Parameterization for ATLAS EMEC

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SLAC ATLAS Forum

13 Dec 2006

Why parameterization?

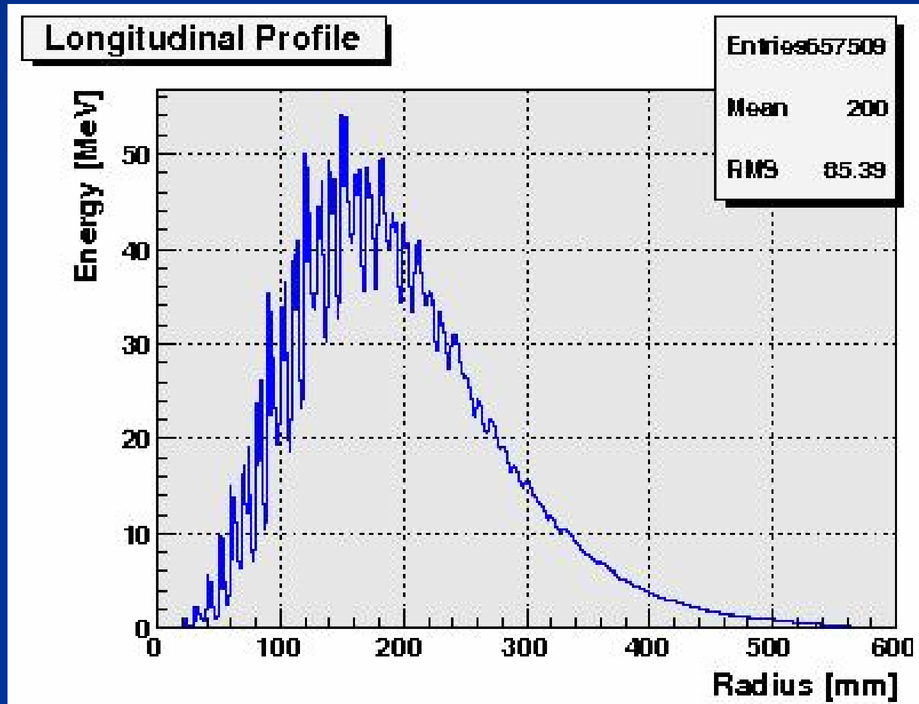
Release	CPU time per event (kSI2K)						
	B4 (jets)	H(130) ->4l	min bias	susy	Z->ee	Z-> mumu	Z-> tautau
12.0.3	765.06	776.72	263.35	921.64	949.58	736.68	668.64

- Full simulation time increases almost **linearly** with the energy.
- Parameterization: generating the energy profile of the shower rather than tracking every secondary particles.

How to parameterize? - longitudinal

- The mean longitudinal profile of a shower is described by a gamma function

$$< \frac{1}{E} \frac{dE(t)}{dt} >= f(t) = \frac{(\beta t)^{\alpha-1} \beta e^{-\beta t}}{\Gamma(\alpha)}$$

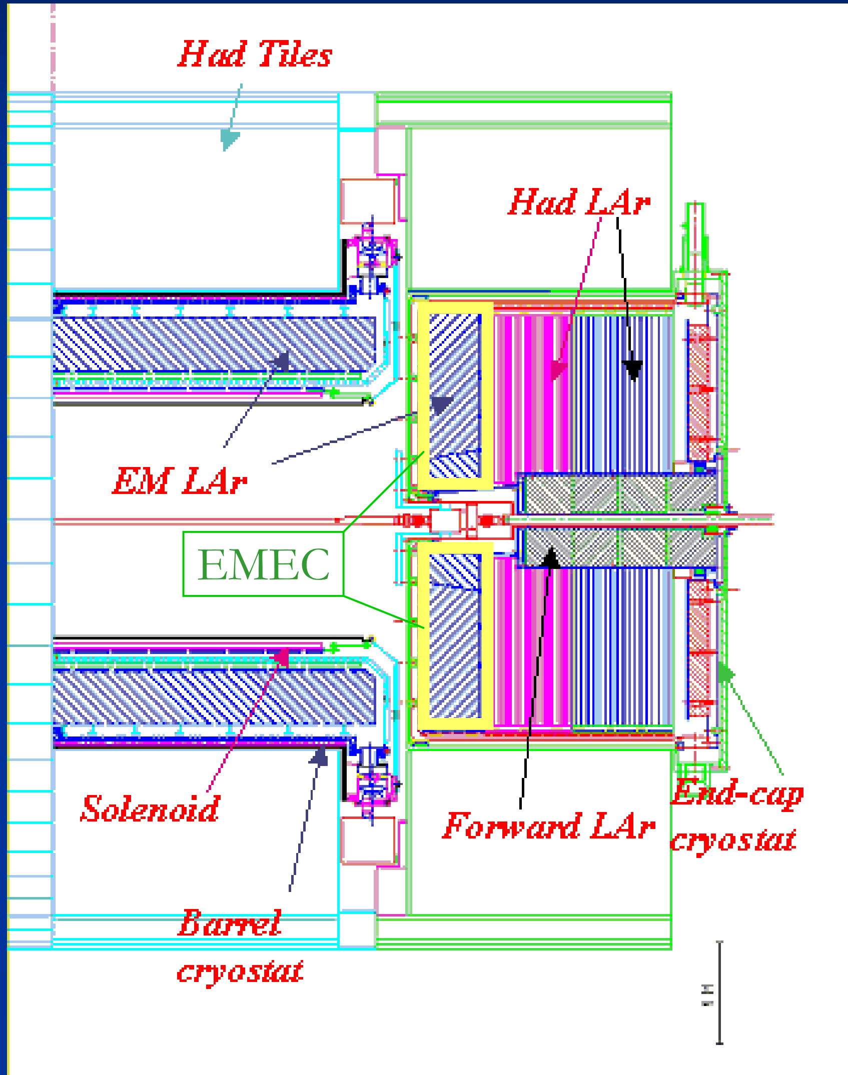


- t – the shower depth in units of radiation length
- T=(α-1)/β – the depth of shower maximum.

How to parameterize? - longitudinal

- T and α are dependent on shower energy and the sampling frequency, which is related to the direction of the incident particle.
- T and α are calculated at each point (energy, direction). And

$$dE(t) = E\int_{t_{j-1}}^{t_j} \frac{(\beta_i t)^{\alpha_i - 1}\beta_i e^{-\beta_i t}}{\Gamma(\alpha_i)}dt$$



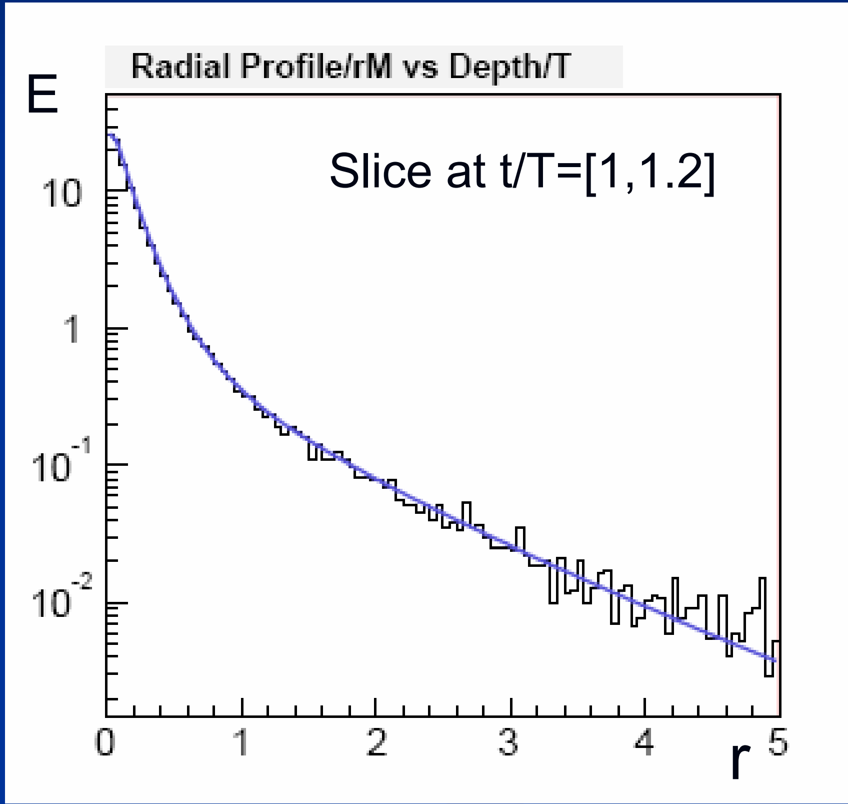
How to parameterize? - radial

- Also described by gamma function. But it consists of two parts, core and tail.
- r is in the unit of Molière radius

