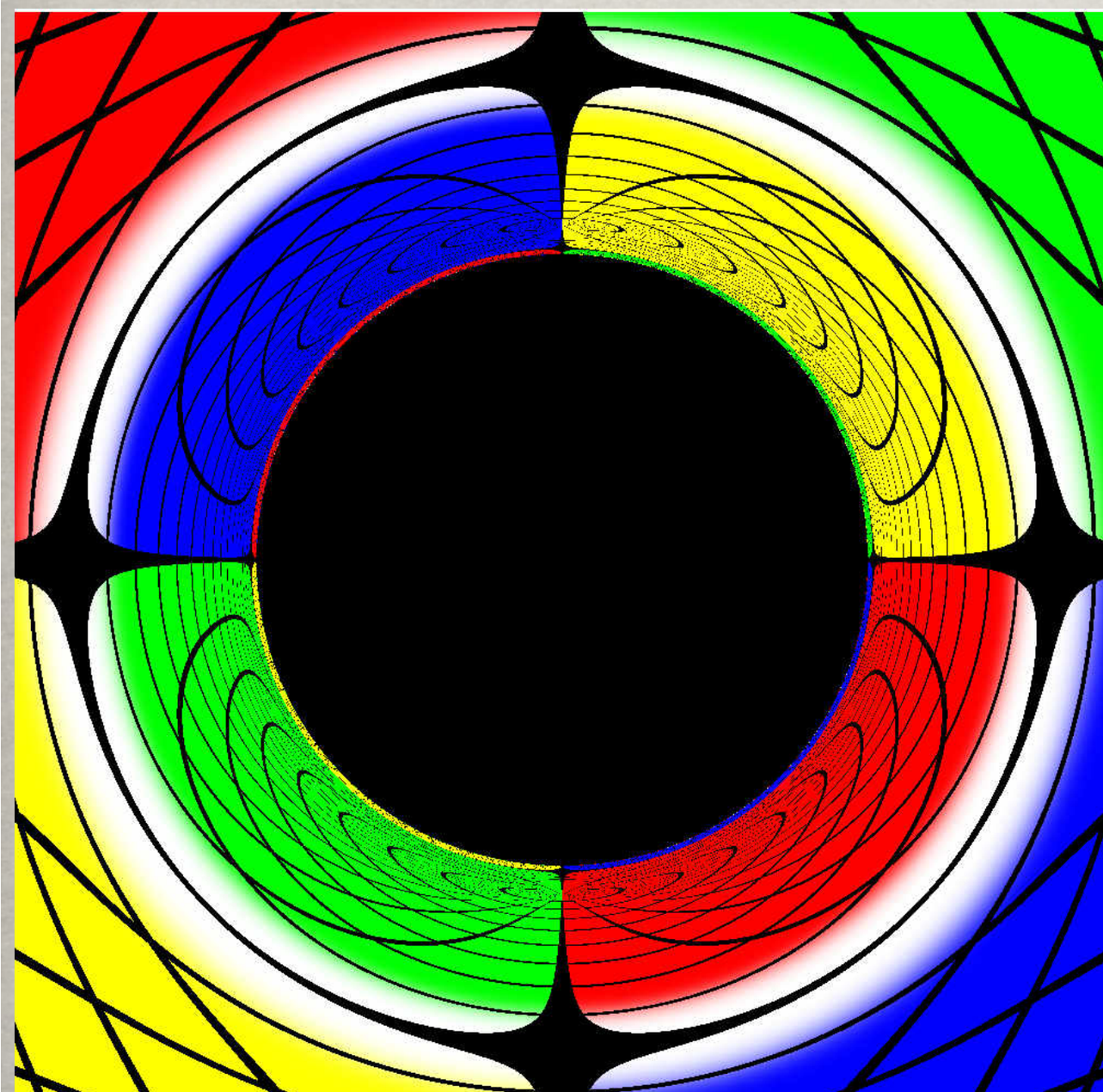
w=0.8402, M=0.3767

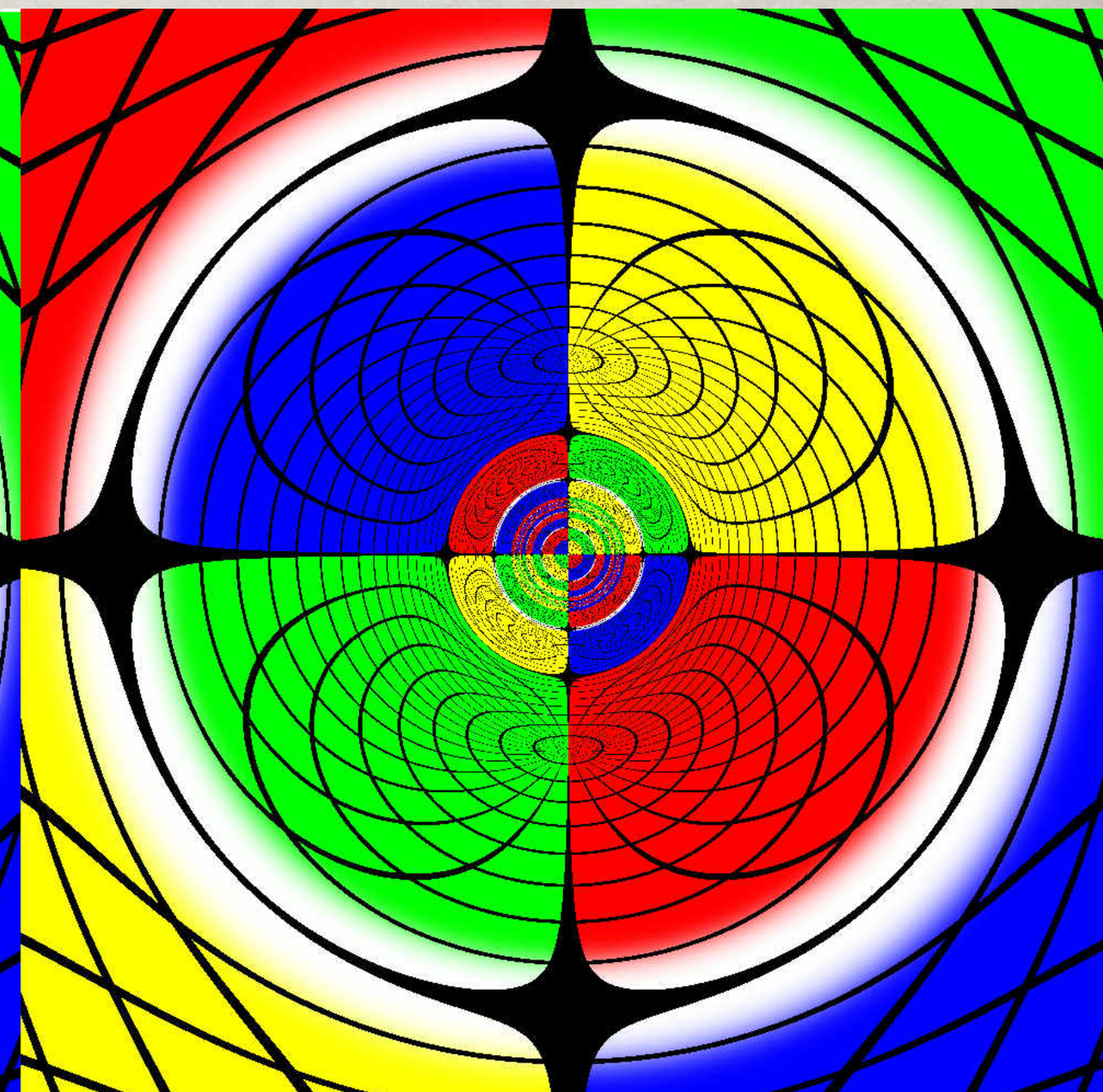
Zoom in



Lensing and shadow of a Schwarzschild BH (left) and a comparable bosonic star (right, Proca star model), in similar observation conditions.

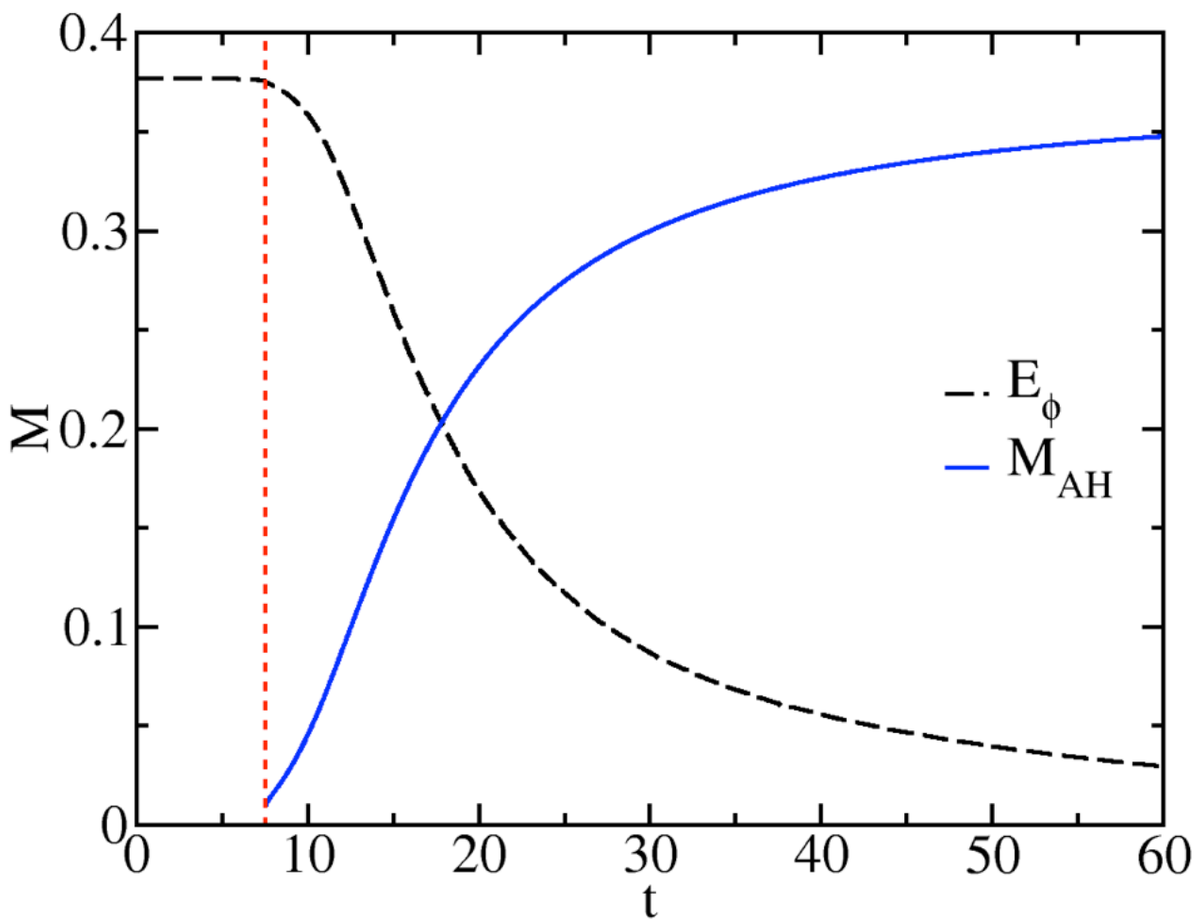
The Einstein ring has a similar dimension (white lensed region), but the strongly lensed region - shadow and near its edge for the BH vs. central rings for the star - is much smaller for the star.

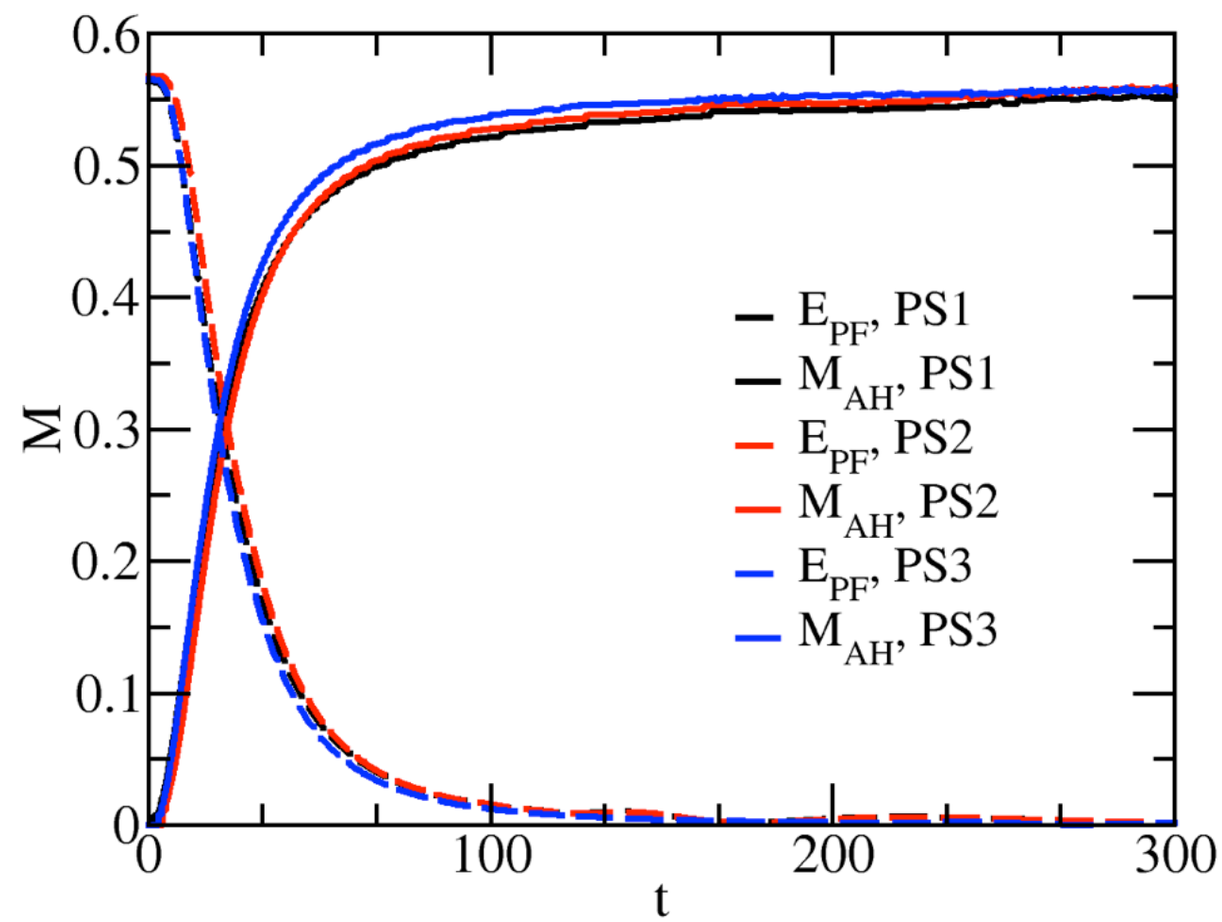




All ultra compact bosonic stars in these models are perturbatively unstable and collapse into a Schwarzschild black hole in a short time scale

Cunha, Font, Herdeiro, Radu, Sanchis-Gual, Zilhão, PRD 96 (2017) 104040





Scalar stars

Vector/Proca stars

Lecture 4

I) Spherical boson (scalar/vector) star solutions

II) Lensing by spherical bosonic stars

III) Rotating bosonic (scalar/vector) star solutions

IV) Lensing by rotating bosonic stars

Rotating boson stars

Axially symmetric solutions ansatz (in quasi-isotropic coordinates) S.Yoshida and Y. Eriguchi, Phys. Rev. D 56 (1997) 762; F. E. Schunck and E. W. Mielke, Phys. Lett. A 249 (1998) 389:

$$ds^2 = -e^{2F_0(r,\theta)}dt^2 + e^{2F_1(r,\theta)}\left(dr^2 + r^2 d\theta^2\right) + e^{2F_2(r,\theta)} r^2 \sin^2\theta \left(d\varphi - W(r,\theta)dt\right)^2 \qquad \Phi = \phi(r,\theta)e^{i(m\varphi - wt)}$$

The solution has three parameters: (w,m,n), but again these do not define solutions uniquely.

Solutions preserved by a single helicoidal Killing vector field:

$$\frac{\partial}{\partial t} + \frac{w}{m}\frac{\partial}{\partial \varphi}$$

The Klein-Gordon plus Einstein equations yield now a system of five coupled PDEs (plus two "constraint" equations which are differential consequences of the others). To solve them:

- one performs an expansion of the unknown functions, both near the origin and asymptotically;

- the equations can be solved using a relaxation method (Newton-Raphson). For each fixed frequency w one can find various or no solutions, corresponding to different ADM masses M.

Rotating boson stars

Details ? Talk to Jorge Delgado

Klein Gordon equation:

$$\phi_{,rr} + \frac{1}{r^2}\phi_{,\theta\theta} + \phi_{,r}(F_{0,r} + F_{2,r}) + \frac{1}{r^2}\phi_{,\theta}(F_{0,\theta} + F_{2,\theta}) + \frac{2}{r}\phi_{,r} + \frac{\cot\theta}{r^2}\phi_{,\theta} - \left(\frac{e^{-2F_2}m^2}{r^2\sin^2\theta} - e^{-2F_0}(w - mW)^2 + \mu^2\right)e^{2F_1}\phi = 0$$

The Einstein equations are combined to have second derivatives of a single function:

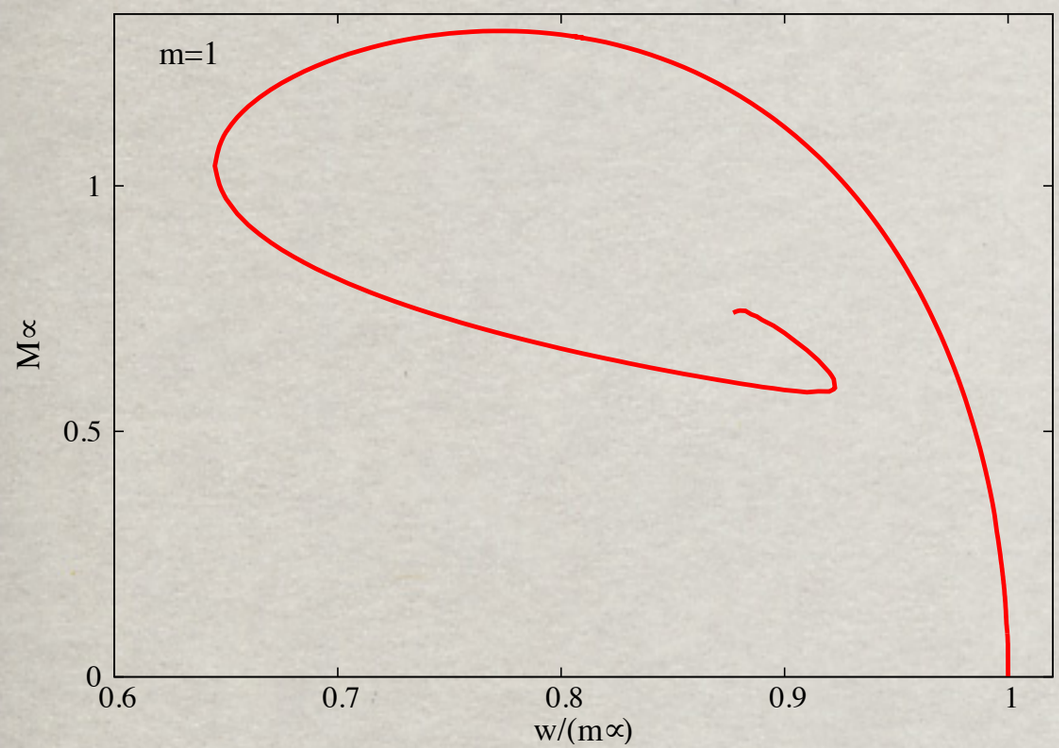
$$F_{1,rr} + \frac{1}{r^2}F_{1,\theta\theta} - \left(F_{0,r}F_{2,r} + \frac{1}{r^2}F_{0,\theta}F_{2,\theta}\right) - \frac{e^{-2F_0+2F_2}r^2\sin^2\theta}{4}\left(W_{,r}^2 + \frac{1}{r^2}W_{,\theta}^2\right) - \frac{F_{0,r}}{r} + \frac{F_{1,r}}{r} - \frac{\cot\theta F_{0,\theta}}{r^2} + 8\pi\left(\phi_{,r}^2 + \frac{1}{r^2}\phi_{,\theta}^2 + e^{2F_1}\left[e^{-2F_0}(w - mW)^2 - \frac{e^{-2F_2}m^2}{r^2\sin^2\theta}\right]\phi^2\right) = 0$$