$$\left\langle \frac{1}{dE(t)} \frac{dE(r,t)}{dr} \right\rangle = p^r g_1(r) + (1 - p^r) g_2(r)$$

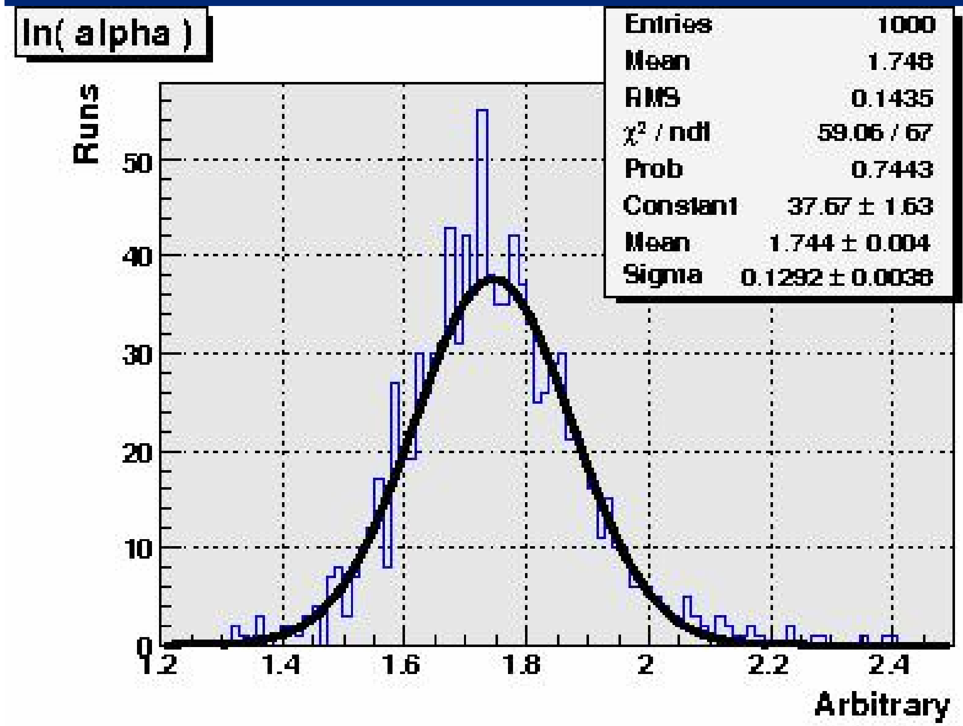
$$g_i(r) = \frac{1}{2\lambda_i^r} \left(\frac{r}{\lambda_i^r} \right)^{\alpha_i^r/2-1} \frac{e^{-\sqrt{r/\lambda_i^r}}}{\Gamma(\alpha_i^r)}$$

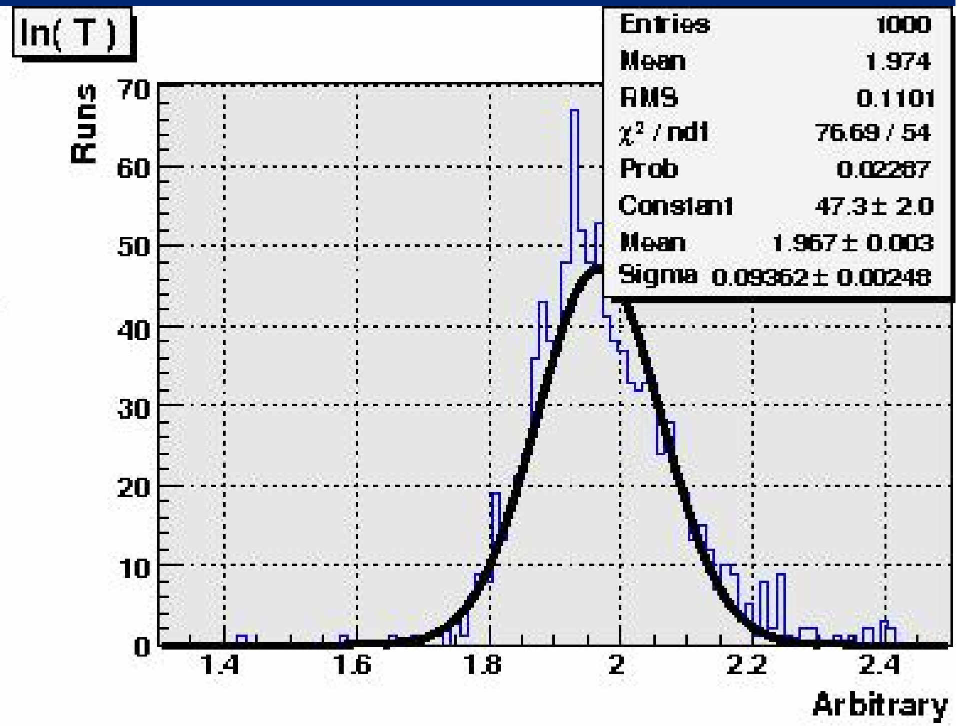
$$\lambda_i^r = \lambda_i^r(t); \; \alpha_i^r = \alpha_i^r(t);$$



Barberio and Straessner, ATL-Com-Phys-2004-015

Shower parameters - longitudinal





- ln α and ln T are Gaussian variables.
- Thus we look into the logarithm rather than α and T.

Shower parameters - longitudinal

- At each point, generate a sample of 1000 showers by full simulation.
- For each shower, calculate the moments. Get T and α from first and second moments.

$$Z_n = \int_m^\infty f(t)t^n dt = \beta^{-n}\Gamma(\alpha+n)/\Gamma(\alpha), \quad T = \frac{2Z_1^{\ 2} - Z_2}{Z_1}, \quad \alpha = \frac{Z_1^{\ 2}}{Z_2 - Z_1^{\ 2}}$$

- Fit the distribution of ln(T) and ln(α) as Gaussian to get mean value and variance.