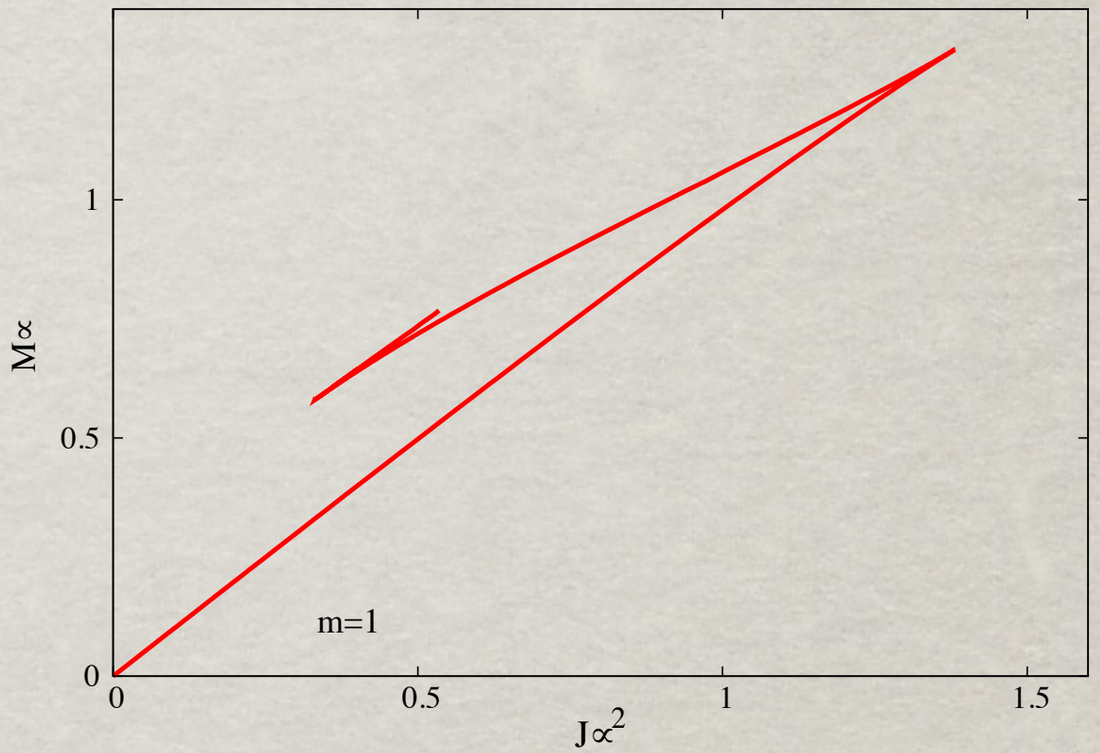
$$F_{2,rr} + \frac{1}{r^2}F_{2,\theta\theta} + F_{2,r}^2 + \frac{1}{r^2}F_{2,\theta}^2 + F_{0,r}F_{2,r} + \frac{1}{r^2}F_{0,\theta}F_{2,\theta} + \frac{e^{-2F_0+2F_2}r^2\sin^2\theta}{2}\left(W_{,r}^2 + \frac{1}{r^2}W_{,\theta}^2\right) + \frac{1}{r}\left(F_{0,r} + \frac{\cot\theta F_{0,\theta}}{r}\right) + \frac{3F_{2,r}}{r} + \frac{2\cot\theta F_{2,\theta}}{r^2} + 8\pi e^{2F_1}\left(\mu^2 + \frac{2e^{-2F_2}m^2}{r^2\sin^2\theta}\right)\phi^2 = 0$$

$$F_{0,rr} + \frac{1}{r^2}F_{0,\theta\theta} + F_{0,r}^2 + \frac{1}{r^2}F_{0,\theta}^2 + F_{0,r}F_{2,r} + \frac{1}{r^2}F_{0,\theta}F_{2,\theta} - \frac{e^{-2F_0+2F_2}r^2\sin^2\theta}{2}\left(W_{,r}^2 + \frac{1}{r^2}W_{,\theta}^2\right) + \frac{2F_{0,r}}{r} + \frac{\cot\theta F_{0,\theta}}{r^2} + \frac{N'F_{2,r}}{2} - 8\pi e^{2F_1}\left(2e^{-2F_0}(w - mW)^2 - \mu^2\right)\phi^2 = 0$$

$$W_{,rr} + \frac{1}{r^2}W_{,\theta\theta} + (3F_{2,r} - F_{0,r})W_{,r} + \frac{1}{r^2}(3F_{2,\theta} - F_{0,\theta})W_{,\theta} + \frac{4}{r}\left(W_{,r} + \frac{3\cot\theta W_{,\theta}}{4r}\right) + 32\pi\frac{e^{2F_1-2F_2}m(w - mW)}{r^2\sin^2\theta}\phi^2 = 0$$

Rotating boson stars





The maximum value for the ADM mass increases with m:

$$M_{\rm ADM}^{\rm max} \simeq \alpha_{\rm BS} \frac{M_{\rm Pl}^2}{\mu} \simeq \alpha_{\rm BS}\, 10^{-19} M_\odot \left(\frac{\rm GeV}{\mu}\right)$$

m=0: $\alpha_{\rm BS} = 0.633$

m=1: $\alpha_{\rm BS} = 1.315$

S.Yoshida and Y. Eriguchi, Phys. Rev. D 56 (1997) 762

m=2: $\alpha_{\rm BS} = 2.216$

P. Grandclement, C. Somé and E. Gourgoulhon, Phys. Rev. D 90 (2014) 2, 024068 [arXiv:1405.4837 [gr-qc]].

For rotating boson stars:

F. E. Schunck and E. W. Mielke, Phys. Lett. A 249 (1998) 389

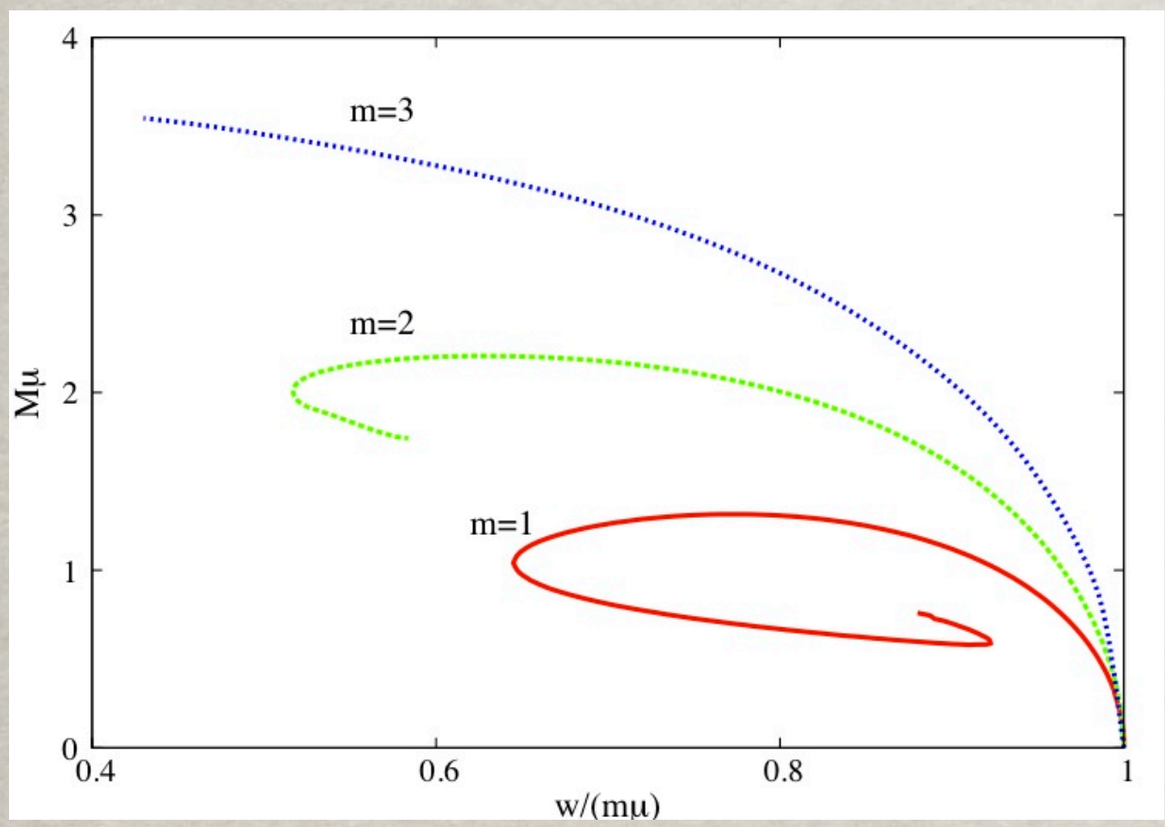
$$J = mQ$$

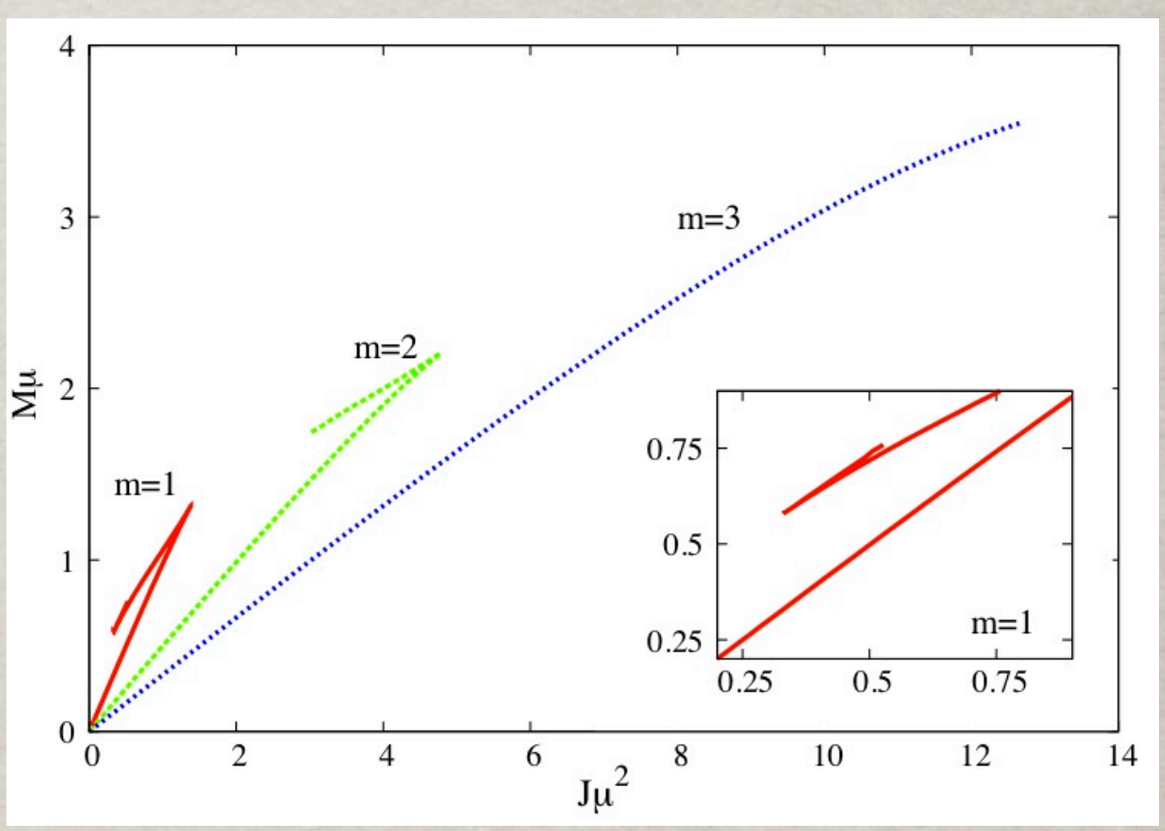
Exercise 4.3

Show this.

Rotating boson stars

ADM mass M vs. frequency w diagram and M vs. J diagram for m=1,2,3:

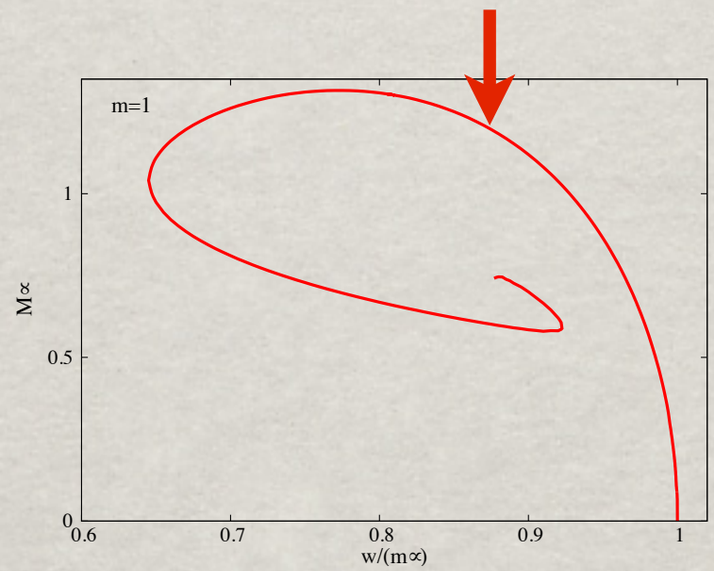


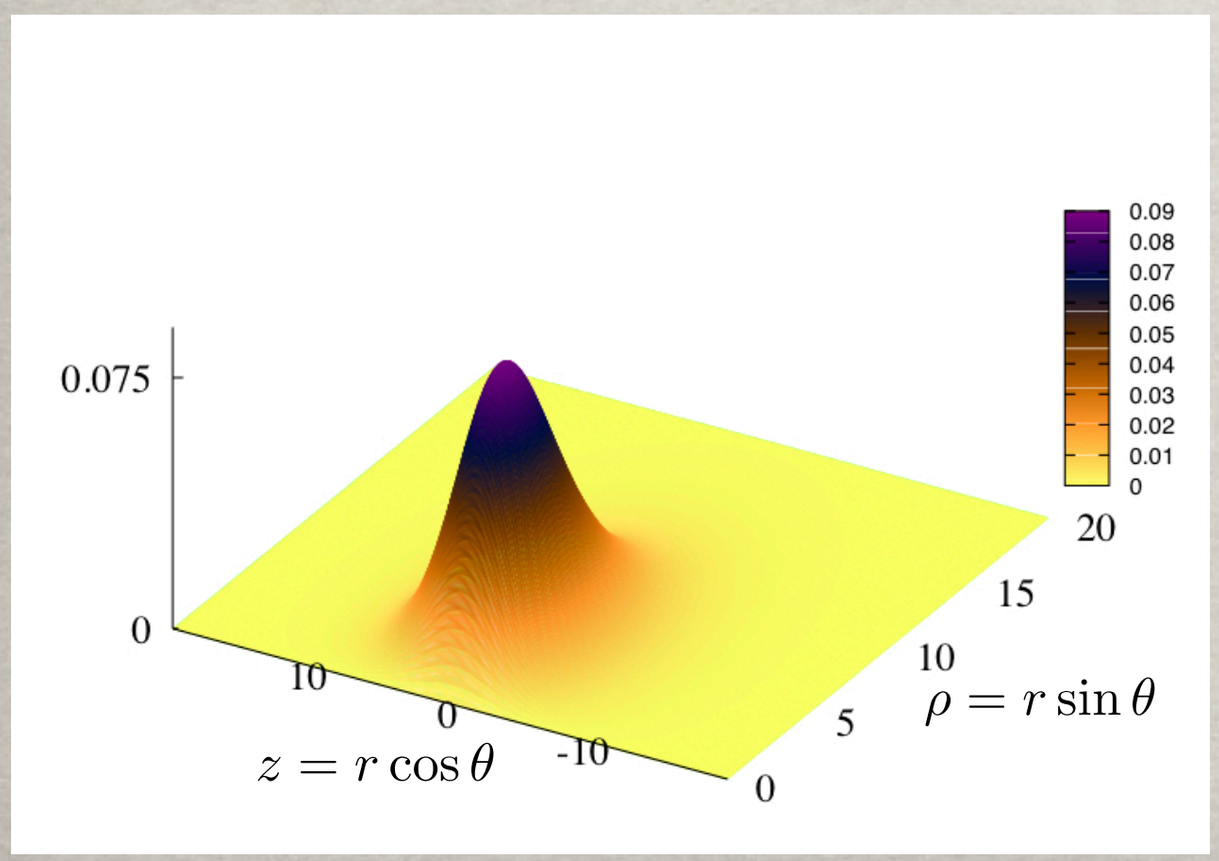


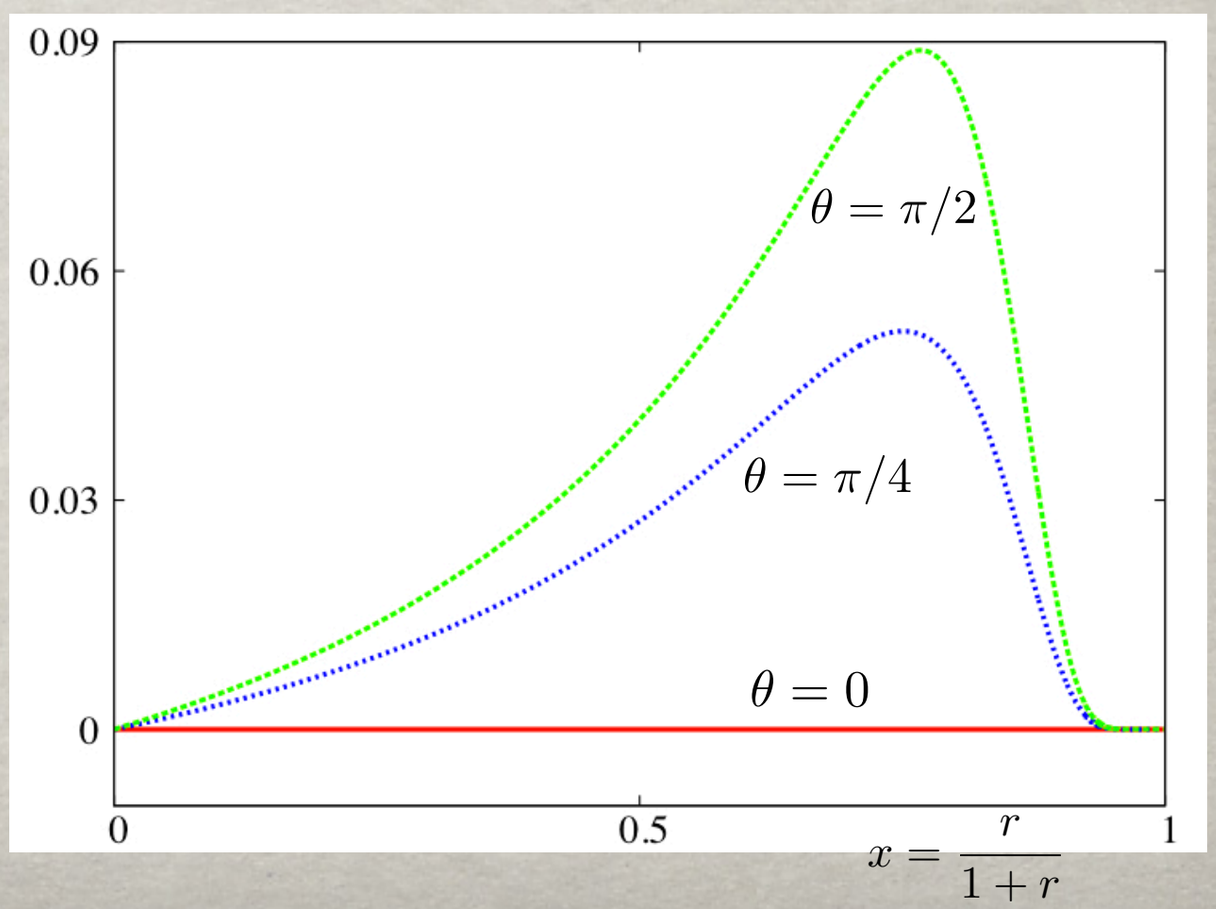
C. Herdeiro and E. Radu, Class. Quant. Grav. 32 (2015) 14, 144001 [arXiv:1501.04319 [gr-qc]]