Shower parameters - longitudinal

- Unfortunately, ln(T) and ln(α) are not independent. We have to deal with another variable, the correlation between ln(T) and ln(α).

$$\rho = \rho(\ln T, \ln\alpha) = \frac{\left\langle \left(\ln\alpha - \langle \ln\alpha \rangle\right)\left(\ln T - \langle \ln T \rangle\right) \right\rangle}{\sqrt{\left(\langle \ln\alpha^2 \rangle - \langle \ln\alpha \rangle^2\right)\left(\langle \ln T^2 \rangle - \langle \ln T \rangle^2\right)}}$$

- For each parameterized shower, T and α are calculated as

$$\begin{pmatrix} \ln T_i \\ \ln\alpha_i \end{pmatrix} = \begin{pmatrix} \langle \ln T \rangle \\ \langle \ln\alpha \rangle \end{pmatrix} + \begin{pmatrix} \sigma_{\ln T} & 0 \\ 0 & \sigma_{\ln\alpha} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1+\rho}{2}} & \sqrt{\frac{1-\rho}{2}} \\ \sqrt{\frac{1-\rho}{2}} & \sqrt{\frac{1+\rho}{2}} \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

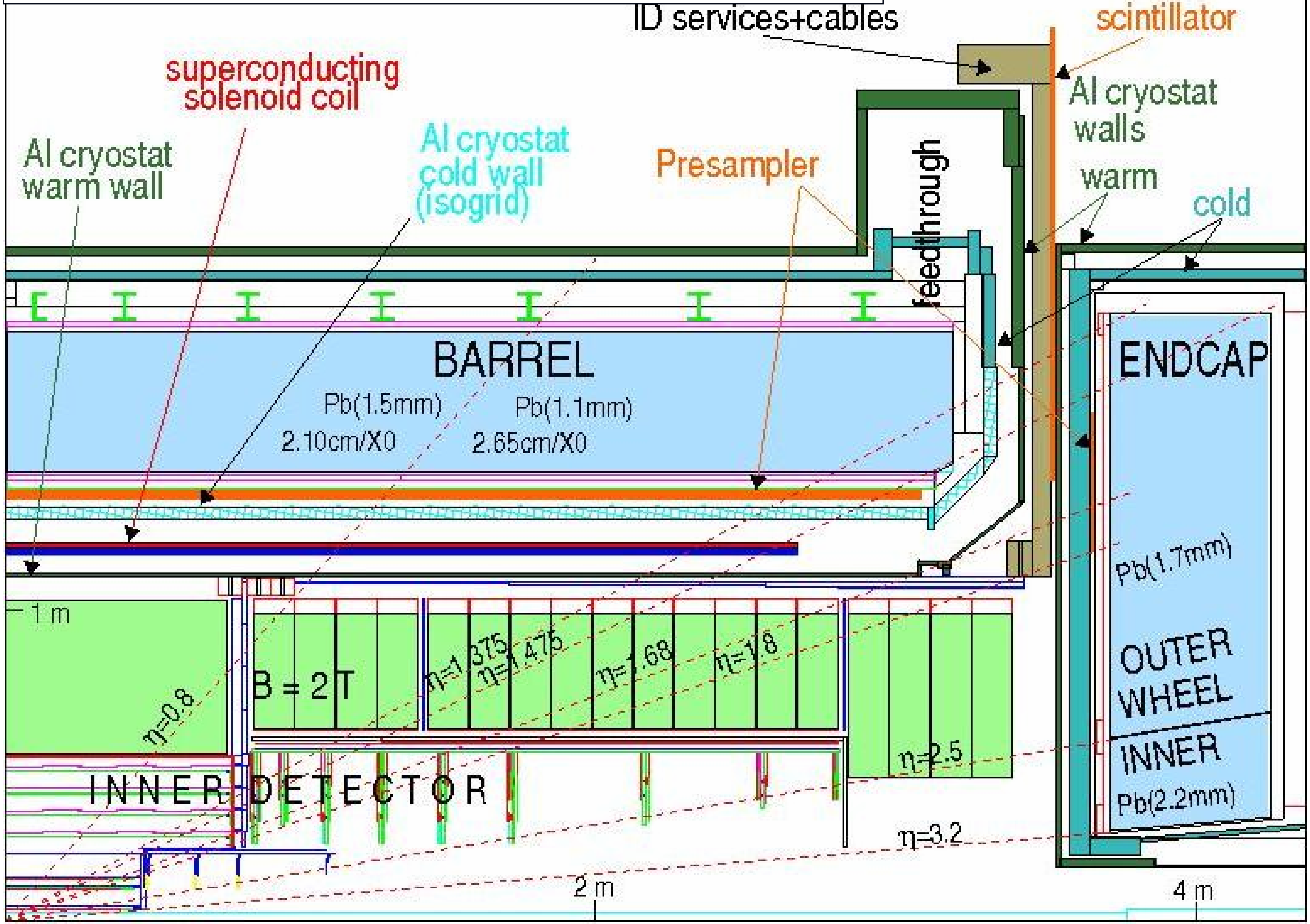
where n_1 and n_2 are normally distributed random numbers

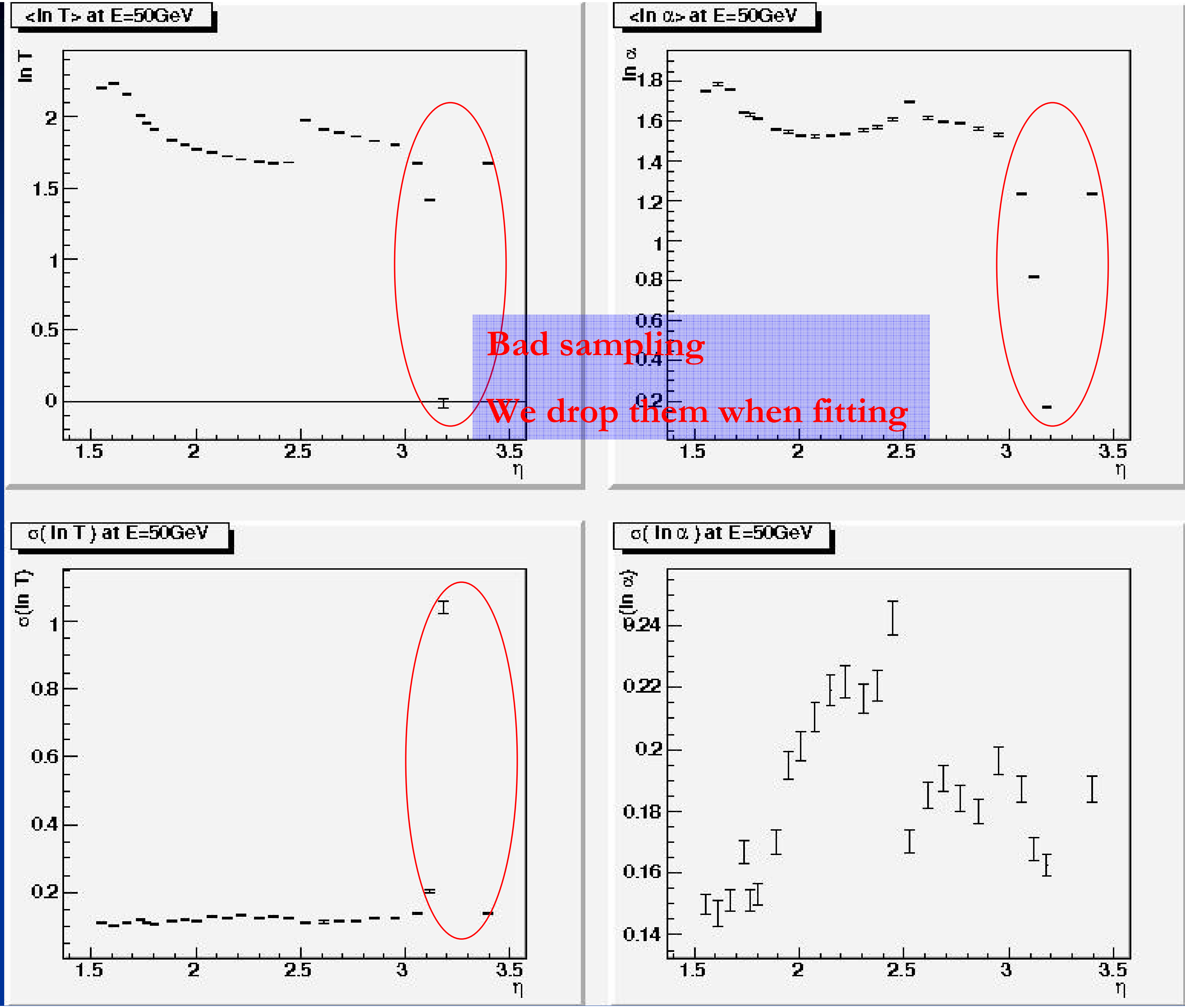
- Deposit energy to every spot of detectors as the profile shows to get a parameterized shower.