Rotating boson stars

Scalar field profile (left) and for a typical rotating boson star, m=1, first branch, w=0.85:

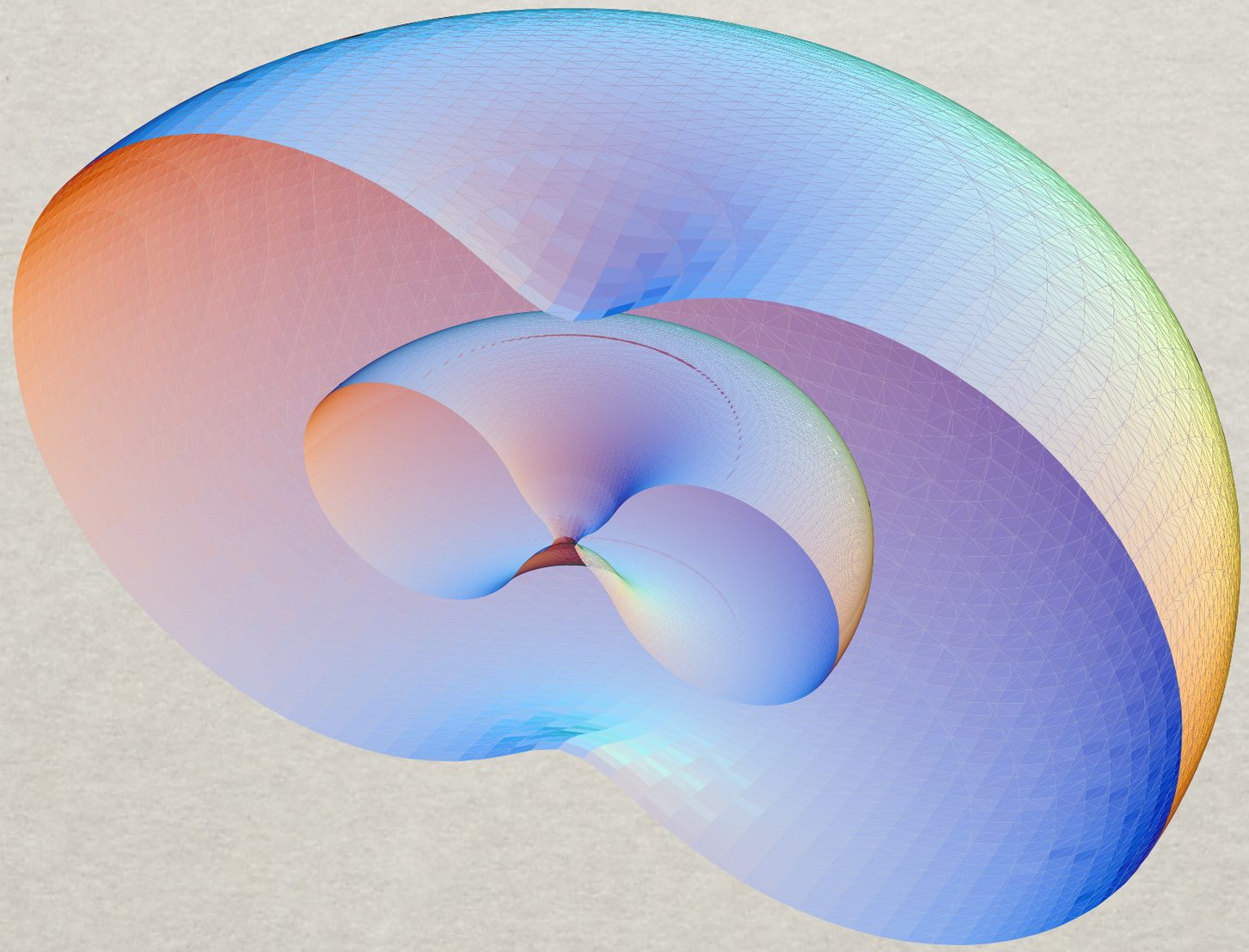






Rotating boson stars

Surfaces of constant scalar energy density:



C. A. R. Herdeiro and E. Radu, Int. J. Mod. Phys. D 23 (2014) 12, 1442014 [arXiv:1405.3696 [gr-qc]]

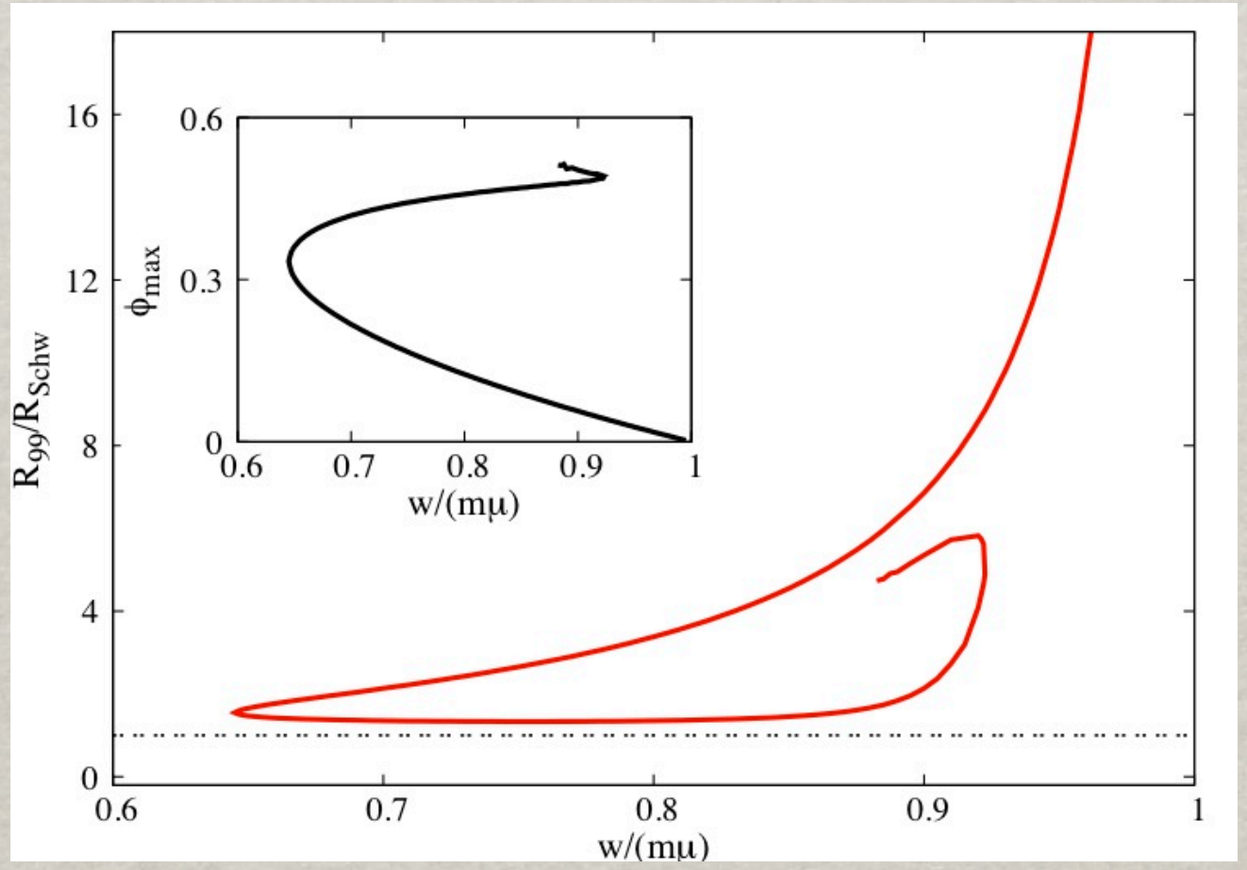
Rotating boson stars are rotating “mass” tori in GR

Rotating boson stars

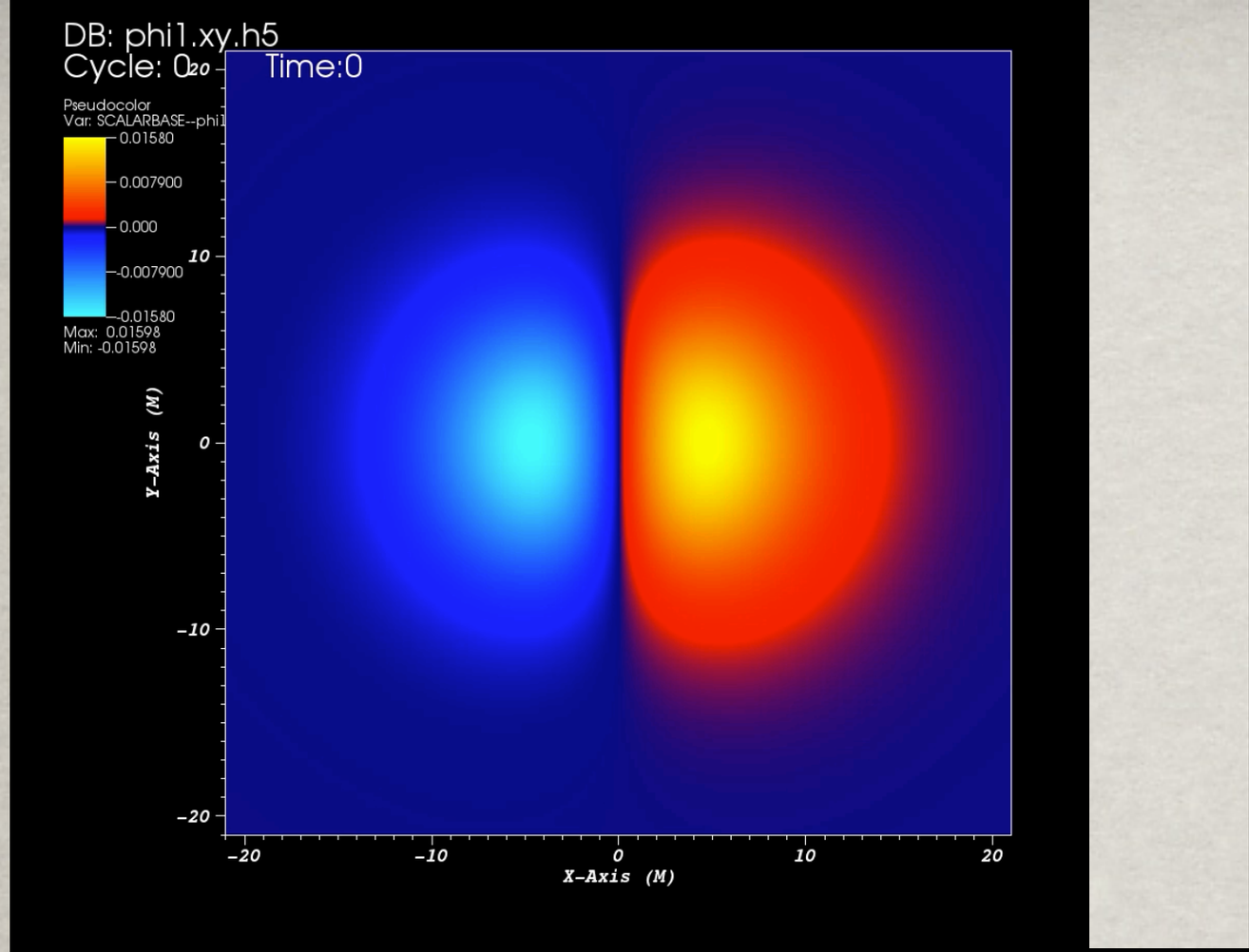
How `compact' are these BSs ?

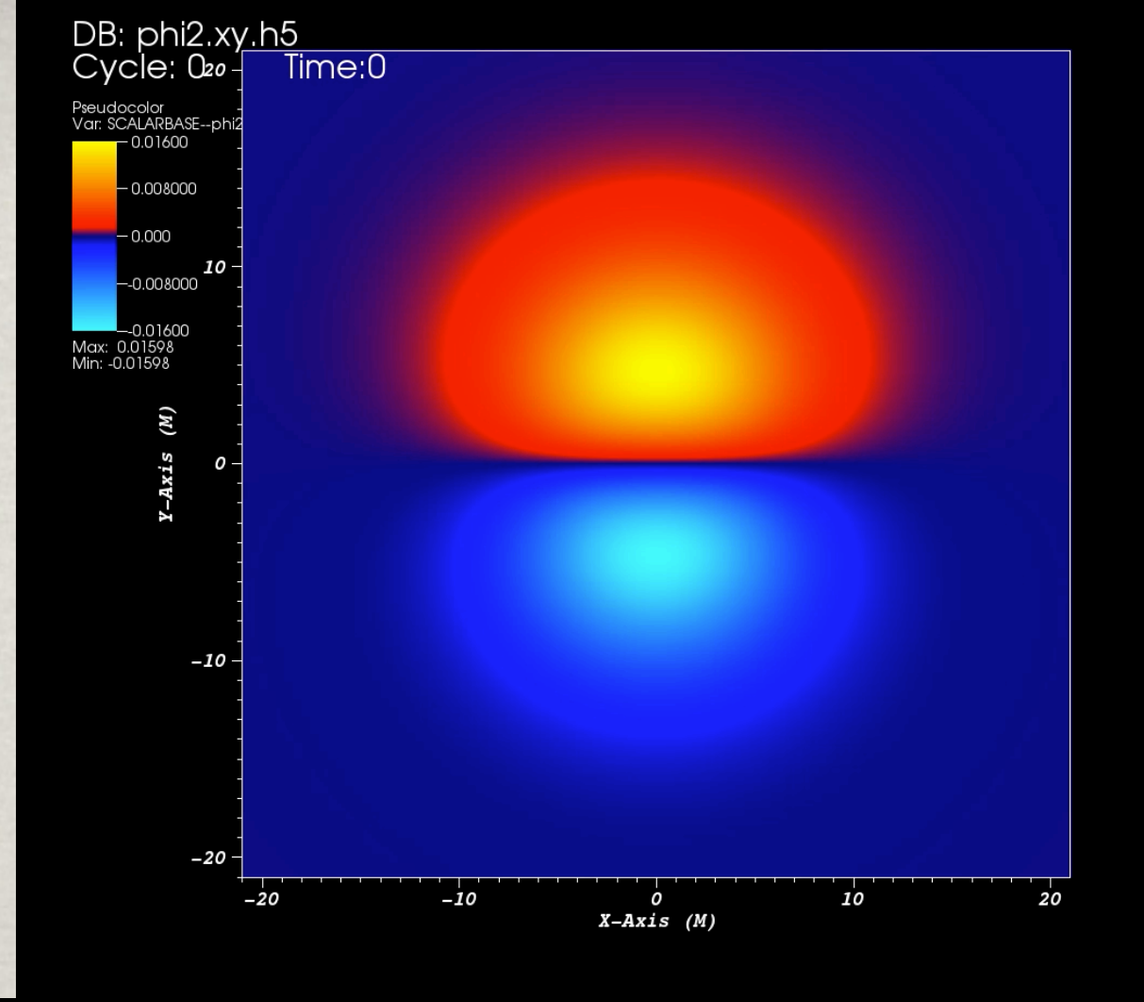
BSs have no surface - the scalar field decays exponentially - thus there is no BS's `radius'.
One estimate is the following:
1) use the `perimeteral' radius, a radial coordinate R such that a circumference along the equatorial plane has perimeter $2\pi R$.
2) compute R_{99}, the perimeteral radius containing 99% of the BS mass, M_{99}.
3) define the **inverse compactness** by comparing R_{99} with the Schwarzschild radius associated to 99% of the BS's mass, $R_{Schw}=2M_{99}$.

$$\text{Compactness}^{-1} \equiv \frac{R_{99}}{2M_{99}}$$

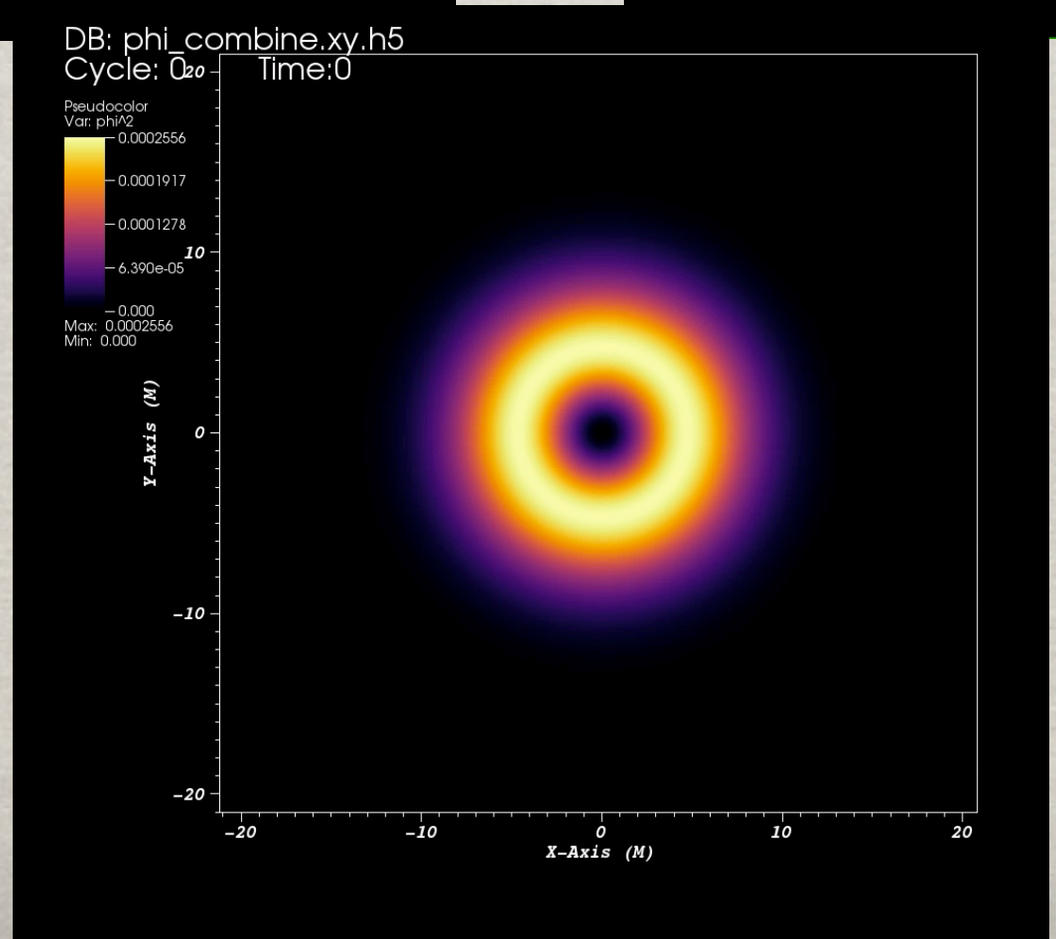


With this measure, the inverse compactness is always greater than unity; in other words, BSs are less compact than BHs, as one would expect.





$\mathcal{R}(\Phi)$



$\mathcal{I}(\Phi)$

Simulations

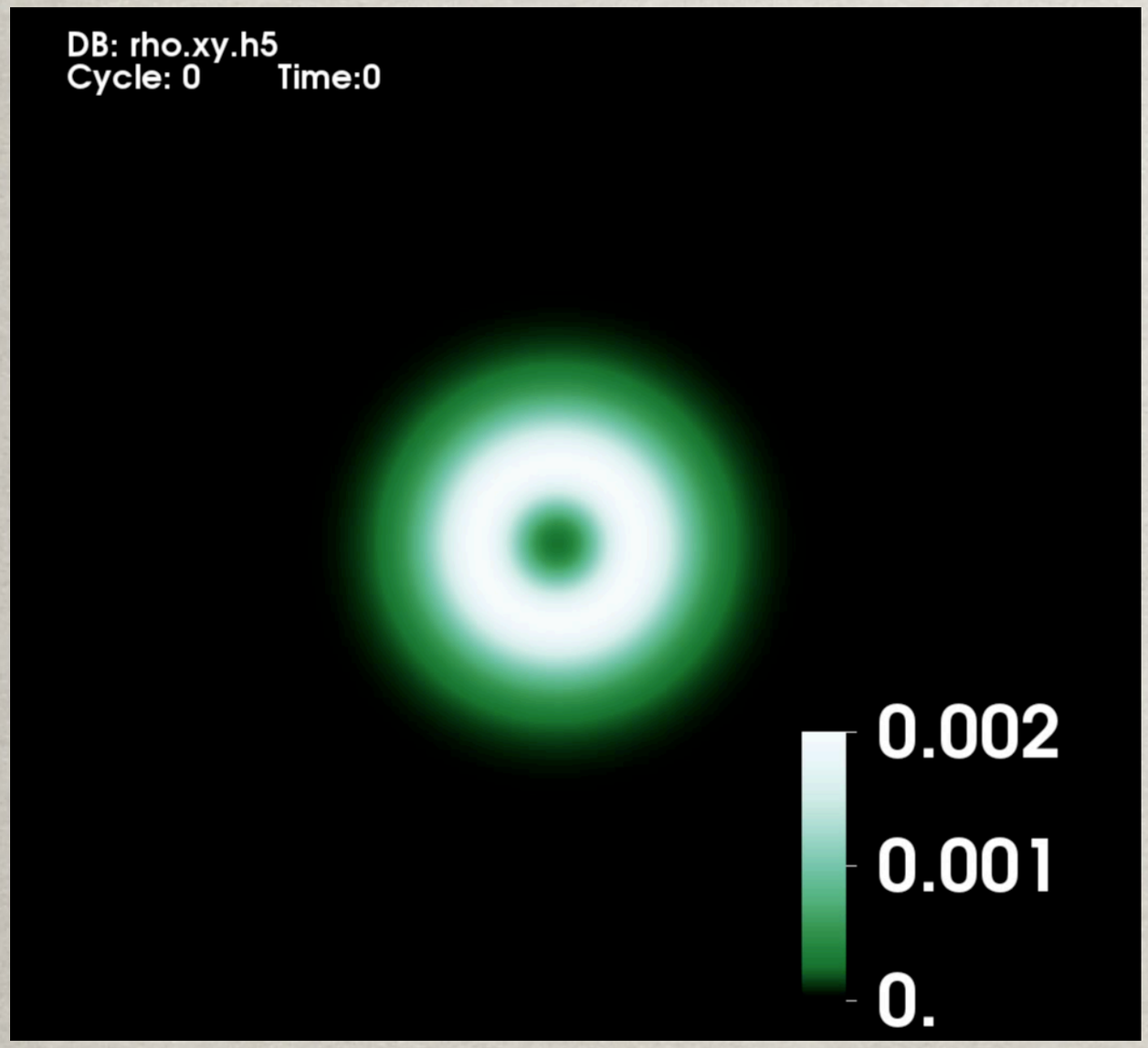
Zilhão (2017)

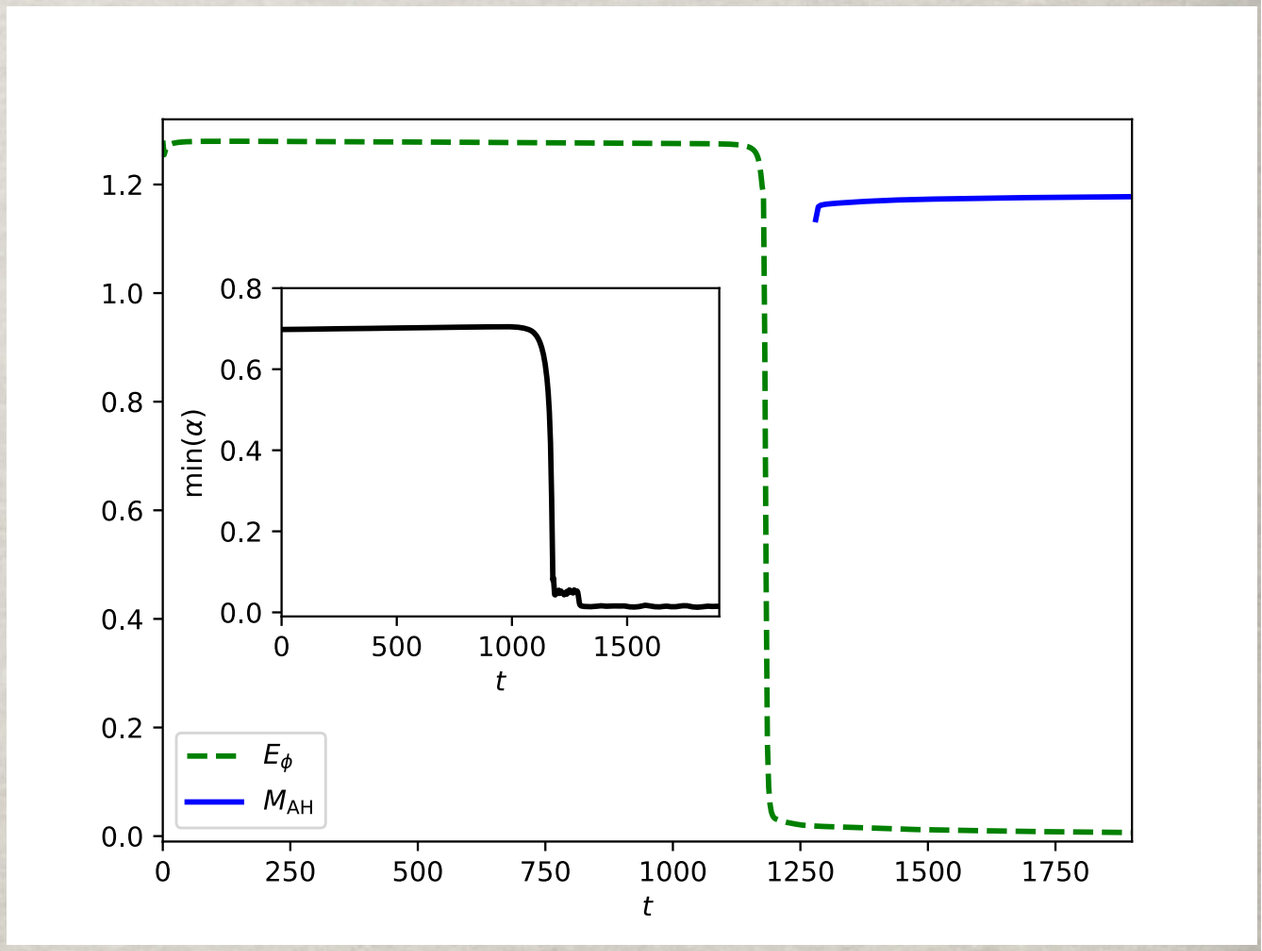
$|\Phi|^2$

Longer evolutions show the appearance of a non-axisymmetric instability

Sanchis-Gual, Di Giovanni, Zilhão, Herdeiro, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740

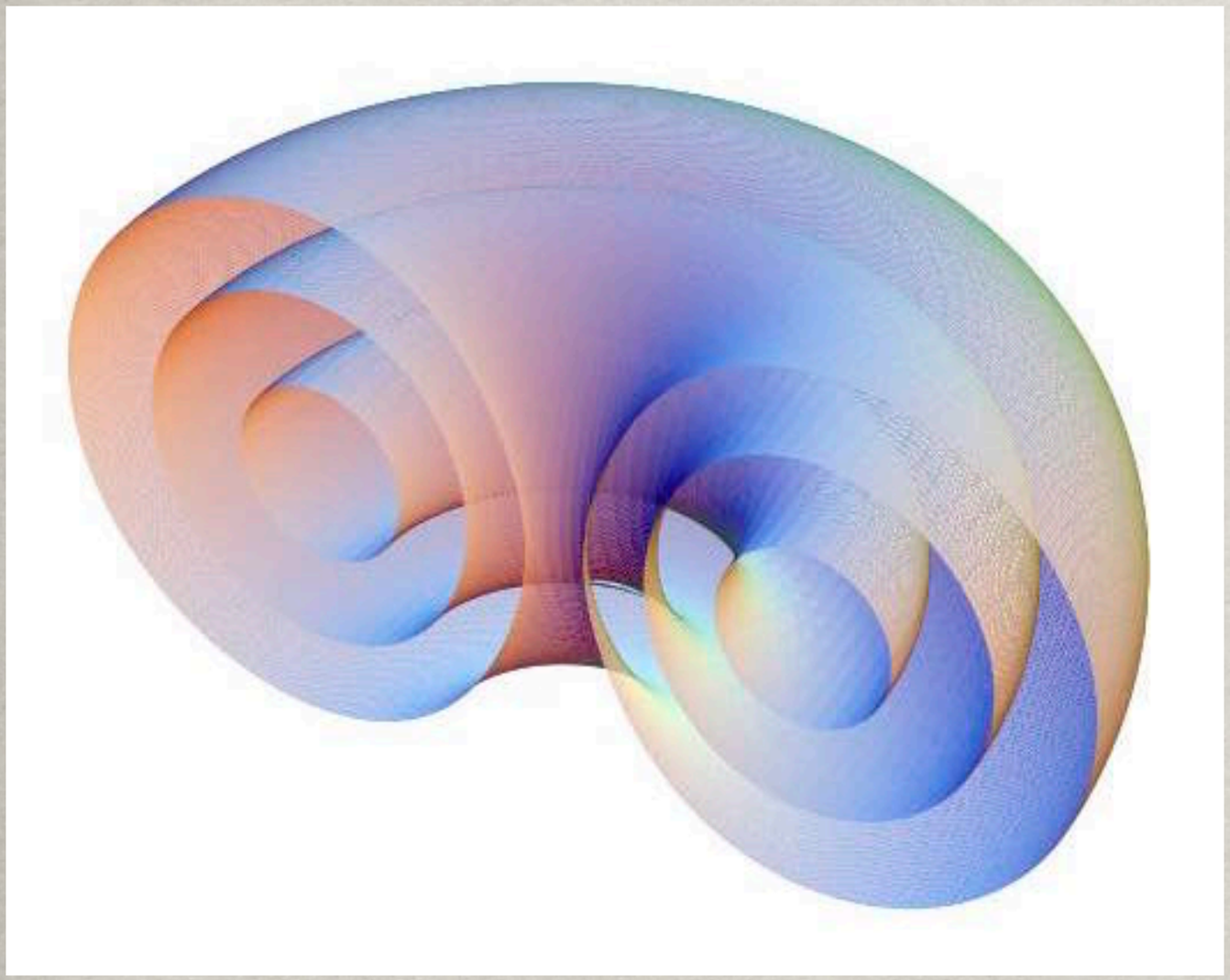




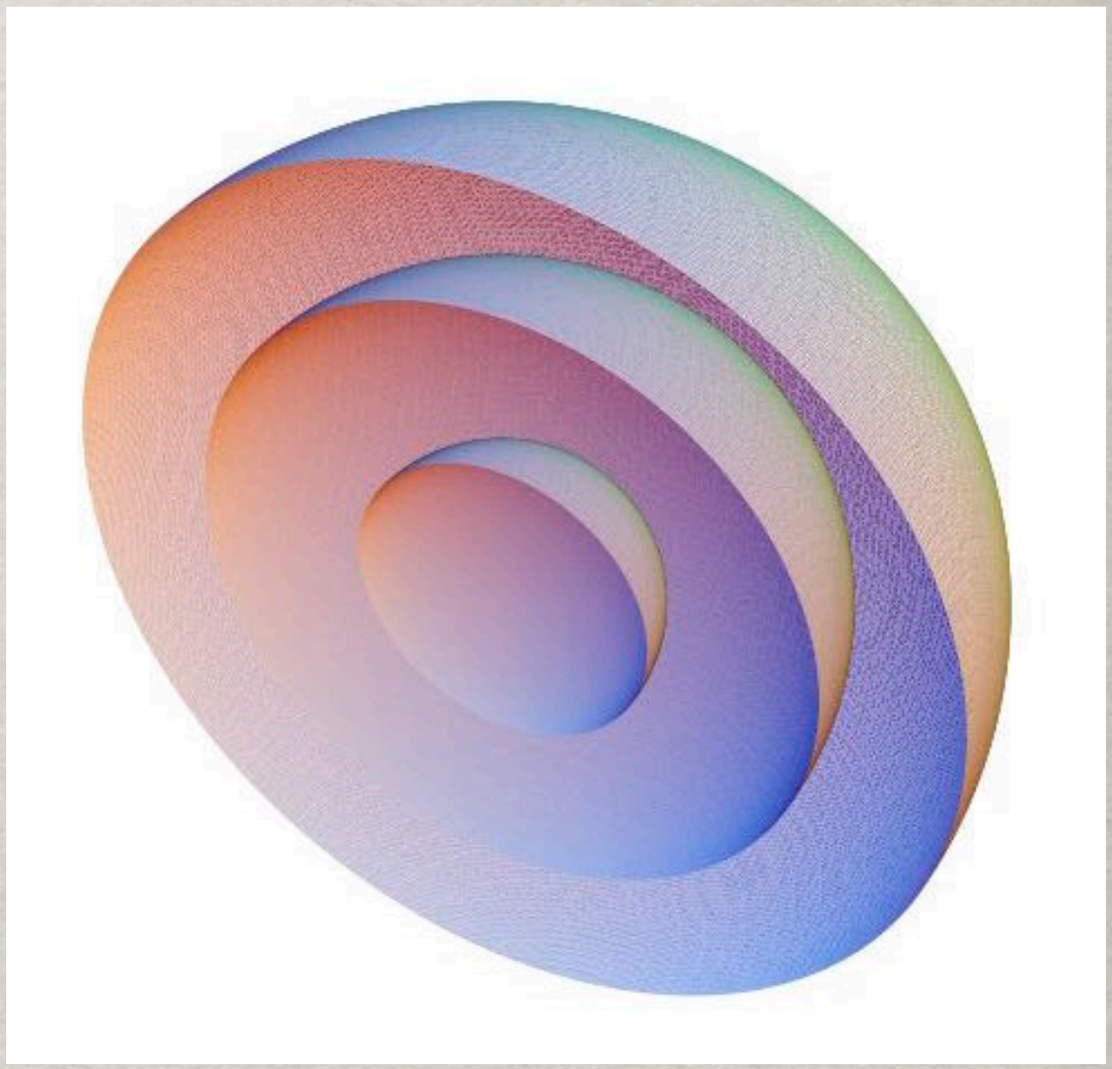
Instability may be associated to toroidal structure and is absent in cousin Proca model

Sanchis-Gual, Di Giovanni, Zilhão, Herdeiro, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740



Rotating boson stars



Rotating Proca stars

Brito, Cardoso, Herdeiro and Radu, PLB 752 (2016) 291

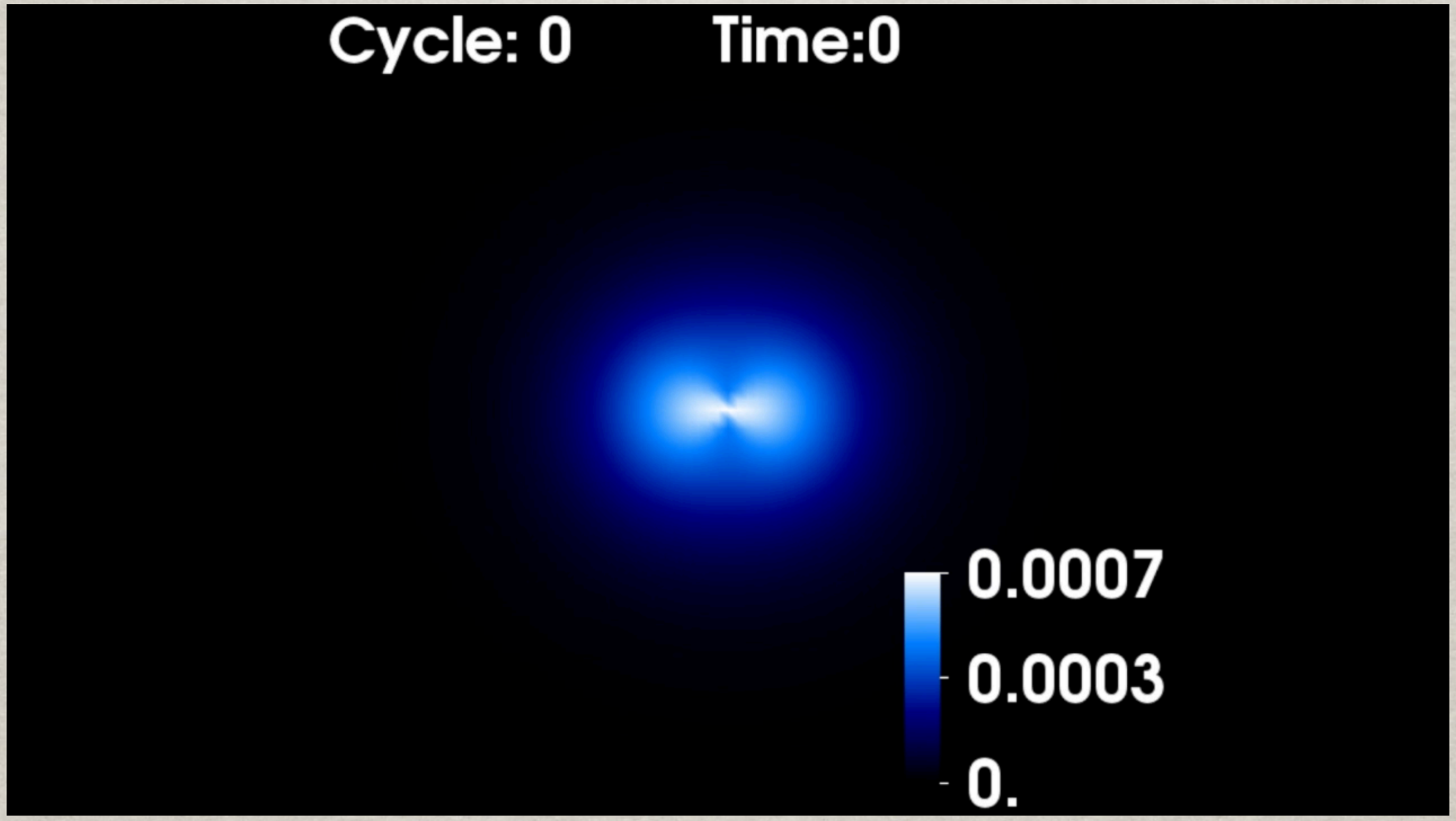
Herdeiro, Radu and Rúnarsson, CQG 33 (2016) 154001

Herdeiro, Perapechka, Radu and Shnir, PLB 797 (2019) 134845

Evolution of a perturbed spinning Proca star

Sanchis-Gual, Di Giovanni, Zilhão, Herdeiro, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

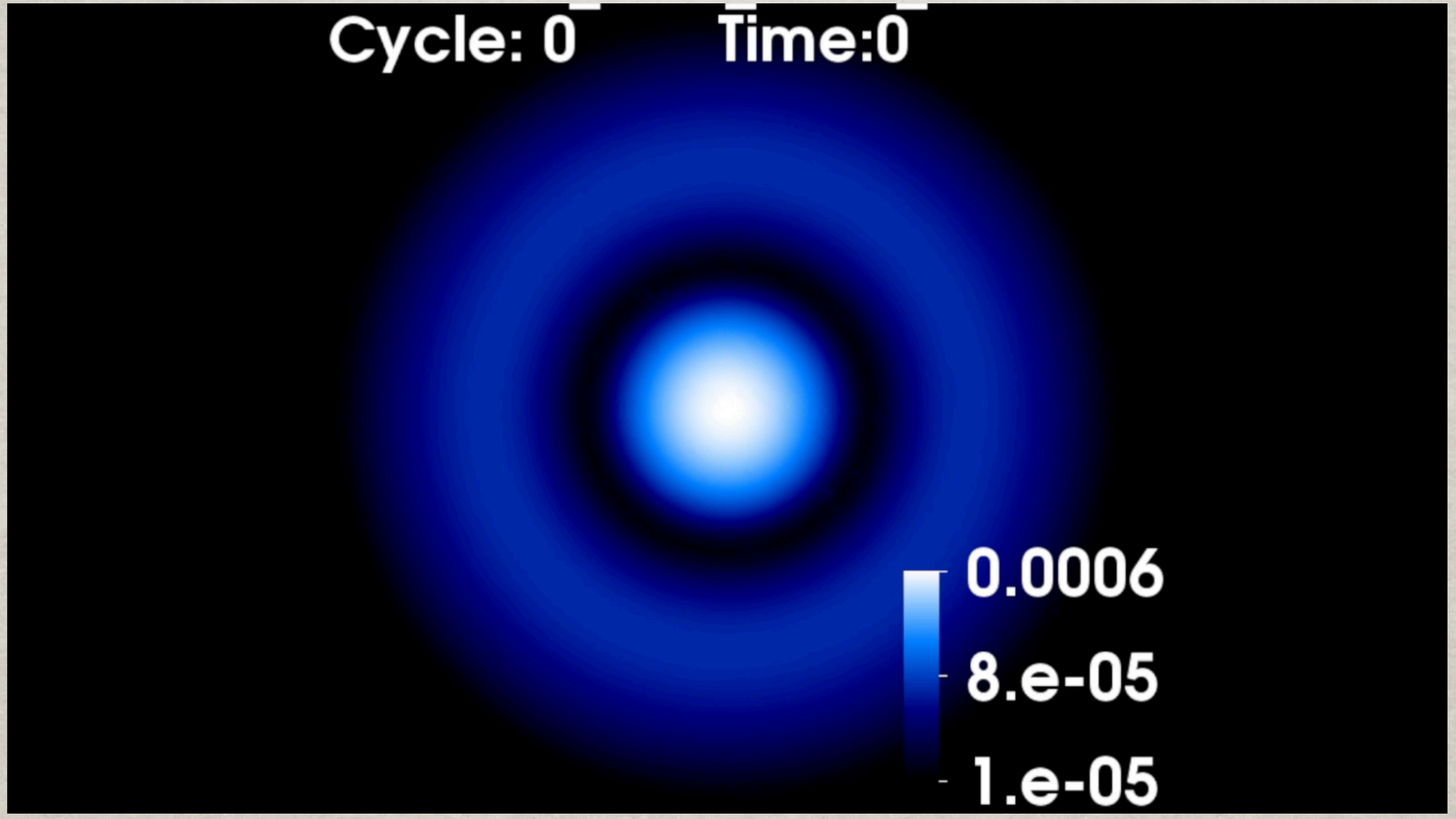
http://gravitation.web.ua.pt/node/1740



Evolution of an excited spinning Proca star

Sanchis-Gual, Di Giovanni, Zilhão, Herdeiro, P. Cerda-Duran, Font and Radu, Phys. Rev. Lett. 123 (2019) 221101

http://gravitation.web.ua.pt/node/1740



The (ultra) light in the dark: A potential vector boson of 8.7×10^{-13} eV from GW190521