Parameters as function of $\ln(E/E_c)$ and η

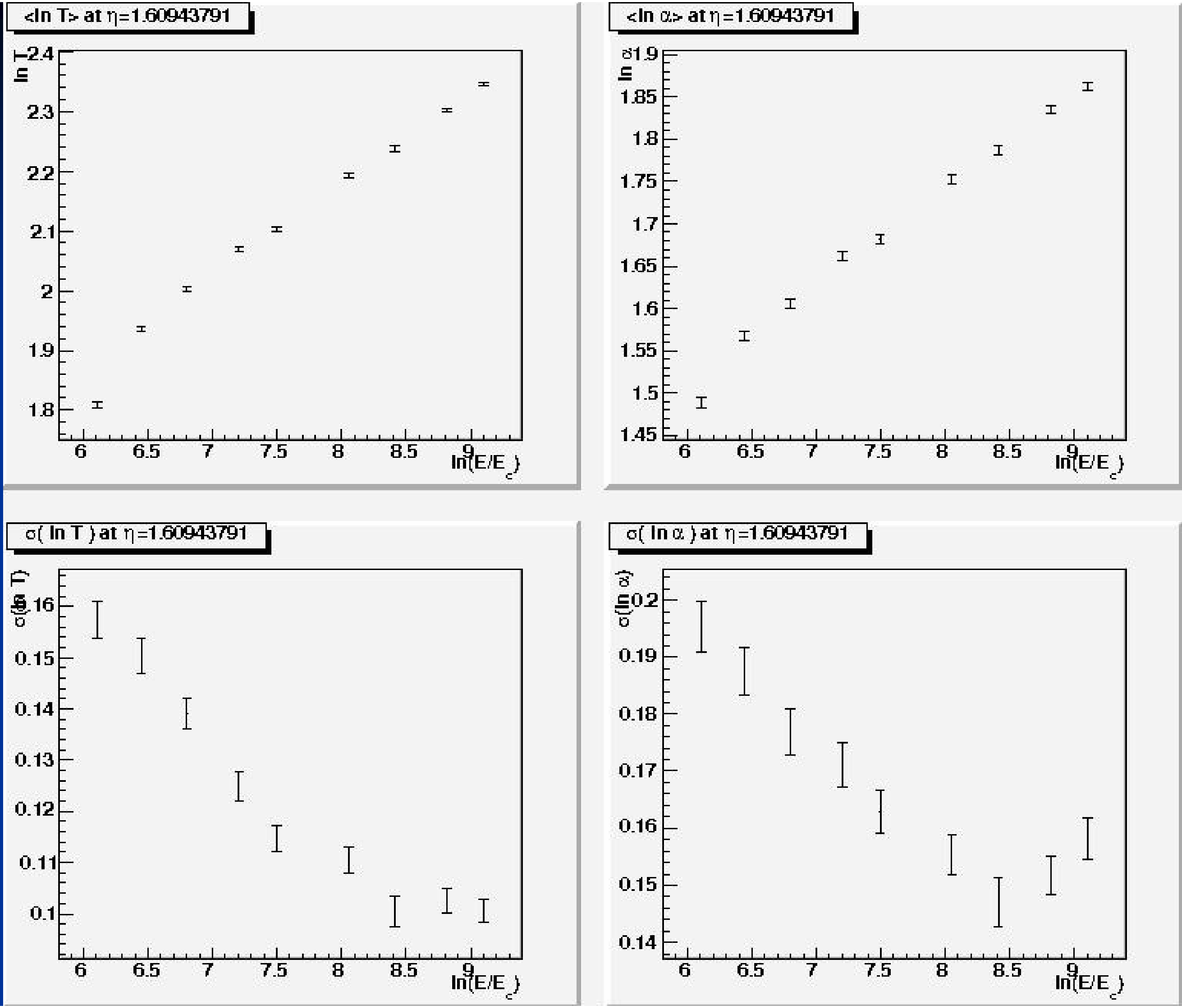
- Use logarithm of shower energy scaled by critical energy and pseudorapidity of incident particle as variables in the ansatz for <lnT>, <ln α>, σ(ln T), σ(ln α) and ρ.
- Critical energy is from TDR: 0.011122GeV
- Pseudorapidity $\eta = -\ln(\tan(\arctan(X/3720)/2))$
- Challenge in parameterization
  - 2-dimensional function
  - Jump at η=2.5
  - Violently fluctuating ρ

Calorimeter Performance Technical Design Report



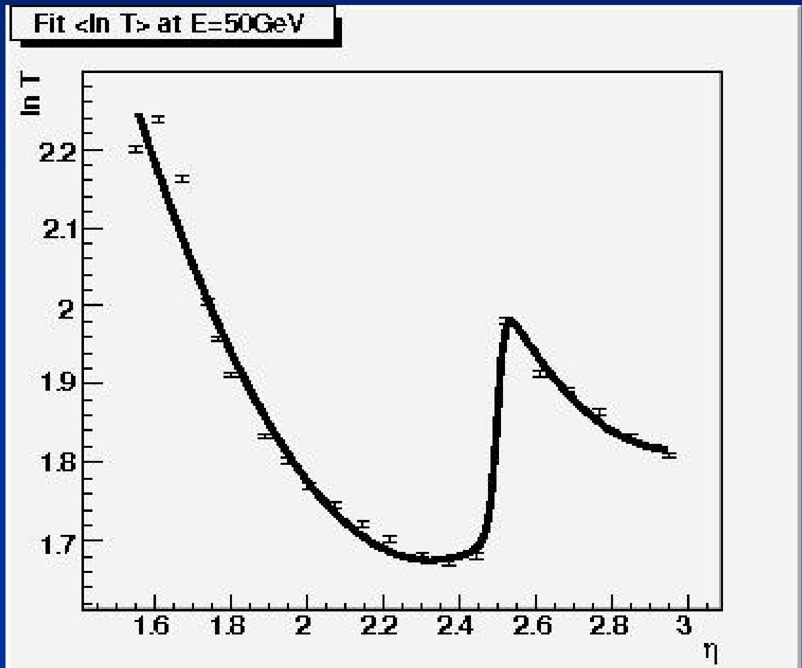


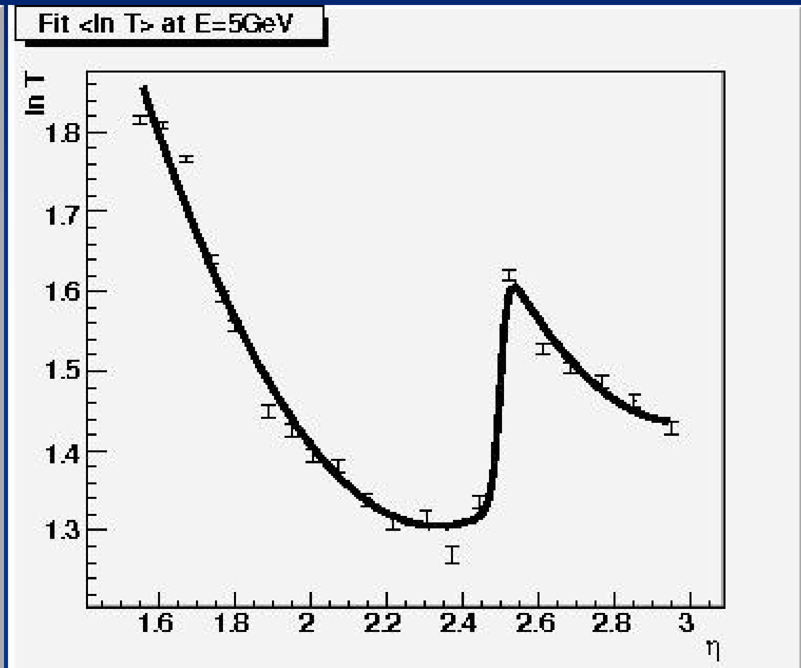
<ln T> at E=50GeV
ln T
η
<ln α> at E=50GeV
ln α
η
σ(ln T) at E=50GeV
σ(ln T)
η
σ(ln α) at E=50GeV
σ(ln α)
η
Bad sampling
We drop them when fitting