Juan Calderón Bustillo,[1,2,3,*] Nicolas Sanchis-Gual,[4,†] Alejandro Torres-Forné,[5] José A. Font,[6,7] Avi Vajpeyi,[2,3] Rory Smith,[2,3] Carlos Herdeiro,[8] Eugen Radu,[8] and Samson H. W. Leong[1]

[1] *Department of Physics, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong*
[2] *Monash Centre for Astrophysics, School of Physics and Astronomy, Monash University, VIC 3800, Australia*
[3] *OzGrav: The ARC Centre of Excellence for Gravitational-Wave Discovery, Clayton, VIC 3800, Australia*
[4] *Centro de Astrofísica e Gravitação - CENTRA, Departamento de Física, Instituto Superior Técnico - IST, Universidade de Lisboa - UL, Avenida Rovisco Pais 1, 1049-001, Portugal*
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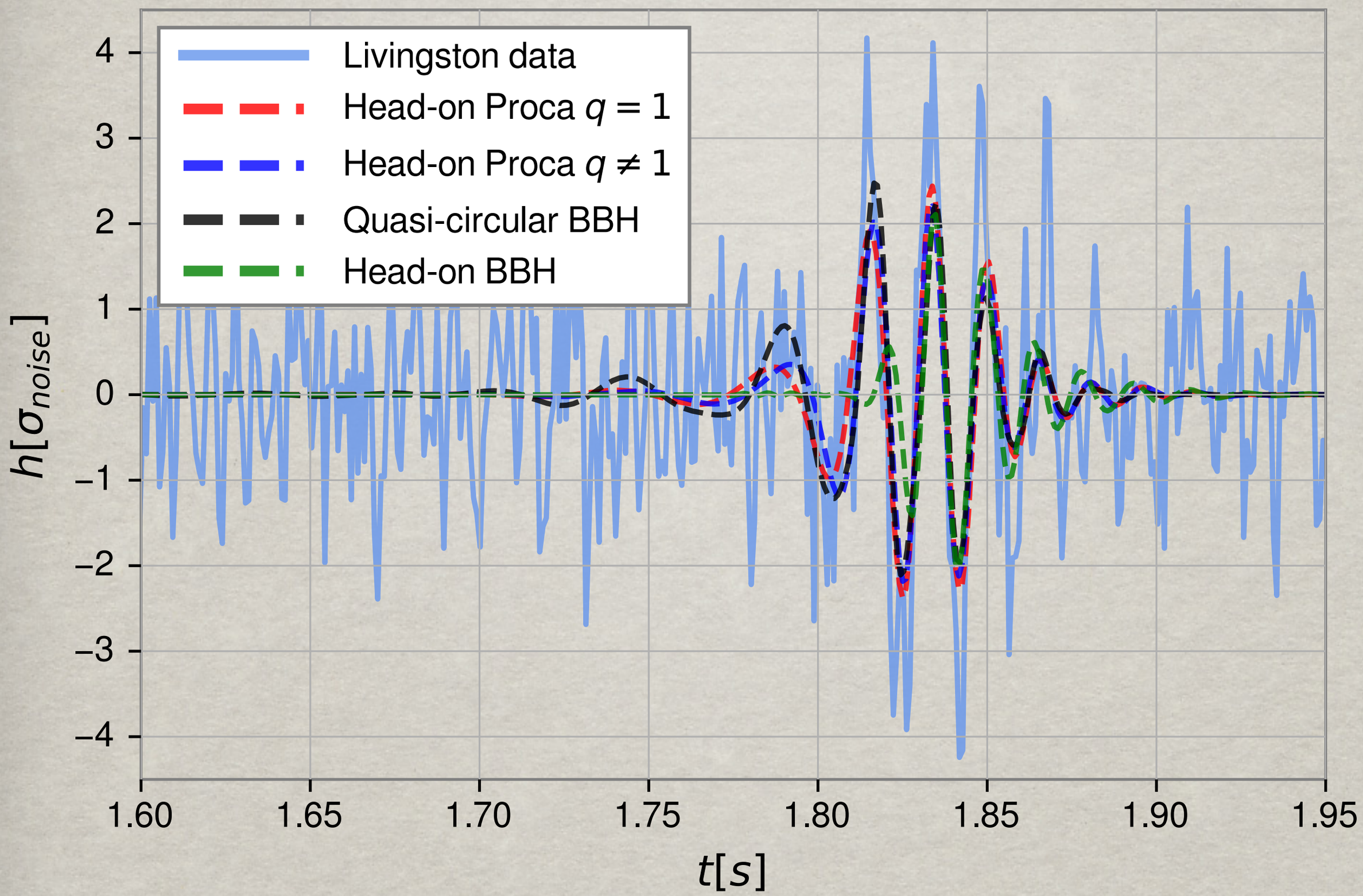
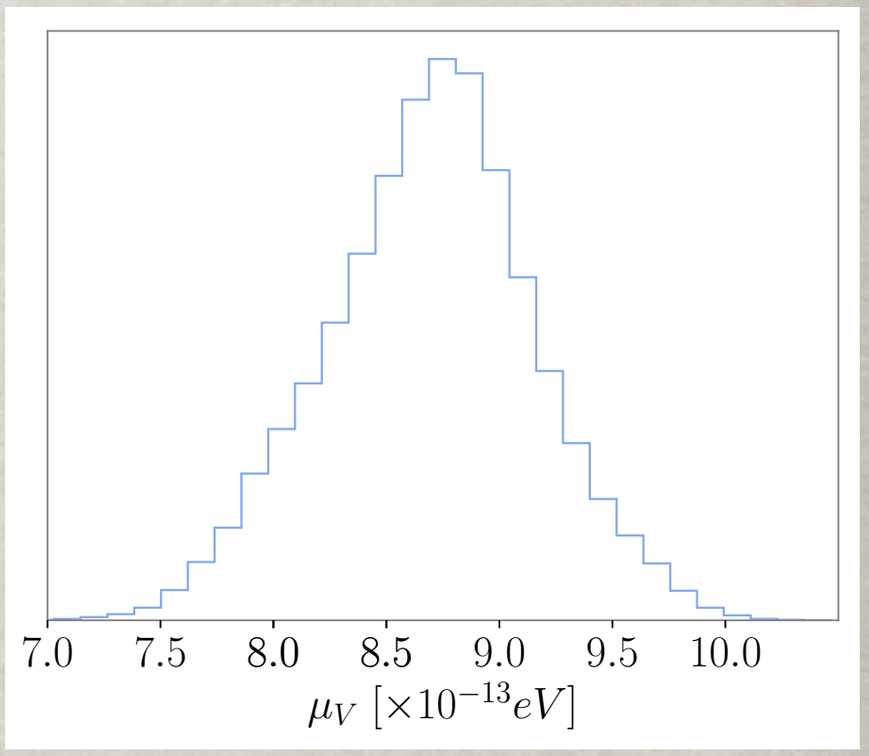
[6] *Departamento de Astronomía y Astrofísica, Universitat de València, Dr. Moliner 50, 46100, Burjassot (València), Spain*
[7] *Observatori Astronòmic, Universitat de València, C/ Catedrático José Beltrán 2, 46980, Paterna (València), Spain*
[8] *Departamento de Matemática da Universidade de Aveiro and Centre for Research and Development in Mathematics and Applications (CIDMA), Campus de Santiago, 3810-183 Aveiro, Portugal*

Advanced LIGO-Virgo reported a short gravitational-wave signal (GW190521) interpreted as a quasi-circular merger of black holes, one populating the pair-instability supernova gap, forming a remnant black hole of $M_f \sim 142 M_\odot$ at a luminosity distance of $d_L \sim 5.3$ Gpc. With barely visible pre-merger emission, however, GW190521 merits further investigation of the pre-merger dynamics and even of the very nature of the colliding objects. We show that GW190521 is consistent with numerically simulated signals from head-on collisions of two (equal mass and spin) horizonless vector boson stars (aka Proca stars), forming a final black hole with $M_f = 231^{+13}_{-17}\, M_\odot$, located at a distance of $d_L = 571^{+348}_{-181}$ Mpc. The favoured mass for the ultra-light vector boson constituent of the Proca stars is $\mu_{\rm V} = 8.72^{+0.73}_{-0.82} \times 10^{-13}$ eV. This provides the first demonstration of close degeneracy between these two theoretical models, for a real gravitational-wave event. Confirmation of the Proca star interpretation, which we find statistically slightly preferred, would provide the first evidence for a long sought dark matter particle.


Livingston data
Head-on Proca $q = 1$
Head-on Proca $q \neq 1$
Quasi-circular BBH
Head-on BBH
$h[\sigma_{noise}]$
$t[s]$
4
3
2
1
0
−1
−2
−3
−4
1.60
1.65
1.70
1.75
1.80
1.85
1.90
1.95

Parameter	$q = 1$ model	$q \neq 1$ model
Primary mass	$115^{+7}_{-8}\ M_\odot$	$115^{+7}_{-8}\ M_\odot$
Secondary mass	$115^{+7}_{-8}\ M_\odot$	$111^{+7}_{-15}\ M_\odot$
Total / Final mass	$231^{+13}_{-17}\ M_\odot$	$228^{+17}_{-15}\ M_\odot$
Final spin	$0.75^{+0.08}_{-0.04}$	$0.75^{+0.08}_{-0.04}$
Inclination $\pi/2 - \lvert\iota - \pi/2\rvert$	$0.83^{+0.23}_{-0.47}$ rad	$0.58^{+0.40}_{-0.39}$ rad
Azimuth	$0.65^{+0.86}_{-0.54}$ rad	$0.78^{+1.23}_{-1.20}$ rad
Luminosity distance	571^{+348}_{-181} Mpc	700^{+292}_{-279} Mpc
Redshift	$0.12^{+0.05}_{-0.04}$	$0.14^{+0.06}_{-0.05}$
Total / Final redshifted mass	$258^{+9}_{-9}\ M_\odot$	$261^{+10}_{-11}\ M_\odot$
Bosonic field frequency ω/μ_V	$0.893^{+0.015}_{-0.015}$	$(*)0.905^{+0.012}_{-0.042}$
Boson mass μ_V $[\times 10^{-13}]$	$8.72^{+0.73}_{-0.82}$ eV	$8.59^{+0.58}_{-0.57}$ eV
Maximal boson star mass	$173^{+19}_{-14}\ M_\odot$	$175^{+13}_{-11}\ M_\odot$



Lecture 4

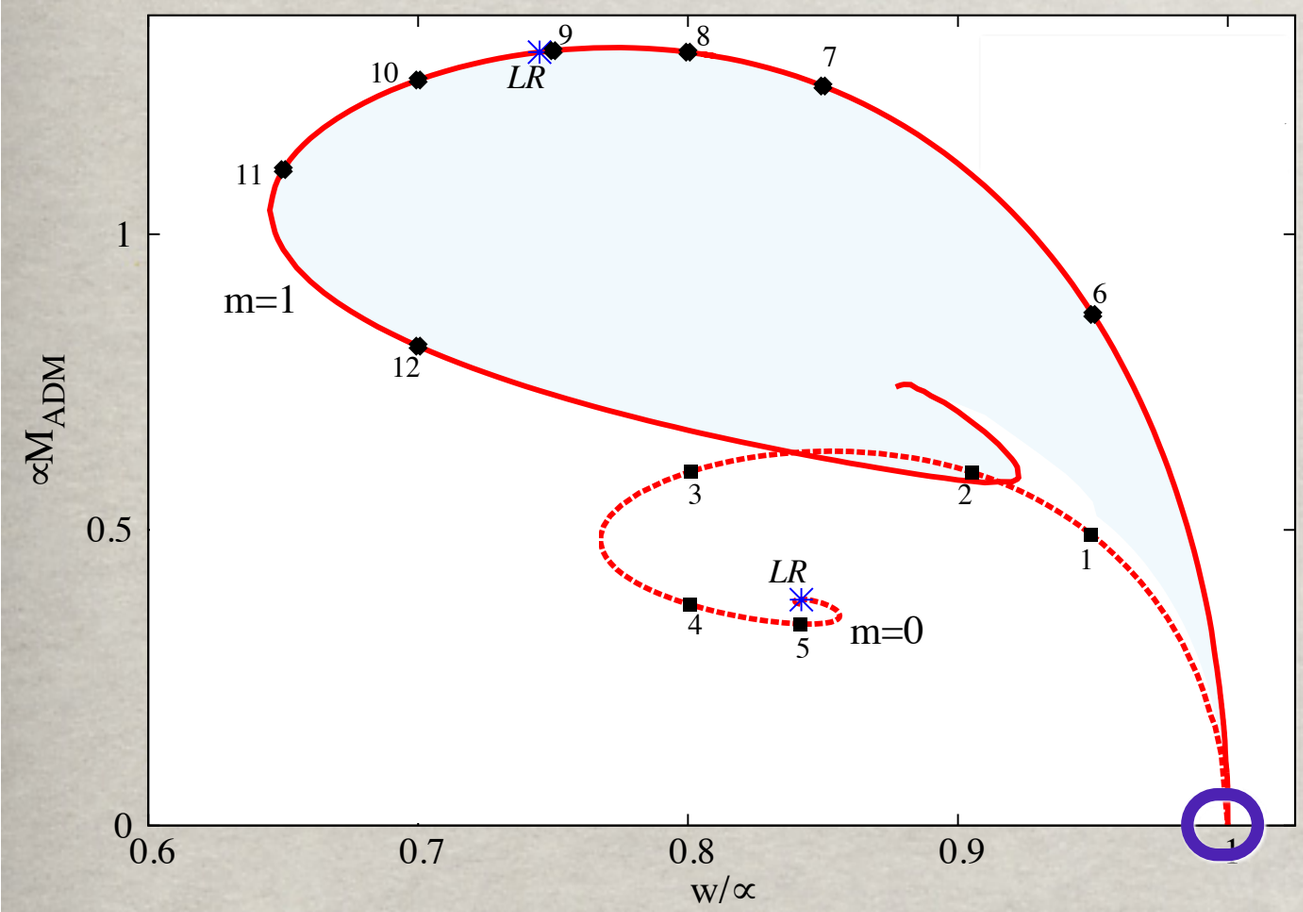
I) Spherical boson (scalar/vector) star solutions

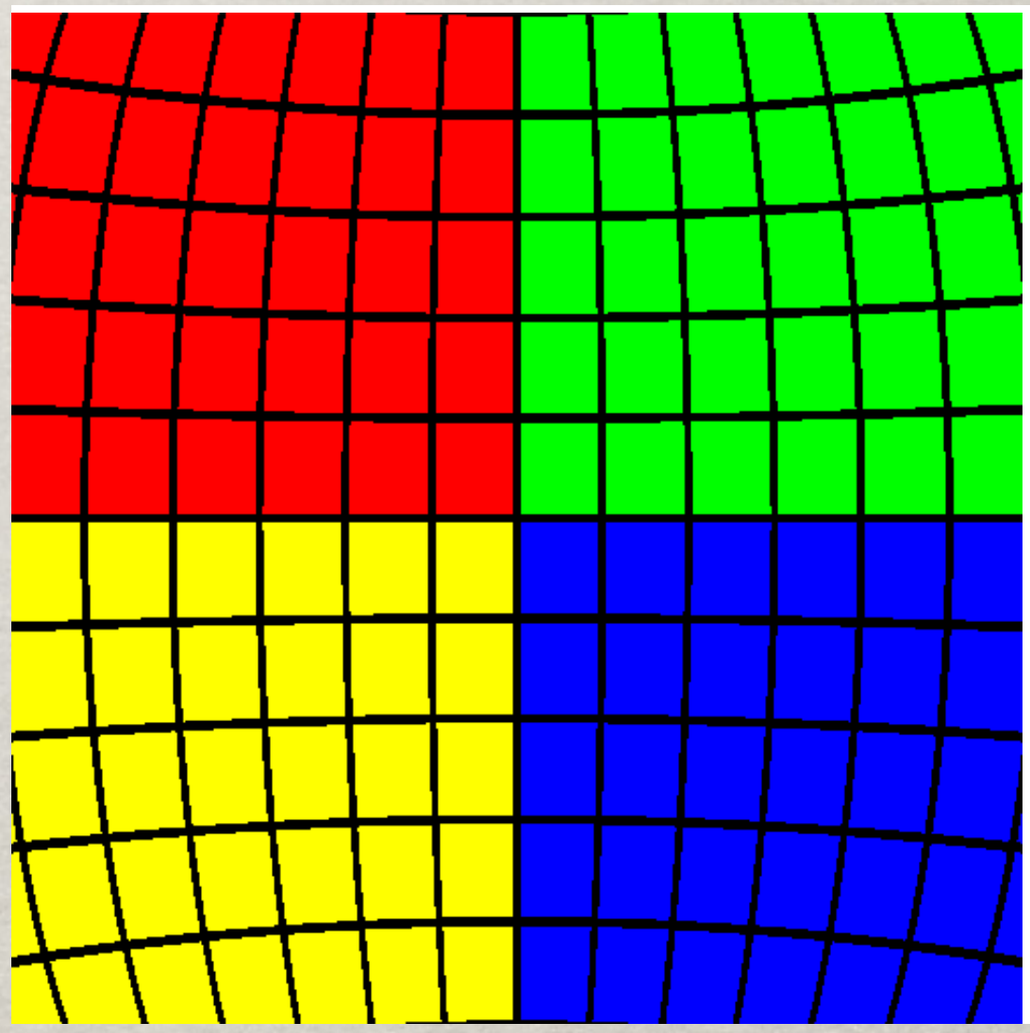
II) Lensing by spherical bosonic stars

III) Rotating bosonic (scalar/vector) star solutions

IV) Lensing by rotating bosonic stars

Configuration 0 (no boson star)