<ln T> at η=1.60943791
ln T
ln(E/E_c)
<ln α> at η=1.60943791
ln α
ln(E/E_c)
σ(ln T) at η=1.60943791
σ(ln T)
ln(E/E_c)
σ(ln α) at η=1.60943791
σ(ln α)
ln(E/E_c)

<ln T> - η dependence

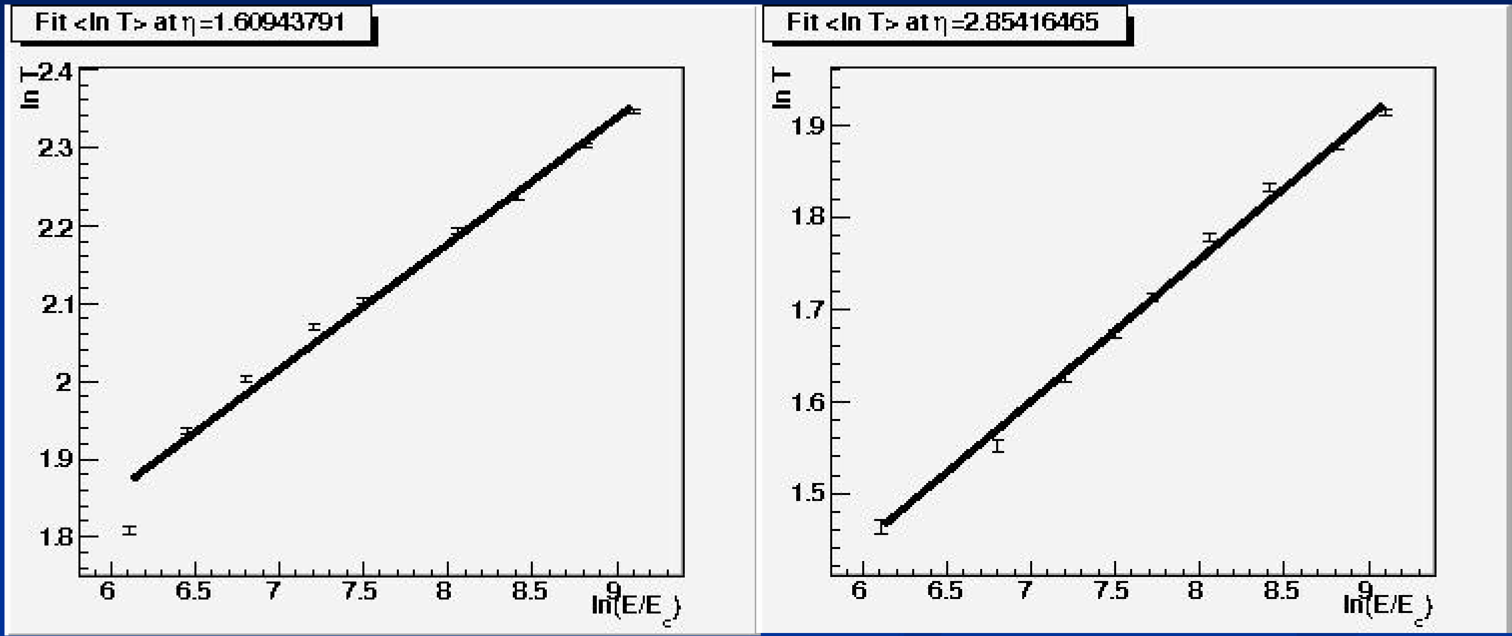




$$s(\eta) = \frac{1}{1+\exp\left(-(\eta-2.5)/0.01\right)}$$

$$<\ln T> = (c_0 + c_1\eta + c_2\eta^2)(1-s(\eta)) + \left[c_4 + c_1(\eta - c_3) + c_2(\eta - c_3)^2\right]s(\eta)$$

<ln T> - energy dependence

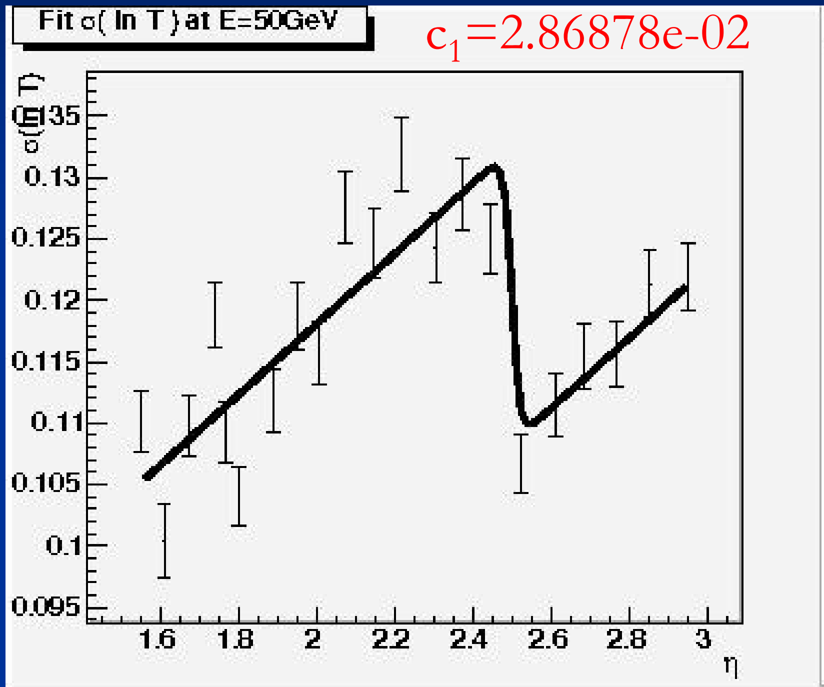


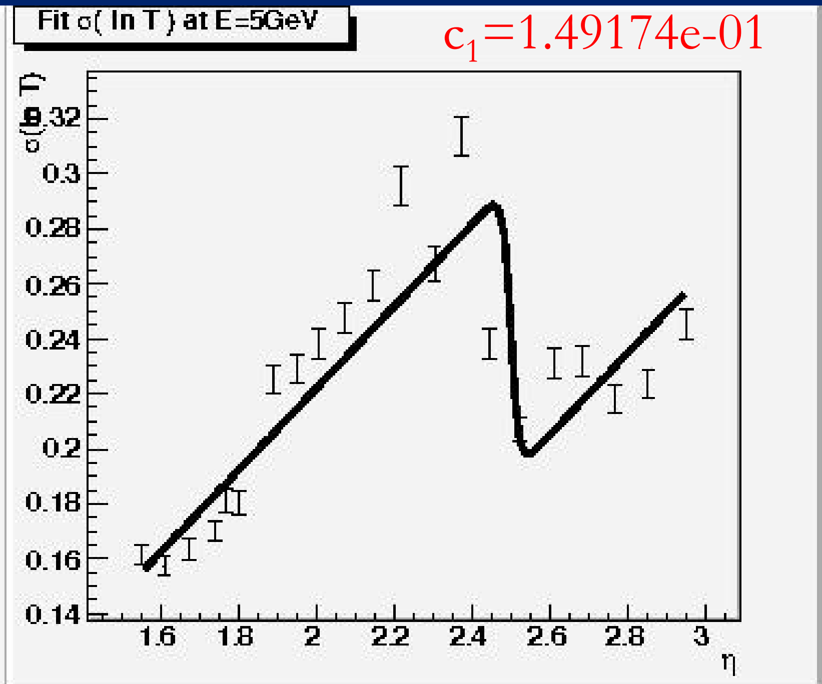
$$< \ln T >= d_1 + d_2 \ln(E / E_c)$$

- 2-dimensional function could be

$$< \ln T >= (c_0 + c_1\eta + c_2\eta^2)(1 - s(\eta)) + \left[c_4 + c_1(\eta - c_3) + c_2(\eta - c_3)^2\right]s(\eta) + c_5 \ln(E / E_c)$$

σ(ln T) - η dependence

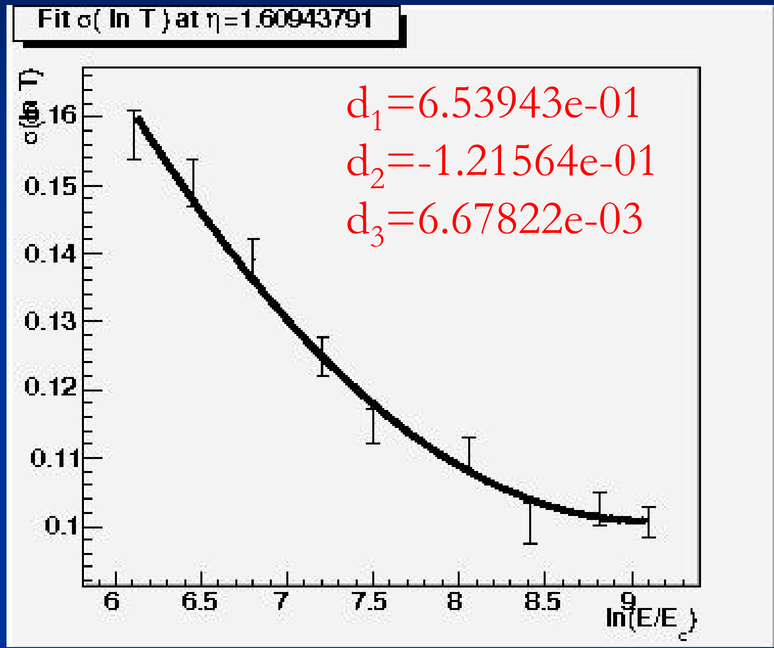


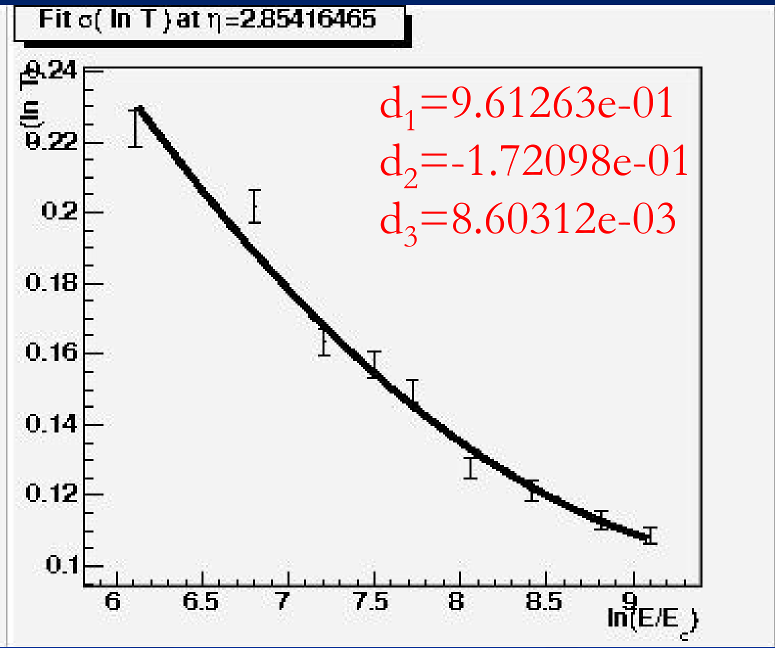


$$s(\eta) = \frac{1}{1+\exp\left(-(\eta-2.5)/0.01\right)} \qquad \sigma\left(\ln T\right) = (c_0 + c_1\eta)(1-s(\eta)) + (c_2 + c_1\eta)s(\eta)$$

Unfortunately, unlike in ln(T), the slope of σ(ln T) varies greatly at different energy, i.e. c_1 is dependent on ln(E/E_c)

σ(ln T) - energy dependence





$$\sigma(\ln T) = d_1 + d_2 \ln(E/E_c) + d_3 \left[\ln(E/E_c)\right]^2$$

- The coefficients in this formula also vary greatly at different η!
- 2-dimensional function could be

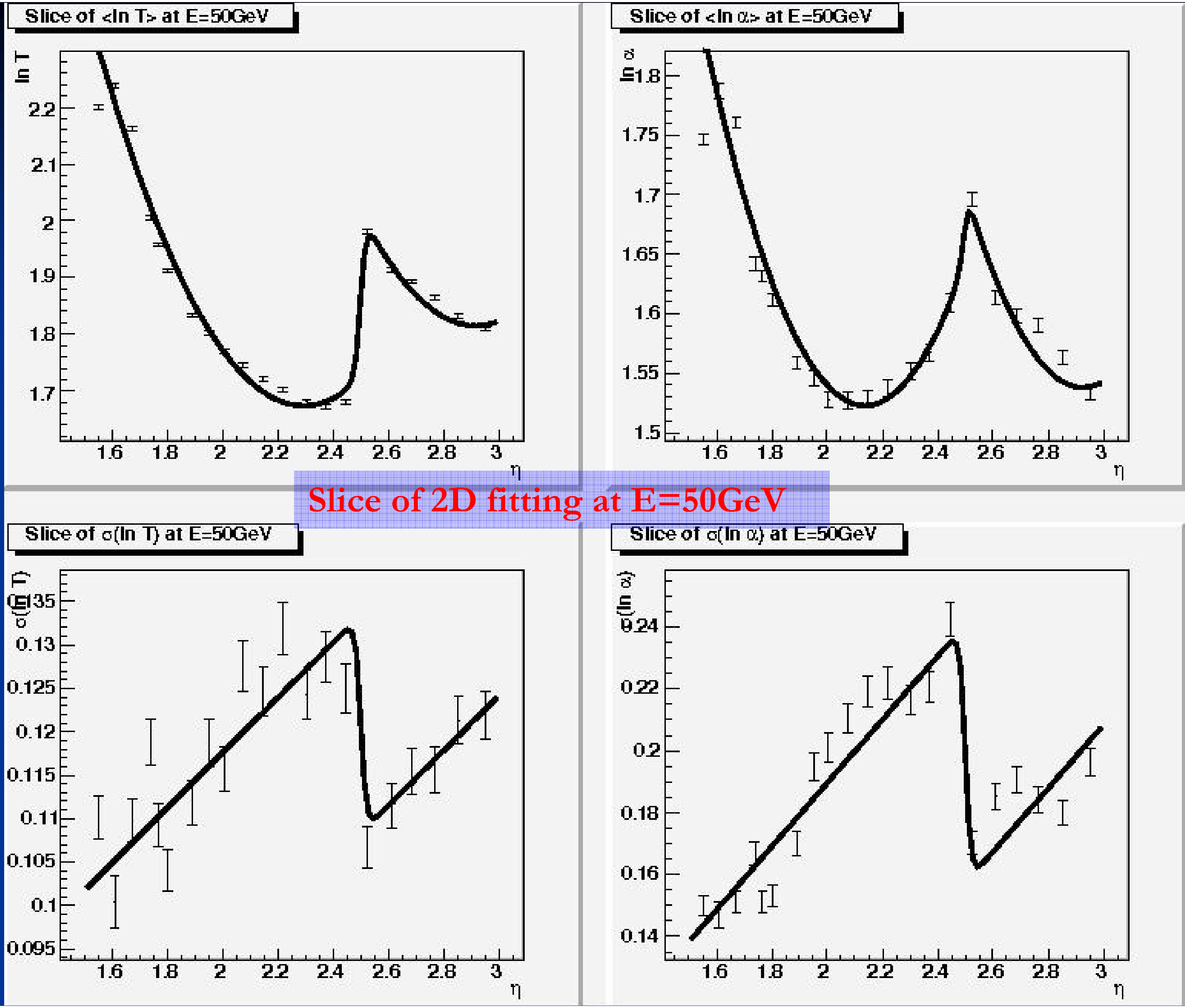
$$\sigma(\ln T) = \left[c_0 + (c_1 + c_2 \ln(E/E_c) + c_3(\ln(E/E_c))^2)\eta + c_4 \ln(E/E_c) + c_5(\ln(E/E_c))^2\right]$$