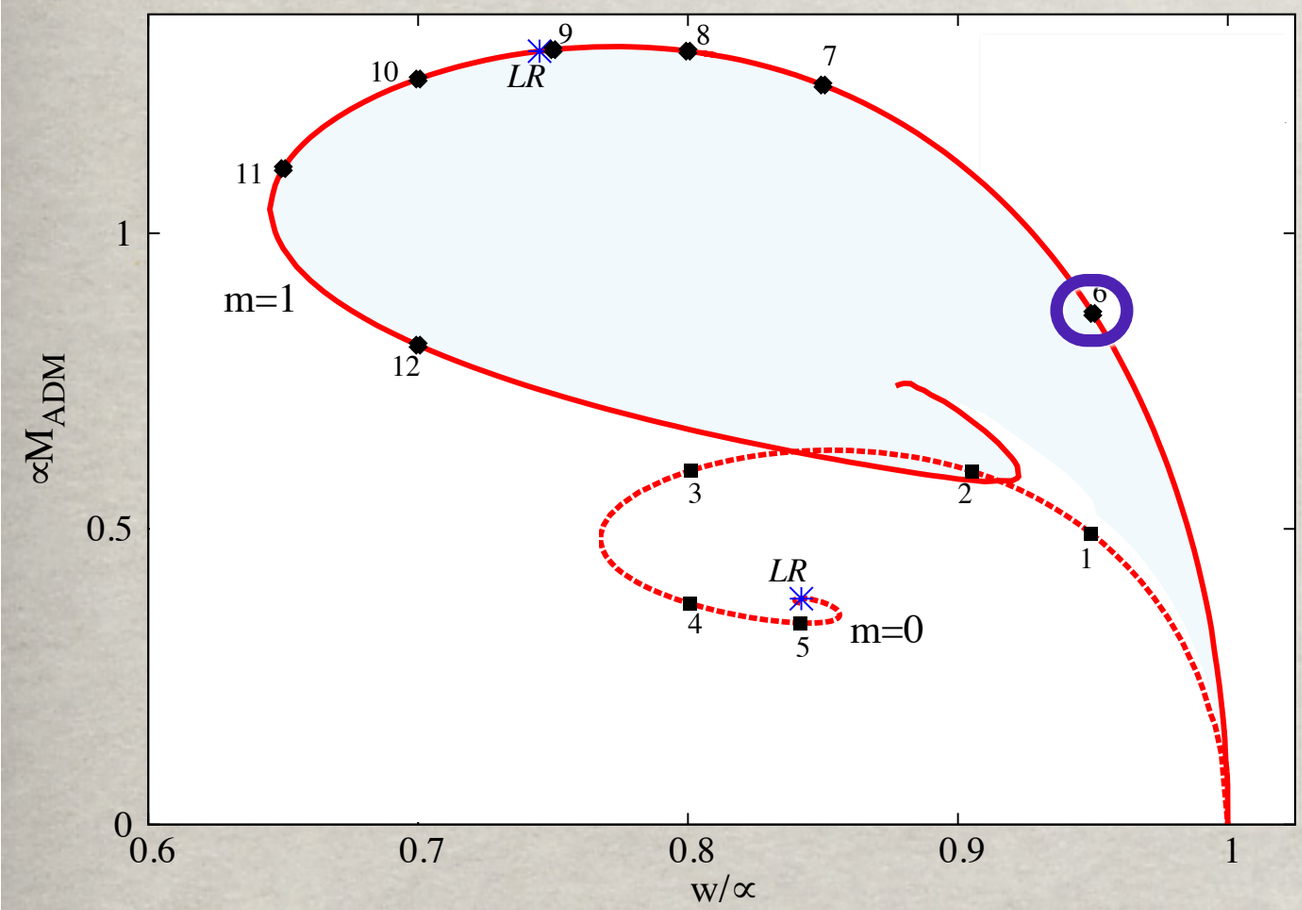


Again take

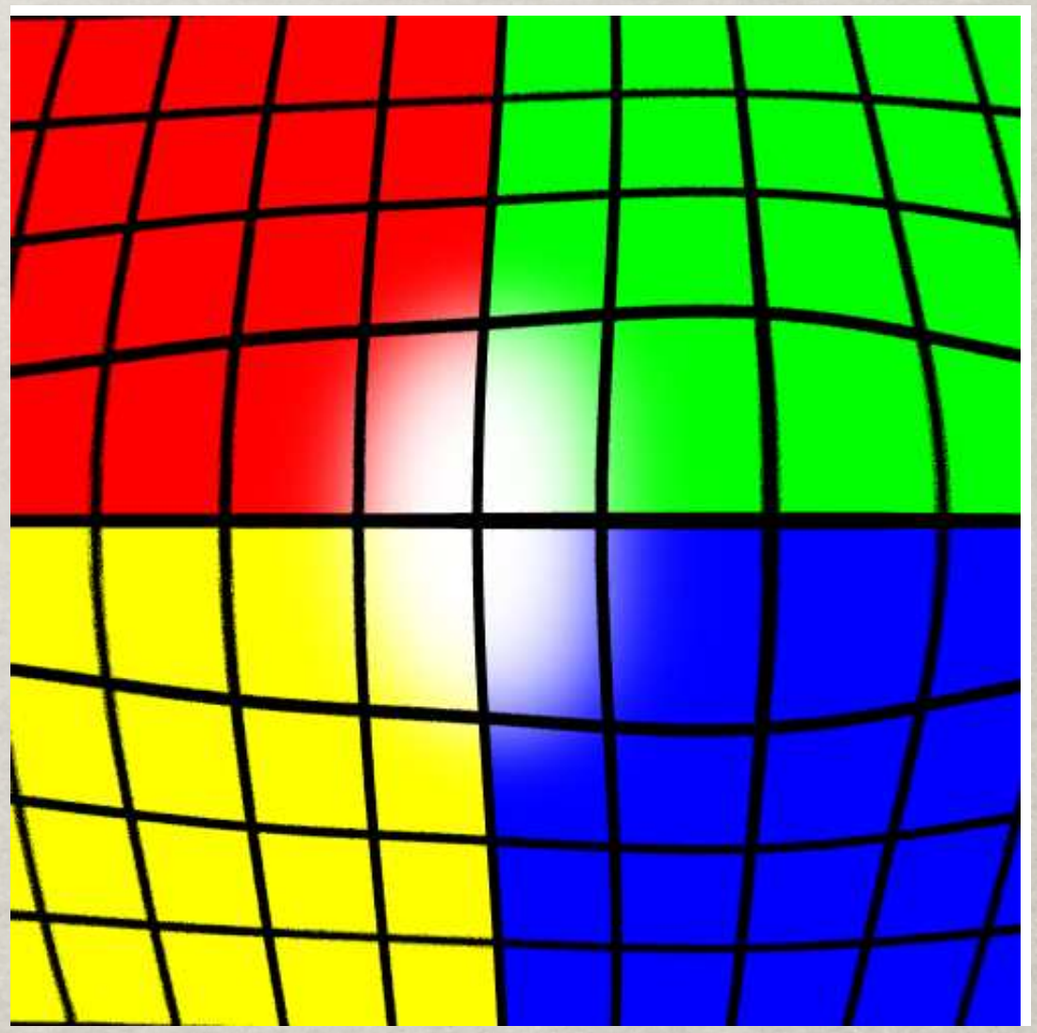
$$\mu = constante$$

so the solutions have varying mass M

Rotating boson stars

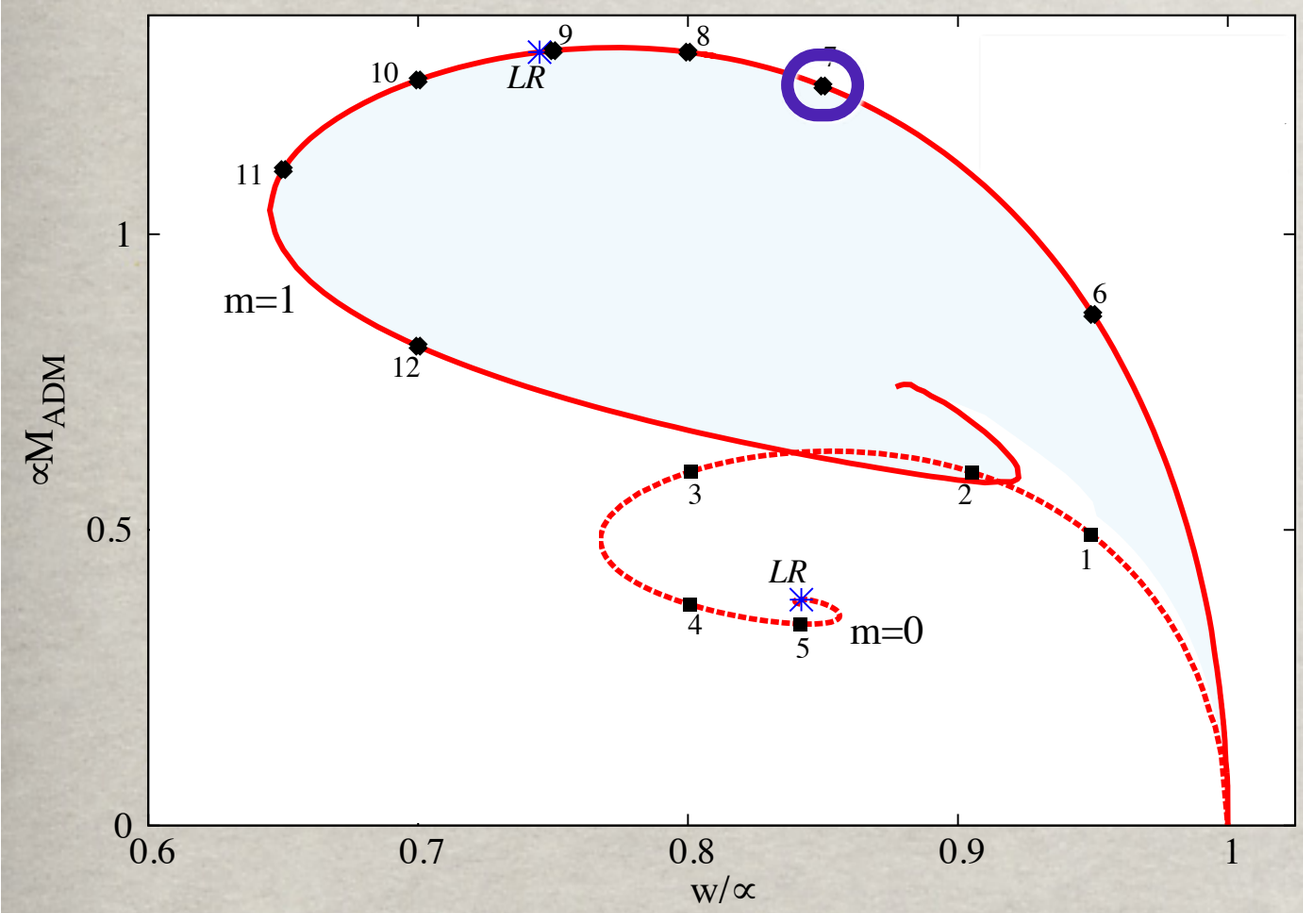


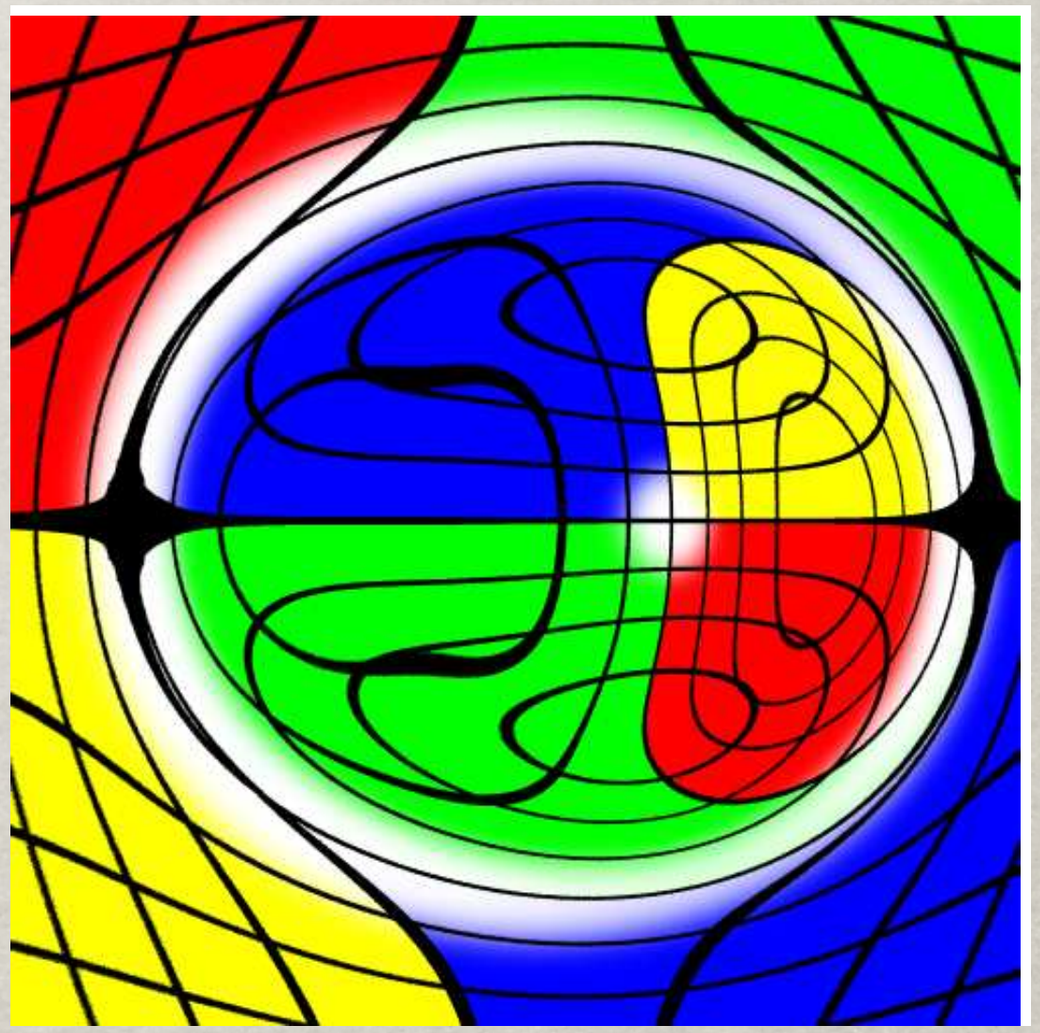
Asymmetric lensing



Non-compact RBS

Rotating boson stars



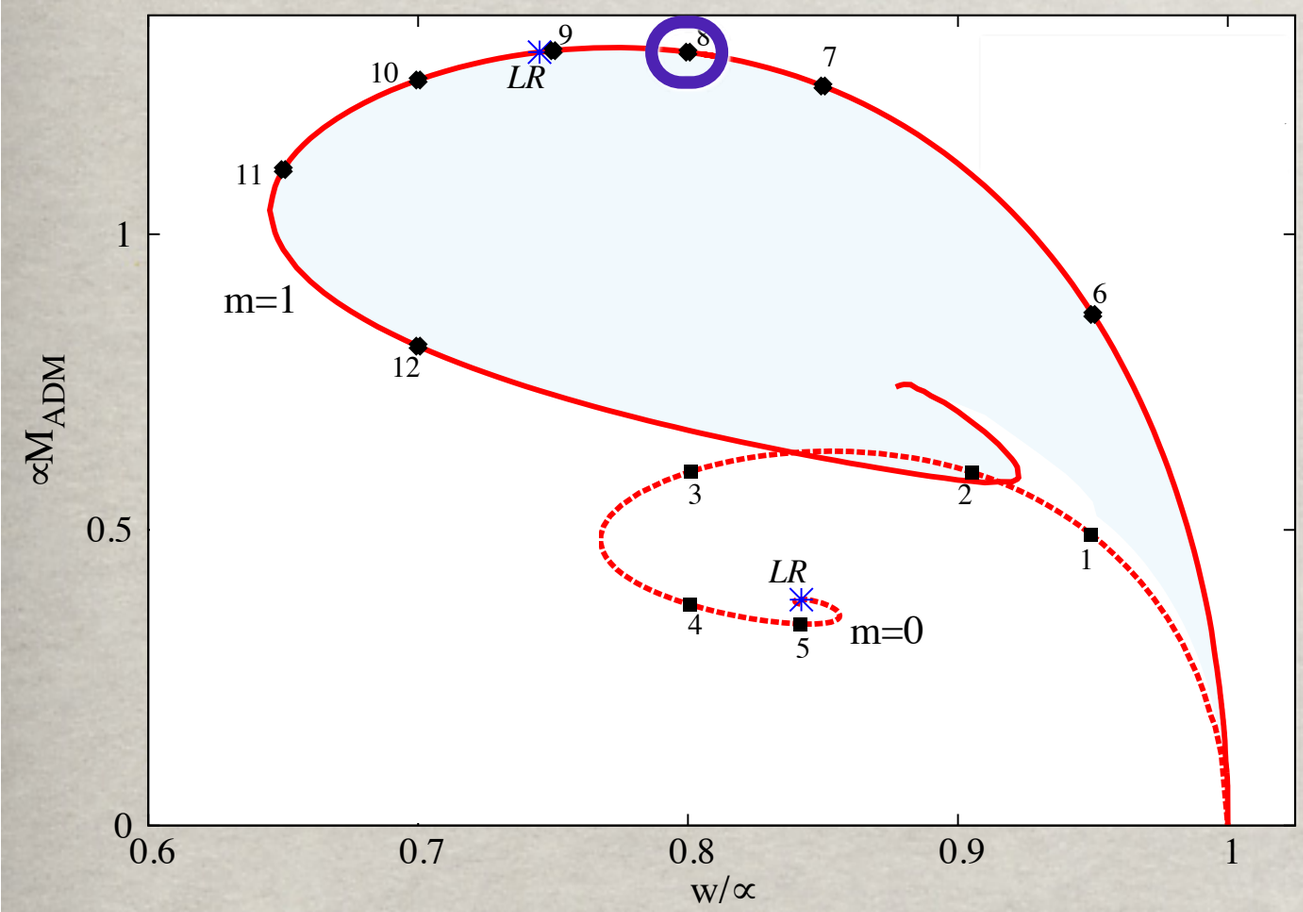


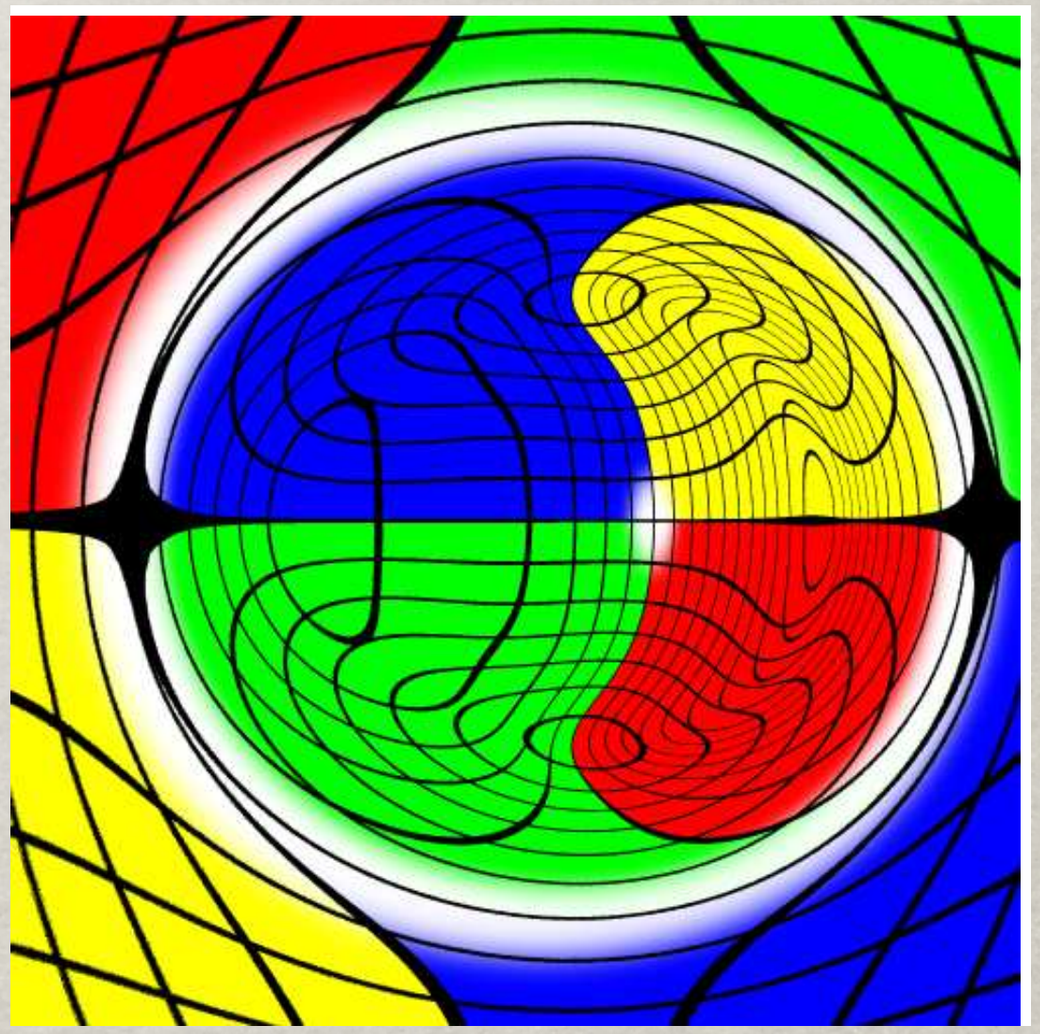
Einstein ring appears at:

$$w_{ER1}^{(b1)} \simeq 0.92$$

Compact RBS

Rotating boson stars

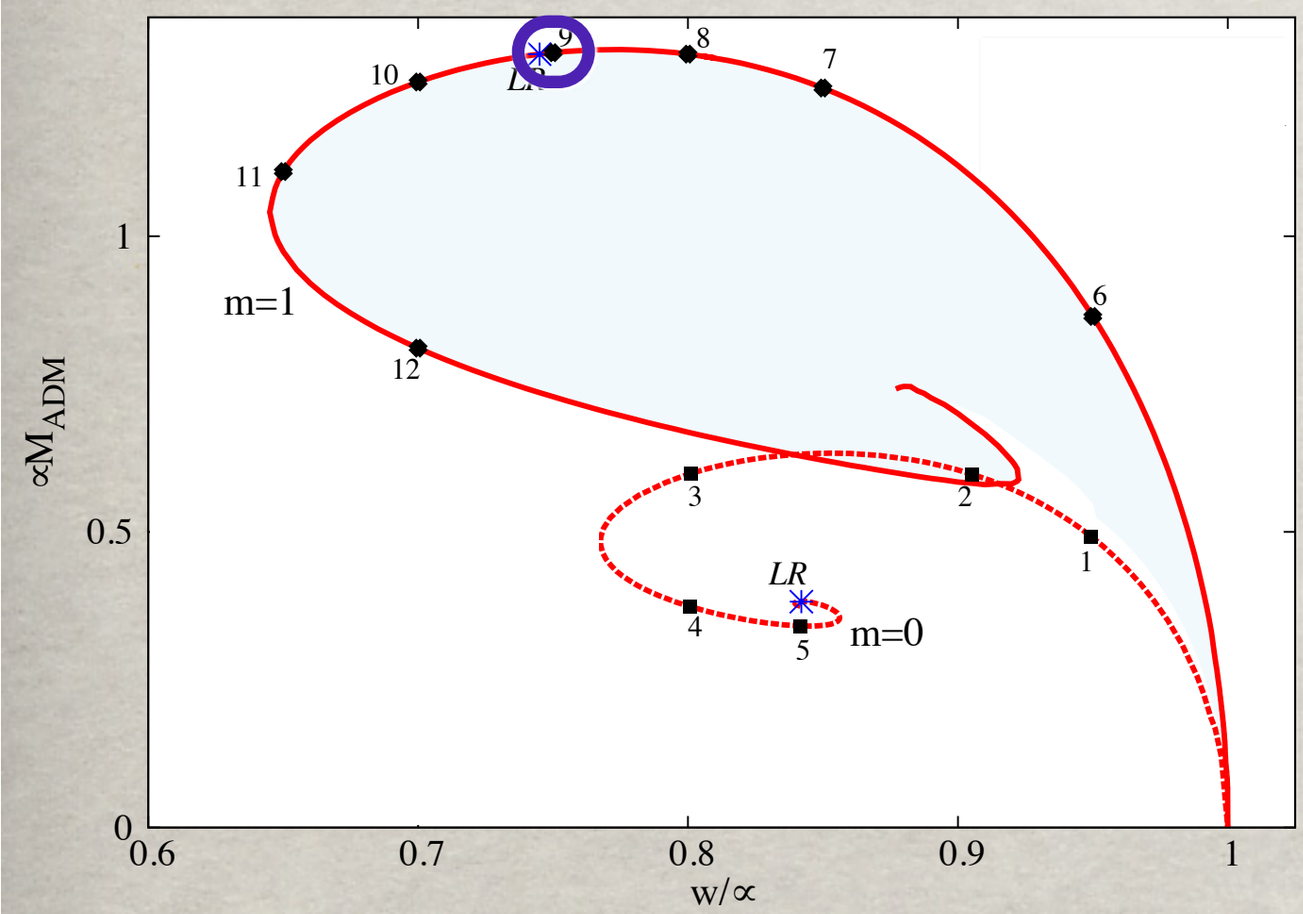


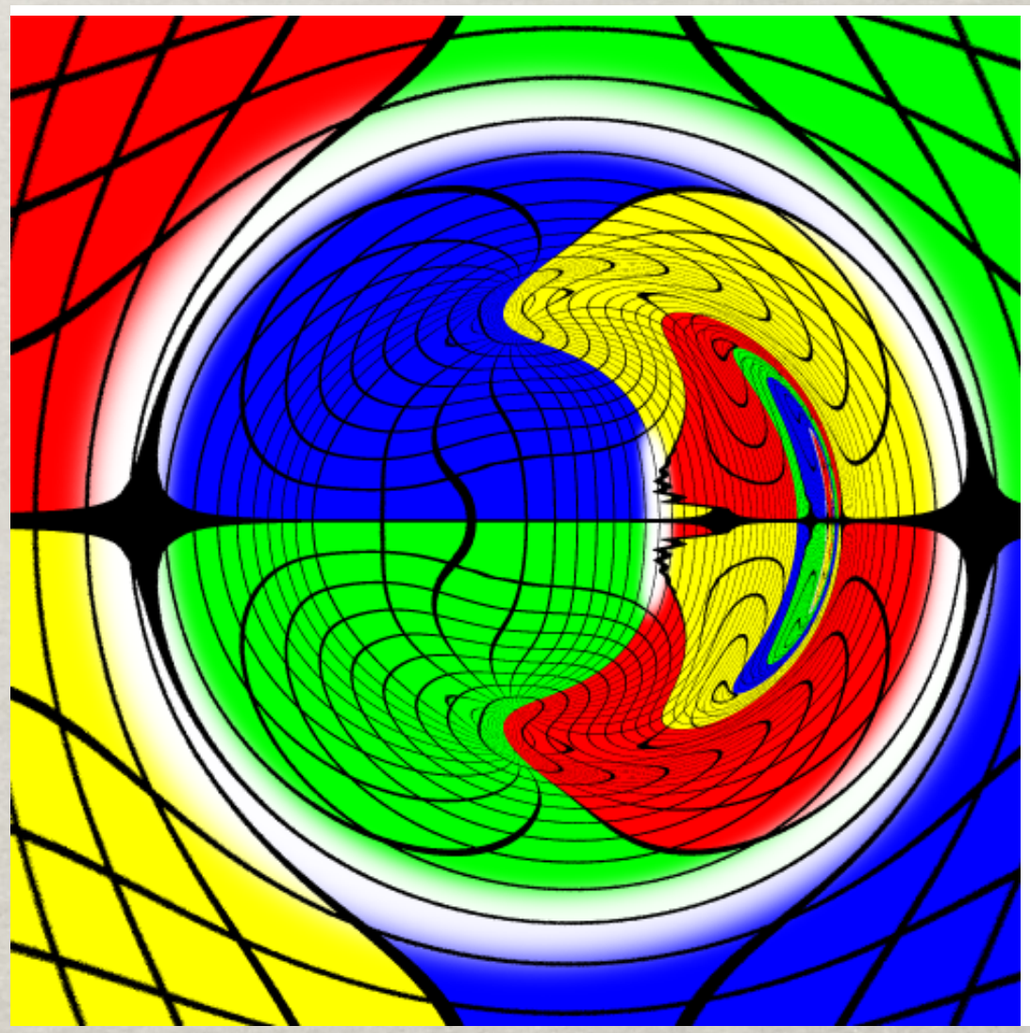


Larger region is “duplicated”

Compact RBS

Rotating boson stars

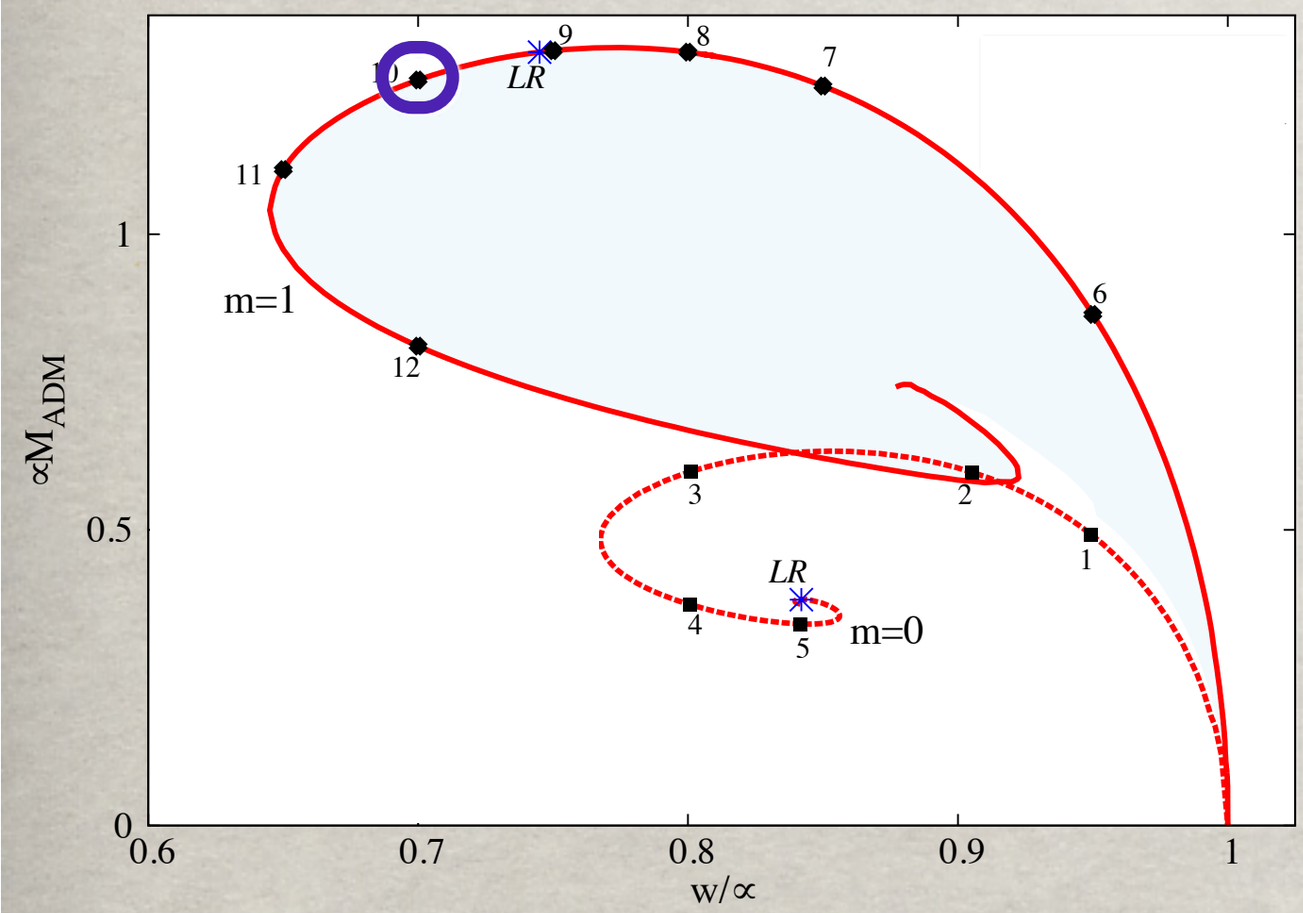


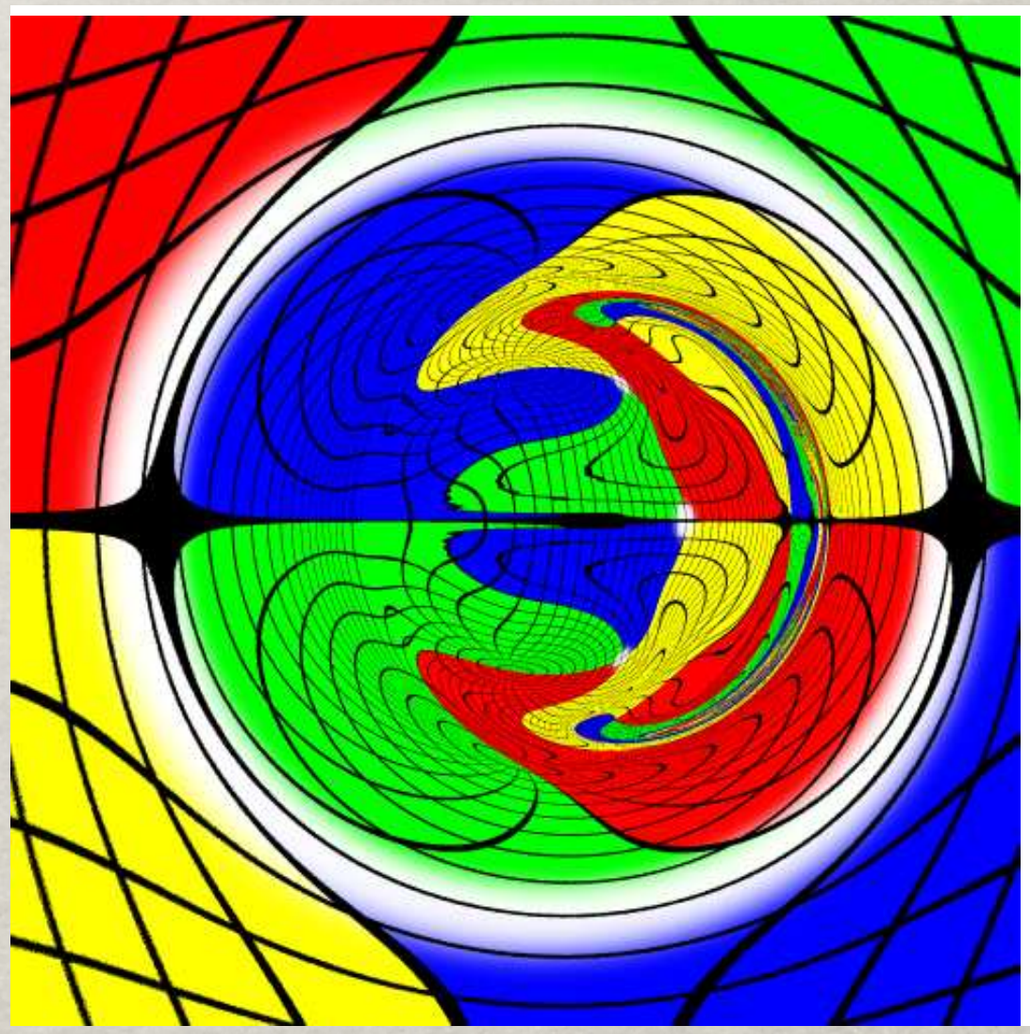


More Einstein rings but squashed D-shaped (not O-shaped)

Compact RBS

Rotating boson stars

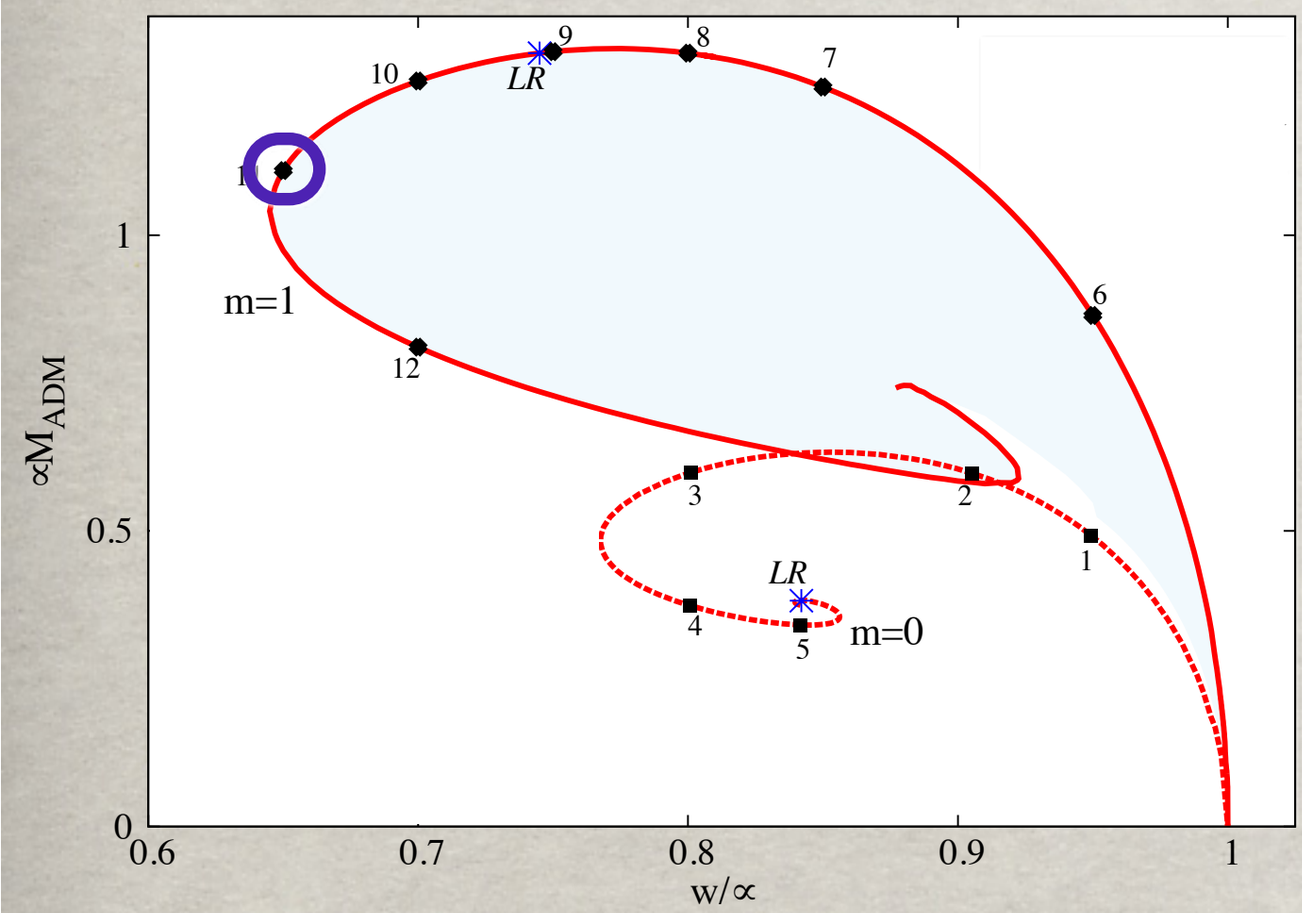


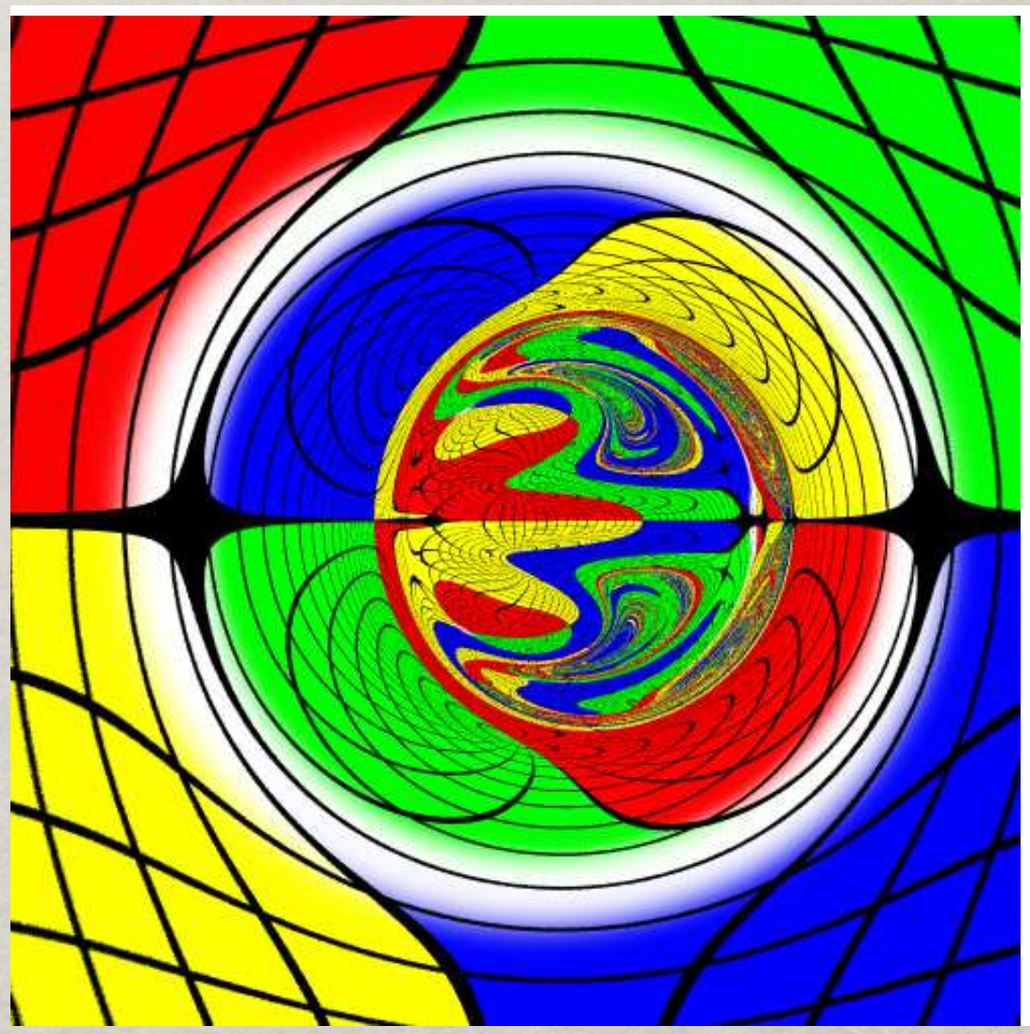


(Presumably)
Infinitely many copies of
the celestial sphere

Ultra-compact RBS

Rotating boson stars

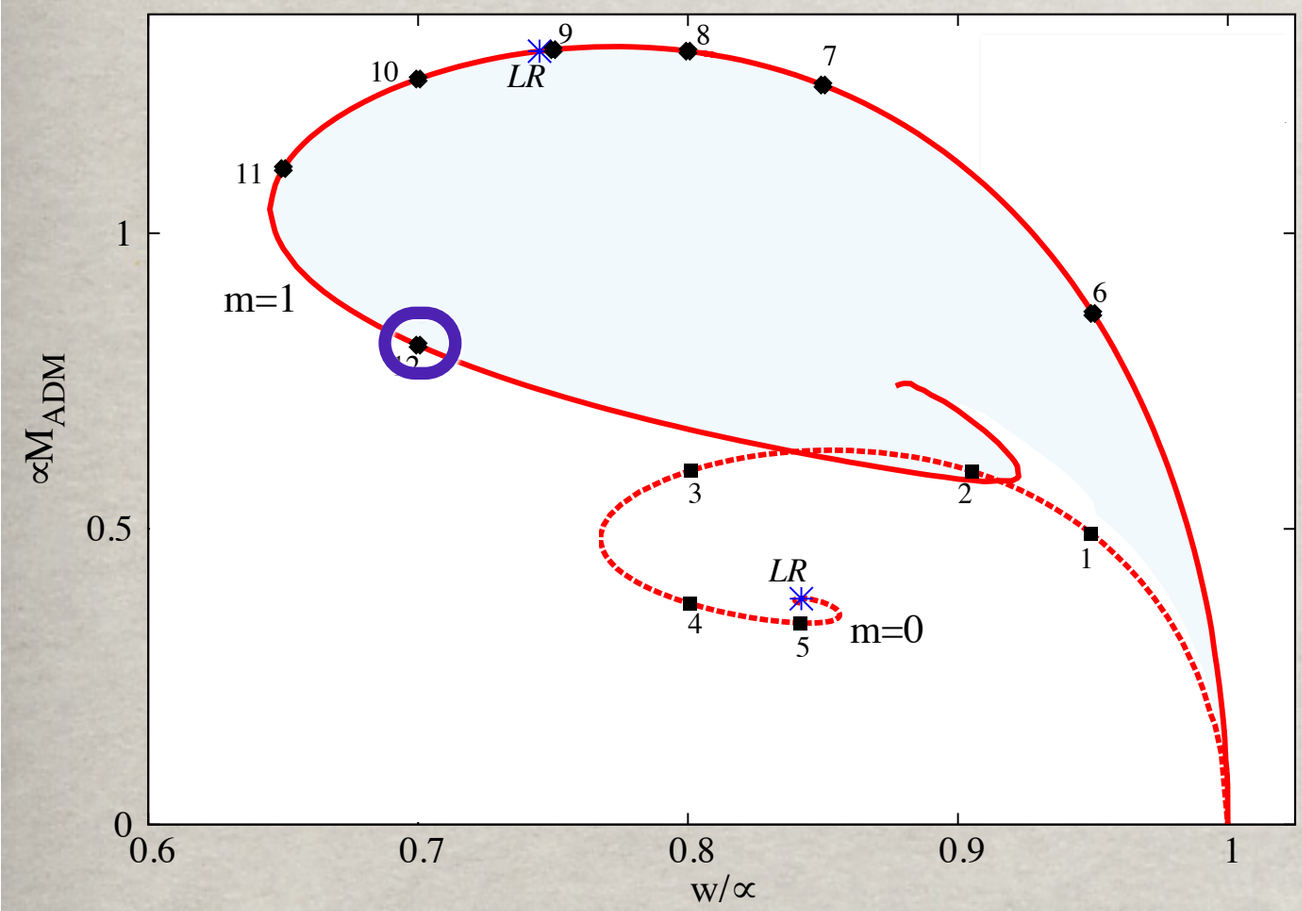




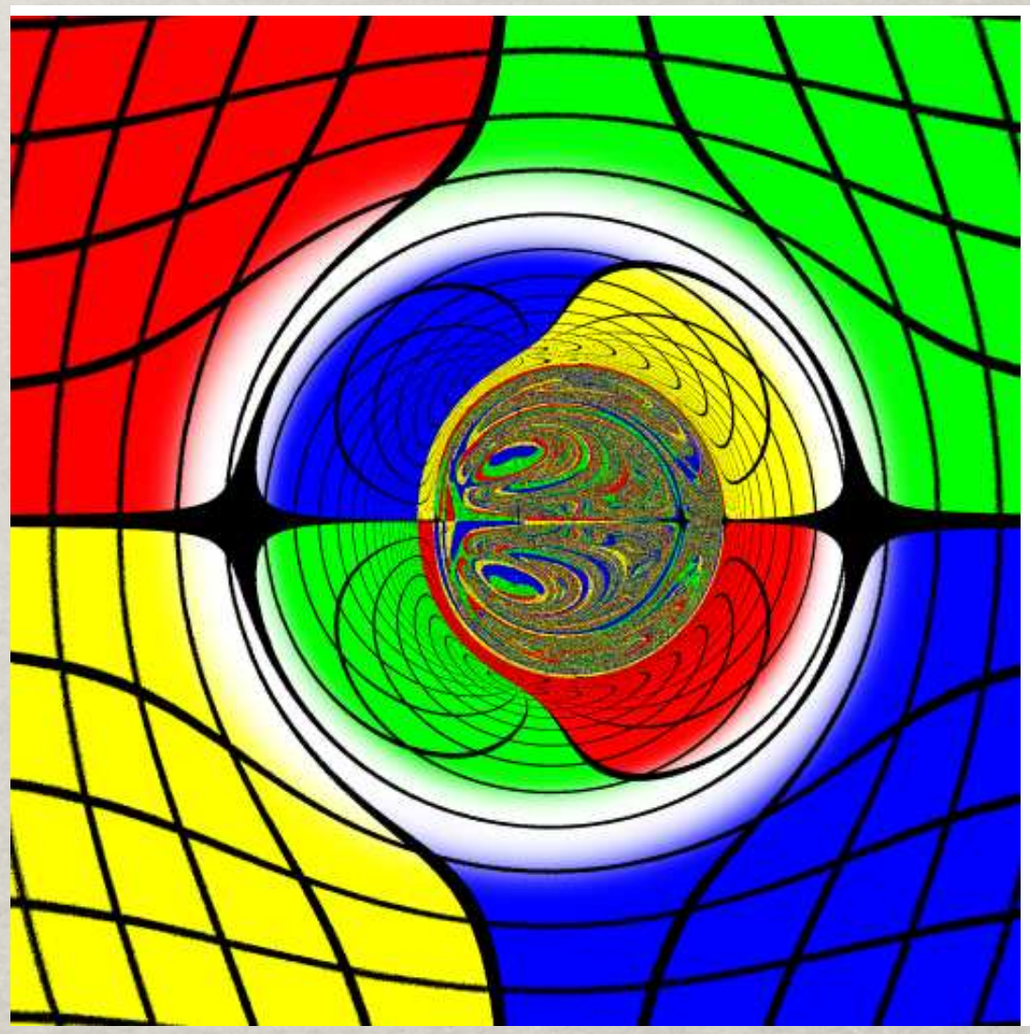
Ultra-compact RBS

Ergo-region appears between configuration 10 and 11

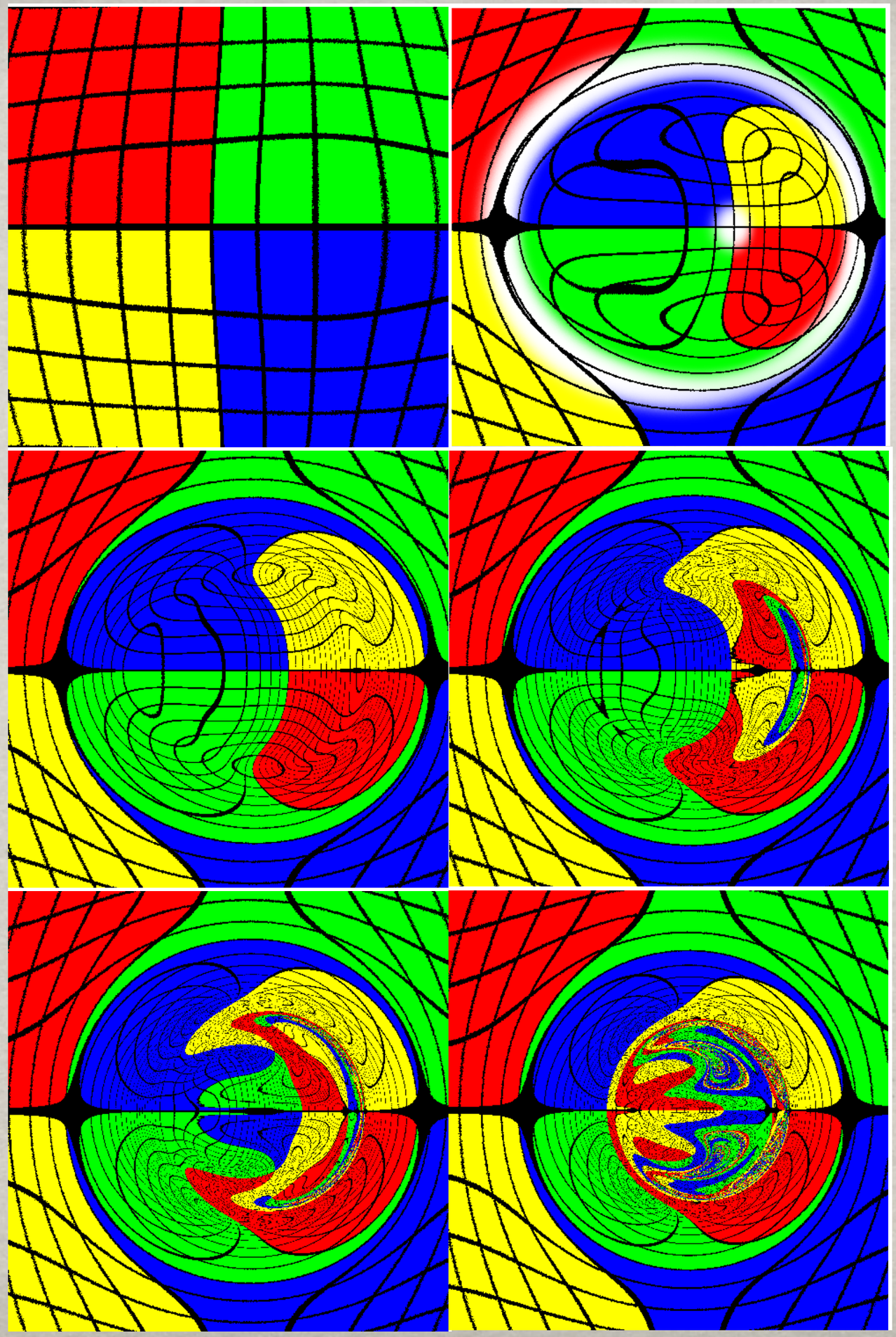
Rotating boson stars

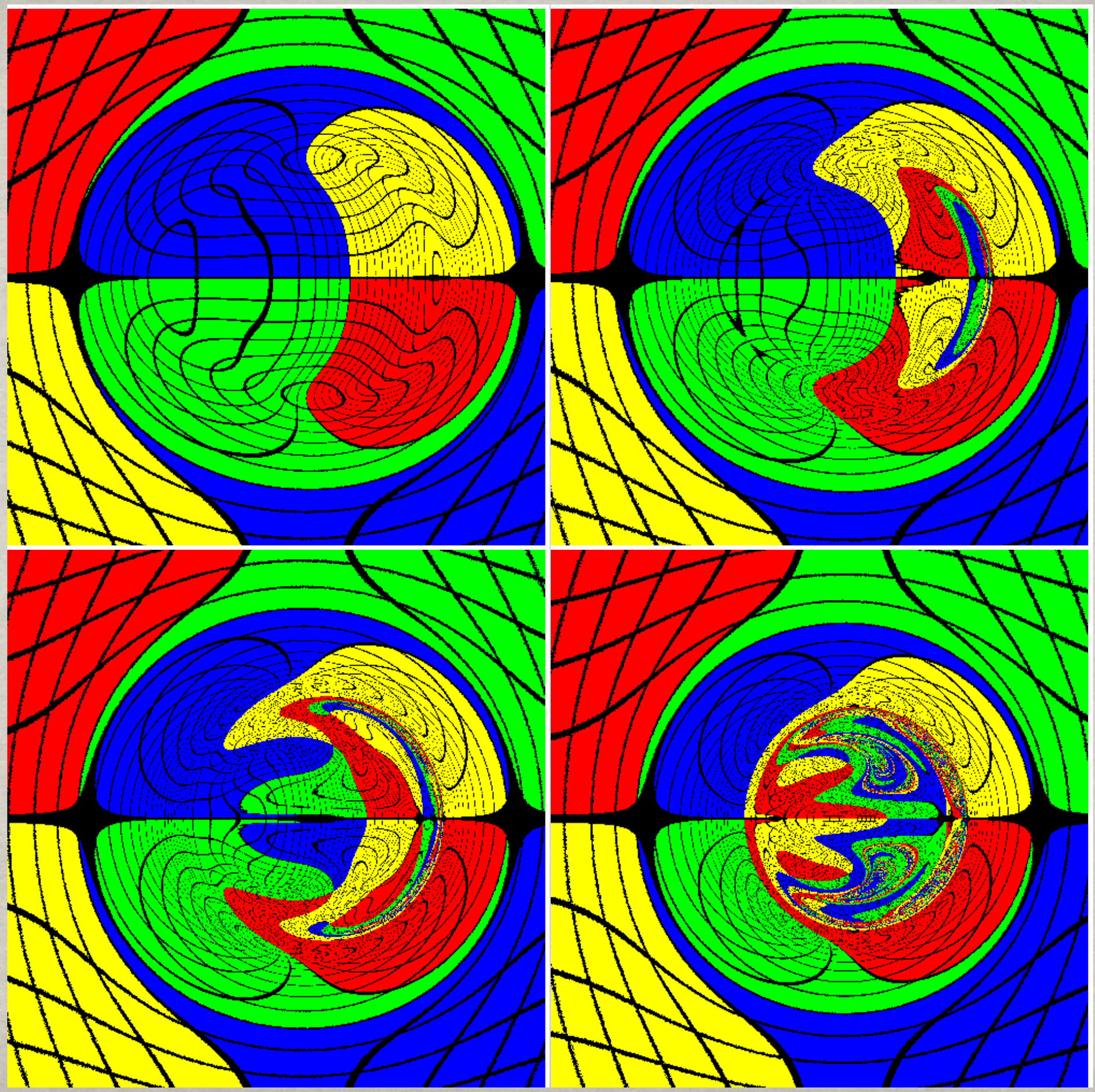


Clear chaotic behaviour

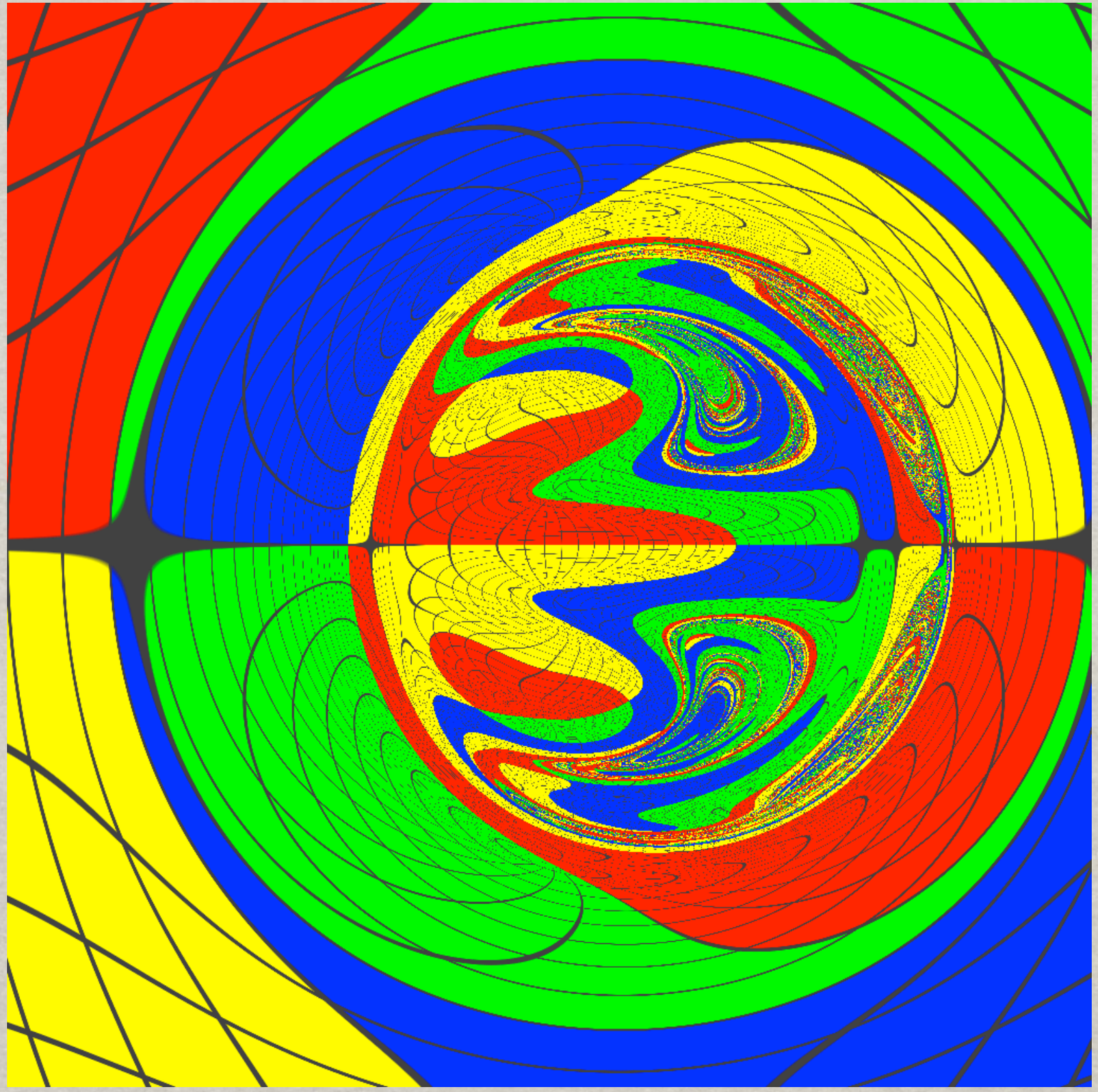


Ultra-compact RBS





There are images with a clear chaotic behaviour in some regions, e.g.:



The emergence of stable light rings turns out to allow the formation of pockets in the effective potentials. This induces chaotic scattering.

Cunha, Grover, Herdeiro, Radu, Runarsson and Wittig, PRD 94 (2016) 104023

Black holes and exotic compact objects

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Thank you for your attention!
Obrigado pela vossa atenção!

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