$$(1 - s(\eta)) + (c_6 + (c_1 + c_2 \ln(E/E_c) + c_3(\ln(E/E_c))^2)\eta + c_7 \ln(E/E_c) + c_8(\ln(E/E_c))^2)s(\eta)$$

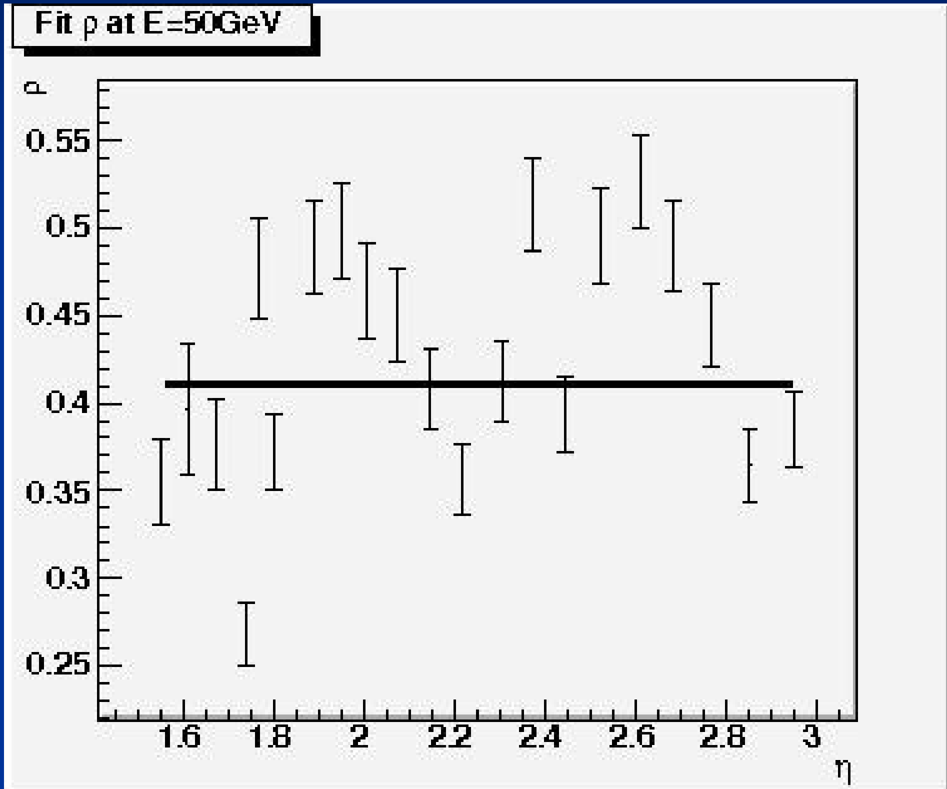
2D fitting

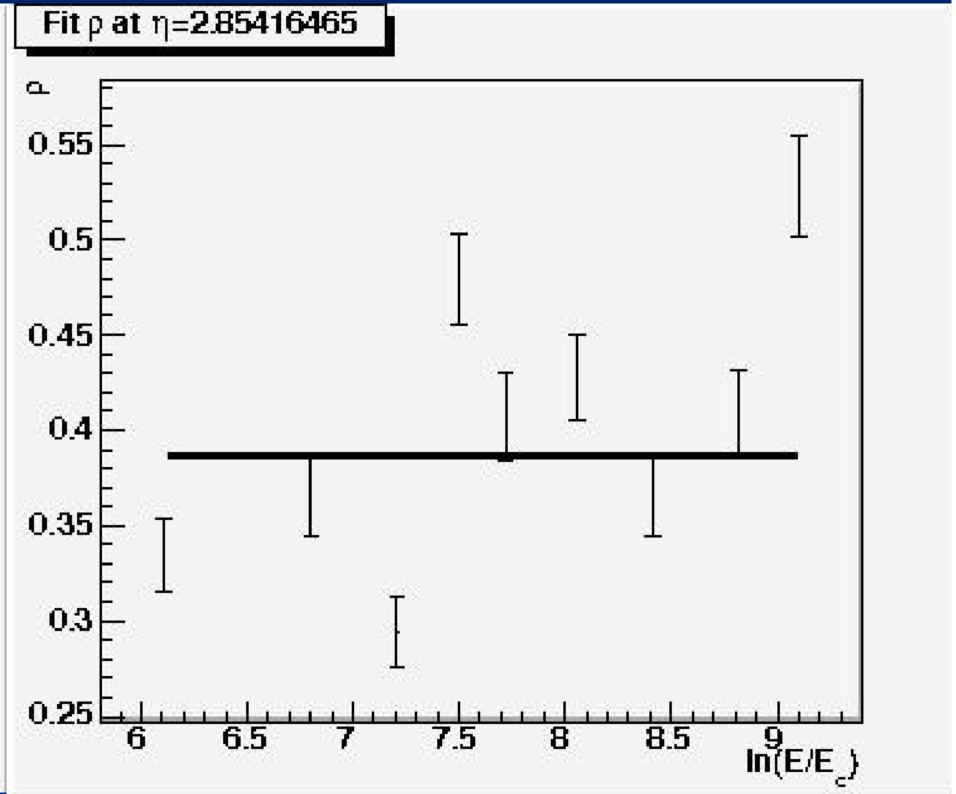
- $\langle \ln \alpha \rangle$ follows the same ansatz as $\langle \ln T \rangle$ does. And $\sigma(\ln \alpha)$ follows the same ansatz as $\sigma(\ln T)$ does.
- We make 2D fits of each quantity as a function of E and eta, using the formula on the previous pages.
- There are 6 parameters in $\langle \ln T \rangle$ and $\langle \ln \alpha \rangle$ fits and 9 parameters in $\sigma(\ln \alpha)$ and $\sigma(\ln T)$ fits.
- We show the quality of the fit by looking at 1D slices.

Slice of 2D fitting at E=50GeV



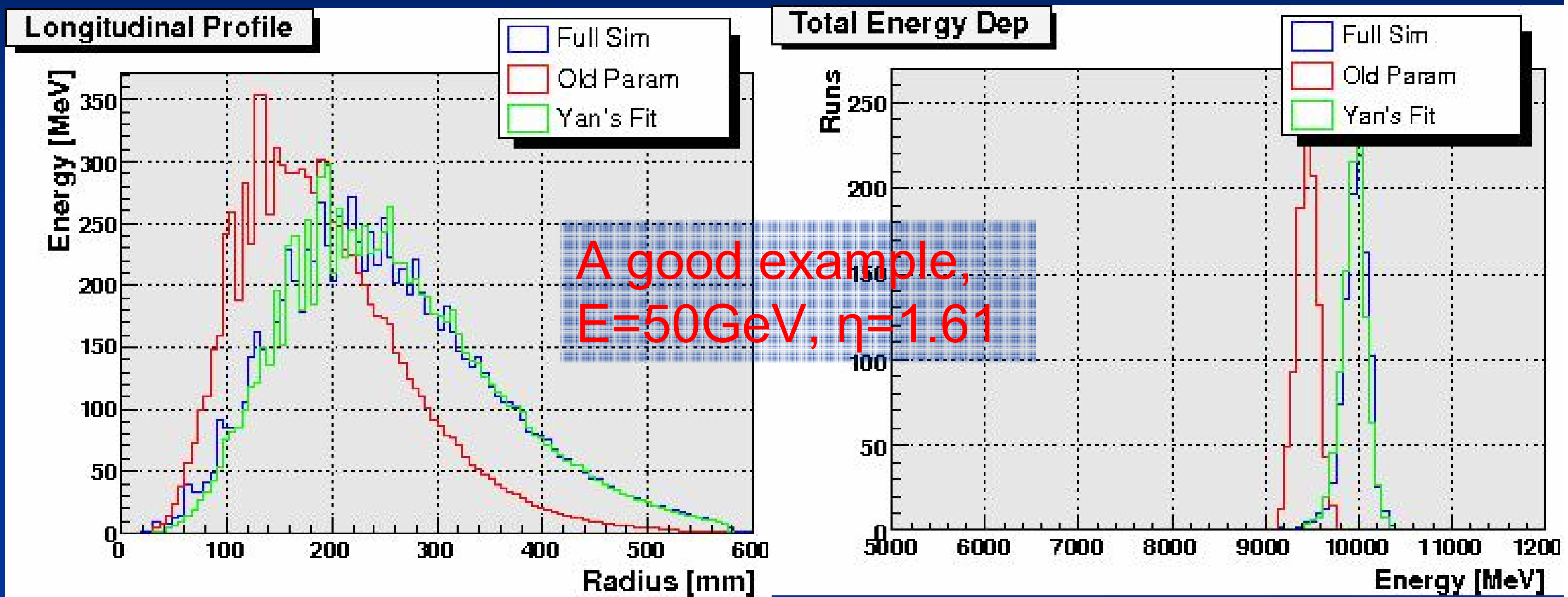
ρ – still a problem





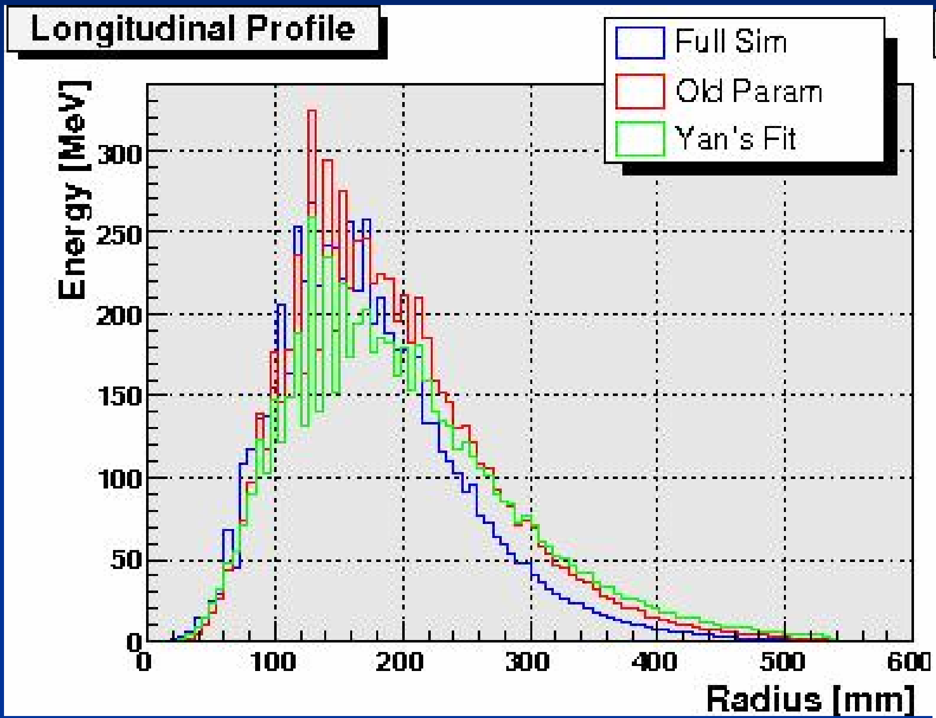
Not satisfactorily solved. We leave it as a constant now.

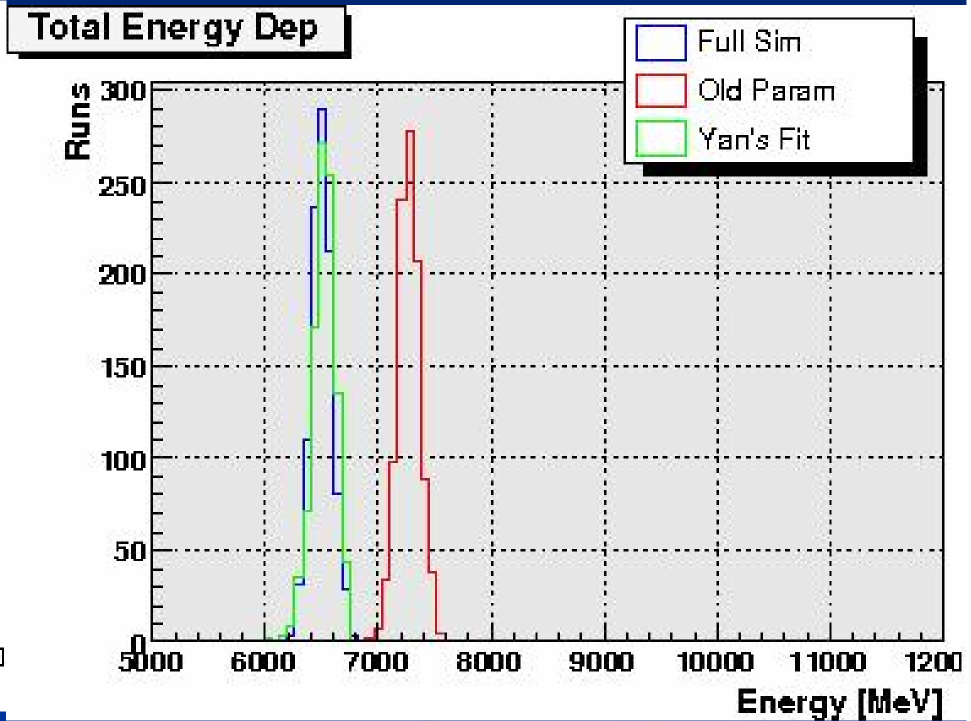
Plots of parameterized shower



- Red and green lines are from parameterized showers using old and new parameters. Only energy in sensitive detectors are recorded here!

Plots of parameterized shower





- E=50GeV, η=2.95
- Not so good now. Maybe due to tuning

Conclusions

- Good approximation using new fits of ln T and ln α.
- Further validation by users.
- Possible improvements
  - Find a reasonable way to fit ρ;
  - Better description of radial profile;
  - Consider the difference in longitudinal profiles of full and parameterized showers is well-behaving, a factor may be found to correct the longitudinal ansatz.